

Article

Planning of Energy Router Siting and Sizing for Load Supply Assurance Under Typical N-1 Scenarios

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Abstract

Ensuring continuous load supply under N-1 contingencies poses a significant challenge for active distribution networks. Traditional fault recovery relies on mechanical tie switches for network reconfiguration; however, such rigid interconnections often fail due to severe terminal voltage drops during long-distance load transfers, leading to forced load shedding. To address this, this paper proposes an optimal planning framework for Electric Energy Routers (EERs) to secure load supply, aiming to maximize load preservation within investment budget constraints. First, a bi-level robust optimization model is constructed: the upper level determines the optimal EER configuration to minimize load shedding in the worst-case N-1 scenarios, while the lower level evaluates the optimal restoration strategies via mixed-integer second-order cone programming. To handle the inherent complexity of discrete–continuous variable coupling and the computational burden of the bi-level structure, a novel topological manifold evolution algorithm based on Wasserstein geometric flow is developed. By embedding the electrical topology into a sensitivity-driven Riemannian metric field, the algorithm effectively enhances global optimization capabilities. Case studies on the IEEE 33-node system demonstrate that the integration of EERs facilitates a transition from rigid to flexible interconnection, circumventing power transfer bottlenecks caused by low voltage through controllable power flow. Compared with baseline schemes, the optimal EER scheme reduces load shedding in the worst-case scenario by 60.3%, providing a robust and economical solution for high-resilience distribution network planning.

Keywords: active distribution network; electric energy router; N-1 contingency; bi-level robust optimization; Wasserstein geometric flow; service restoration



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1. Introduction

Driven by the global carbon-neutrality agenda, modern power systems are undergoing a profound transition toward active distribution networks (ADNs) that deeply integrate distributed renewable energy sources and flexible load demands [1,2]. The large-scale penetration of distributed photovoltaic generation, coupled with the rapid growth of electric vehicle (EV) charging fleets, has introduced significant spatial–temporal uncertainties into distribution networks, posing new challenges to power quality and supply reliability [3]. Furthermore, the increasing frequency and intensity of extreme weather events, together

with the progressive aging of distribution infrastructure, have increasingly exposed the vulnerability of conventional distribution systems to line faults [4]. Consequently, enhancing the resilience and supply assurance capability of distribution networks under N-1 contingencies—where any single line may be disconnected—has become a critical research priority [5,6].

Traditionally, distribution networks have relied primarily on network reconfiguration to restore supply after faults, whereby sectionalizing and tie switches are toggled to redirect power flow along alternative paths [7]. This approach has proven effective for moderate load transfer scenarios. However, a fundamental limitation arises during long-distance load transfer: the high resistance-to-reactance ratio characteristic of distribution lines induces substantial voltage drop along the transfer path [8]. As a result, terminal nodes at the far end of the transfer path frequently violate the lower voltage limit, making it physically impossible to deliver power to those loads even though a connected path exists through the tie switches. This “rigid interconnection” through mechanical switches cannot provide continuous power flow regulation or reactive power compensation, leaving a significant portion of the network’s transfer potential underutilized and forcing operators to resort to load shedding to maintain voltage security [9,10]. Therefore, the traditional tie-switch-based reconfiguration strategy suffers from a structural bottleneck: the available transfer capacity is severely constrained by voltage limitations rather than thermal limits.

With the advancement of power electronics technology, the Electric Energy Router (EER) has emerged as a novel flexible interconnection device that fundamentally overcomes the voltage bottleneck of traditional load transfer schemes [11,12]. Unlike mechanical tie switches, the EER employs voltage source converters connected in a back-to-back configuration through a common DC bus, enabling precise, continuous, and bidirectional control of active power flow between feeders [10]. More critically, each converter terminal of the EER can independently regulate reactive power, providing active voltage support at its point of connection [13]. This reactive power compensation capability effectively counteracts the voltage drop caused by long-distance line impedance, thereby eliminating the low-voltage blind zones that constrain conventional transfer capacity and significantly releasing the latent transfer potential of distribution networks. Recent studies have demonstrated the superior performance of soft open points and EERs in enhancing distribution network resilience under fault conditions, including multi-period restoration with networked microgrids [5], islanding partition optimization [14], sequential load restoration [15], extreme weather risk-averse planning [16], and coordinated planning with mobile energy storage for resilience improvement [17].

Despite the demonstrated operational advantages of EERs, their high investment cost renders full-network deployment economically infeasible. Therefore, strategic siting and sizing of EERs must be optimized to maximize their contribution to N-1 security within a constrained budget [18,19]. To defend against the worst-case fault scenario, this planning problem is naturally formulated as a bi-level robust optimization model: the upper level determines the EER installation scheme to minimize investment cost and worst-case load shedding, while the lower level evaluates the fault recovery performance under each N-1 contingency [20]. The resulting formulation involves the coupling of discrete siting variables and continuous capacity/power flow variables, nonconvex AC power flow constraints, and a nested min–max–min structure, making it an exceptionally challenging mixed-integer nonlinear programming (MINLP) problem. While mixed-integer second-order cone programming (MISOCP) relaxation has been widely adopted to convexify the lower-level subproblem [21], the upper-level combinatorial optimization over the network topology still poses significant difficulties for conventional meta-heuristic

algorithms, which are prone to premature convergence and the curse of dimensionality in high-dimensional search spaces.

To address the above challenges, this paper proposes a bi-level robust planning framework for EER siting and sizing oriented toward N-1 load supply assurance. The main contributions of this paper are summarized as follows:

- (1) A min–max bi-level planning model is established for EER siting and sizing. The upper level optimizes EER locations and capacities to minimize investment cost and worst-case load shedding, while the lower level formulates the fault recovery optimization as an MISOCP model to evaluate the supply assurance capability under each N-1 contingency.
- (2) A topological manifold evolution algorithm based on Wasserstein geometric flow (WGF-TME) is developed. By constructing a sensitivity-driven Riemannian metric field from the distribution network topology, the algorithm maps the discrete–continuous hybrid search space onto a low-dimensional manifold, effectively guiding population evolution and overcoming the premature convergence inherent in conventional evolutionary algorithms.
- (3) The mechanism by which EER integration breaks the voltage bottleneck of traditional load transfer is systematically revealed. Through comparative analysis, it is demonstrated that EERs transform the paradigm from rigid physical interconnection to flexible active defense, eliminating low-voltage blind zones and fully releasing the latent transfer potential of the distribution network.

The remainder of this paper is organized as follows. Section 2 introduces the N-1 security challenges in distribution networks and the EER application framework. Section 3 formulates the bi-level planning model for EER siting and sizing. Section 4 proposes the WGF-TME algorithm for solving the multi-objective robust optimization problem. Section 5 presents and discusses the case study results on the IEEE 33-bus system. Section 6 concludes the paper.

2. Problem Description and EER Application Principle

During the transition of distribution networks toward active systems with coupled supply and demand, the traditional closed-loop design and open-loop operating mode reveal significant rigidity and regulation bottlenecks under N-1 contingencies. As shown in Figure 1a, when a single distribution line is disconnected, the system usually relies on tie switches for load transfer. Although this mechanical topological reconfiguration establishes a physical connection, long-distance supply causes terminal voltage drops and tie-line thermal constraints, forcing part of the non-faulted area to shed load. Existing grid-upgrade schemes mainly rely on line reinforcement or additional interconnection channels. However, these schemes are constrained by scarce urban corridor resources, and redundant construction for low-probability N-1 faults leads to low asset utilization. In addition, traditional schemes lack continuous power-flow regulation capability and cannot effectively smooth power fluctuations during transfer. This passive response mechanism based on rigid physical interconnection is therefore insufficient to meet the stringent requirements of high-reliability power supply for system resilience and flexibility.

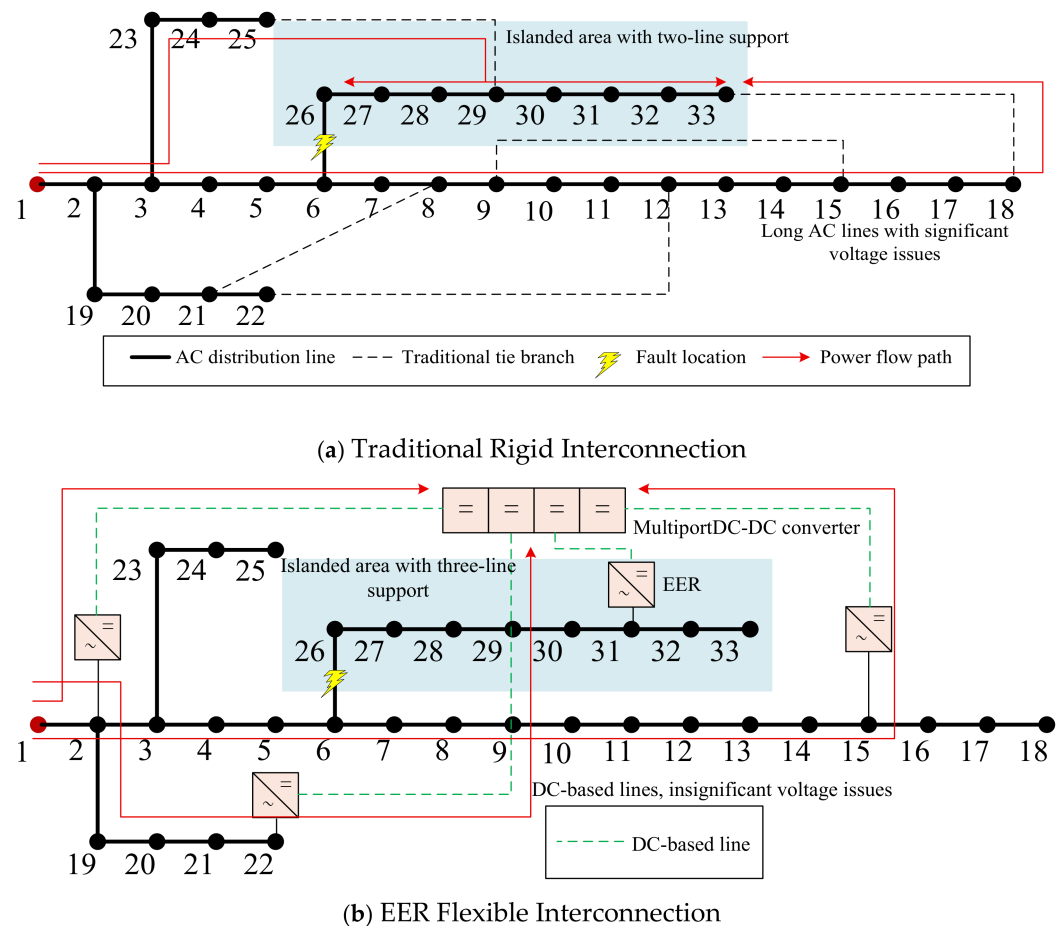


Figure 1. Comparative analysis of interconnection modes in distribution networks.

Against this background, the electric energy router (EER), enabled by power-electronic technology, provides a new flexible interconnection solution for overcoming the N-1 security bottleneck of distribution networks. EERs feature a millisecond-level response and accurate four-quadrant power control, allowing them to break physical isolation between feeders, actively redirect power flow, and dynamically support voltage. Compared with traditional schemes, as shown in Figure 1b, the core of the EER application framework is to upgrade rigid load transfer into flexible mutual support. When a fault occurs, the multi-port characteristics of the EER can be used to rapidly construct a temporary power channel, while reactive-power compensation maintains nodal voltage levels and maximizes the release of transfer capability. Considering the high investment cost of EERs, the proposed planning framework does not pursue full-network redundancy. Instead, it adopts a min-max robust-planning concept and focuses on typical fault sets other than important trunk lines. By optimizing the access locations of EERs, the total load shedding under the worst N-1 contingency is minimized, thereby achieving a qualitative improvement in distribution network supply assurance at the lowest economic cost.

3. Bi-Level Robust Planning Model for EERs

To meet the supply-restoration requirement of distribution networks under N-1 contingencies, this paper proposes an EER siting and sizing method that accounts for the worst fault scenario. The problem involves two coupled time scales: planning and operation. Therefore, a bi-level planning model is constructed to identify an optimal configuration that balances economy and robustness.

3.1. Bi-Level Planning Solution Framework

The proposed bi-level planning model consists of an upper-level siting-and-sizing model and a lower-level fault recovery operation model, as shown in Figure 2.

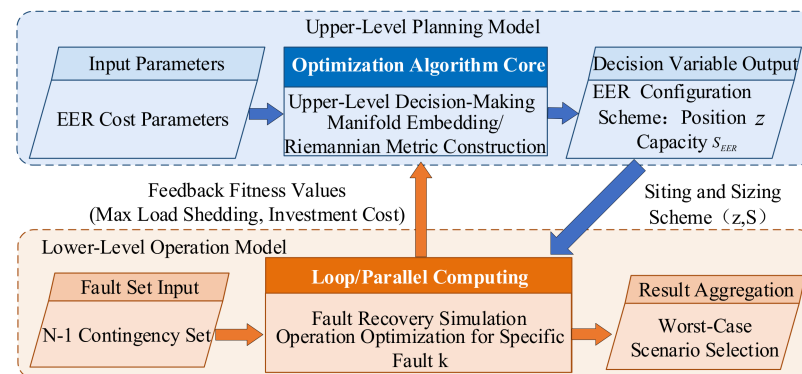


Figure 2. Logic framework of bi-level planning for EER siting and sizing oriented to N-1 security.

The upper-level model is responsible for long-term planning decisions. Taking EER installation locations and capacities as decision variables, it aims to minimize equipment investment cost and the maximum load shedding over the N-1 fault set. The configuration generated by the upper level is then passed to the lower level. The lower-level model is responsible for short-term operational assessment. After receiving the EER configuration, it solves the distribution network fault recovery problem for each N-1 fault scenario while considering the support provided by EERs. This layer adopts mixed-integer second-order cone programming (MISOCP) to calculate the minimum load shedding under each fault scenario and feeds the corresponding loss back to the upper level. Through this bi-level interaction mechanism, the planning scheme can fully anticipate operational risks and effectively defend against extreme N-1 contingencies.

3.2. Upper-Level Model for EER Siting and Sizing

The upper-level model aims to determine the optimal siting and sizing of EERs. As a multi-objective optimization problem, its decision-making process must consider both the investment economy and the ultimate survival capability of the system under N-1 contingencies.

3.2.1. Decision Variables

(1) Siting variables

$$\mathbf{z} = [z_1, z_2, \dots, z_N]^T : z_i \in \{0, 1\}, \forall i \in \Omega_{\text{node}} \quad (1)$$

where the binary siting variable indicates whether an EER is installed at a candidate node, and the candidate-node set denotes the available installation locations in the distribution network.

(2) Sizing variables

$$\mathbf{S}^{\text{EER}} = [S_1^{\text{EER}}, S_2^{\text{EER}}, \dots, S_N^{\text{EER}}]^T \quad (2)$$

where the sizing variable denotes the rated capacity of the EER installed at the corresponding node, in MW.

3.2.2. Objective Function

The model contains two mutually conflicting optimization objectives: minimizing investment cost and minimizing load loss under the worst fault scenario.

(1) Minimization of EER investment cost

The investment cost is mainly determined by EER capacity, and the unit capacity cost is assumed to be constant:

$$\min F_1(\mathbf{z}, \mathbf{S}^{\text{EER}}) = \sum_{i \in \Omega_{\text{node}}} C_{\text{unit}} \cdot S_i^{\text{EER}} \quad (3)$$

where the cost coefficient denotes the unit construction cost of an EER, expressed in 10,000 CNY/MW.

(2) Minimization of maximum load shedding over the N-1 fault set

To ensure robustness against uncertain faults, this paper does not use expected load shedding as the objective, but instead adopts a min–max criterion. The EER configuration is optimized so that the maximum value among the minimum load shedding amounts caused by any line outage in the fault set is minimized:

$$\min F_2(\mathbf{z}, \mathbf{S}^{\text{EER}}) = \max_{k \in \Omega_{\text{fault}}} (L_k^*(\mathbf{z}, \mathbf{S}^{\text{EER}})) \quad (4)$$

where the load shedding term is obtained by the lower-level operation model for the corresponding fault scenario, and the fault set denotes the predefined N-1 faulted-line set. This objective reflects the defensive capability of the planning scheme under the worst case.

3.2.3. Constraints

(1) Installation number constraint

Considering investment budget and operation-and-maintenance difficulty, the total number of installed EERs is limited to a fixed value:

$$\sum_{i \in \Omega_{\text{node}}} z_i = N_{\text{num}} \quad (5)$$

where the total number of EERs to be installed is set to four in this paper.

(2) Capacity selection constraint

The capacity of each EER must satisfy the allowable engineering lower and upper bounds. If no EER is installed at a node, its capacity is forced to zero:

$$z_i \cdot S^{\min} \leq S_i^{\text{EER}} \leq z_i \cdot S^{\max} \quad (6)$$

where the lower and upper bounds represent the minimum and maximum selectable capacities of a single EER.

3.3. Lower-Level Fault Recovery Optimization Model

To evaluate the power supply guarantee capability of the EER siting and sizing scheme provided by the upper layer under N-1 fault scenarios, this section establishes a distribution network fault recovery operation model. To illustrate the operation strategy and its mathematical processing in a more direct fashion, this section first introduces its physical model representation, and subsequently reformulates it into an easy-to-solve mixed-integer second-order cone programming (MISOCP) framework.

3.3.1. Physical Model Representation of the Fault Recovery Strategy

Under a specific fault scenario, this physical strategy aims to minimize the system's load loss by coordinating the active and reactive power control of EERs with the load shedding strategy.

(1) Objective Function

Given the EER installation locations and capacity configurations, the operation optimization objective for a specific line fault scenario is to minimize the total load shedding amount during the fault recovery period, which is expressed as follows:

$$L_k^* = \min \sum_{t \in T} \sum_{i \in \Omega_{\text{node}}} P_{i,t}^{\text{shed}} \quad (7)$$

where L_k^* denotes the minimum total load shedding amount under the fault scenario k ; t is the time index; T is the set of scheduling periods during the fault recovery time; i is the node index; Ω_{node} is the set of all nodes in the distribution network; and $P_{i,t}^{\text{shed}}$ represents the active load shedding amount at node i at time t .

(2) Power Balance and Network Constraints

The DistFlow branch flow model is adopted to constrain the physical power distribution. For the set of healthy lines after excluding the faulty line, the following nodal power balance and voltage drop constraints must be satisfied:

$$\sum_{j \in \delta(i)} (P_{ji,t} - r_{ji} l_{ji,t}) + P_{i,t}^{\text{inj}} = \sum_{k \in \pi(i)} P_{ik,t} + P_{i,t}^{\text{load}} - P_{i,t}^{\text{shed}} \quad (8)$$

$$\sum_{j \in \delta(i)} (Q_{ji,t} - x_{ji} l_{ji,t}) + Q_{i,t}^{\text{inj}} = \sum_{k \in \pi(i)} Q_{ik,t} + Q_{i,t}^{\text{load}} - Q_{i,t}^{\text{shed}} \quad (9)$$

$$v_{k,t} = v_{i,t} - 2(r_{ik} P_{ik,t} + x_{ik} Q_{ik,t}) + (r_{ik}^2 + x_{ik}^2) l_{ik,t} \quad (10)$$

where $\delta(i)$ and $\pi(i)$ denote the sets of connected nodes with node i as the parent node and child node in the radial topology, respectively; $P_{ji,t}$ and $Q_{ji,t}$ are the active and reactive power flowing from node j to node i at time t , respectively; $P_{ik,t}$ and $Q_{ik,t}$ are the active and reactive power flowing from node i to node k at time t , respectively; r_{ji} and x_{ji} (as well as r_{ik} and x_{ik}) denote the resistance and reactance of the corresponding lines, respectively; $l_{ji,t}$ and $l_{ik,t}$ represent the square of the current magnitude of the corresponding lines at time t ; $P_{i,t}^{\text{inj}}$ and $Q_{i,t}^{\text{inj}}$ are the net injected active and reactive power from devices such as EERs at node i at time t , respectively, ignoring the voltage drop caused by the DC resistance [22]; $P_{i,t}^{\text{load}}$ and $Q_{i,t}^{\text{load}}$ are the original active and reactive load demands at node i at time t , respectively; $Q_{i,t}^{\text{shed}}$ is the reactive load shedding amount at node i at time t ; and $v_{i,t}$ and $v_{k,t}$ represent the square of the voltage magnitude at node i and node k at time t , respectively.

(3) EER Operation Constraints

Each EER is modeled as a flexible node with four-quadrant power regulation capability. For nodes equipped with EERs, their operating power must satisfy:

$$0 \leq P_{i,t}^{\text{EER,in}} \leq S_i^{\text{EER}} \cdot \mu_{i,t} \quad (11)$$

$$0 \leq P_{i,t}^{\text{EER,out}} \leq S_i^{\text{EER}} \cdot (1 - \mu_{i,t}) \quad (12)$$

$$Q_{i,t}^{\text{EER}} \leq S_i^{\text{EER}} \quad (13)$$

$$P_{i,t}^{\text{inj}} = P_{i,t}^{\text{EER,out}} - P_{i,t}^{\text{EER,in}} \quad (14)$$

Furthermore, all EERs are interconnected via the DC bus and must satisfy the system-level power balance and energy conversion efficiency constraints:

$$\eta_{\text{eer}} \sum_{i \in \Omega_{\text{EER}}} P_{i,t}^{\text{EER,in}} = \sum_{i \in \Omega_{\text{EER}}} P_{i,t}^{\text{EER,out}} \quad (15)$$

where $P_{i,t}^{\text{EER,in}}$ and $P_{i,t}^{\text{EER,out}}$ denote the active power absorbed from and injected into the distribution network by the EER at node i at time t , respectively; S_i^{EER} is the apparent power capacity upper limit of the EER at node i ; $\mu_{i,t}$ is a binary state indicator variable, which takes 1 when the EER is absorbing power and 0 when injecting power, used to prevent simultaneous charging and discharging; $Q_{i,t}^{\text{EER}}$ is the reactive power output of the EER at node i at time t ; η_{eer} is the energy conversion efficiency of the EER network, set to 98% in this paper; and Ω_{EER} represents the set of nodes equipped with EERs.

(4) Load Shedding Constraints

The amount of load shedding cannot exceed the original nodal load, and the power factor of the node must remain constant when the load is shed:

$$0 \leq P_{i,t}^{\text{shed}} \leq P_{i,t}^{\text{load}} \quad (16)$$

$$Q_{i,t}^{\text{shed}} \cdot P_{i,t}^{\text{load}} = P_{i,t}^{\text{shed}} \cdot Q_{i,t}^{\text{load}} \quad (17)$$

3.3.2. MISOCP Mathematical Reformulation for Solvability Guarantee

The physical model defined above contains nonlinear power flow relationships, as well as logical topology discontinuities caused by fault isolation. To guarantee global optimality and computational efficiency, the model is reformulated into a MISOCP form through the following relaxation and linearization techniques.

(1) Second-Order Cone Relaxation

To handle the inherent nonlinear term ($l_{ik,t} \cdot v_{i,t} = P_{ik,t}^2 + Q_{ik,t}^2$) in the exact branch flow equations, the second-order cone relaxation technique is introduced to transform the equality into a convex inequality:

$$l_{ik,t} \cdot v_{i,t} \geq P_{ik,t}^2 + Q_{ik,t}^2 \quad (18)$$

$$\forall ik \in \mathcal{L}_{\text{healthy}} \quad (19)$$

where $\mathcal{L}_{\text{healthy}}$ denotes the set of non-faulty, healthy branches after excluding the faulty line.

(2) Big-M Method for Fault Isolation and Safe Operation

When an N-1 line fault occurs, the faulty line is disconnected, and its power flow is completely interrupted. To prevent the model from becoming unsolvable due to voltage loss in non-faulty areas or islanded regions, the big-M method is introduced to logically relax the voltage constraints:

$$P_{k,t} = 0, Q_{k,t} = 0, l_{k,t} = 0 \quad (20)$$

$$v_{\min} - M \cdot u_{i,t}^{\text{zero}} \leq v_{i,t} \leq v_{\max} + M \cdot u_{i,t}^{\text{zero}} \quad (21)$$

$$0 \leq P_{i,t}^{\text{shed}} \leq P_{i,t}^{\text{load}} \cdot u_{i,t}^{\text{zero}} \quad (22)$$

$$u_{1,t}^{\text{zero}} = 0 \quad (23)$$

where k herein specifically refers to the faulty line, and therefore $P_{k,t}$, $Q_{k,t}$, and $l_{k,t}$ are forcibly constrained to zero, indicating no power flows through the faulty line; $v_{\min}$ and

$v_{\max}$ denote the safety lower and upper limits of the squared nodal voltage magnitude, respectively; M is a sufficiently large positive real constant; $u_{i,t}^{\text{zero}}$ is an introduced binary de-energization indicator variable, which takes 1 if node i is in a completely de-energized (loss of voltage) state at time t , and 0 if it is in normal operation; in Equation (23), $u_{1,t}^{\text{zero}}$ represents the state of the root node (power source), which is strictly constant at 0. Equations (20)–(23) mathematically allow the voltage of islanded nodes in a completely de-energized state to drop to zero without causing numerical singularity issues.

By solving the above MISOCP model, the minimum load shedding loss under a specific fault scenario can be obtained, serving as the input basis for the objective function of the upper-level planning model.

4. Model Solving Based on the WGF-TME Algorithm

The min–max bi-level planning model for EER siting and sizing is characterized by mixed-integer nonconvexity, a nested bi-level structure, and high N-1 simulation cost. Conventional evolutionary algorithms often suffer from the curse of dimensionality and local traps when handling such discrete–continuous coupled problems. Therefore, the WGF-TME method does not treat optimization as a simple point-set search. Instead, based on optimal transport theory, it models population evolution as a gradient-flow process of probability measures in Wasserstein space and constructs a Riemannian metric field using the physical topology of the distribution network. In this way, the robust Pareto front can be captured accurately with a limited number of simulations.

4.1. Population Evolution in Wasserstein Space

In the EER planning problem, the distribution of the solution set contains not only statistical information on locations and capacities but also the topological coupling relationships among decision variables. To describe the evolutionary dynamics of the population toward the Pareto front, the Wasserstein distance is adopted as the metric basis of the evolution process.

4.1.1. Probability-Measure Modeling of Population Distribution

Let the empirical probability measure corresponding to the population of a given generation be defined by Dirac distributions. The optimization process is then equivalent to finding the optimal transport path from the initial distribution to the target distribution.

The second-order Wasserstein distance is defined as follows:

$$W_2^2(\rho, \mu) = \inf_{\pi \in \Pi(\rho, \mu)} \int_{\mathcal{X} \times \mathcal{X}} \|x - y\|^2 d\pi(x, y) \quad (24)$$

where the transport-plan set contains all joint distributions with the specified marginal distributions. Physically, this distance represents the minimum work required to transport the current population shape to the optimal population shape. Compared with KL divergence, the Wasserstein distance can still provide effective gradient signals when the supports do not overlap, thereby avoiding gradient disappearance and ensuring a clear direction during the early stage of evolution.

4.1.2. Particle-Evolution Update Under the JKO Scheme

Based on the Jordan–Kinderlehrer–Otto theory, population evolution can be regarded as the steepest descent flow of a free-energy functional in Wasserstein space. This paper adopts an approximate particle-discretization solution scheme, in which the update di-

rection of particles is no longer the Euclidean gradient of the objective function but the Wasserstein natural gradient:

$$x_i^{t+1} = x_i^t - \eta \nabla_W F(\rho_t) \Big|_{x_i} = x_i^t - \eta (\nabla f(x_i) + \beta \nabla \cdot (\rho_t * \mathcal{K})) \quad (25)$$

where the first term denotes the Wasserstein gradient and the second term denotes the repulsive kernel function. This mechanism has two functions. The first term drives particles toward low-loss regions, corresponding to better objective-function values. The second term introduces repulsive forces generated by particle interactions, automatically maintaining population diversity on the Pareto front without the complicated crowding-distance calculation used in traditional multi-objective optimization.

4.2. Topological Manifold Embedding for Distribution Networks

EER siting involves the discrete graph structure of distribution network nodes. Direct continuous relaxation in Euclidean space ignores the physical connection characteristics of the grid; for example, nodes that are spatially close may not be electrically close. Therefore, this paper introduces topological manifold embedding to map the search space onto a Riemannian manifold.

4.2.1. Riemannian Metric Tensor Induced by Electrical Distance

The search space is modeled as a Riemannian manifold with a location-dependent metric tensor. Unlike conventional information geometry, which constructs metrics using the Fisher information matrix, this paper constructs an electrical topological metric based on the sensitivity matrix of the distribution network:

$$g_{ij}(x) = \delta_{ij} + \alpha \sum_{k \in \Omega_{\text{fault}}} p_k \left| \frac{\partial P_{\text{loss},k}}{\partial x_i} \frac{\partial P_{\text{loss},k}}{\partial x_j} \right| \quad (26)$$

where the sensitivity vector is derived from the Jacobian matrix, and the scenario probability represents the occurrence probability of each N-1 fault scenario. The physical meaning of this metric tensor is to curve the search space. In regions with strong electrical connections and high sensitivity to faults, the space is stretched so that the search step becomes smaller for refined search. In electrically unrelated regions, the space is compressed, allowing the algorithm to move rapidly across the search domain and improve optimization efficiency.

4.2.2. Exponential Mapping in the Discrete–Continuous Hybrid Space

To handle discrete node indices on a continuous manifold, graph embedding is used to map node coordinates into a low-dimensional manifold space. New solutions are generated through exponential mapping on the manifold:

$$x_{\text{new}} = \text{Exp}_x(v) = x + v - \frac{1}{2} \Gamma_{jk}^i v^j v^k + O(\|v\|^3) \quad (27)$$

where the update vector lies in the tangent space, and the Christoffel symbols are used to correct the direction deviation caused by space curvature. Finally, inverse mapping projects the continuous coordinates back to the nearest electrical node, ensuring the physical feasibility of the siting scheme.

4.2.3. Overall Algorithm Procedure

The WGF-TME algorithm integrates the above geometric-flow evolution with manifold embedding. Its overall procedure is shown in Figure 3.

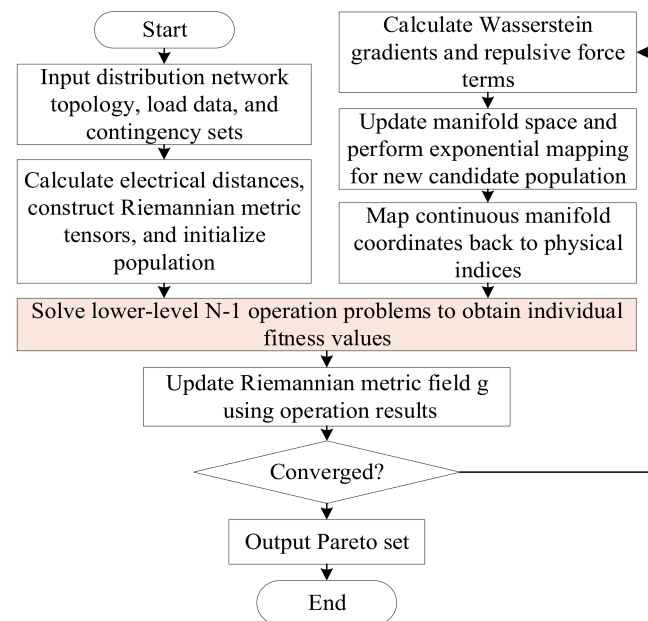


Figure 3. Flowchart of the WGF-TME algorithm.

5. Case Study

To verify the effectiveness of the proposed EER siting-and-sizing planning model and the WGF-TME solution algorithm, simulations were conducted on the IEEE 33-bus distribution system. The software environment was Python 3.11 with Pyomo, and the commercial solver Gurobi 12.0.3 was used to solve the lower-level problem. The hardware environment was an AMD Ryzen 9 9950X CPU at 4.30 GHz with 64 GB of memory.

5.1. Case Parameters and Scenario Settings

The simulation system adopts the standard IEEE 33-bus distribution network with a rated voltage of 12.66 kV. The basic system topology is shown in Figure 4. Solid lines represent conventional distribution lines, while dashed lines represent the five existing tie branches, namely 8–21, 9–15, 12–22, 18–33, and 25–29. These tie switches are open during normal operation and can be closed during faults to provide load transfer paths.

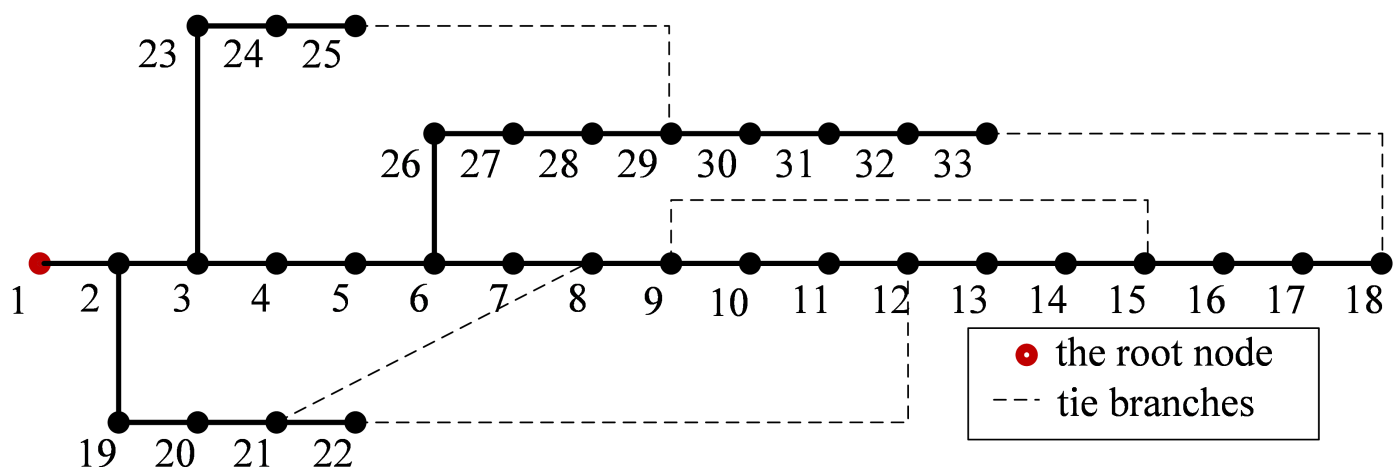


Figure 4. Initial parameters and topology of the IEEE 33-bus system.

To quantitatively analyze the performance of the proposed method, the following three comparative scenarios were established:

Scenario 1 (baseline scheme): no EER is connected, and the system relies only on its own distribution network structure to handle N-1 contingencies.

Scenario 2 (traditional comparison): tie switches are configured in the network, and traditional network reconfiguration is used to handle N-1 contingencies.

Scenario 3 (proposed scheme): the siting-and-sizing scheme is obtained using the bi-level robust planning model and algorithm proposed in this paper.

5.2. Planning Results and Pareto-Front Analysis

The proposed planning model is a typical multi-objective optimization problem that aims to balance investment cost and system robustness. Figure 5 shows the Pareto-optimal front obtained by the algorithm.

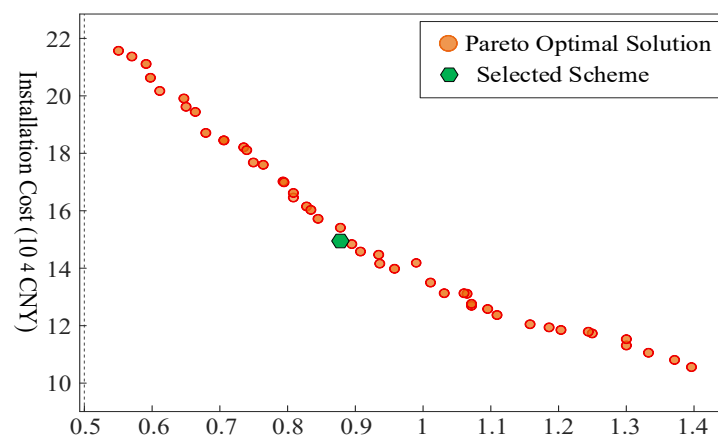


Figure 5. Pareto optimal front of EER siting planning.

As shown in Figure 5, as installation cost increases, the maximum load loss of the system decreases markedly, confirming the positive role of EERs in improving distribution network resilience. Three typical solutions are marked in the figure. Point A represents the lowest-cost scheme but has relatively poor reliability. Point B represents the highest-reliability scheme but involves high investment. Point C is the compromise scheme selected in this paper, which achieves a favorable security level within an acceptable cost range.

For the compromise solution, C, the corresponding spatial distribution of optimal EER siting is shown in Figure 6. The final optimal EER installation locations are nodes 2, 18, 25, and 32, with capacities of 1.19 MW, 0.77 MW, 0.51 MW, and 0.54 MW. These nodes are mostly located at feeder terminals or key interconnection channels, which is beneficial for voltage support and power transfer during faults. This spatial distribution verifies the effectiveness of the WGF-TME algorithm in handling complex topological constraints and shows that it can automatically identify key nodes that contribute the most to N-1 robustness.

Table 1 lists the planning performance indicators under the three scenarios.

Table 1. Comparison of performance indicators for different planning schemes.

Scheme	Worst-Case Load Shedding/MW	Average Load Shedding/MW	Worst-Case Faulted Line
Without EER	2.225	1.344	Line 22
Tie switches	2.163	0.997	
Proposed scheme	0.883	0.349	

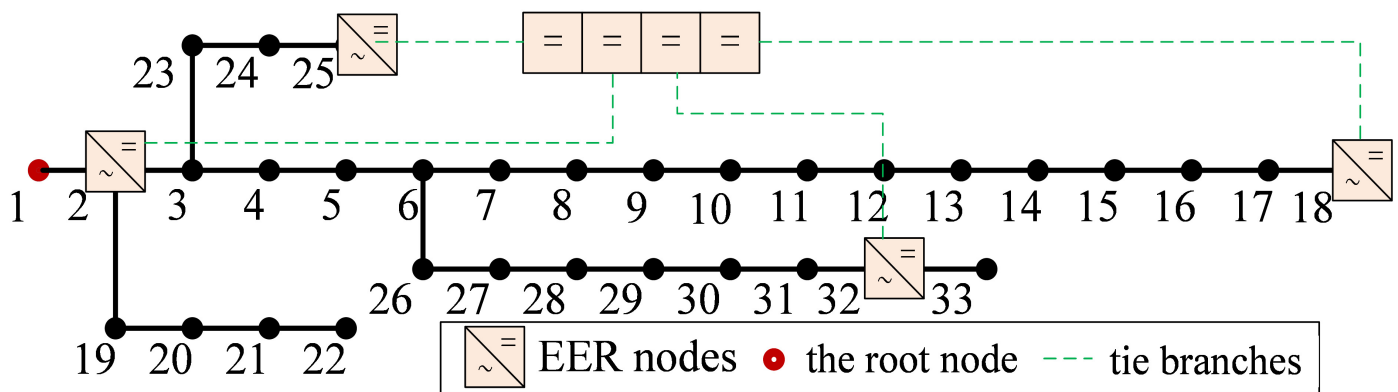


Figure 6. Optimal siting scheme of EER.

Compared with the baseline scenario without EERs, the proposed scheme reduces load shedding under the worst fault scenario from 2.225 MW to 0.883 MW, corresponding to a reduction of 60.3%. Compared with the tie-switch scheme, the worst-case load shedding is reduced by 59.2%. These results indicate that EER integration substantially alleviates the supply pressure caused by N-1 faults and compensates for the limitations of the original distribution network structure.

5.3. Fault Recovery Performance Evaluation

5.3.1. Global Fault Recovery Capability

Figure 7 compares the total system load shedding of the three schemes when N-1 faults occur on different lines. The proposed EER scheme produces significantly lower load shedding than the baseline and tie-switch schemes in most fault scenarios. This verifies that EERs provide superior critical support compared with traditional physical switches under extreme fault conditions.

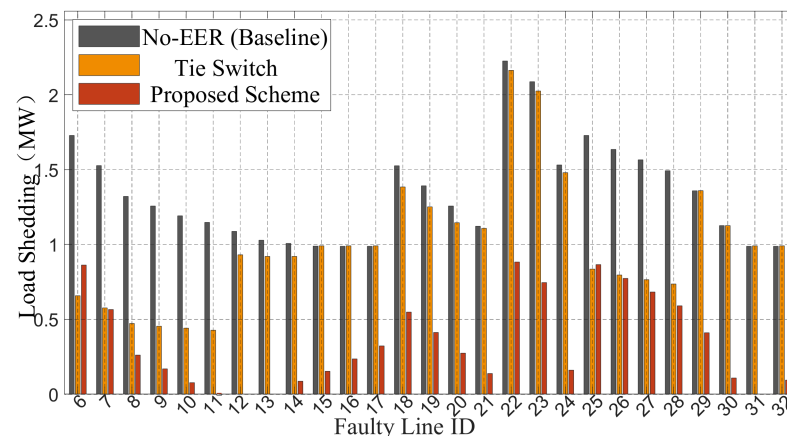


Figure 7. Comparison of load shedding distribution under N-1 contingencies.

Figure 8 reveals the load shedding distribution caused by faults at the microscopic nodal level. Comparing the three groups of views, the baseline and tie-switch schemes exhibit obvious large-area high-risk regions under specific fault scenarios, indicating severe local supply bottlenecks. In contrast, the proposed EER scheme effectively suppresses these high-loss regions and keeps the supply deficit of each node at a relatively low level, verifying the effectiveness of EERs in providing local power support.

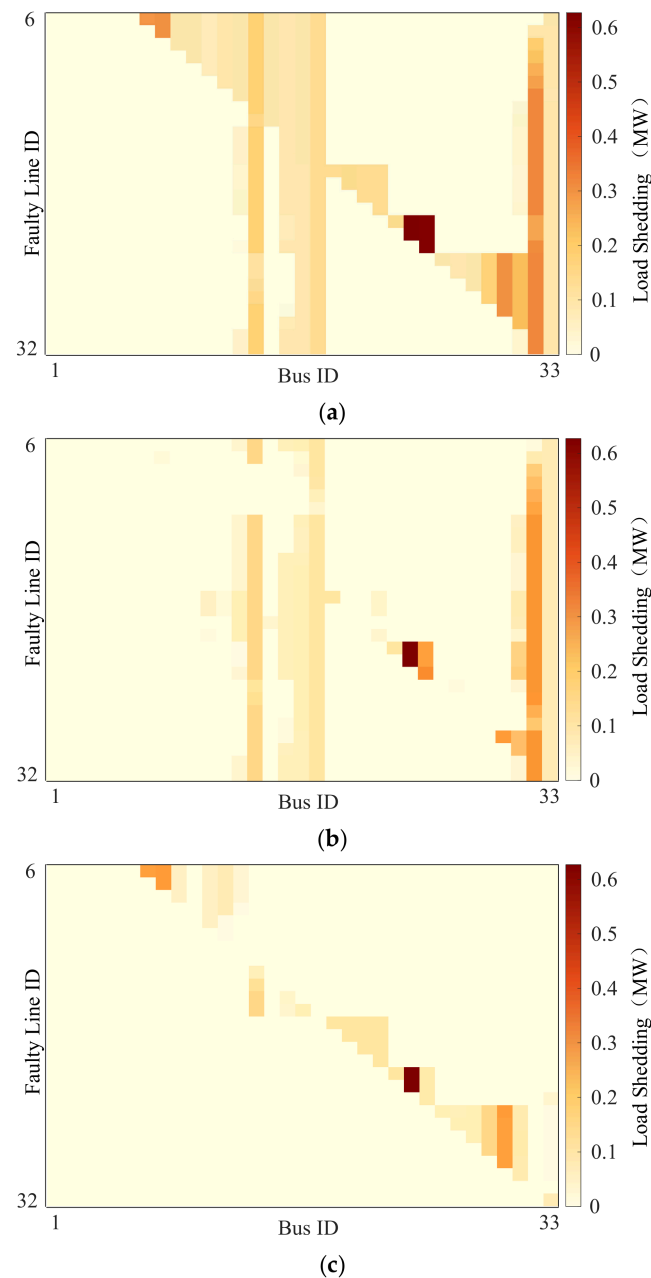


Figure 8. Heat maps of nodal load shedding under various fault scenarios for three schemes (MW). (a) Heat map of nodal load shedding under various fault scenarios without EER; (b) Heat map of nodal load shedding under various fault scenarios with tie switches; (c) Heat map of nodal load shedding under various fault scenarios with EER.

5.3.2. Global Nodal Voltage Distribution Under Different Schemes

Voltage violation is the direct cause of forced load shedding under large disturbances. Figure 9 presents global nodal-voltage heat maps of the baseline, tie-switch, and EER schemes under the N-1 fault set. The vertical axis represents the faulted line, and the horizontal axis represents system nodes. For visualization, some islanded EER access points are treated as grid-forming points with their voltage set to 1 p.u., while the voltages of the remaining islanded nodes are calculated through power-flow analysis.

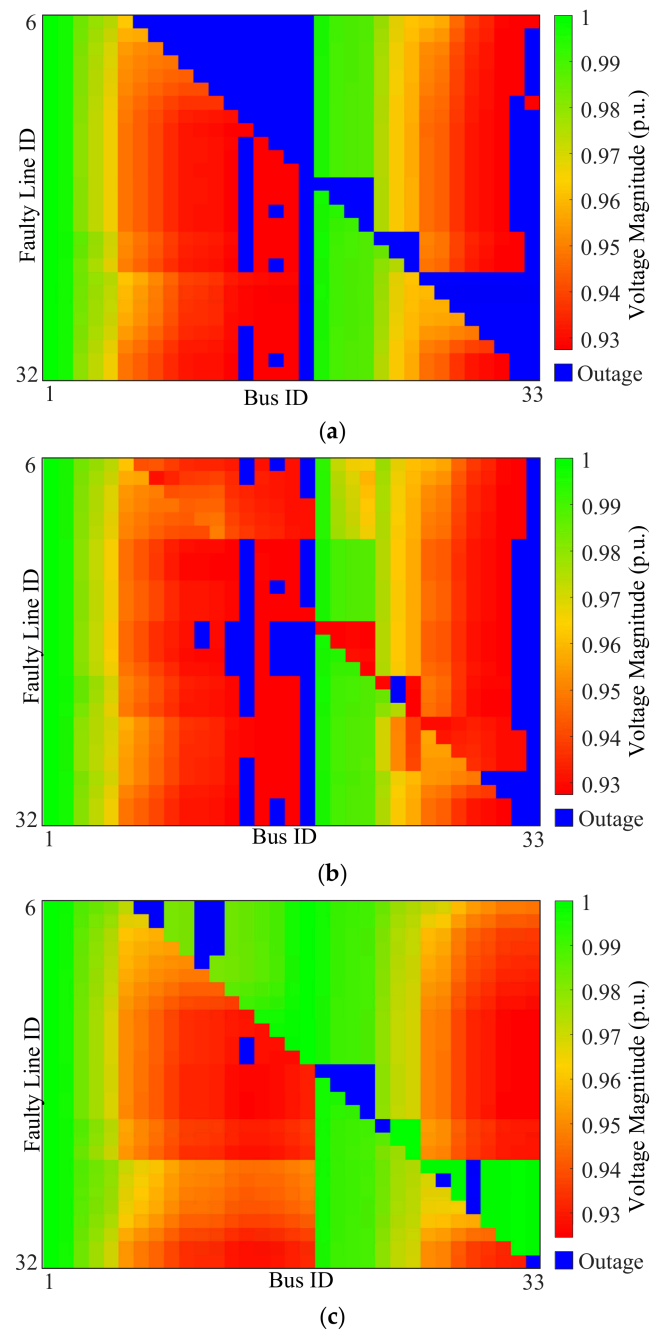


Figure 9. Heat maps of nodal voltage distribution under various fault scenarios for three schemes. (a) Nodal voltage distribution under various fault scenarios without EER (p.u.); (b) Nodal voltage distribution under various fault scenarios with tie switches (p.u.); (c) Nodal voltage distribution under various fault scenarios with EER (p.u.).

Figure 9a,b show large dark-red low-voltage regions. In particular, in the row corresponding to the outage of line 22, the red region extends across multiple terminal nodes. This indicates that even though tie switches provide a physical transfer path, the voltage drop caused by long-distance line impedance is too large for terminal-node voltages to remain within the secure range. By contrast, EER integration effectively improves the voltage distribution of the entire network and mitigates the load shedding risk caused by voltage violations.

5.4. Algorithm Performance Analysis

As shown in Figure 10, the convergence trajectories of the proposed WGF-TME algorithm and the benchmark algorithm are visually compared. The three-dimensional evolution surface shows that WGF-TME presents a steeper decline in the early iterations, indicating that it can use the Wasserstein natural gradient to rapidly approach the optimal-solution region and converge significantly faster than the benchmark algorithm. In addition, the final distribution of WGF-TME on the Pareto front is more uniform and extensive, effectively covering the complete decision space from minimum cost to maximum reliability. This verifies the advantages of the algorithm in avoiding premature convergence and maintaining population diversity.

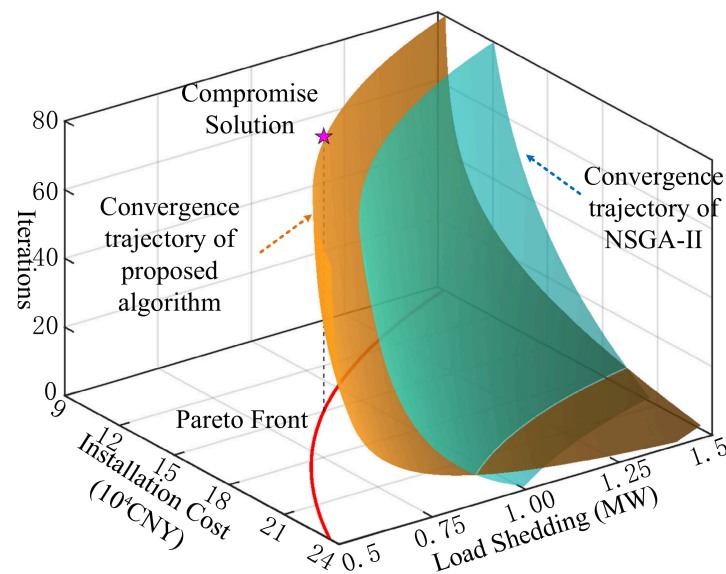


Figure 10. Convergence trajectory of the multi-objective optimization algorithm.

The total running time of the entire planning process is approximately 6537.3 s. Considering that the bi-level optimization model embeds a large number of complex MISOCP subproblems, the current computational efficiency can fully satisfy offline calculation requirements in the distribution network planning and design stage.

6. Conclusions

Based on robust optimization theory, this paper establishes a bi-level EER siting-and-sizing planning model for defense against typical N-1 fault sets, providing a new method for improving the supply-restoration capability of distribution networks. The simulations on the IEEE 33-bus system led to the following conclusions:

- (1) The proposed planning scheme improves both system robustness and economy. The simulation results show that when the system faces the worst fault, the traditional tie-switch scheme improves the supply deficit by only about 2.8%, whereas the proposed scheme reduces load shedding from the baseline value of 2.225 MW to 0.883 MW, a reduction of 60.3%, thereby effectively mitigating load loss in extreme scenarios.
- (2) The proposed WGF-TME algorithm demonstrates excellent convergence performance in solving the siting-and-sizing problem. The case-study results show that the algorithm can converge to a stable Pareto front within 3000 lower-level simulations. Compared with traditional evolutionary algorithms, the method effectively maintains population diversity and overcomes premature convergence in a high-dimensional search space.

- (3) The key value of EERs in overcoming the voltage bottleneck of traditional load transfer is revealed. The comparative analysis shows that traditional tie switches are constrained by the line impedance of long-distance supply and therefore suffer from voltage limitations. The proposed scheme uses the active voltage-support capability of EERs to effectively eliminate low-voltage blind zones at the terminals of the distribution network and convert the transfer potential of physical interconnection into actual supply-assurance capability.

Subsequent studies will examine the expected load supply capability and site selection planning of energy storage systems or EERs with energy storage capacity during island power outages, within the context of photovoltaic substations.

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References

1. Escalera, A.; Prodanović, M.; Castronuovo, E.D.; Roldan-Perez, J. Contribution of active management technologies to the reliability of power distribution networks. *Appl. Energy* **2020**, *267*, 114919. [\[CrossRef\]](#)
2. Xiao, J.; Liu, S.; Li, Z.; Li, F. Loadability formulation and calculation for interconnected distribution systems considering N-1 security. *Int. J. Electr. Power Energy Syst.* **2016**, *77*, 70–76. [\[CrossRef\]](#)
3. Alobaidi, A.H.; Fazlhashemi, S.S.; Khodayar, M.; Wang, J.; Khodayar, M.E. Distribution service restoration with renewable energy sources: A review. *IEEE Trans. Sustain. Energy* **2023**, *14*, 1151–1168. [\[CrossRef\]](#)
4. Wang, C.; Pang, K.; Shahidehpour, M.; Wen, F.; Duan, S. Two-stage robust design of resilient active distribution networks considering random tie line outages and outage propagation. *IEEE Trans. Smart Grid* **2023**, *14*, 2630–2644. [\[CrossRef\]](#)
5. Ding, T.; Wang, Z.; Jia, W.; Chen, B.; Chen, C.; Shahidehpour, M. Multiperiod distribution system restoration with routing repair crews, mobile electric vehicles, and soft-open-point networked microgrids. *IEEE Trans. Smart Grid* **2020**, *11*, 4795–4808. [\[CrossRef\]](#)
6. Ding, T.; Li, C.; Liu, X.; Xie, H.; Tang, Y.; Huang, C. Optimally allocating energy storage for active distribution networks to reduce the risk under N-1 contingencies. *IEEE Syst. J.* **2021**, *15*, 1518–1527. [\[CrossRef\]](#)
7. Baran, M.E.; Wu, F.F. Network reconfiguration in distribution systems for loss reduction and load balancing. *IEEE Trans. Power Deliv.* **1989**, *4*, 1401–1407. [\[CrossRef\]](#)
8. Zu, G.; Wang, Y.; Jiang, X.; Hao, Z.; Zhang, X. Total supply capability of electricity distribution networks considering flexible interconnection of low-voltage service transformers. *IET Smart Grid* **2024**, *7*, 454–469. [\[CrossRef\]](#)
9. Cao, W.; Wu, J.; Jenkins, N.; Wang, C.; Green, T. Operating principle of soft open points for electrical distribution network operation. *Appl. Energy* **2016**, *164*, 245–257. [\[CrossRef\]](#)
10. Fan, P.; Yu, T.; Wang, Z.; Pan, Z.; Wu, Y. Resilience-oriented soft open point deployment with topology-variable-based frequency stability constraints for distribution networks. *Appl. Energy* **2025**, *394*, 126120. [\[CrossRef\]](#)
11. Huang, J.; Han, W.; Tian, H.; Qiu, J.; Chan, C.C. The application and challenge of energy router in energy internet. *Energy Internet* **2024**, *1*, 14–26. [\[CrossRef\]](#)
12. Cao, W.; Wu, J.; Jenkins, N.; Wang, C.; Green, T. Benefits analysis of soft open points for electrical distribution network operation. *Appl. Energy* **2016**, *165*, 36–47. [\[CrossRef\]](#)

13. Lan, L.; Liu, G.; Zhu, S.; Hou, M.; Liu, X. Fault recovery strategy for urban distribution networks using soft open points. *Energy Convers. Econ.* **2024**, *5*, 42–53. [\[CrossRef\]](#)
14. Ji, H.; Wang, C.; Li, P.; Song, G.; Wu, J. SOP-based islanding partition method of active distribution networks considering the characteristics of DG, energy storage system and load. *Energy* **2018**, *155*, 312–325. [\[CrossRef\]](#)
15. Wang, Y.; Su, X.; Song, M.; Jiang, W.; Shahidehpour, M.; Xu, Q. Sequential load restoration with soft open points and time-dependent cold load pickup for resilient distribution systems. *IEEE Trans. Smart Grid* **2023**, *14*, 3427–3438. [\[CrossRef\]](#)
16. Chen, Z.; Hu, Z.; Wan, Y.; Li, J. An extreme weather risk-averse planning method for soft open points considering distribution network reconfiguration. *J. Mod. Power Syst. Clean Energy* **2025**, *14*, 1014–1026. [\[CrossRef\]](#)
17. Luan, W.; Zhang, C.; Xia, X. Soft open point and mobile energy storage system coordination planning for improving the resilience of distribution systems. In Proceedings of the 2024 36th Chinese Control and Decision Conference (CCDC), Xi'an, China, 25–27 May 2024; pp. 4398–4404.
18. Wang, R.; Ji, H.; Li, P.; Yu, H.; Zhao, J.; Zhao, L.; Zhou, Y.; Wu, J.; Bai, L.; Yan, J.; et al. Multi-resource dynamic coordinated planning of flexible distribution network. *Nat. Commun.* **2024**, *15*, 4535. [\[CrossRef\]](#) [\[PubMed\]](#)
19. Ehsanbakhsh, M.; Sepasian, M.S. Simultaneous siting and sizing of Soft Open Points and the allocation of tie switches in active distribution network considering network reconfiguration. *IET Gener. Transm. Distrib.* **2023**, *17*, 1126–1142. [\[CrossRef\]](#)
20. Ehsanbakhsh, M.; Sepasian, M.S. Bi-objective robust planning model for optimal allocation of soft open points in active distribution network: A flexibility improvement approach. *Electr. Power Syst. Res.* **2023**, *224*, 109780. [\[CrossRef\]](#)
21. Zhang, T.; Mu, Y.; Jia, H.; Wang, X.; Pu, T. Successive MISOCP algorithm for islanded distribution networks with soft open points. *CSEE J. Power Energy Syst.* **2023**, *9*, 209–220. [\[CrossRef\]](#)
22. Cao, H.; Shao, C.; Jia, J.; Yan, X.; He, J.; Liu, Y. A novel electric energy router for enhancing the resilience of building distribution networks under extreme cold wave conditions. *Energy Build.* **2026**, *362*, 117533. [\[CrossRef\]](#)

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