

Article

Numerical Investigation of a Novel Hybrid Strategy Combining Obstacles and Nanofluids for Enhanced Corrugated Channel Performance

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Abstract

This study presents a numerical investigation of heat transfer enhancement in a corrugated channel equipped with concave-up obstacles and hybrid nanofluids. The novelty of the present work lies in the combined evaluation of a new obstacle configuration with five different nanoparticles and hybrid nanofluids at two volume concentrations using both single-phase and two-phase numerical models. Numerical simulations were carried out using ANSYS Fluent 19.2 with a two-phase mixture model. Five types of nanoparticles (SiO_2 , TiO_2 , Al_2O_3 , ZnO , and CuO) were tested at volume fractions of 1% and 2%, with obstacles optimized in size and position to enhance fluid mixing, over a Reynolds number range of 10,000–30,000. The combined application of concave-up obstacles and nanofluids increased the heat-transfer performance by approximately 244% in terms of percentage enhancement (PE) relative to the baseline corrugated channel using water without obstacles. Despite a considerable pressure drop (up to 15.5 times the baseline pressure ratio (PR)), the performance evaluation coefficient (PEC) indicates an effective trade-off, with the Al_2O_3 – ZnO (50:50) hybrid nanofluid (Case 2) achieving a balanced thermal–hydraulic performance with a PEC of 1.28. These findings demonstrate that the combined application of corrugated channels, obstacles, and hybrid nanofluids is a highly effective strategy for improving heat exchanger efficiency in practical applications.

Keywords: corrugated channel; obstacles; hybrid nanofluid; passive heat transfer enhancement

Received: 29 May 2026

Revised: 4 July 2026

Accepted: 5 July 2026

Published: 9 July 2026

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1. Introduction

Passive heat transfer enhancement methods are widely used because they are simple, inexpensive, and capable of providing good performance. These methods are applied to systems without the need for any external energy input, unlike active methods. Common passive techniques include the use of corrugated channels, obstacles, and nanofluids. In many cases, two or more of these methods can be combined to achieve even greater heat transfer improvement [1,2].

Figure 1 illustrates the overall passive heat transfer enhancement strategy, where corrugated channel geometry, obstacles, and nanofluids are individually introduced and then combined to achieve improved thermal performance.

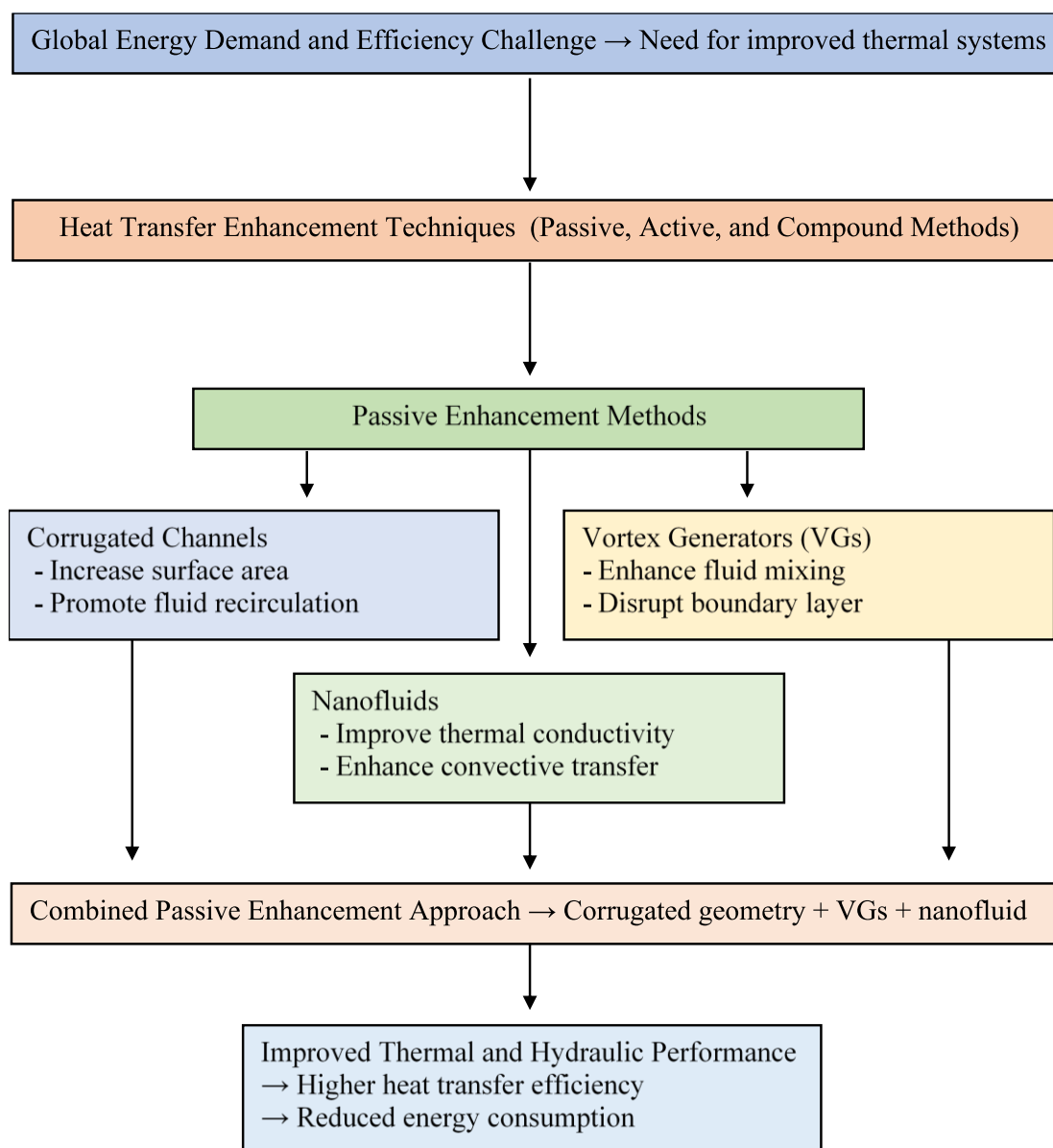


Figure 1. Schematic diagram of passive heat transfer enhancement techniques.

Corrugated channels are commonly employed to enhance heat transfer due to their strong ability to improve thermal performance. Numerous experimental and numerical investigations have examined their characteristics, and the findings consistently show that corrugated channels are effective in increasing heat transfer efficiency [3–6]. Ahmed et al. [7] conducted numerical and experimental studies on three types of channel configurations: trapezoidal, sinusoidal, and straight. Their findings indicated that the corrugated channels provided superior heat transfer performance compared to the straight configuration, with the trapezoidal geometry showing the most significant improvement. However, this enhancement was associated with a higher pressure drop. Similarly, Ajeel et al. [8] performed both numerical and experimental investigations on a half-sinusoidal corrugated channel using nanofluids. Their comparison between corrugated and straight channels revealed that the corrugated design achieved better thermal performance, highlighting its effectiveness for heat transfer enhancement applications.

The vortex generator is a solid body that can be designed in multiple ways. They help control and manipulate fluid flow by directing it toward specific regions, increasing the contact between the fluid and the surface, and improving thermal efficiency [9–12]. Depending on their geometry (e.g., winglet, cylindrical, wavy, and perforated forms), VGs

can significantly alter flow structures and enhance mixing, although often accompanied by an increase in pressure drop. Extensive research has demonstrated their effectiveness in a wide range of thermal and aerodynamic applications. Numerous studies have investigated the role of VGs both numerically and experimentally in different thermal systems. Ruifang et al. [13] investigated the optimal configuration and inflow angle of vortex generators considering three-dimensional rotational effects. Their study demonstrated that the use of vortex generators enhanced the aerodynamic performance of the wind turbine by delaying flow separation along the blade section. Thier et al. [14] analyzed two-dimensional steady convective heat transfer in a corrugated channel equipped with V-type vortex generators. The introduction of longitudinal vortex generators (winglets) aligned with the corrugation direction disrupted the stationary vortical structures. As a result, fluid mixing and thermal uniformity were markedly improved due to the formation of beneficial secondary flows that redirected cooler core fluid toward the heated channel walls. Leylakoochi and Deylami [15] proposed corrugated rectangular winglet pairs (CRWP) as a modified vortex-generator design in a turbulent minichannel. Their results showed that corrugating the vortex generators improved the thermal performance factor by up to 15% and enhanced heat transfer by approximately 35–36% compared with a smooth channel. The study highlighted the importance of optimizing the heat-transfer enhancement and pressure-drop trade-off when designing passive enhancement techniques. Kolenchukov et al. [16] studied hemispherical protrusions in tubular channels and showed that surface roughness enhances heat transfer by up to 2.23 times compared to a smooth channel. They also reported an optimal protrusion height (~ 2 mm), where heat transfer improvement is balanced with pressure-drop effects, while excessive roughness reduces performance due to stagnant zones and higher hydraulic losses. Other studies have also reported that different VG geometries, such as curved, cylindrical, and perforated designs, can further enhance heat transfer by reducing recirculation zones and thinning the thermal boundary layer, while their performance strongly depends on geometric parameters such as size, spacing, and arrangement [17,18]. These vortex generators have been widely applied in heat exchangers, electronic cooling systems, and internal channel flows to improve thermal performance and flow mixing, demonstrating their practical importance in compact heat transfer devices.

Nanofluids are engineered by modifying the thermophysical properties of a base fluid through the addition of different types of nanoparticles with specific characteristics. This modification enhances the thermal conductivity of the fluid and increases its viscosity, which leads to improved heat transfer performance but also results in a higher pressure drop. Due to these advantages, nanofluids are widely used in various thermal applications such as heat exchangers, automotive radiators, electronic cooling systems, solar collectors [19–22]. Ajeel et al. [23] investigated heat transfer and flow behavior of SiO_2 -water nanofluid in a symmetric semicircle-corrugated channel using numerical simulations. Their results showed that the height-to-width ratio had a stronger influence on thermal performance than the pitch-to-length ratio, despite increasing pressure drop. An optimal geometry was identified at $h/W = 0.05$ and $p/L = 0.075$, and new correlations for the Nusselt number and friction factor were proposed. Ebrahimi et al. [24] investigated nanofluid flow in a rectangular microchannel heat sink with longitudinal vortex generators using numerical simulations. Their results showed that Al_2O_3 -water and CuO -water nanofluids improved heat transfer and overall efficiency compared to pure water, with a corresponding increase in pressure drop, and that CuO -water nanofluid achieved the best performance.

Hybrid nanofluids are formed by combining two different types of nanoparticles, which allows the material properties of the nanoparticles to be tailored and better controlled. This approach enables adjustment of their effects on the thermal and hydraulic

behavior of the fluid. Hybrid nanofluids have been widely explored in previous studies, where they demonstrated promising results and practical applicability in heat transfer enhancement [25–28]. Alawi et al. [29] investigated heat transfer enhancement using MWCNTs–TiO₂ hybrid nanofluids flowing over a corrugated surface through numerical simulations. Their results showed that the hybrid nanofluid significantly improved thermal performance compared to water, with heat transfer enhancement increasing as the nanoparticle weight concentration increased. The study also confirmed that the hybrid nanofluid altered thermophysical properties, promoted stronger turbulence and secondary flows, and effectively disrupted thermal boundary layers, leading to improved heat transfer efficiency. Selimefendigil and Oztop [30] investigated the combined use of surface corrugation and hybrid nanofluids to enhance the performance of a thermoelectric generator module through numerical simulations. Their results showed that hybrid nanofluids, together with corrugated channel surfaces, significantly increased power output by intensifying heat transfer, with corrugation height having a stronger effect than the number of waves. The study concluded that this combined approach is a promising method for improving thermoelectric generator performance.

Two main approaches are commonly used to simulate nanofluid flow. The first is the single-phase method, which is the most widely applied approach. In this method, the nanofluid is assumed to be homogeneous, and its thermophysical properties are calculated using appropriate mixture correlations based on the base fluid and the suspended nanoparticles. The second approach is the two-phase method, which is more advanced and considers distinct behaviors of the base fluid and nanoparticles. This method allows the software to account for differences in velocity and interaction between the two phases, making it more representative of real physical conditions; however, it requires significantly higher computational cost [31–33]. Moghari et al. [34] investigated laminar-mixed convection of Al₂O₃–water nanofluid in an annular channel using a two-phase mixture model. Their results showed that accounting for nanoparticle Brownian motion improved the prediction of thermophysical properties, and increasing nanoparticle volume fraction enhanced heat transfer at both inner and outer walls without significantly affecting friction. The study also highlighted the important role of secondary flows in strengthening thermal transport. Sabaghan et al. [35] investigated nanofluid flow in a rectangular micro-channel with longitudinal vortex generators using a two-phase Eulerian–Eulerian model. The study showed that TiO₂-based nanofluids significantly enhanced heat transfer and overall efficiency, especially when combined with LVGs, while the EG–water mixture provided better performance than pure water. The results also confirmed that increasing nanoparticle concentration improved heat transfer despite changes in friction behavior. Tanougast et al. [36] numerically investigated heat transfer enhancement in a trapezoidal channel equipped with winglet-type vortex generators using single and hybrid nanofluids within a two-dimensional single-phase framework. Their results demonstrated that combining passive enhancement techniques improved the overall thermal–hydraulic performance. However, the study was limited to the single-phase approach and a limited range of nanofluids, motivating further investigation using more advanced two-phase modeling and a broader selection of nanoparticles and hybrid nanofluids.

Previous studies have examined three different passive techniques for heat transfer enhancement and their individual effects. However, the combined application of these three methods has not yet been fully investigated. These methods include the use of a corrugated channel, which defines the channel geometry and improves thermal efficiency; obstacles, which control and manipulate the fluid flow to enhance heat absorption; and nanofluids, which modify the working fluid properties to improve heat transfer performance. By integrating these three approaches, a higher-efficiency heat exchanger can be achieved. Unlike previous studies, the present work considers a different corrugated

channel geometry and employs an obstacle configuration specifically selected to be consistent with the corrugation characteristics and to maximize the interaction between the generated vortices and the corrugated surface. In the present study, the channel geometry and obstacle design are selected based on previously validated high-performance configurations to form a unique combined arrangement [37]. Furthermore, the study extends previous investigations by examining both single and hybrid nanofluids based on five different nanoparticles (SiO_2 , TiO_2 , Al_2O_3 , ZnO , and CuO), allowing a comprehensive assessment of the influence of nanoparticle type and hybridization on thermohydraulic performance. Simulations are carried out using both single-phase and two-phase models to provide more accurate predictions. The results offer a detailed assessment of the thermal and hydraulic behavior of the proposed configuration. This combined enhancement strategy shows strong potential for industrial heat exchanger applications.

2. Numerical Methodology

2.1. Mathematical Formulation

In this study, water was modeled as a viscous, Newtonian, and incompressible fluid with constant thermophysical properties. The flow was assumed to be steady [38] and turbulent. To simplify the analysis while capturing the essential flow characteristics, a two-dimensional (2D) simulation was carried out in the x – y plane. This approach allowed for a detailed investigation of the primary flow direction along the x -axis, the formation of recirculation zones, and the development of the thermal boundary layer, in accordance with previous two-dimensional studies [14,17,39]. Body forces were neglected because their influence on the flow field was considered insignificant. The governing equations for the conservation of mass, momentum, and energy in Cartesian coordinates are expressed as follows [40].

- Mass conservation equation:

$$\frac{\partial u_i}{\partial x_i} = 0. \quad (1)$$

- Momentum conservation equation:

$$\frac{\partial(u_j \rho u_i)}{\partial x_j} = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \left(\frac{\partial u_i}{\partial x_j} - \rho \overline{u'_i u'_j} \right) \right). \quad (2)$$

- Energy conservation equation:

$$\frac{\partial(\rho u_i T)}{\partial x_j} = \frac{\partial}{\partial x_i} \left(\left(\frac{\mu}{Pr} + \frac{\mu_t}{Pr_t} \right) \frac{\partial T}{\partial x_i} \right). \quad (3)$$

Due to the presence of turbulence, the Reynolds-averaged Navier–Stokes (RANS) equations introduce additional unknown terms, known as Reynolds stresses, represented by $-\rho \overline{u'_i u'_j}$. These terms arise from the averaging process and require a turbulence model to close the system of equations. In the present study, the shear stress transport (SST) k – ω model was adopted to provide closure [41]. The SST k – ω model was selected due to its proven capability to accurately predict near-wall flow behavior, adverse pressure gradients, and flow separation/reattachment phenomena commonly encountered in corrugated channels with obstacles.

$$-\rho \overline{u'_i u'_j} = \mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \rho k \delta_{ij} - \frac{2}{3} \mu_t \frac{\partial u_k}{\partial x_k} \delta_{ij}, \quad (4)$$

- The turbulent kinetic energy (k):

$$\frac{\partial u_i k}{\partial x_i} = \frac{\partial}{\partial x_i} \left[(\mu + \mu_t \sigma_k) \frac{\partial k}{\partial x_i} \right] + G_k - Y_k. \quad (5)$$

- The specific dissipation rate (ω):

$$\frac{\partial u_i \omega}{\partial x_i} = \frac{\partial}{\partial x_j} \left(\Gamma_\omega \frac{\partial \omega}{\partial x_j} \right) + G_\omega - Y_\omega + D_\omega. \quad (6)$$

In these equations, G_k denotes the rate of turbulent kinetic energy generation resulting from the mean velocity gradients, whereas G_ω represents the production rate of ω associated with the mean flow. The terms Y_k and Y_ω account for the dissipation of k and ω due to turbulence effects. The turbulent viscosity μ_t is incorporated into the momentum and energy equations, influencing the Reynolds stresses and enhancing the effective thermal diffusion.

2.2. Thermophysical Properties of Nanofluid

- The density [42]:

$$\rho_{nf} = (1 - \phi) \rho_{bf} + \phi \rho_{np}. \quad (7)$$

- The effective heat capacity [43]:

$$(\rho C_p)_{nf} = (1 - \phi) (\rho C_p)_{bf} + \phi (\rho C_p)_{np}. \quad (8)$$

- The effective viscosity [44]:

$$\frac{\mu_{nf}}{\mu_{bf}} = \frac{1}{1 - 34.87 \left(\frac{d_{np}}{d_{bf}} \right)^{-0.3} \phi^{1.03}}. \quad (9)$$

The equivalent diameter of base molecule:

$$d_{bf} = 0.1 \left(\frac{6M}{N\pi\rho_{bf}} \right)^{1/3}, \quad (10)$$

where the Avogadro number is $N = 6.022 \times 10^{23} \text{ mol}^{-1}$.

Using the empirical correlation below, the influence of Brownian motion will be taken into account when computing the k_{nf} while employing nanoparticles in the corrugated channel [45]:

$$k_{nf} = k_{static} + k_{brownian}, \quad (11)$$

$$\frac{k_{static}}{k_{bf}} = \frac{k_{np} + 2k_{bf} + 2(k_{np} - k_{bf})\phi}{k_{np} + 2k_{bf} - (k_{np} - k_{bf})\phi}, \quad (12)$$

$$k_{brownian} = 5 * 10^4 * \beta * \phi * \rho_f * C p_f * \sqrt{\frac{k_B T}{2}} * f(T, \phi), \quad (13)$$

where Boltzmann constant: $k_B = 1.3807 \times 10^{-23} \text{ J/K}$; and the values of $\beta = 8.4407(100\phi)^{-1.07304}$, the function $f(T, \phi)$ is given by

$$f(T, \phi) = (2.8217 \times 10^{-2} \phi + 3.917 \times 10^{-3}) \left(\frac{T}{T_0} \right) + (-3.0669 \times 10^{-2} \phi - 3.391123 \times 10^{-3}). \quad (14)$$

The above correlations were used only for the single-phase simulations.

2.3. Nanoparticle Characteristics

In this study, five metal-oxide nanoparticles, SiO_2 , TiO_2 , Al_2O_3 , ZnO , and CuO , were selected due to their widespread use, well-documented thermophysical properties [25,46,47], and strong relevance in heat-transfer applications. These nanoparticles are

among the most commonly investigated materials in nanofluid research, primarily because they combine high stability, chemical inertness, and enhanced thermal performance compared with base fluids. An additional advantage is that these metal-oxide nanoparticles are relatively inexpensive and among the most widely available and frequently used in both experimental and numerical nanofluid studies, which makes them practical and cost-effective choices for engineering applications [46].

SiO_2 is frequently used as a benchmark nanoparticle owing to its excellent stability in aqueous suspensions and its low density, which helps reduce sedimentation a key aspect highlighted in several reviews on nanofluid reliability [48]. TiO_2 is similarly known for its chemical stability and moderate thermal conductivity, making it suitable for assessing the influence of material properties on heat-transfer mechanisms [49]. Al_2O_3 remains one of the most studied nanoparticles because of its good thermal conductivity, high availability, and proven stability, with many experimental works confirming that Al_2O_3 –water nanofluids maintain consistent dispersion over long periods [50].

On the other hand, ZnO and CuO were included because they provide a contrasting behavior characterized by much higher thermal conductivities, offering insight into how highly conductive nanoparticles influence thermal enhancement and pressure-drop penalties. ZnO , in particular, has been reported to provide one of the strongest increases in effective thermal conductivity, while CuO is known for its high density and its ability to significantly modify the hydrodynamic characteristics of the flow [51]. Including materials with different conductivities, densities, and stability characteristics allows for a comprehensive evaluation of how nanoparticle type affects heat transfer, pressure drop, mixing intensity, and overall performance. This diverse selection ensures that the conclusions drawn from the simulations are robust and consistent with the trends reported in the nanofluid literature.

After selecting the nanoparticle types, their thermophysical properties were collected from previously published studies available in the literature. These properties include density, specific heat capacity, thermal conductivity, and dynamic viscosity, which are required for defining the nanofluid behavior in the numerical model. The selected nanoparticles (SiO_2 , TiO_2 , Al_2O_3 , ZnO , and CuO) present a wide range of thermal conductivities and densities, allowing a meaningful comparison of their effects on heat transfer and pressure drop. In the present study, a uniform nanoparticle diameter of 30 nm was assumed for all nanoparticle types, as commonly adopted in the literature [52,53]. The properties adopted in the present work are summarized in Table 1 and were taken directly from literature sources to ensure consistency and reliability [54–58]. These nanoparticle properties are subsequently used in Equations (7)–(14) to determine the effective thermophysical properties of the nanofluid, which are then implemented in ANSYS Fluent assuming a homogeneous single-phase flow model.

Table 1. The thermophysical properties of base fluid (water) and different types of nanoparticles.

Properties	Water	SiO_2	TiO_2	Al_2O_3	ZnO	CuO
Density (ρ)	998.2	2200	4250	3970	5610	6500
Heat capacity (C_p)	4182	703	686.2	765	502	535.6
Thermal conductivity	0.6	1.2	8.95	40	110	20
Viscosity (μ)	0.00103	–	–	–	–	–

2.4. Two-Phase Model

To apply the two-phase model, the mixture's momentum equations are solved for a single velocity field, while the secondary phase velocity is estimated using algebraic formulas. Both phases are interpenetrating, with separate velocity fields based on volume fraction, but they share the same pressure. The interaction between phases is modeled

through drag forces and momentum exchange. This approach allows the relative motion (slip velocity) between nanoparticles and base fluid to be considered without fully solving separate momentum equations for each phase, making it computationally efficient while still capturing particle–fluid dynamics. The governing equations are solved separately: for the primary phase (base fluid), the continuity, momentum, and energy equations are used; for the secondary phase (nanoparticles), only the scalar transport equation for volume concentration is solved, accounting for phase slip effects [59]. Thus, the mixture (two-phase) model provides a practical balance between accuracy and computational cost for simulating nanofluid flow and heat transfer. In the present study, the slip velocity was calculated using the Manninen et al. formulation, while the interphase momentum exchange was modeled using the Schiller–Naumann drag law. The Brownian force and turbulent dispersion options available in the mixture model were not activated because of the low nanoparticle volume fractions (1–2%) and the assumption of a uniformly dispersed suspension. In the numerical setup, the base fluid and nanoparticle properties are defined separately in ANSYS Fluent, which subsequently computes the effective mixture properties and flow behavior according to the adopted two-phase mixture formulation.

- Continuity Equation:

The continuity equation for the mixture can be expressed as [60]:

$$\nabla \cdot (\rho_m \vec{V}_m) = 0, \quad (15)$$

$$V_m = \sum_{np=1}^n \frac{\varphi_{np} \rho_{np} \vec{V}_{np}}{\rho_m}, \quad (16)$$

where ρ_m is the density of the nanofluid, V_m is the mass-averaged velocity of the nanofluid, V_{np} is the velocity of the nanoparticles, φ_{np} is the volume fraction of nanoparticles, np refers to the solid nanoparticles, and n is the number of phases (with $n = 1$ for a simple nanofluid and $n = 2$ for a hybrid nanofluid).

- Momentum Equation [61]:

$$\nabla \cdot (\rho_m \vec{V}_m \otimes \vec{V}_m) = -\nabla P + \nabla \cdot (\mu_m (\nabla \vec{V}_m)) + \nabla \cdot \sum_{np=1}^n (\varphi_{np} \rho_{np} \vec{V}_{dr,np} \vec{V}_{dr,np}), \quad (17)$$

where V_{dr} is the drift velocity of the nanoparticle, which can be written as:

$$\vec{V}_{dr,np} = \vec{V}_{np} - \vec{V}_m. \quad (18)$$

The drift velocity and the slip velocity can be related as follows:

$$\vec{V}_{dr,np} = \vec{V}_{np,bf} - \sum_{np=1}^n \frac{\varphi_{np} \rho_{np} \vec{V}_{np}}{\rho_m}. \quad (19)$$

The slip velocity is defined as the velocity of the second phase (np) relative to the velocity of the first phase (bf), as follows:

$$\vec{V}_{np,bf} = \vec{V}_{np} - \vec{V}_{bf}. \quad (20)$$

Manninen et al. [62] proposed Equation (22) for the relative velocity, while Equation (23), which was proposed by Schiller and Naumann [63], is manipulated to calculate the drag function.

$$\vec{V}_{np,bf} = \frac{\rho_{np} d_{np}^2}{18 \mu_{bf} f_{drag}} \frac{\rho_{np} - \rho_m}{\rho_{np}} \vec{a}, \quad (21)$$

d_{np} and a denote the nanoparticle diameter and acceleration, respectively.
Where

$$\vec{a} = \vec{g} - (\vec{V}_m \cdot \nabla) \vec{V}_m, \quad (22)$$

g denotes the gravitational acceleration.

$$f_{drag} = \begin{cases} 1 + 0.15 Re_{np}^{0.687}, & Re_{np} \leq 1000 \\ 0.0183 Re_{np}, & Re_{np} > 1000 \end{cases} \quad (23)$$

$$Re_{np} = \frac{V_m d_{np}}{\nu_m}. \quad (24)$$

- Energy Conservation [64]:

$$\nabla \cdot \sum_{np=1}^n (\varphi_{np} \rho_{np} \vec{V}_{np} C p_{np} T) = \nabla \cdot (k_m \nabla T). \quad (25)$$

- Volume Fraction Equation:

The volume fraction of the secondary phase can be derived from the continuity equation of the secondary phase as follows:

$$\nabla \cdot (\varphi_{np} \rho_{np} \vec{V}_m) = -\nabla \cdot (\varphi_{np} \rho_{np} \vec{V}_{dr,np}). \quad (26)$$

The present study employs the mixture (two-phase) model in ANSYS Fluent to simulate the nanofluid flow. In this framework, particle–fluid interactions are represented in an averaged (non-particle-resolved) manner through slip velocity and drag forces rather than explicit particle tracking. It should be noted that the Brownian force model available in the mixture formulation was not activated. This should not be confused with the Brownian conductivity term used in the empirical thermal conductivity correlation of the single-phase model, which is employed solely to estimate the effective thermal conductivity and is independent of the Brownian force option in the mixture model. These assumptions are consistent with commonly adopted mixture-model approaches in nanofluid CFD studies [65]. Furthermore, detailed phase-resolved quantities, such as nanoparticle volume-fraction distribution and slip-velocity fields, were not extracted, representing a limitation of the present mixture-model approach.

2.5. Evaluation Parameters

The pressure drop across the channel was determined using the following relation:

$$\Delta P = P_{in} - P_{out}. \quad (27)$$

The average Nusselt number (Nu) is obtained as follows [66]:

$$Nu = \frac{h D_h}{K}, \quad (28)$$

where h represents the convective heat transfer coefficient ($W/(m^2 \cdot K)$), K is the thermal conductivity of the working fluid ($W/(m \cdot K)$), and D_h denotes the hydraulic diameter (m) of the corrugated passage, expressed as [67]:

$$D_h = H + h, \quad (29)$$

where H and h denote the maximum and minimum corrugation heights, respectively, as shown in Figure 2.

The friction factor (f) is calculated using the following equation [68]:

$$f = \frac{2 D_h \Delta P}{L \rho u^2}. \quad (30)$$

To quantify the increase in pressure loss caused by the obstacles (nanofluid) relative to that of a plain wavy channel, the pressure drop ratio (PR) is defined as [69]:

$$PR = \frac{\Delta P_{obs(nf)}}{\Delta P_{obs}}, \quad (31)$$

where $\Delta P_{obs(nf)}$ and ΔP_{obs} are the pressure drops in channels obstacles (nanofluid) and obstacles (water), respectively.

The percentage enhancement (PE) quantifies the improvement in heat transfer when obstacles (nanofluid) are applied and is given by [70]:

$$PE (\%) = \frac{(Nu_{obs(nf)} - Nu_{obs})}{Nu_{obs}} 100, \quad (32)$$

where $Nu_{obs(nf)}$ and Nu_{obs} denote the Nu for channels obstacles (nanofluid) and obstacles (water), respectively.

The overall thermal-hydraulic performance was assessed using the performance evaluation criterion (PEC), which was calculated according to the formulation proposed in Ref. [71]. In this formulation, the hydraulic performance is evaluated using the pressure-drop ratio ($\Delta P_{obs(nf)} / \Delta P_{obs}$), where ΔP_{obs} corresponds to the selected baseline configuration at the same Reynolds number. The reference configuration used in each analysis is explicitly identified in the corresponding figures and discussion:

$$PEC = \frac{Nu_{obs(nf)} / Nu_{obs}}{\left(\Delta P_{obs(nf)} / \Delta P_{obs} \right)^{\frac{1}{3}}}. \quad (33)$$

Table 2 summarizes the configurations used in the present study. Two reference baselines are employed: (G1) corrugated channel with water and obstacles for evaluating nanofluid and hybrid nanofluid enhancement and (G2) corrugated channel with water without obstacles for assessing the combined obstacle + nanofluid effect. This ensures consistent and clear performance evaluation across all cases.

Table 2. Summary of investigated configurations and reference baselines.

Case Group	Channel Configuration	Working Fluid	Obstacle	Nanofluids	Purpose
G1	Corrugated channel	Water	Yes	No	Baseline for nanofluid and hybrid nanofluid cases
G2	Corrugated channel	Water	No	No	Baseline for combined obstacle + nanofluid comparison

2.6. Geometry Description

The semi-circular corrugated channel examined in this work is shown in Figure 2. The channel has an overall length of 500 mm and is divided into three sections: an inlet region ($L1 = 200$ mm), a central test section ($L2 = 200$ mm), and an outlet region ($L3 = 100$ mm). To guarantee fully developed flow conditions and eliminate any possibility of back-flow, the inlet and outlet sections were designed with heights larger than the main channel height (H), as recommended in [14]. The channel geometry features a maximum height (H) of 10 mm and a minimum height (h) of 5 mm. The corrugated surface follows a sinusoidal-like wave pattern defined by a wavelength (P) of 10 mm and a pitch diameter (D) of 5 mm. In addition, concave-up obstacles (CUOs) are used. Their dimensions, including the size (S) and base width (B), were fixed at 5 mm, and the obstacles were positioned at a constant distance of $d = -2$, which corresponds to the configuration that yielded the best performance in previous investigations.

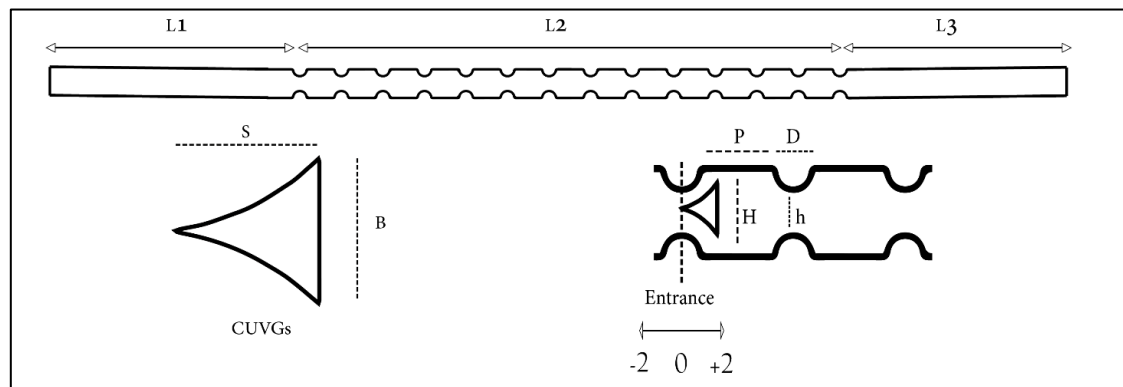


Figure 2. The physical model.

2.7. Boundary Conditions

- Inlet: A uniform velocity inlet is applied with temperature $T = 298$ K. The inlet velocity is calculated based on the Reynolds number as $u = \frac{Re\mu}{\rho D_h}$, corresponding to $Re = 10,000$ – $30,000$. The turbulence intensity is set to 5% with appropriate hydraulic diameter-based specification.
- Walls: No-slip boundary condition is applied on all solid surfaces ($u = v = 0$). The thermal conductivity of the wall material is $K_{cu} = 387.6$ W/m·K. A constant heat flux condition is applied only on the corrugated section ($q_w = 10,000$ W/m²), while the remaining walls are considered adiabatic.
- Outlet: A pressure outlet boundary condition is imposed with $P = P_{atm}$, and zero-gradient conditions are applied for velocity and temperature ($\frac{\partial u}{\partial x} = 0$; $\frac{\partial v}{\partial x} = 0$; $\frac{\partial T}{\partial x} = 0$).

2.8. Numerical Procedure

All numerical simulations in this work were carried out using ANSYS Fluent, a commercial computational fluid dynamics (CFD) package based on the finite volume method, which guarantees strict conservation of mass, momentum, and energy within each control volume. The pressure-based coupled solver [72] was adopted to solve the continuity, momentum, and energy equations simultaneously within a fully coupled solution framework, improving numerical stability and convergence. The solution was considered converged when the residuals of the energy equation dropped below 10^{-6} , while the residuals for the continuity, momentum, and turbulence equations were maintained below 10^{-4} . To enhance solution accuracy, second-order upwind discretization schemes [73] were employed for all governing equations, and the least-squares cell-based approach was used for gradient reconstruction. The main computational setup and numerical methodology adopted in the present simulations are summarized in Table 3.

Table 3. Computational setup and numerical methodology.

Item	Description
Solver type	Pressure-based coupled solver
Governing equations	Continuity, momentum, energy, turbulence
Turbulence model	SST k- ω
Working fluid	Water/nanofluid/hybrid nanofluid
Multiphase model	Mixture model
Discretization scheme	Second-order upwind
Gradient method	Least-squares cell-based
Pressure–velocity coupling	Coupled scheme

Convergence criteria	10^{-6} (energy), 10^{-4} (momentum, continuity, turbulence)
Inlet condition	Velocity inlet
Outlet condition	Pressure outlet

3. Mesh Independence and Validation

Mesh quality has a significant influence on the accuracy of the numerical predictions. Therefore, a mesh independence analysis was performed to determine the minimum grid resolution that still provides reliable results. Five different mesh arrangements were tested, with the total number of computational nodes ranging from 98,037 to 557,168. To accurately resolve near-wall flow behavior, 10 inflation layers were applied along the channel walls (Figure 3) as well as around the obstacles, allowing for precise capture of boundary layer development and steep velocity gradients in these critical regions. The relative deviation between consecutive mesh solutions was evaluated using Equation (34), following the methodology reported in [5], where M_{i+1} represents the current solution, and M_i corresponds to the previously obtained average Nu :

$$\text{Error}(\%) = \left| \frac{M_i - M_{i+1}}{M_{i+1}} \right| * 100. \quad (34)$$

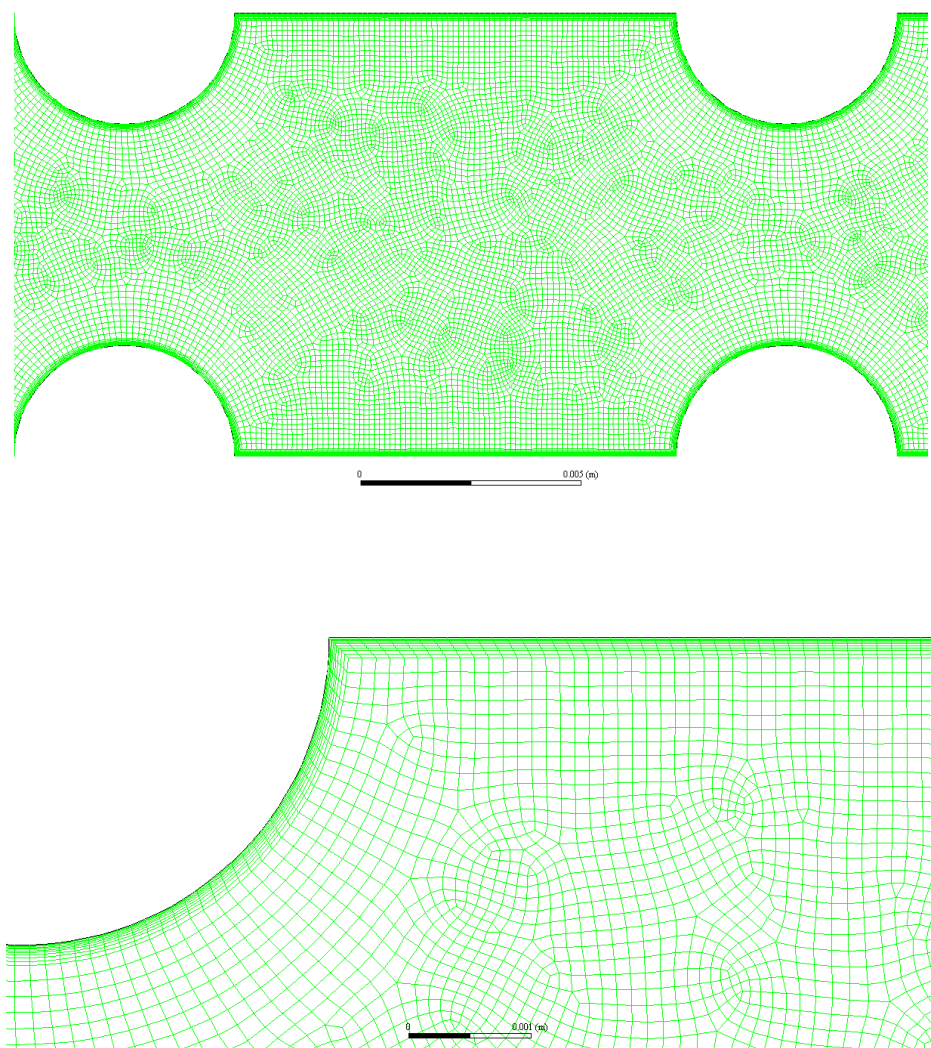


Figure 3. Computational mesh.

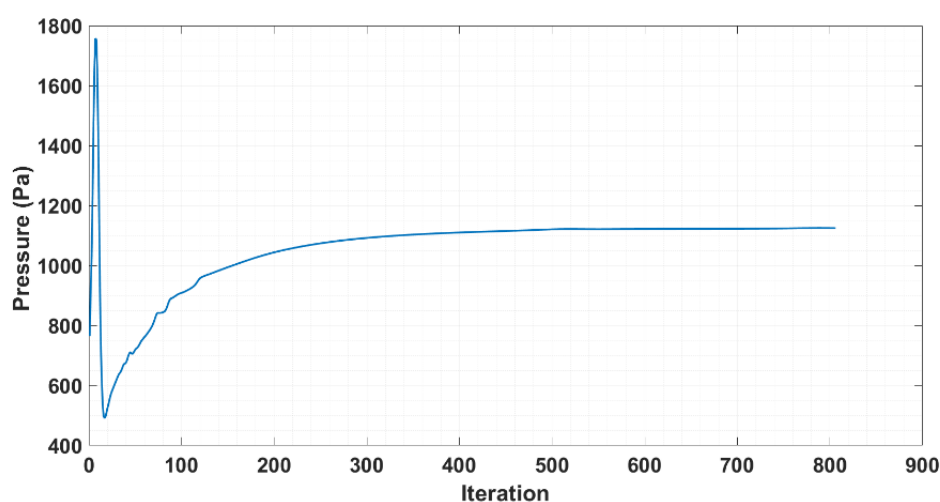
To further assess the numerical uncertainty associated with mesh refinement, the grid convergence index (GCI) method proposed by Roache [74] was also considered. Three representative meshes (coarse, medium, and fine) containing 98,037, 226,732, and 557,168 nodes, respectively, were selected for the analysis. Based on the average Nusselt number, the calculated fine-grid GCI was approximately 2.12%, indicating low mesh-related numerical uncertainty and confirming mesh-independent behavior of the numerical solution.

A notable reduction in error was observed in the final configuration, in which a mesh comprising 392,205 nodes (see Table 4) provided sufficient accuracy and was therefore adopted for the numerical analysis.

Table 4. Mesh independence.

Mesh	Number of Nodes	Nusselt Number	Error (%)
1	98,037	198.41	—
2	148,639	196.72	0.85
3	226,732	195.46	0.64
4	392,205	194.45	0.51
5	557,168	193.46	0.50

The convergence behavior of the numerical solution was assessed by monitoring the variation of the heat transfer coefficient (h) and pressure drop (ΔP) during the iterations. As shown in Figure 4, both h and ΔP reach stable values with negligible fluctuations as the solution progresses, indicating that a steady-state condition has been achieved. Since the Nusselt number is directly related to the local heat transfer coefficient, the stabilized behavior of h confirms the convergence of Nu . Similarly, the steady behavior of ΔP ensures reliable prediction of the flow resistance. These results demonstrate that the numerical solution is stable and independent of further iterations.



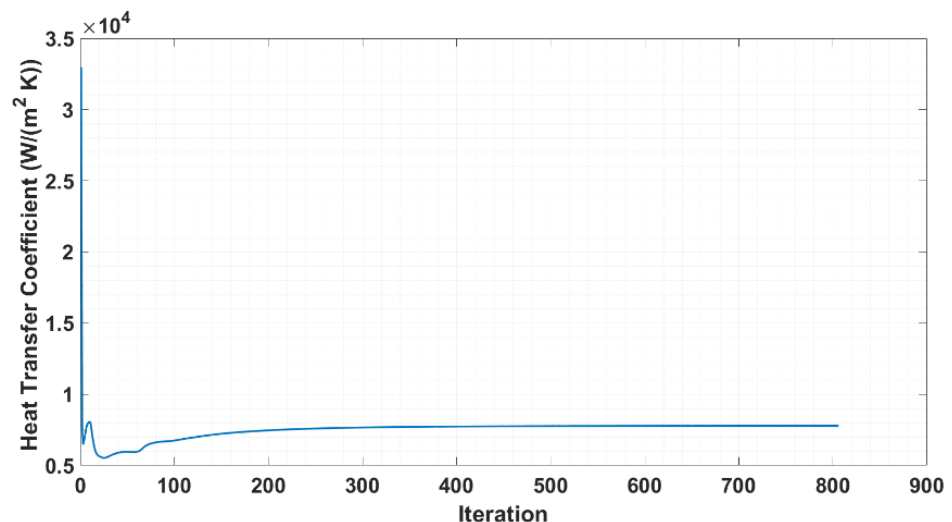
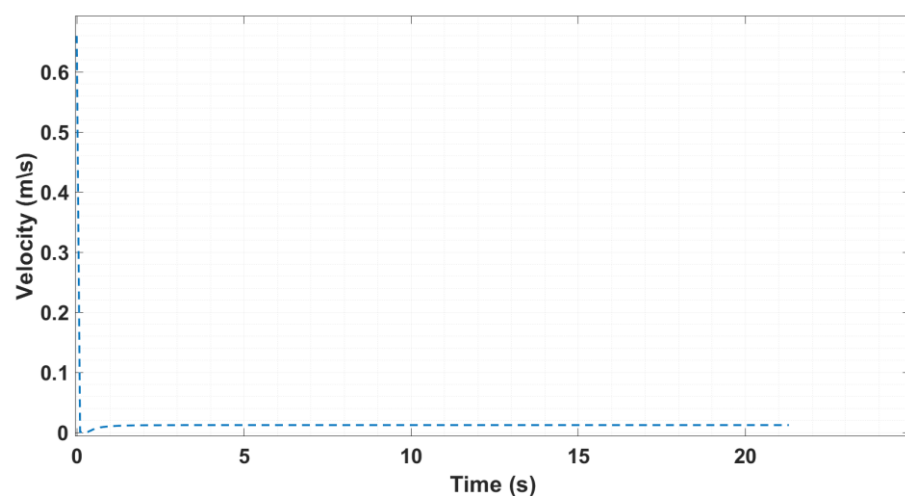


Figure 4. Convergence of residuals during the simulation.

3.1. Steady-State Verification

To verify the steady-state assumption in this study, two monitoring points were selected one located at the wall of the channel and the other at the centerline. The velocity and temperature variations with time were recorded at these points to examine how the hydrodynamic and thermal fields evolved during the simulation.

Figure 5 presents the velocity and temperature profiles at the channel wall. It can be observed that the velocity remains nearly constant over time, with only a minor fluctuation at the very beginning of the simulation. The velocity value at the wall is essentially zero, which confirms the validity of the no-slip boundary condition; this is expected since viscous effects cause the fluid velocity to vanish at the wall. The temperature, on the other hand, shows a slight transient variation during the first second due to the imposed wall heat flux, but the difference is less than 1 K. After this initial adjustment, it stabilizes and remains constant throughout the rest of the simulation period.



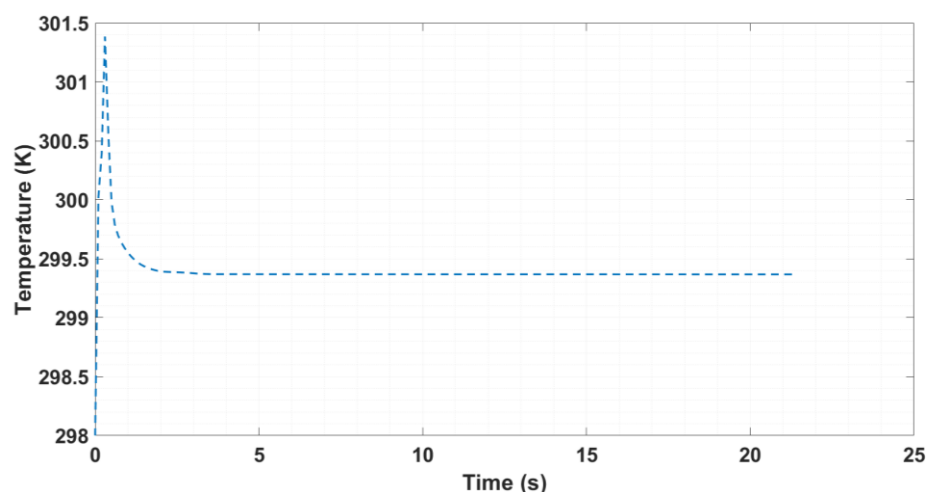
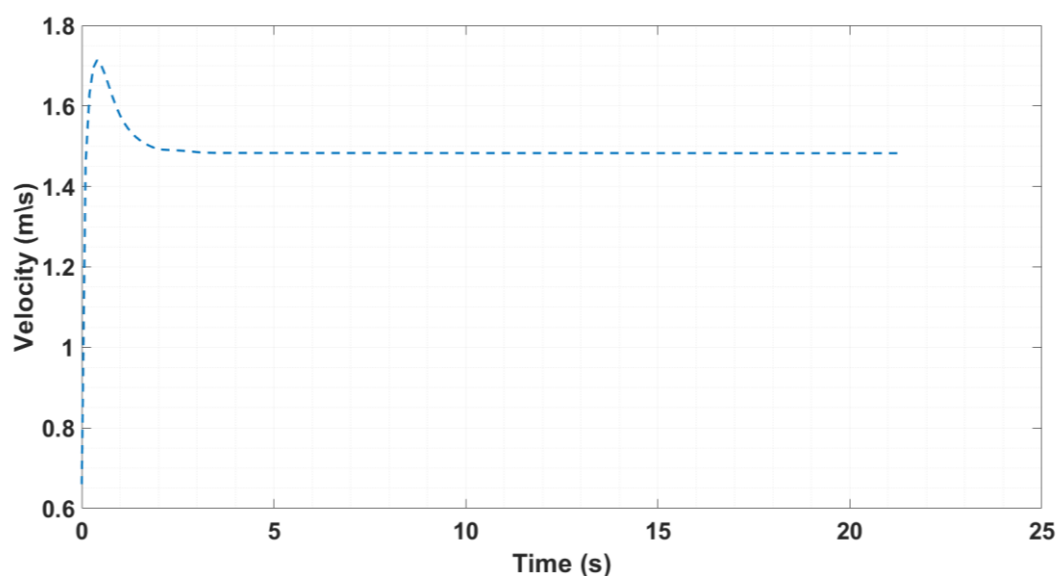


Figure 5. Velocity and temperature profiles with time at the wall of the corrugated channel.

Figure 6 shows the same profiles at the midline of the channel. Here, the velocity initially varies by about 0.2 m/s during the first second before reaching a steady value, while the temperature fluctuates by approximately 0.06 K before becoming stable. This small transient behavior occurs as the flow develops and the solver begins to converge toward a steady solution.

In the numerical setup, each time step was set to 0.1 s with 20 iterations per step. During the initial time steps, the residuals of the governing equations had not yet reached their convergence criteria, resulting in minor variations in the monitored parameters. Once the residuals dropped below the predefined thresholds (after approximately the first second), both velocity and temperature remained constant with time. This confirms that the flow and heat transfer fields are independent of time, and therefore the system can be accurately treated as steady-state in the subsequent analysis.



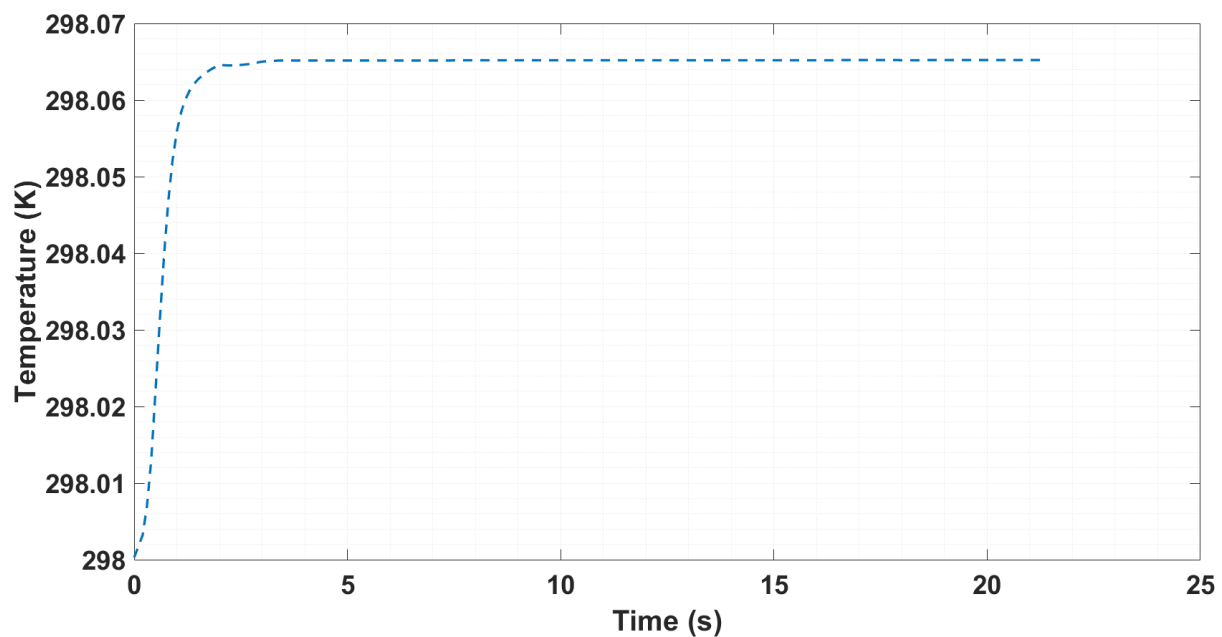


Figure 6. Velocity and temperature profiles with time at the centerline of the corrugated channel.

3.2. Evaluation of 3D Model Effects

To evaluate whether a full three-dimensional analysis was necessary, an additional 3D simulation was performed by extruding the channel 50 mm along the z -axis, as shown in Figure 7. The main flow direction remained along the x -axis. The obstacles were extruded uniformly along the channel width, resulting in a geometry that is invariant in the spanwise (z) direction and therefore does not generate fully developing 3D longitudinal vortex structures. Instead, their primary function is to modify the x - y plane flow field by disrupting and eliminating recirculation zones induced by the corrugated geometry. Velocity contours were examined at several cross-sectional planes located at $z = 10, 20, 30$, and 40 mm. The results showed very similar flow patterns to those obtained from the 2D model, with no significant variation along the z -direction, indicating a uniform flow throughout the channel depth. A slight difference was observed at the channel centerline, where the 3D model exhibited slightly higher velocity concentration due to the influence of four confining walls, whereas the 2D model was limited to interaction with only two walls.

It should be noted that the present model is two-dimensional and therefore cannot reproduce all three-dimensional flow structures, such as longitudinal vortex stretching and secondary flows that may develop in practical devices at high Reynolds numbers. The additional 3D check was used only to assess the spanwise uniformity of the present geometry and not to claim full equivalence between 2D and 3D turbulent flow.

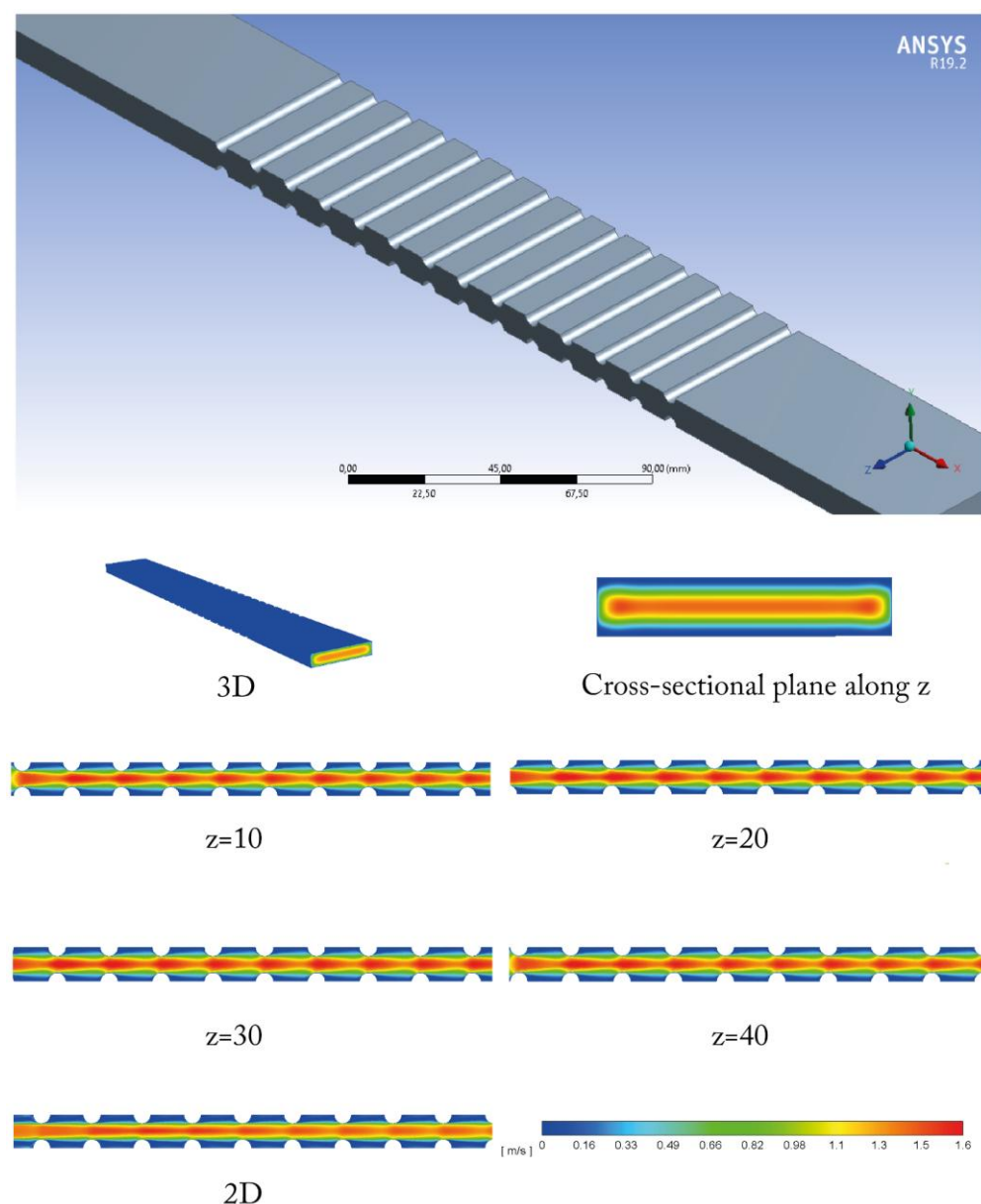


Figure 7. Three-dimensional model of the corrugated channel extruded along the z-axis for comparison with the 2D simulation.

3.3. Validation

To ensure the reliability of the simulation results, a validation study was conducted prior to the main analysis. The model was compared with the experimental work of Ajeel et al. [8], who investigated a semi-circular corrugated channel under similar conditions. Figure 8 shows the variation in the average Nu over a Re range of 10,000–30,000, indicating that the numerical predictions are in good agreement with the experimental data. The average relative error between the numerical and experimental Nusselt number values was approximately 5.31%, while the maximum deviation remained below 8.55%, confirming the reliability of the present numerical methodology for predicting heat-transfer behavior in the corrugated channel without obstacles. This close agreement validates the precision of the numerical setup and the suitability of the selected modeling method for estimating heat transfer in corrugated channels. It is important to note that this validation was conducted for the baseline case (channel without obstacles). An additional comparison with a numerical study incorporating obstacles [14] further supported the robustness and accuracy of the proposed method (Figure 8). The comparison with the numerical

study involving obstacles showed an average relative error of approximately 4.04%, while the maximum deviation remained below 6.18%, further confirming the capability of the present numerical methodology to predict the thermal–hydraulic behavior of corrugated channels equipped with obstacles. Moreover, the ΔP results obtained from the simulation were validated against experimental data [8], with deviations remaining below 13%, demonstrating the reliability of the hydraulic performance prediction (Table 5). For consistency with the experimental study, the pressure-drop values reported in the validation section were calculated using the friction-factor correlation provided in Ref. [8], whereas the pressure-drop values presented in the results section were extracted directly from the ANSYS Fluent simulations. The validation data used in this study are taken from literature for the same corrugated channel geometry and boundary conditions.

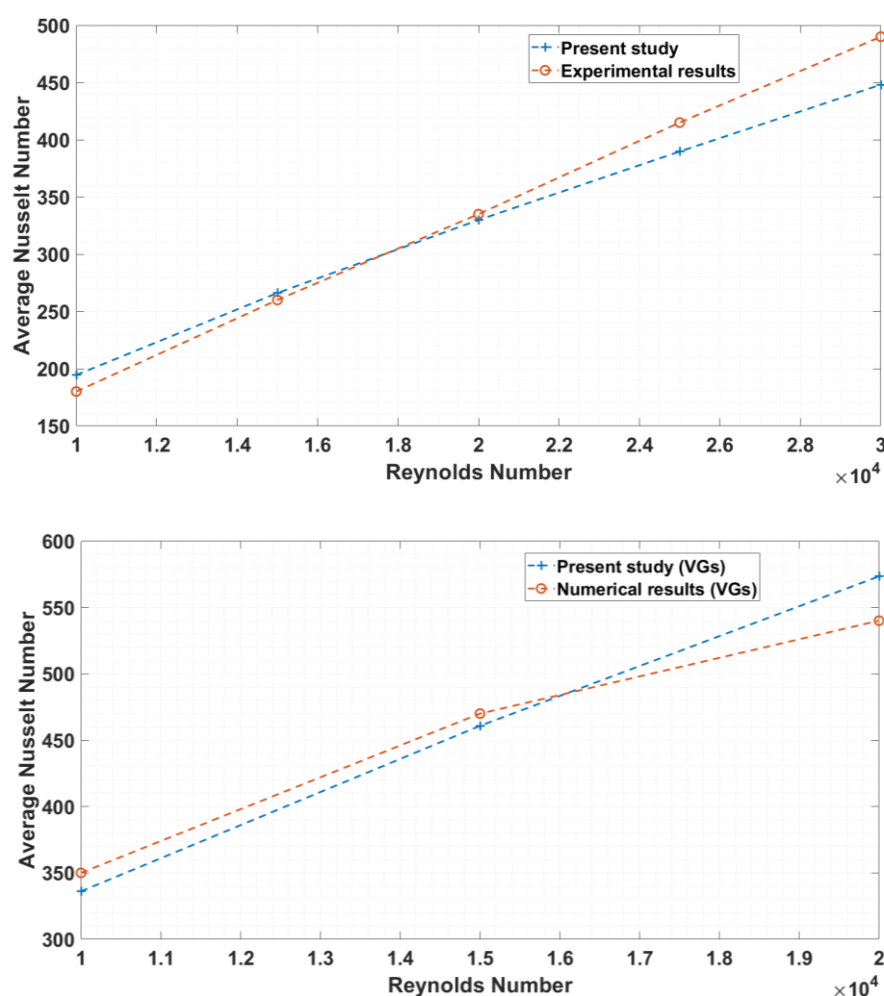


Figure 8. Validation of present CFD results against literature data: average Nu for channels with and without obstacles.

Table 5. Validation of CFD pressure drop against experimental data for the baseline channel, showing percentage deviations.

Re	Friction Factor f	ΔP Numerical (Pa)	ΔP Experimental (Pa)	Difference %
10,000	0.0204	148	132	12.1%
20,000	0.0153	450	400	12.5%
30,000	0.0131	872	800	9.0%

4. Results and Discussion

Figure 9 presents the isotherm contours and streamlines illustrating the thermal and flow characteristics within the corrugated channel equipped with obstacles. The presence of obstacles strongly influences the thermal–hydraulic behavior of the flow by disrupting the thermal boundary layer, particularly near the corrugation corners. The resulting flow structures enhance fluid mixing and increase turbulence intensity, leading to a noticeable improvement in heat transfer performance, although this is accompanied by an increase in pressure drop across the channel. A more detailed analysis of the obstacle configuration and the associated flow behavior can be found in our previous study [37].

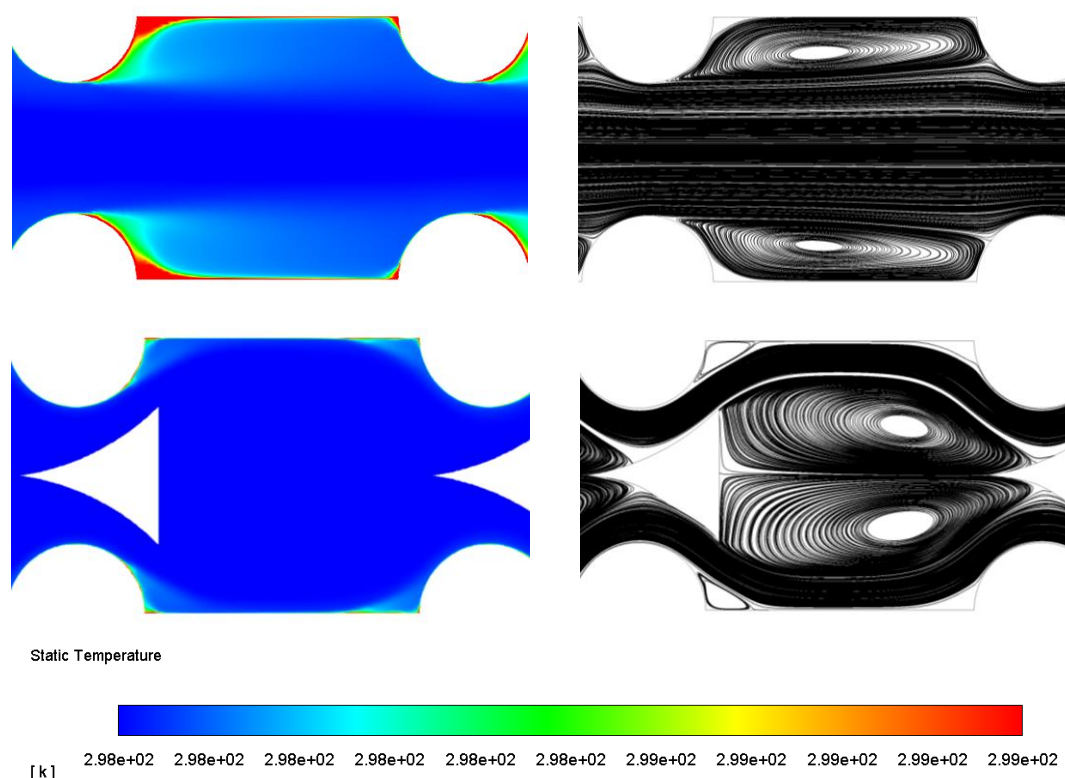


Figure 9. Isotherm contours and streamlines showing the effect of different obstacle positions.

4.1. Validation of Nanofluid Model

Figure 10 presents a comparison between the results of the present numerical study and the experimental data available in the literature for nanoparticle concentrations of 1% and 2%, respectively. The comparison is performed for the channel without obstacles. In both concentrations, the numerical predictions follow the experimental trends closely, with only minor deviations, particularly at higher Re where the enhancement becomes more pronounced. The good agreement observed at all tested Re confirms the reliability of the current numerical approach. It is also noted that the single-phase model produced results nearly identical to those obtained using the two-phase mixture model for this ge-

ometry without obstacles, indicating that single-phase modeling is adequate for baseline channel configurations.

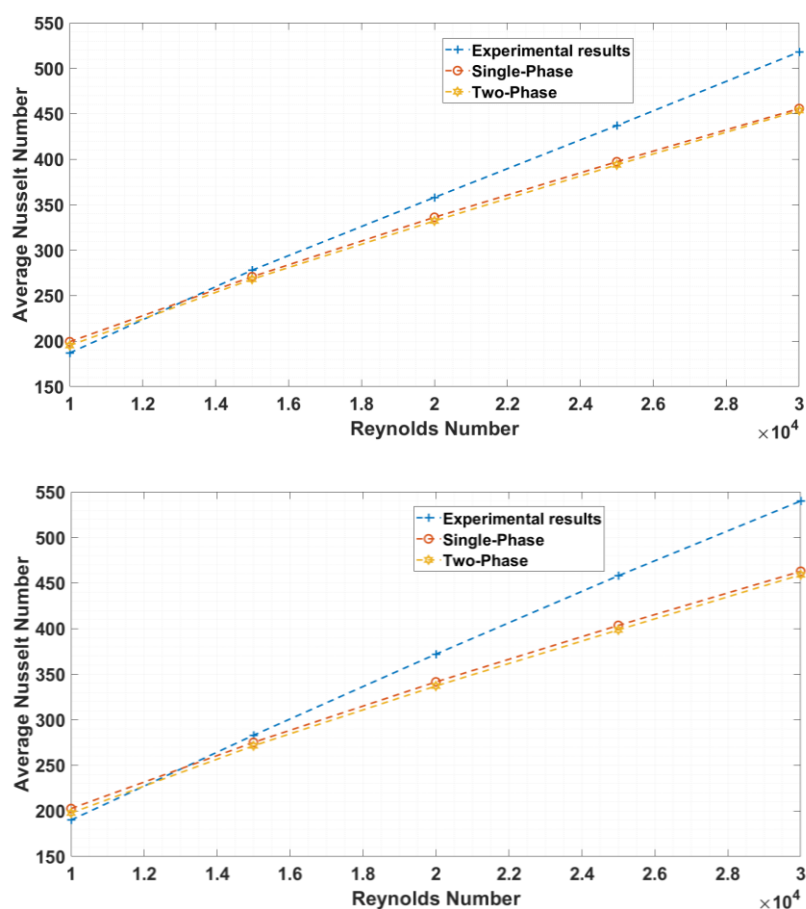


Figure 10. Validation of single-phase and two-phase nanofluid models at nanoparticle concentrations of 1% and 2%.

4.2. Single-Phase vs. Two-Phase Nanofluid Modeling

The Nusselt number and pressure drop results for water and the different nanoparticle types at volume fractions of 1% and 2% are summarized in Tables 6–9.

Table 6. Overview of the average Nu and pressure drop for different nanoparticle types at various Re using the single-phase model (1% concentration).

Parameter	Fluid	Re = 10,000	Re = 15,000	Re = 20,000	Re = 25,000	Re = 30,000
Nu	Water (without obstacle)	194.45	266.09	329.86	389.69	448.08
	Water (with obstacle)	399.09	555.01	700.26	845.48	993.64
	SiO ₂	407.74	562.05	710.29	858.49	1005.56
	TiO ₂	407.78	562.36	710.66	858.83	1005.95
	Al ₂ O ₃	409.19	564.33	713.12	861.79	1009.45
	ZnO	405.98	560	707.61	855.06	1001.51
	CuO	404.57	558.09	705.22	852.17	998.13

ΔP	Water (without obstacle)	1123.93	2503.78	4336.47	6637.79	9457.5
	Water (with obstacle)	14,394.38	31,703.37	54,411.41	82,593.7	117,250.8
	SiO ₂	15,459.23	33,396.27	57,604.27	87,699.88	123,588.70
	TiO ₂	15,155.59	32,746.37	56,480.6	85,996.17	121,183.5
	Al ₂ O ₃	15,208.07	32,862.73	56,675.93	86,288.35	121,602.2
	ZnO	14,949.11	32,300.31	55,703.57	84,815.68	119,510.8
	CuO	14,849.48	32,084.99	55,331.7	84,248.87	118,712.5

Table 7. Overview of the average Nu and pressure drop for different nanoparticle types at various Re using the single-phase model (2% concentration).

ParameterFluid		Re = 10,000	Re = 15,000	Re = 20,000	Re = 25,000	Re = 30,000
Nu	Water (without obstacle)	194.45	266.09	329.86	389.69	448.08
	Water (with obstacle)	399.09	555.01	700.26	845.48	993.64
	SiO ₂	415.06	572.06	722.92	873.94	1023.58
	TiO ₂	414.38	571.49	722.14	872.54	1022.01
	Al ₂ O ₃	418.11	576.70	728.72	880.60	1031.43
	ZnO	411.87	568.31	718.15	867.62	1016.26
	CuO	408.72	564.04	712.72	861.03	1008.54
ΔP	Water (without obstacle)	1123.93	2503.78	4336.47	6637.79	9457.5
	Water (with obstacle)	14,394.38	31,703.37	54,411.41	82,593.7	117,250.8
	SiO ₂	16,779.6	36,254.98	62,523.58	95,198.65	134,141.8
	TiO ₂	16,029.41	34,628.33	59,730.41	90,938.84	128,151.9
	Al ₂ O ₃	16,193.9	34,989.7	60,341.53	91,877.12	129,461.3
	ZnO	15,702.75	33,928.64	58,519.85	89,091.04	125,547.2
	CuO	15,465.04	33,415.01	57,625.51	87,731.46	123,633.7

Table 8. Overview of the average Nu and pressure drop for different nanoparticle types at various Re using the two-phase model (1% concentration).

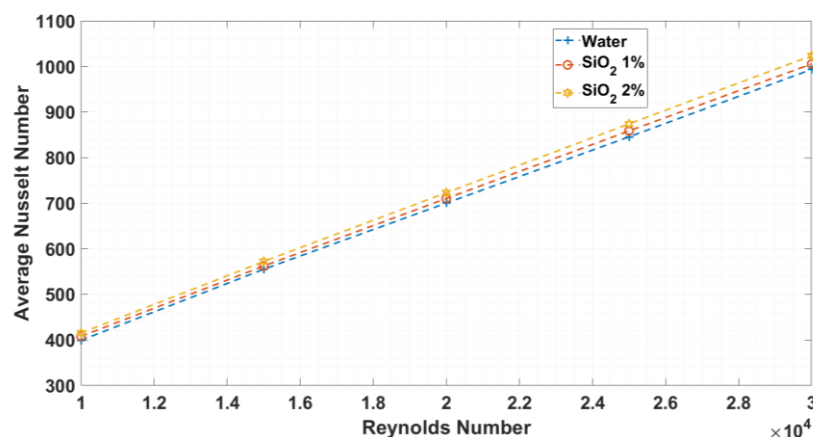
ParameterFluid		Re = 10,000	Re = 15,000	Re = 20,000	Re = 25,000	Re = 30,000
Nu	Water (without obstacle)	194.45	266.09	329.86	389.69	448.08
	Water (with obstacle)	399.09	555.01	700.26	845.48	993.64
	SiO ₂	419.18	574.355	717.78	859.89	1005.145
	TiO ₂	434.58	595.795	744.63	891.77	1041.94
	Al ₂ O ₃	481.69	660.86	825.335	986.655	1149.96
	ZnO	561.26	772.275	964.915	1152.185	1340.085
	CuO	454.415	623.47	779.32	932.905	1089.355
ΔP	Water (without obstacle)	1123.93	2503.78	4336.47	6637.79	9457.5
	Water (with obstacle)	14,394.38	31,703.37	54,411.41	82,593.7	117,250.8
	SiO ₂	16,188.45	36,001.46	62,210.08	95,055.26	135,651.1
	TiO ₂	16,491.63	36,670.84	63,360.5	96,792.74	138,203.6
	Al ₂ O ₃	16,450.38	36,579.47	63,203.11	96,556.16	137,811

ZnO	16,692.79	37,114.23	64,122.18	97,942.85	139,810.9
CuO	16,823.87	37,404	64,620.09	98,694.23	140,940.9

Table 9. Overview of the average Nu and pressure drop for different nanoparticle types at various Re using the two-phase model (2% concentration).

Parameter	Fluid	$Re =$ 10,000	$Re =$ 15,000	$Re =$ 20,000	$Re =$ 25,000	$Re =$ 30,000
Nu	Water (without obstacle)	194.45	266.09	329.86	389.69	448.08
	Water (with obstacle)	399.09	555.01	700.26	845.48	993.64
	SiO ₂	420.8	576.875	721.21	864.23	1010.505
	TiO ₂	450.725	618.64	773.625	926.435	1082.42
	Al ₂ O ₃	531.75	731.18	913.735	1091.65	1271.005
	ZnO	649.965	898.41	1124.74	1343.27	1561.805
	CuO	487.215	669.835	838.04	1002.955	1171.07
ΔP	Water (without obstacle)	1123.93	2503.78	4336.47	6637.79	9457.5
	Water (with obstacle)	14,394.38	31,703.37	54,411.41	82,593.7	117,250.8
	SiO ₂	16,353.75	36,363.81	62,828.61	95,979.57	137,044.7
	TiO ₂	16,958.78	37,697.68	65,124.27	99,442.66	141,987
	Al ₂ O ₃	16,876.01	37,515.99	64,811.28	98,970	141,306.1
	ZnO	17,359.4	38,579.5	66,641.33	101,745.5	145,296.8
	CuO	17,620.91	39,153.35	67,630.47	103,258.5	147,459

Figures 11 and 12 present the average Nu for the channel equipped with obstacles, obtained using the single-phase model and the two-phase mixture model, respectively. The single-phase approach predicts only a modest increase in heat transfer and, interestingly, suggests that Al₂O₃ provides the highest enhancement. This outcome contradicts the expected behavior based on thermal conductivity, since ZnO possesses a significantly higher thermal conductivity. The reason is inherent to the single-phase method, where the fluid properties are recalculated and assigned as uniform effective values. In flows with obstacles where strong mixing, secondary vortices, and complex interaction between the fluid and dispersed particles occur, the single-phase assumption cannot fully capture the underlying physics.



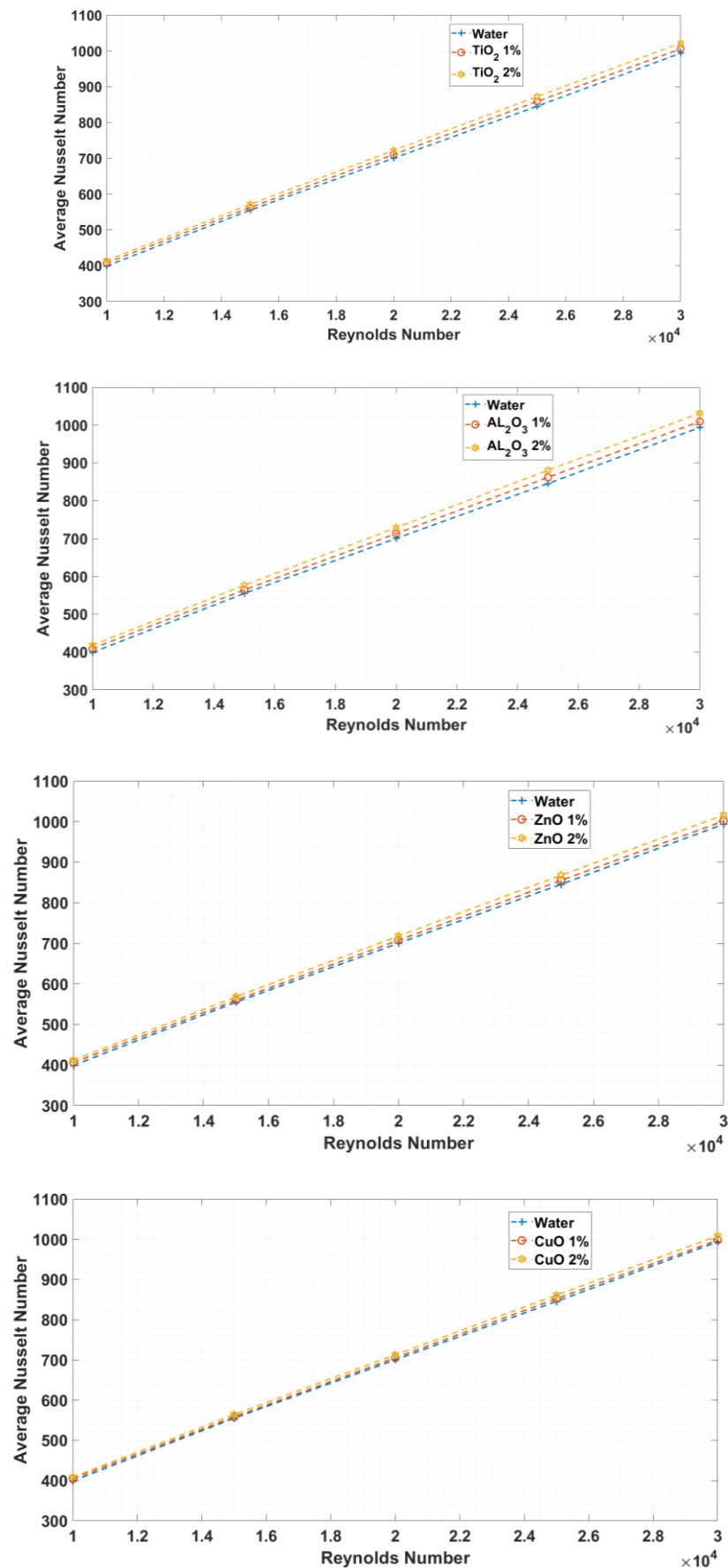
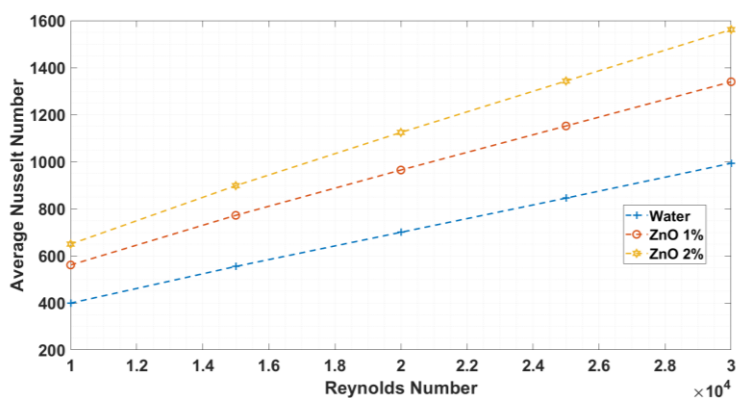
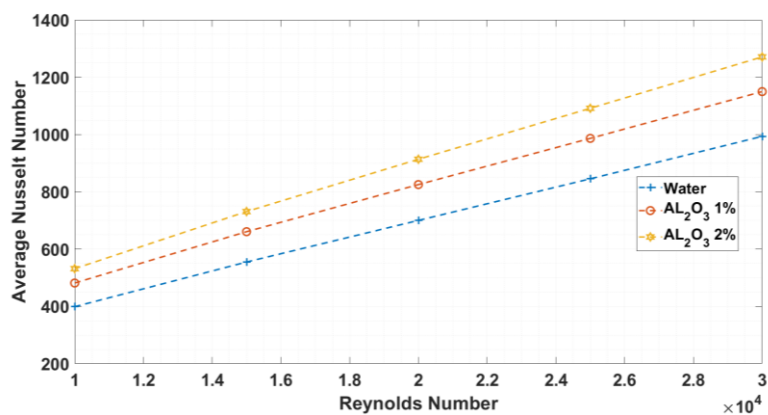
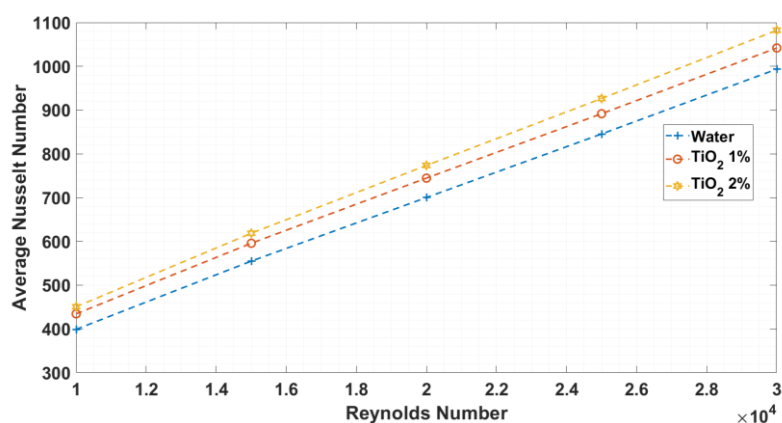
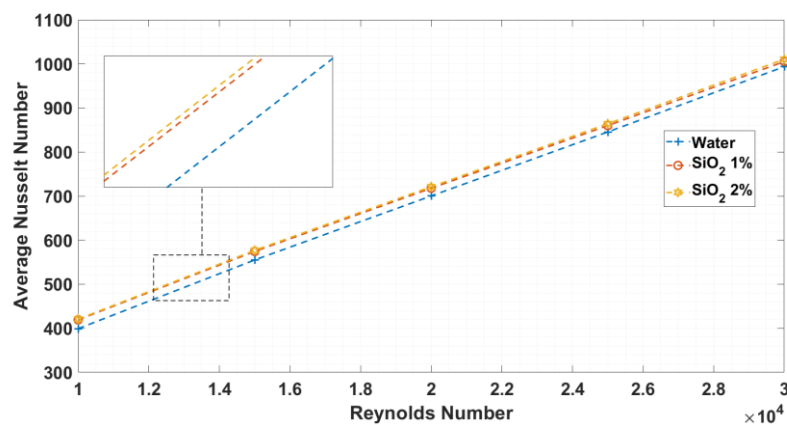


Figure 11. Variation of the average Nu with Re for different nanoparticle types using the single-phase model.



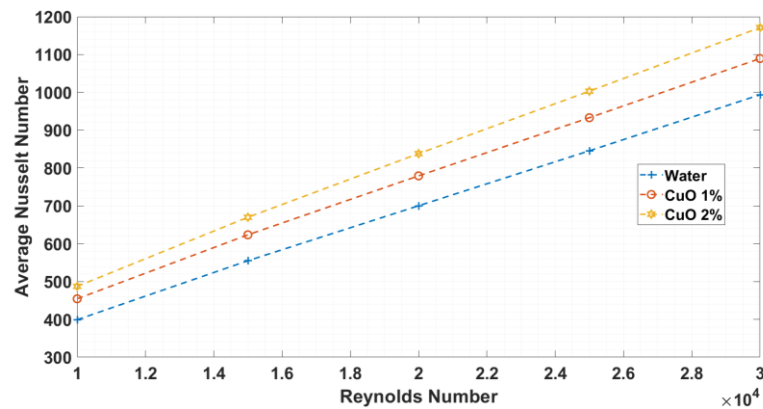
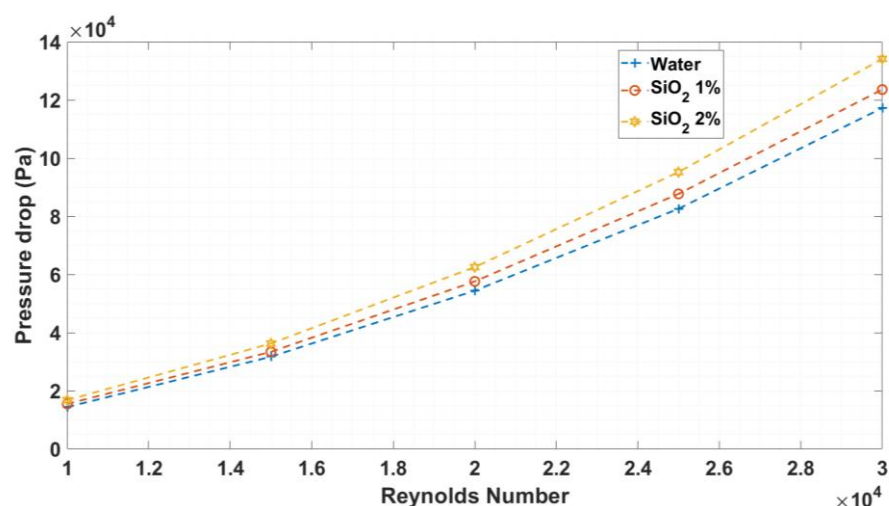
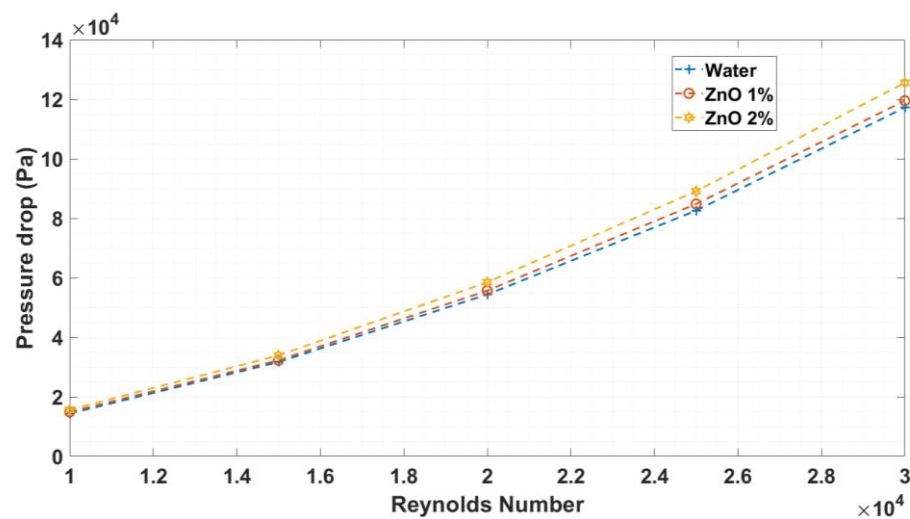
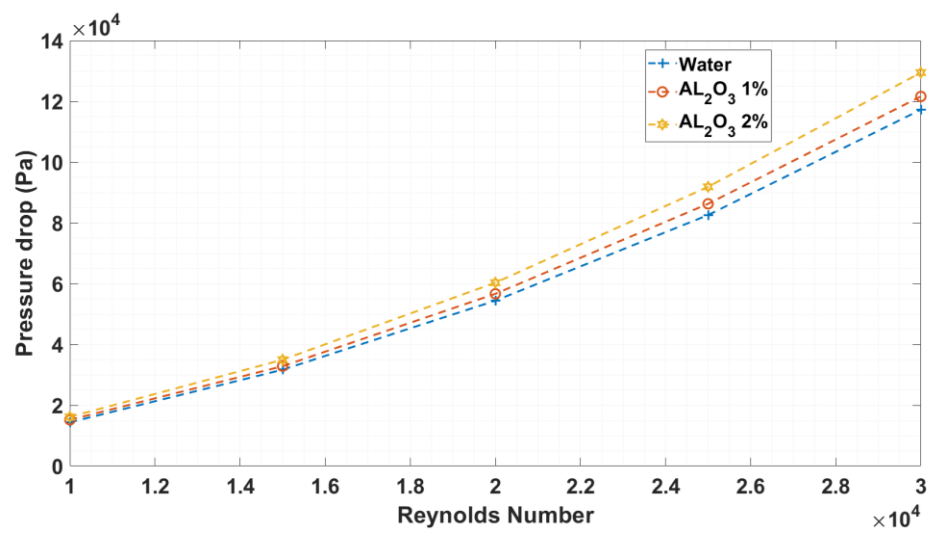
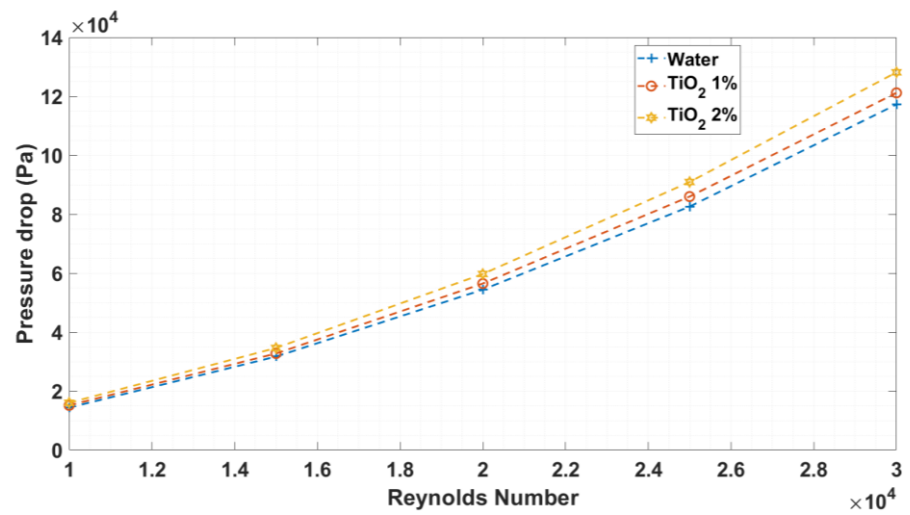


Figure 12. Variation of the average Nu with Re for different nanoparticle types using the two-phase model.

In contrast, the two-phase mixture model yields results that are more physically realistic. It correctly predicts that ZnO produces the greatest heat-transfer enhancement, consistent with its superior thermal conductivity. By explicitly accounting for the transport of solid nanoparticles within the flow, the mixture model better represents particle–fluid interactions, turbulence modification, and the effects induced by the obstacles. Therefore, the two-phase results provide a more reliable description of heat-transfer augmentation in such complex flow regimes.

Figures 13 and 14 illustrate the ΔP in the channel equipped with obstacles, predicted using the single-phase and two-phase mixture models, respectively. In the single-phase model, the variation in pressure drop is barely noticeable, even when the nanoparticle concentration changes. This model also incorrectly suggests that SiO_2 produces the highest ΔP . However, ΔP in nanofluids is strongly influenced by the density of the dispersed nanoparticles, and among the considered materials, CuO has the highest density. Because the single-phase approach homogenizes all properties into a single effective fluid, it fails to reflect this physical behavior.





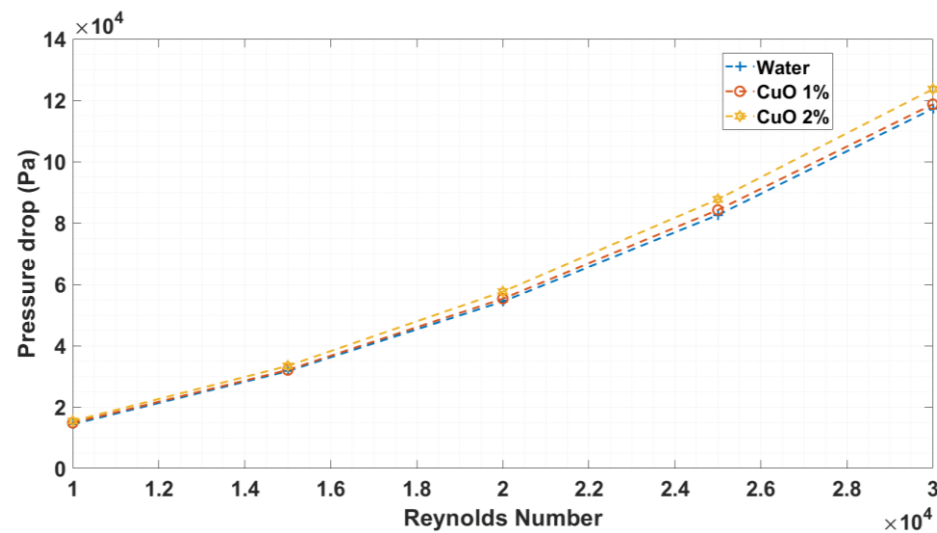
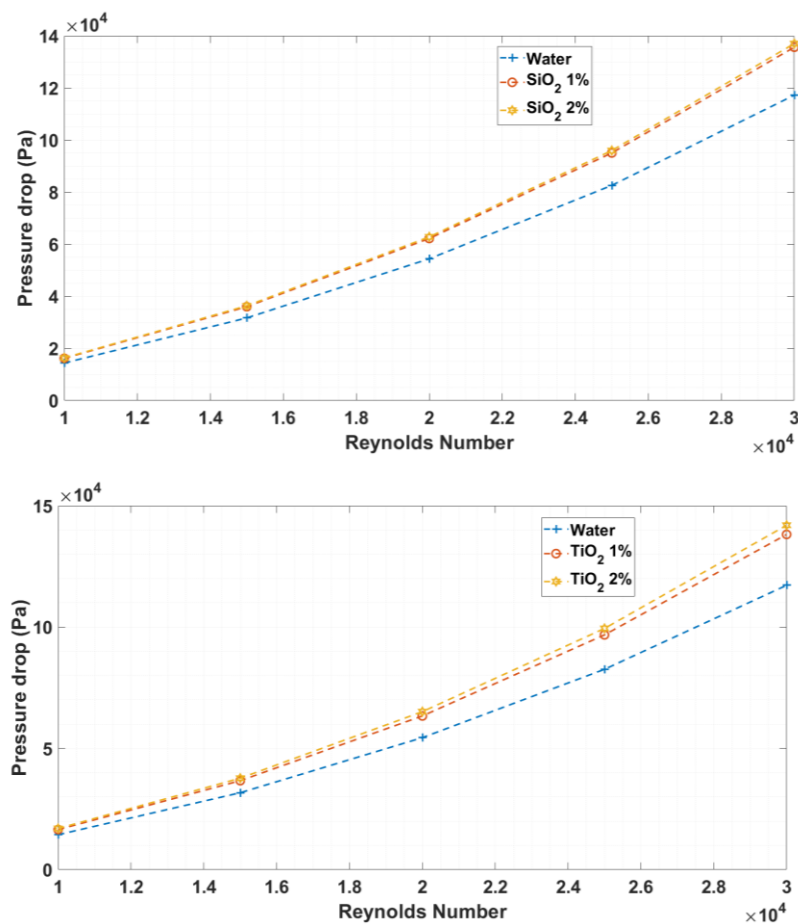


Figure 13. Variation of pressure drop with Re for different nanoparticle types using the single-phase model.



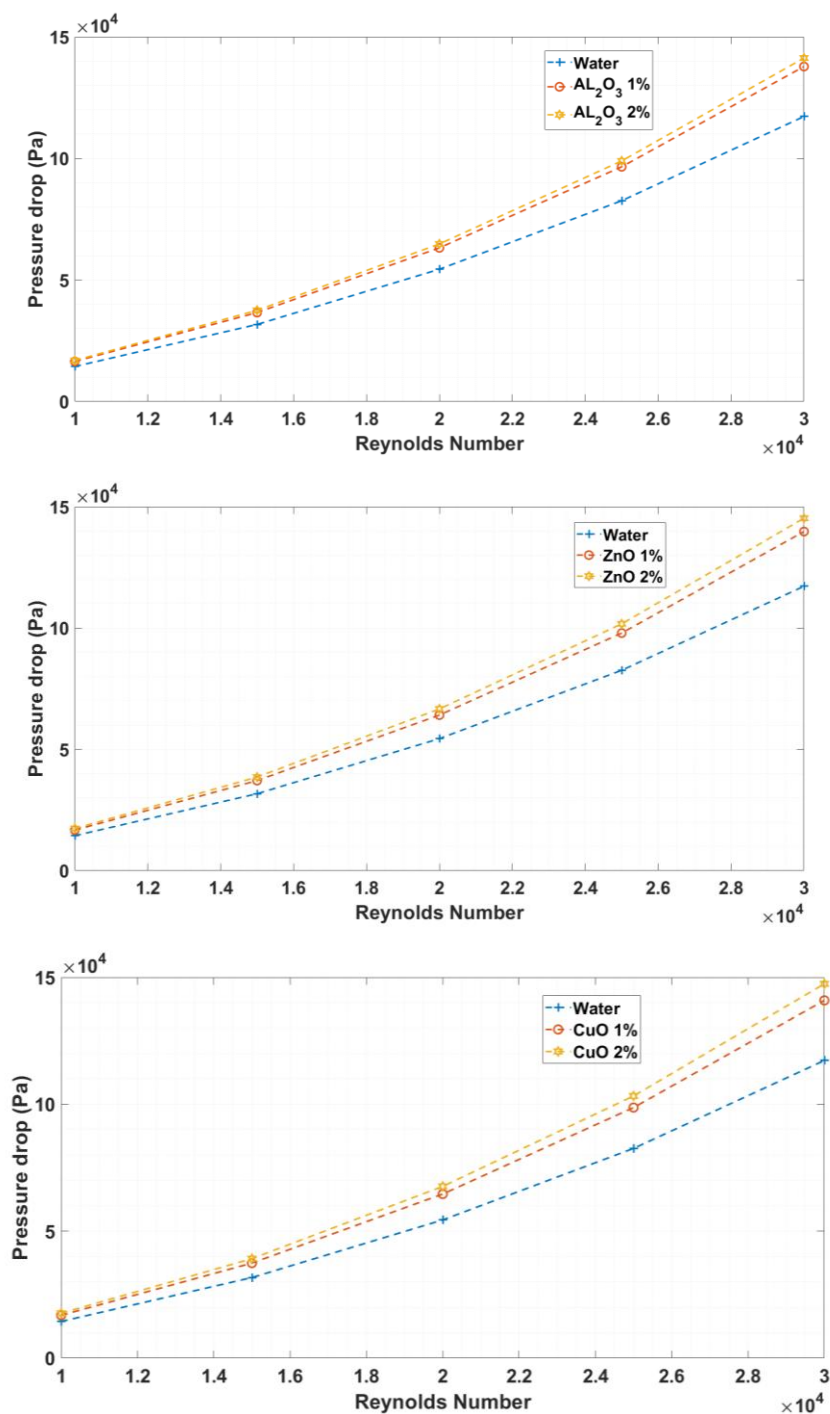


Figure 14. Variation of pressure drop with Re for different nanoparticle types using the two-phase model.

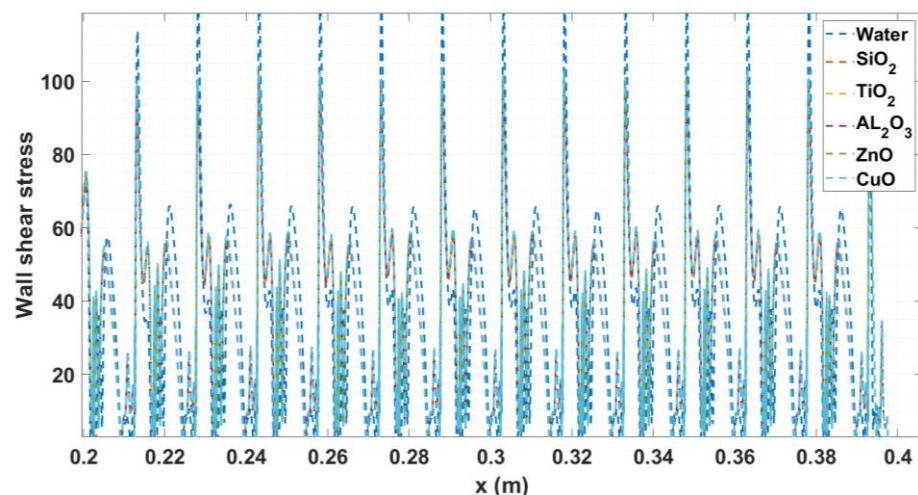
In contrast, the two-phase mixture model clearly differentiates the ΔP levels between the various nanofluids and correctly captures the trend governed by particle density. The results show that CuO produces the highest ΔP , while SiO₂ yields the lowest consistent with their respective densities. This demonstrates that the mixture model provides a more realistic and physically consistent prediction, particularly in flows involving obstacles where momentum exchange between phases influences the overall flow behavior.

The two-phase model offers a clearer and more physically consistent representation of the flow behavior in this configuration; therefore, it is selected as the main reference for

further analysis. In contrast, the single-phase model treats the nanofluid as a homogeneous fluid with modified properties, which leads to higher predicted velocities compared to the base fluid and results in only minor differences in heat transfer among the various nanoparticle types. This approach does not adequately capture the distinct effects of nanoparticle thermal conductivity and density, making it less capable of distinguishing the thermal–hydraulic behavior of different nanofluids. In addition, the 2% nanoparticle concentration exhibits the most distinguishable trends among the tested cases, making it suitable for comparing the different nanoparticle types in the following sections.

4.3. Near-Wall and Local Transport Characteristics

Figure 15 presents the wall shear stress along the corrugated section as a function of x coordinate for the different nanoparticle types at a concentration of 2%. The results show that the introduction of nanoparticles increases the wall shear stress, particularly near the corners of the corrugation where strong velocity gradients occur. In contrast, the water case exhibits its highest shear stress more toward the central region of the corrugated wall, consistent with the earlier velocity profiles, which showed higher near-wall velocities for the base fluid. Since wall shear stress is directly linked to the local velocity gradient at the wall, the higher near-wall velocities of water produce a stronger shear in that region, while the nanofluids due to their modified viscosity and altered flow structure show enhanced shear mainly in the high-curvature corner zones. However, only small differences are observed between different nanoparticle types because the dynamic viscosity of the nanofluids remains very close for all cases, as they are modeled at the same volume fraction (2%) and with the same nanoparticle size (30 nm), resulting in similar momentum resistance. The slight variations that still appear among nanoparticle types are mainly attributed to differences in density, which affect inertia and secondary flow strength, leading to minor changes in local velocity gradients and thus wall shear stress distribution.



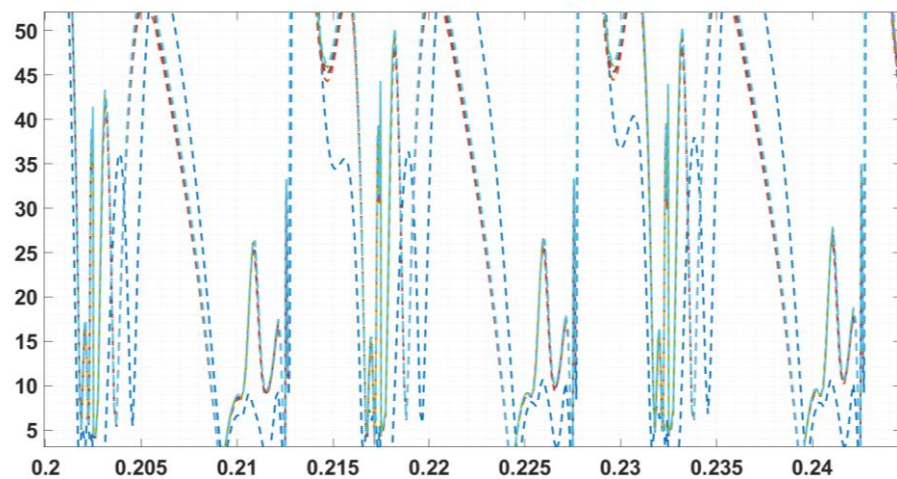
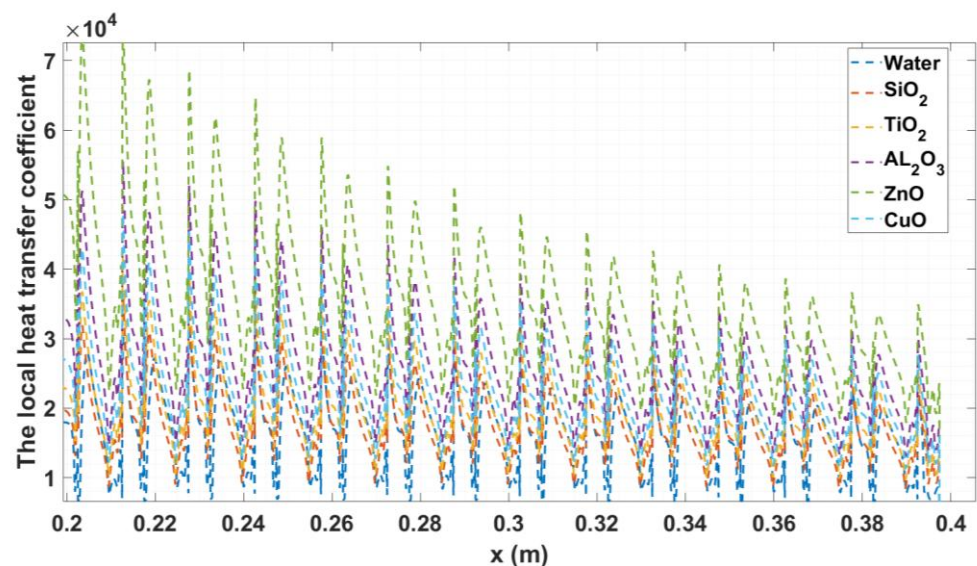


Figure 15. Distribution of the wall shear stress for different nanoparticle types.

Figure 16 presents the local h along the corrugated section as a function of the stream-wise coordinate for the different nanoparticle types at a concentration of 2%. The results show that adding nanoparticles increases the heat transfer coefficient, and the magnitude of enhancement follows the order of their thermal conductivities ZnO yields the highest values, while SiO_2 provides the lowest. An interesting observation appears near the middle of the corrugation wall: in this region, the base fluid (water) exhibits a higher heat transfer coefficient compared to SiO_2 and TiO_2 nanofluids. This behavior is explained by the local velocity distribution; water has a higher near-wall velocity at this location, and the effect of velocity becomes more dominant than the contribution of nanoparticle thermal conductivity, particularly for nanofluids with relatively low conductivity such as SiO_2 and TiO_2 . Moreover, the noticeable differences between nanoparticle types are primarily attributed to their distinct thermal conductivities, where higher conductivity nanoparticles enhance thermal diffusion and energy transport more effectively within the boundary layer, leading to stronger local heat transfer rates compared to lower conductivity nanoparticles.



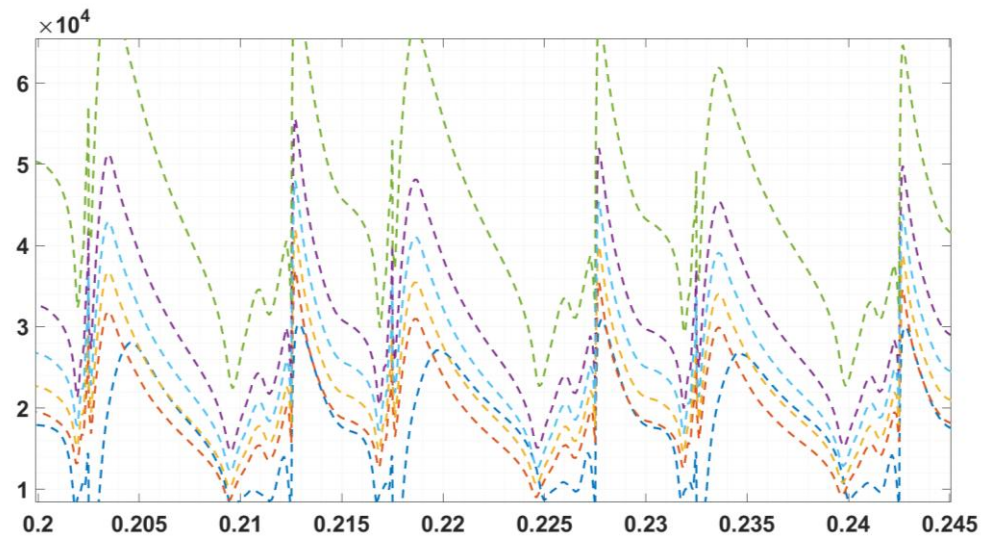
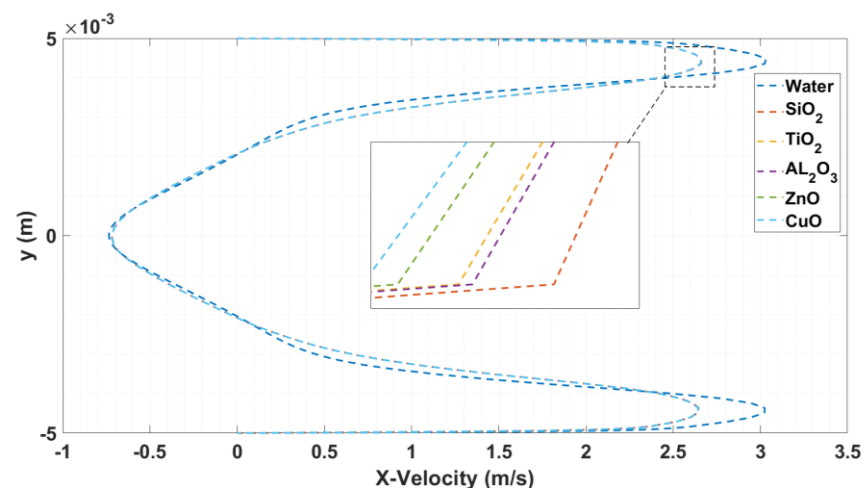


Figure 16. Distribution of the heat transfer coefficient for different nanoparticle types.

4.4. Velocity and Turbulence Characteristics

Figure 17 shows the x-velocity distribution and the turbulent kinetic energy for different nanoparticle types compared to the base fluid (water) at a concentration of 2% in the channel equipped with obstacles. The presence of nanoparticles slightly reduces the near-wall velocity relative to water, which is expected because the nanofluid has higher viscosity and density, causing stronger momentum diffusion close to the walls. In contrast, along the centerline, particularly inside the main recirculation zone generated by the obstacles, the velocity becomes higher for the nanofluid. This occurs because the vortex-induced secondary flow redistributes momentum from the main stream into the recirculating region, where the suspended nanoparticles further enhance momentum transport through increased fluid inertia effects. As a result, the recirculation zone becomes more energetic, leading to a local increase in velocity compared to the base fluid case.



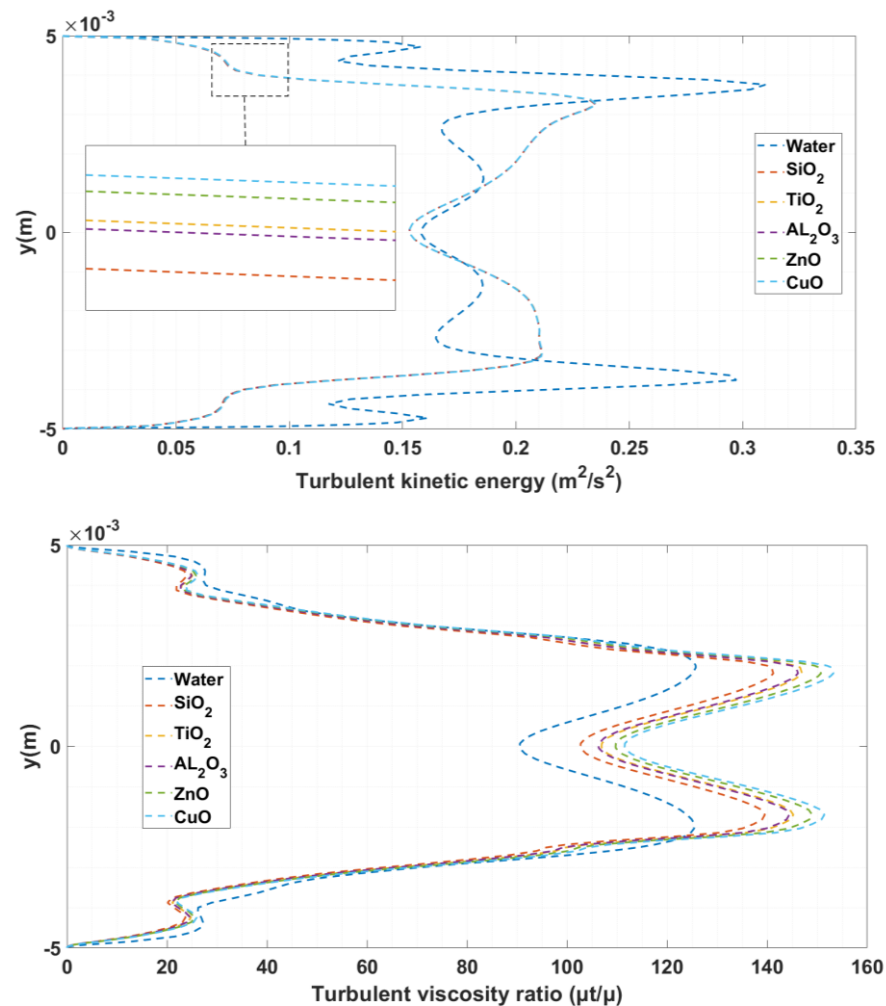


Figure 17. Profiles of x-velocity, turbulent kinetic energy, and turbulent viscosity ratio along the vertical line at the mid-length of the corrugated section, where the horizontal axis represents the y-coordinate.

The turbulent kinetic energy profiles show a clear difference as well. Compared to the water case, all nanofluids exhibit higher turbulent kinetic energy in the centerline region, especially inside the recirculation bubble. This is attributed to the interaction between nanoparticles and vortical structures generated by the obstacles, where the presence of dispersed particles enhances momentum exchange and promotes stronger velocity fluctuations at small scales. The interaction between the nanoparticles and the vortical structures intensifies the small-scale fluctuations, which increases turbulence levels and enhances local mixing. Moreover, the magnitude of both the x-velocity and turbulent kinetic energy variations follows the order of nanoparticle density, with denser nanoparticles producing stronger momentum redistribution and higher turbulence levels than lighter ones. This is because higher-density nanoparticles increase the effective inertia of the nanofluid, strengthening its response to vortex-induced flow instabilities in separation and reattachment regions. This confirms that nanoparticles have a stronger influence in regions of complex flow such as separation and reattachment zones compared to the more uniform near-wall region.

In addition, the turbulent viscosity ratio (μ_t/μ) exhibits a pronounced peak in the central recirculation region generated by the obstacles, indicating intensified turbulent mixing and momentum diffusion. This behavior confirms that the obstacles play a dominant role in producing localized turbulence, while the nanofluid further amplifies this ef-

fect due to increased effective inertia. The magnitude of the turbulent viscosity ratio follows the same trend as nanoparticle density, where CuO, having the highest density, exhibits the largest values, followed by ZnO, Al_2O_3 , TiO_2 , and SiO_2 . This demonstrates that higher-density nanoparticles enhance turbulence production and mixing intensity more effectively within vortex-dominated regions.

4.5. Thermal–Hydraulic Performance of Single Nanoparticle Nanofluids

Figure 18 presents the PE and PR (G1) for different nanoparticle types, evaluated relative to the base channel with obstacles across several Re . The results show that all nanoparticles provide noticeable enhancement, with ZnO achieving the highest PE of up to 62%. As the Re increases, the enhancement gradually decreases, which is expected because higher flow speeds reduce nanoparticle residence time and make particle stability more challenging.

The pressure ratio increases at higher Re for all cases due to the stronger influence of viscosity and inertial effects. Among the tested nanoparticles, CuO produces the highest PR, consistent with its relatively large density, reaching approximately 1.26 times that of water. This implies a greater pumping-power requirement and reduced flow stability. In contrast, Al_2O_3 delivers strong enhancement while maintaining the second-lowest PR after SiO_2 , making it a balanced option in terms of performance and flow stability.

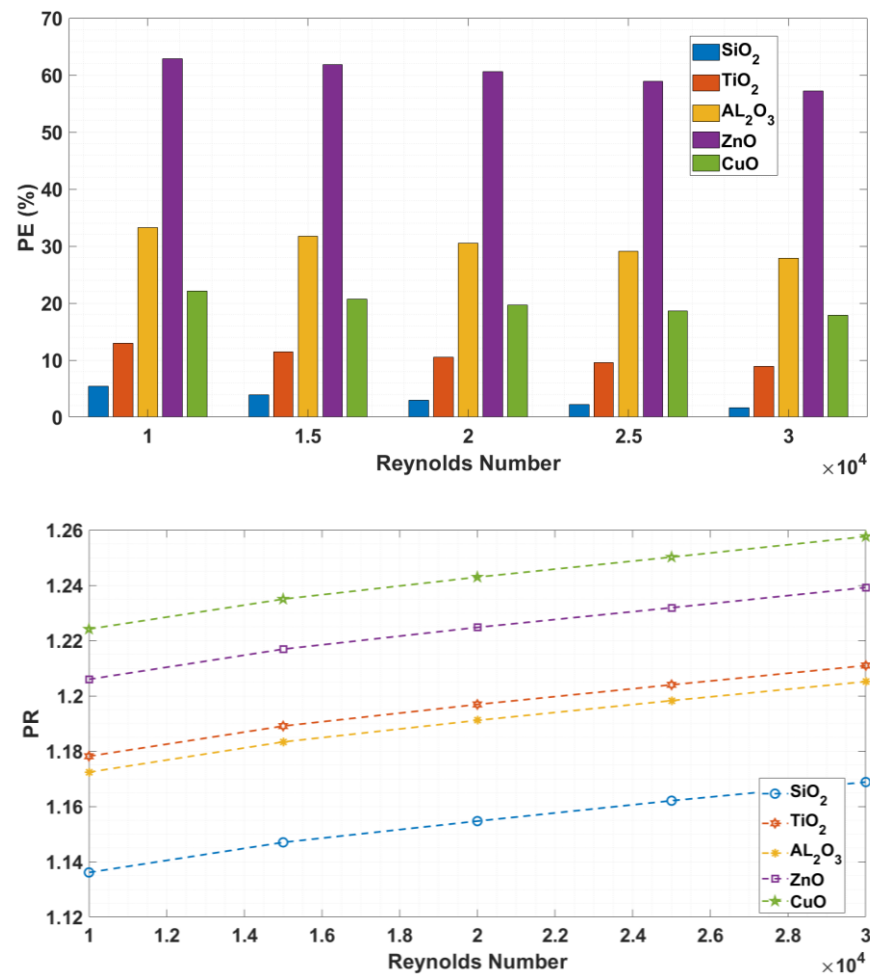


Figure 18. Variation of PE and PR with Re for different nanoparticle types.

Figure 19 presents the PEC (G1) for the different nanoparticle types at 2% concentration, compared with the baseline channel equipped with obstacles. The PEC values remain

nearly constant across the range of Re , with only a slight decrease at higher Re . This behavior is expected, as the heat-transfer enhancement provided by the nanoparticles becomes less pronounced when the flow velocity increases.

All nanoparticle types demonstrate a favorable balance between heat-transfer enhancement and pressure penalty, since their PEC values remain above unity indicating a net performance improvement except for SiO_2 , whose low thermal enhancement leads its PEC to fall below 1. ZnO exhibits the highest PEC, approximately 1.5, due to its strong heat-transfer enhancement, followed by Al_2O_3 with a PEC of around 1.3. These results show that nanoparticles can provide meaningful thermal benefits while maintaining reasonable pressure requirements, especially in cases where their thermal conductivity and density are well-balanced.

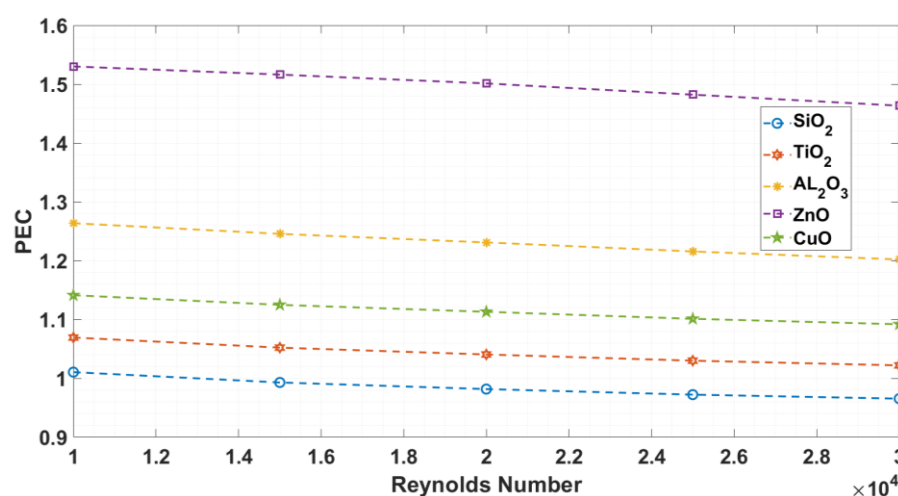


Figure 19. Variation of the PEC with Re for different nanoparticle types.

The nanofluid analysis in this work is purely numerical, relying on governing equations and computational algorithms to predict the thermal hydraulic behavior. However, to bring the study closer to realistic conditions, assumptions commonly adopted in the literature are considered. According to the available literature, ZnO nanofluids are generally assumed to exhibit relatively low dispersion stability [75,76], whereas Al_2O_3 nanofluids are generally assumed to possess better dispersion stability [77]. Both nanoparticle types demonstrate strong thermal performance, but their different stability characteristics highlight the need for improved formulations.

Based on these literature-based assumptions, a hybrid nanofluid is proposed for further investigation, with three possible mixing scenarios (see Table 10). The aim of this suggestion is to enhance overall stability while maintaining or even improving the thermal efficiency of the working fluid, making it more suitable for real-world applications. It should be noted that the stability discussion is based solely on assumptions reported in the literature and was not evaluated within the present numerical study.

Table 10. Composition of hybrid nanofluids used in the present study at a fixed volume concentration.

Hybrid Nanofluid	Al_2O_3	ZnO	ϕ (%)
Case 1	30%	70%	2
Case 2	50%	50%	2
Case 3	70%	30%	2

4.6. Thermal–Hydraulic Performance of Hybrid Nanofluids

Figure 20 presents the average Nu and ΔP for the different hybrid nanofluid (HNF) cases, compared to the single-nanoparticle nanofluids at a fixed concentration of 2%. The results show a consistent and physically reasonable trend: the performance of each hybrid nanofluid lies between that of the two individual nanoparticles that compose it. When the mixture is dominated by one nanoparticle type, the hybrid nanofluid properties shift closer to that dominant component. In Case 2, where equal proportions of each nanoparticle are used, the thermal–hydraulic behavior falls nearly at the midpoint between the two single-nanoparticle curves.

The differences in Nu among the hybrid formulations can be clearly explained by the variation in the effective thermal conductivity of each mixture. In contrast, the changes in ΔP are smaller, since the density and viscosity variations between the cases are less pronounced than the differences in thermal conductivity. Overall, the HNF behavior follows expected physical trends, confirming the consistency of the model.

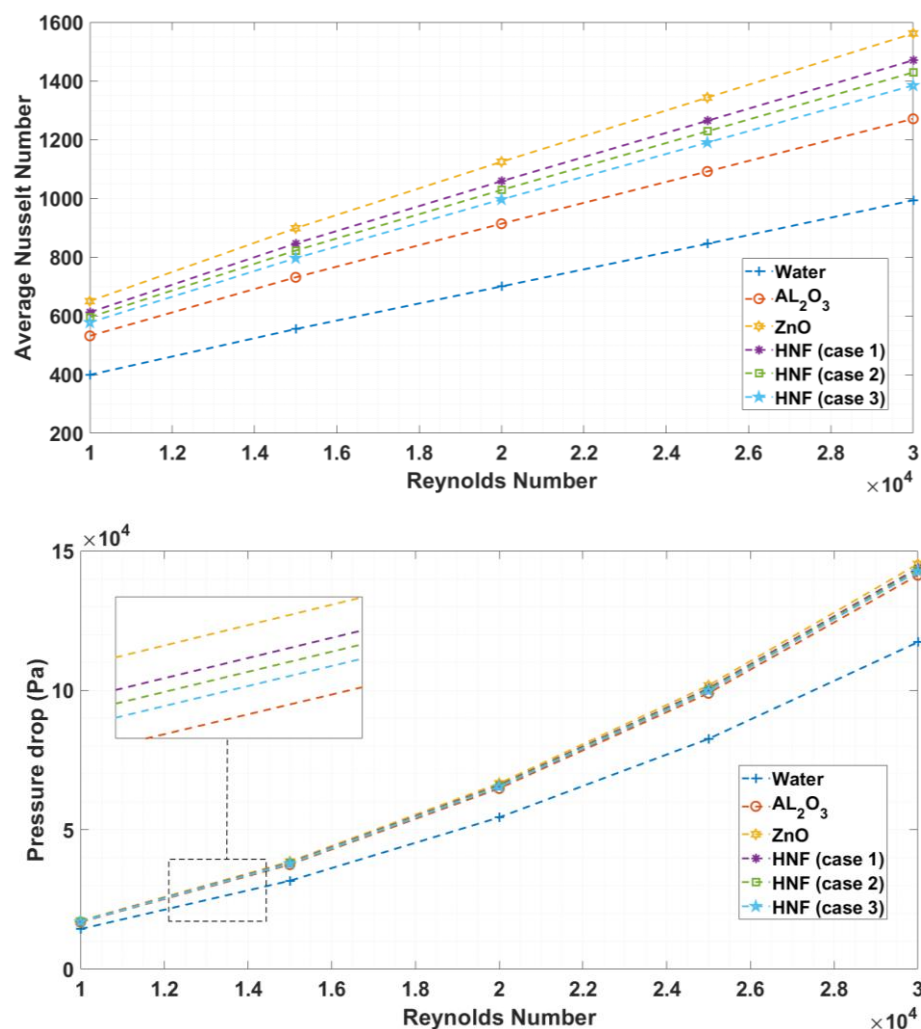


Figure 20. Variation of the average Nu with Re for different hybrid nanofluid cases.

Figure 21 presents the PE and the PR (G1) for the different hybrid nanofluid (HNF) cases, as well as for the single-nanoparticle nanofluids at a fixed concentration of 2%, all compared to the baseline channel with obstacles. The trends in PE and PR are consistent with those observed for the Nu and ΔP . In all cases, the HNFs exhibit intermediate be-

havior between the corresponding single-nanoparticle fluids, confirming the physical coherence of the mixtures.

The HNFs provide noticeable enhancement, particularly in Case 2, where the balanced mixing of the two nanoparticles results in approximately 50% performance improvement while maintaining a moderate pressure ratio of around 1.22. This combination indicates a favorable compromise between heat-transfer enhancement and pressure penalty, while the potential stability advantages of hybrid nanofluids are assumed based on previous literature and were not evaluated in the present numerical study.

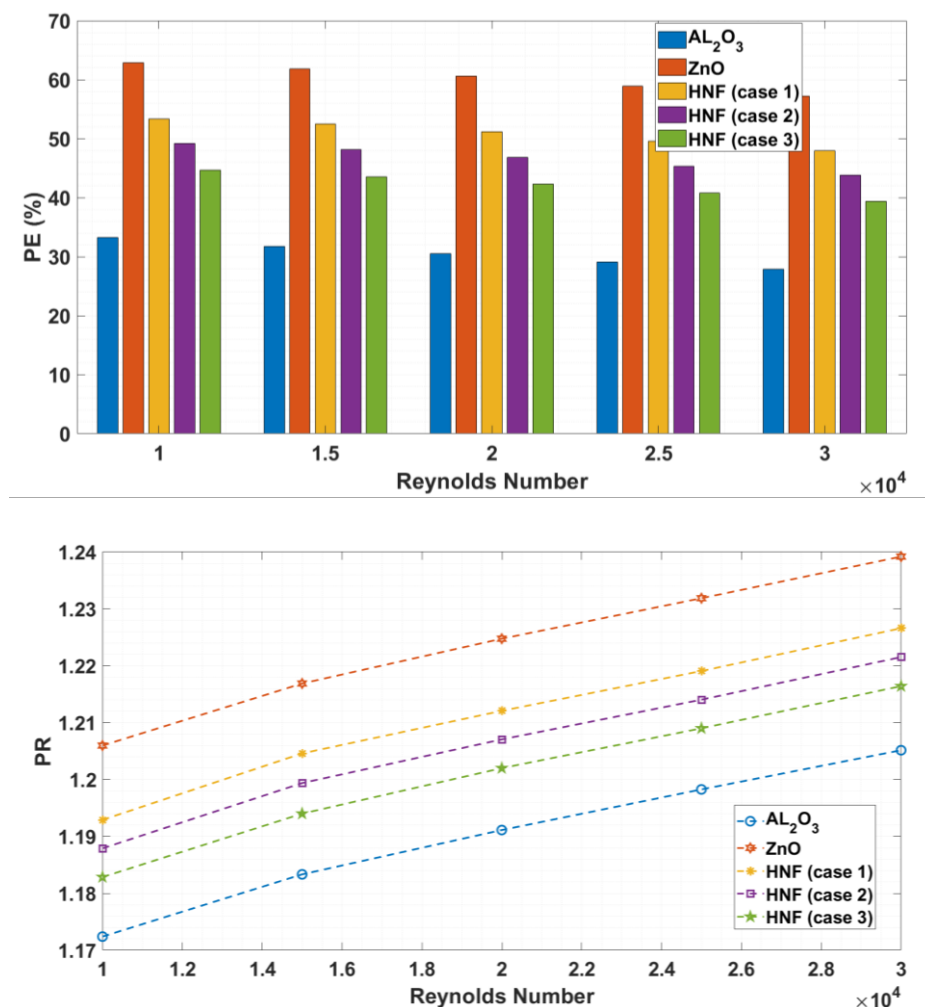


Figure 21. Variation of PE and pressure ratio PR with Re for different hybrid nanofluid cases.

Figure 22 presents the PEC (G1) for the different hybrid nanofluid (HNF) cases and the single-nanoparticle nanofluids at a concentration of 2%, all compared to the channel with obstacles. The PEC values of the three HNF cases fall within a narrow range of approximately 1.35–1.45, positioned between the corresponding single-nanoparticle PEC values. Each HNF case naturally shifts toward the behavior of the nanoparticle type that dominates its mixture ratio.

Among the tested configurations, Case 2 achieves a PEC of around 1.4, indicating a strong balance between heat-transfer enhancement and pressure penalty. This makes it a promising option for practical applications, while any potential stability advantages associated with hybrid formulations are assumed based on the available literature and were not evaluated in the present numerical study.

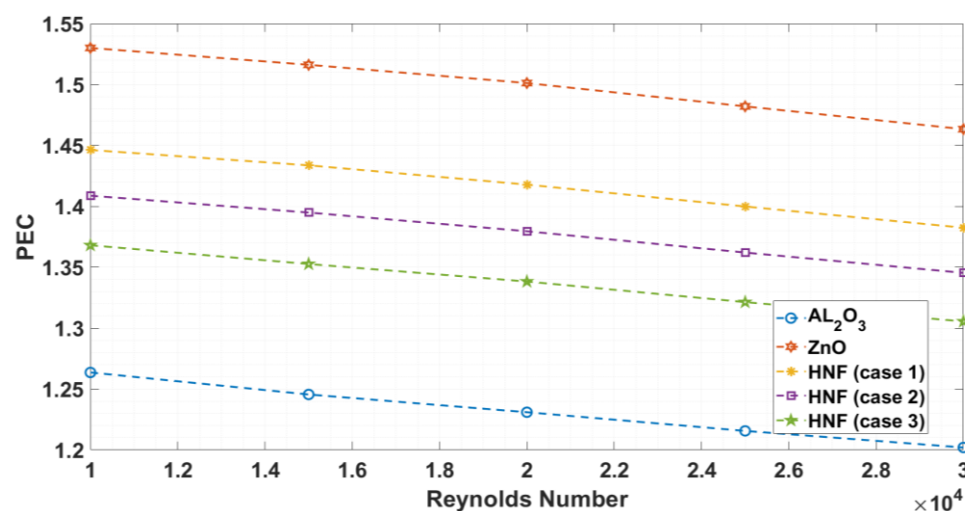
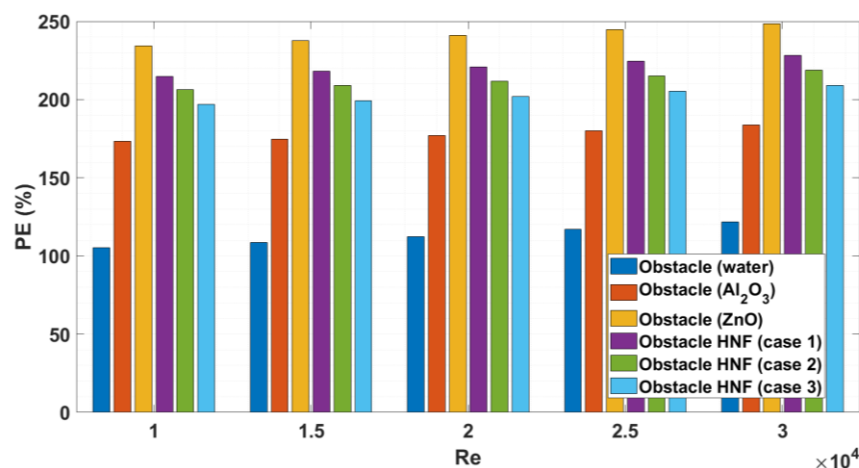


Figure 22. Variation of the PEC with Re for different hybrid nanofluid cases.

4.7. Combined Effect of Obstacles and Nanofluids

Figure 23 presents the PE and PR (G2) for the corrugated channel equipped with obstacles, using both water and nanofluids at a concentration of 2%. The combination of nanoparticles with obstacles shows a substantial improvement in heat transfer performance. The enhancement reaches approximately 244% compared with the smooth channel operating with pure water. However, this improvement comes with a significant pressure penalty, with the pressure drop ratio reaching nearly 15.5 times that of the baseline channel. To assess whether this thermal improvement justifies the associated hydraulic cost, the PEC for the combined effect of both enhancement techniques will be evaluated in the following section.



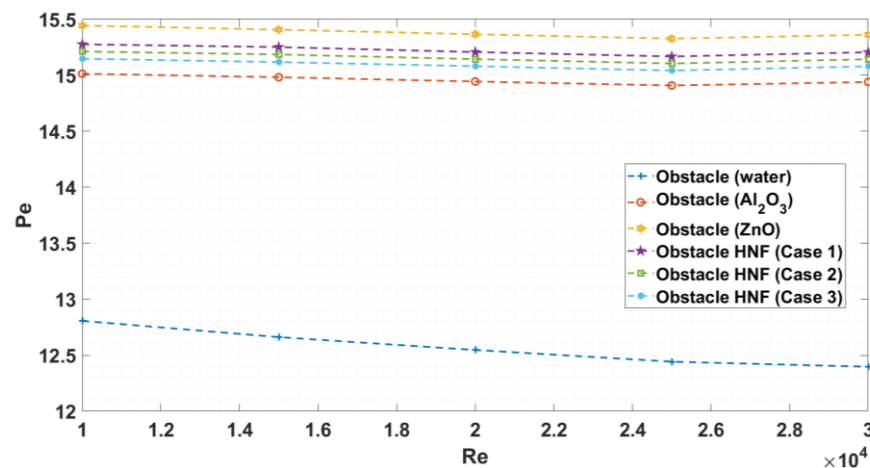


Figure 23. Heat transfer enhancement and pressure ratio of channels with obstacles and nanofluids compared to a baseline channel without obstacles using water.

Figure 24 shows the PEC (G2) of the channel with obstacles using water and using nanofluids at 2% concentration, compared against the baseline channel without obstacles. When nanoparticles are combined with obstacles, the overall performance increases significantly. The PEC rises from approximately 0.96 (water with obstacles) to values greater than 1 when nanofluids are used, indicating that the passive techniques obstacles and nanoparticles reinforce each other and provide a more balanced trade-off between heat-transfer enhancement and pressure drop.

Among all cases, Hybrid Nanofluid Case 2 achieves a PEC of about 1.28, demonstrating a strong improvement while maintaining reasonable pressure penalties. This value is higher than that reported in previous related studies [78], such as the work of water–DWCNT–TiO₂ hybrid nanofluid in vortex-generator channels, which reported a maximum PEC of approximately 1.14, indicating that the present configuration provides superior thermohydraulic performance. Under these conditions, the configuration consisting of this obstacle type, size, position, and the Case 2 hybrid nanofluid can be considered a suitable and effective choice for enhancing the performance of heat exchangers.

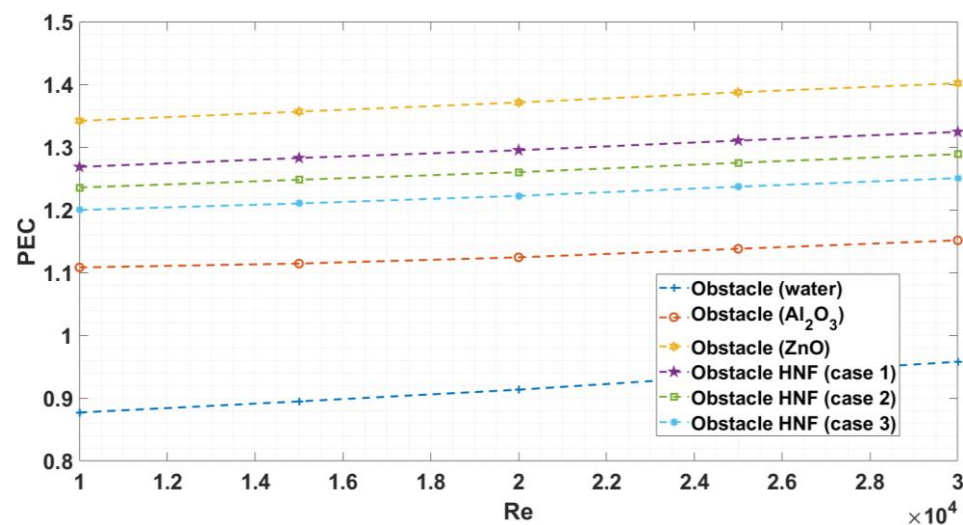


Figure 24. Comparison of the PEC for obstacles and nanofluids relative to a baseline channel using water.

5. Conclusions

This study presents a detailed numerical investigation of heat transfer enhancement in a corrugated channel by integrating multiple passive techniques, including obstacles and nanofluids, under turbulent flow conditions.

The results indicate that the two-phase mixture model provides improved predictive capability compared to the single-phase approach, as it includes particle–fluid interactions in an averaged manner and accounts for the influence of nanoparticle thermal conductivity and density on both heat transfer and pressure drop behavior. The thermal–hydraulic performance is strongly influenced by the type of nanoparticles: nanofluids with higher thermal conductivity yield greater heat transfer enhancement, while denser nanoparticles contribute to increased pressure losses. When combined with obstacles, the overall heat transfer enhancement reaches approximately 244% compared to a smooth channel with water, although the pressure drop rises to nearly 15.5 times the baseline.

To optimize the balance between heat transfer enhancement and pressure-drop penalty, a hybrid Al_2O_3 –ZnO nanofluid (Case 2) was investigated. The combined application of the corrugated channel geometry, strategically placed obstacles, and the hybrid nanofluid resulted in a performance evaluation coefficient (PEC) of 1.28, demonstrating an effective overall thermal–hydraulic performance compared with the selected baseline configuration. These findings highlight the effectiveness of this integrated passive enhancement strategy and its potential for practical implementation in high-efficiency heat exchanger systems.

These results demonstrate a significant potential to enhance the thermal efficiency of heat exchangers and can be applied to a wide range of industrial systems for improved energy performance.

5.1. Limitations

This study is limited to a specific type of obstacle and selected nanoparticles, analyzed under particular boundary conditions and turbulent flow regimes, with water used as the base fluid. Therefore, the results and conclusions are specific to these configurations and operating conditions. In addition, practical implementation aspects such as nanofluid cost, long-term operational stability, and potential issues including erosion and channel clogging are not considered in the present numerical model. Furthermore, the relatively high pressure drop observed in the enhanced configurations may increase pumping power requirements, which could affect real-world applicability.

The present study is limited to a two-dimensional formulation of flow over spanwise-uniform concave-up obstacles. While this approach is suitable for identifying comparative trends between water, nanofluids, and hybrid nanofluids under identical geometric conditions, it does not resolve fully three-dimensional turbulent mechanisms such as longitudinal vortex stretching and secondary flow structures. Therefore, the reported quantitative values of Nu , ΔP , and PEC should be interpreted within the framework of the adopted 2D model, and future work should include fully three-dimensional simulations and experimental validation.

5.2. Future Work

For future work, the integration of machine learning and artificial intelligence techniques could be explored to optimize the design of corrugated channels, vortex generator arrangements, and nanofluid compositions. Data-driven models, such as neural networks or genetic algorithms, can predict thermal–hydraulic performance under a wide range of operating conditions, reducing the need for extensive numerical simulations. Such approaches may enable rapid identification of optimal configurations, improve predictive

accuracy, and support the development of advanced heat exchanger systems with enhanced efficiency and adaptability for industrial applications.

Author Contributions: Conceptualization and methodology, K.H.; software and visualization, A.T. and I.O.; investigation, A.T.; validation, I.O.; resources and data curation, A.T., I.O. and K.H.; writing—original draft preparation, A.T.; writing—review and editing, A.T., I.O. and K.H.; supervision, K.H.; formal analysis and project administration, A.T. and K.H. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Data available on request from the first author.

Conflicts of Interest: The authors declare no conflicts of interest.

Nomenclature

SYMBOLS	GREEK LETTERS
k Turbulent kinetic energy [m^2/s^2]	ω Specific dissipation rate [s^{-1}]
K Thermal conductivity [$\text{W}/(\text{m K})$]	μ Dynamic viscosity [$\text{N s}/\text{m}^2$]
f Friction factor	μ_t Turbulent dynamic viscosity [$\text{N s}/\text{m}^2$]
q_w Heat flux [W/m^2]	σ_k, σ_ω Empirical constant for turbulence model
Re Reynolds number	
Nu Nusselt number	ρ Density [kg/m^3]
h Heat transfer coefficient [$\text{W}/(\text{m}^2 \text{K})$]	ΔP Pressure drop [Pa]
P Pressure [Pa]	τ Shear stress [Pa]
T Temperature [K]	
u, v Velocity components [m/s]	SUBSCRIPTS
x, y Cartesian coordinates [mm]	w Wall
G The production term in various transport equations	av Average
C_p Specific heat [$\text{J}/(\text{kg K})$]	np Nanoparticles
	nf Nanofluid
	bf Base fluid

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