

Article

Effect of Normalization Approaches on Shear Modulus Degradation Curve of Saturated Sand in Shaking Table Tests

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Abstract

Shear modulus degradation curves are fundamental inputs in nonlinear site response analyses and are conventionally normalized by the small-strain shear modulus, $G_{\max}$, defined at shear strains on the order of $\gamma \approx 10^{-4}\%$. In shaking table experiments, however, reliable measurements at very small strains are often unattainable due to instrumentation resolution and strain demand limitations. Consequently, normalization is frequently performed using the shear modulus at a higher reference strain ($\gamma = 0.01\%$). The impact of this alternative normalization on the resulting shear modulus degradation relationship has not been systematically evaluated. This study investigates the influence of normalization strain level on shear modulus degradation behavior using stress–strain relationships reconstructed from shaking table acceleration records. The shear modulus values were computed from individual hysteresis loops. The shear modulus normalized by $G_{\max}$ estimated from empirical correlations was compared with the shear modulus normalized by its value at $\gamma = 0.01\%$ directly obtained from shaking table measurements. Results indicate that normalization at $\gamma = 0.01\%$ produces slightly lower normalized modulus values for shear strains exceeding 0.01% compared with the curve normalized by $G_{\max}$. Normalization using $G_{\gamma=0.01\%}$ resulted in reduced scatter and uncertainty.

Keywords: stress–strain behavior; shear modulus; normalization; saturated sand; shaking table test

1. Introduction

Seismic events in saturated sandy deposits can become a critical phenomenon when it triggers liquefaction. This phenomenon can lead to significant ground deformation, lateral spreading, and foundation instability, often resulting in severe structural damage and loss of life, as evidenced during historic earthquakes such as the 1969 Niigata event in Japan and the 1971 San Fernando earthquake in California. Under cyclic loading, loose saturated or partially saturated sands typically exhibit contractive behavior, generating excess pore water pressure and causing a progressive reduction in effective stress toward zero. Strong seismic events can substantially decrease the soil strength and stiffness of saturated sandy soils, particularly under liquefaction conditions [1].

The cyclic stress–strain response of soils is fundamental to the assessment of seismic site behavior. In particular, shear modulus degradation ($G-\gamma$), commonly obtained from element testing, is a key input parameter for dynamic numerical analyses. Accurate eval-

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uation of shear modulus over a strain range of approximately 0.0001–1% is essential for geotechnical earthquake design [2]. The shear modulus is commonly expressed as the secant stiffness, represented by the slope of the stress–strain hysteresis loop [3]. Physical modeling offers an alternative framework for investigating soil behavior under controlled conditions. By employing real soil specimens, it reduces dependence on assumed constitutive parameters and provides a more realistic representation of stress–strain transfer compared to conventional element tests [4]. However, measurements in such tests are generally limited to time-history data recorded at discrete locations, and the accuracy of the results depends strongly on the instrumentation layout and data acquisition quality.

The primary factors influencing shear modulus were systematically identified by Hardin and Drnevich [5], who highlighted the roles of shear strain amplitude, effective confining stress, and void ratio in clean sands. Subsequent studies have extensively utilized cyclic triaxial and resonant column tests to quantify these relationships [6,7]. Field studies were conducted to evaluate soil stiffness nonlinearity using recorded earthquake acceleration [8–10], although such studies are limited to a relatively small number of well-instrumented sites. In comparison, physical modeling enables controlled evaluation of soil behavior across a broader range of conditions while avoiding some of the inherent limitations of both element tests and field observations. The application of physical model tests for assessing the shear modulus of saturated sands remains relatively limited.

In shaking table studies, the shear modulus is commonly normalized with respect to the modulus corresponding to a shear strain of $\gamma = 0.01\%$, primarily due to the absence of direct $G_{\max}$ measurements and the relatively large strains induced during testing [1,11]. On the other hand, some studies have employed the bender elements in physical modeling instrumentation to directly determine $G_{\max}$ through shear wave velocity measurements or empirical correlations [12,13]. However, the influence of different normalization approaches on the resulting shear modulus degradation curves has not yet been investigated.

To address this issue, the present study compares shear modulus degradation curves normalized using $G_{\max}$ and $G_{\gamma=0.01\%}$ based on shaking table test results. Acceleration time-history records were utilized to compute shear stress and shear strain at the midpoint between accelerometer pairs. The value of $G_{\max}$ was estimated using empirical correlations, whereas $G_{\gamma=0.01\%}$ was determined from best-fit trendline relationships developed from the G – γ response curves for each test. Following the determination of the reference shear modulus values, the shear modulus corresponding to each loading cycle during weak and strong shakings in Tests 1 through 9 was normalized accordingly.

2. Experimental Program

2.1. Physical Model Preparation

A 1 g shaking table applying unidirectional motion was used in this experimental investigation [14]. A laminar box consisting of stacked hollow rectangular aluminum frames was employed, allowing relative motion between layers using a roller system. Figure 1 shows dimensions of the laminar box mounted on the shaking table ($L = 122$ cm, $W = 51$ cm, and $H = 91$ cm). The effects of boundary conditions of the laminar box on the experimental outcomes were assessed and determined to be insignificant, as discussed comprehensively by Farzalizadeh et al. [1].

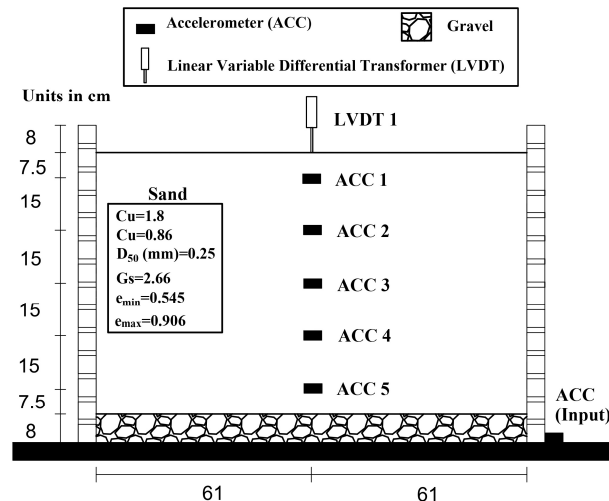


Figure 1. Sensor layout and the properties of the used sand.

Per the Unified Soil Classification System (USCS), the used sand was identified as poorly graded sand (SP). The properties of the used sand are presented in Figure 1. A geomembrane liner was installed inside the laminar container to retain the saturated soil specimen. The samples were prepared using a sand raining technique to achieve target relative densities. A range of sensors were installed at different elevations within the specimen to monitor its response under seismic loading. These included accelerometers (ACC1–ACC5), pore water pressure transducers (PP1–PP4), and displacement transducers for measuring settlement (LVDT1). An additional accelerometer (ACC Input) positioned on the shaking table recorded the input motion. To ensure uniform saturation, a layer of gravel was placed at the base of the container to facilitate even water distribution. After specimen preparation, carbon dioxide (CO_2) was introduced to displace entrapped air within the pore spaces. Subsequently, water was gradually introduced from the bottom through the gravel layer at a regulated rate to achieve full saturation. The water level was raised slightly above the ground surface and allowed to rest so that gravity-driven drainage could eliminate possible unsaturated zones. Afterward, the excess water was removed from the specimen surface [15–17].

2.2. Testing Plan

Given the ongoing debate regarding the influence of loading frequency on liquefaction potential, the present study adopts input motions derived from earthquake records that are commonly cited in liquefaction research. These records suggest that the dominant frequency content of many earthquakes associated with significant damage typically lies within the range of 1–2 Hz. In this study, four seismic loading scenarios were considered, comprising two weak shakings (Events A and B) and two strong shakings (Events C and D) characterized by different peak base accelerations (PBAs), as shown in Figure 2. In general, weak (small) pre-shaking corresponds to a seismic excitation that does not induce liquefaction, whereas strong pre-shaking involves a higher-intensity seismic loading capable of triggering liquefaction ($R_u = 1$) or producing a shear strain of approximately 5% in double amplitude or 3.75% in single amplitude. Events A and B, each consisting of five loading cycles, were classified as low-intensity excitations ($\text{PBA} = 0.1 \text{ g}$ and 0.06 g) and were not sufficient to trigger liquefaction. In contrast, Events C and D, with PBAs of 0.17 g and 0.25 g and a total of fifteen cycles, were categorized as high-intensity loadings capable of inducing liquefaction. The number of loading cycles was selected to represent earthquake magnitude, where five cycles (Events A and B) approximate a magnitude of

about 6, while fifteen cycles (Events C and D) correspond to a magnitude of approximately 7.5 [18,19].

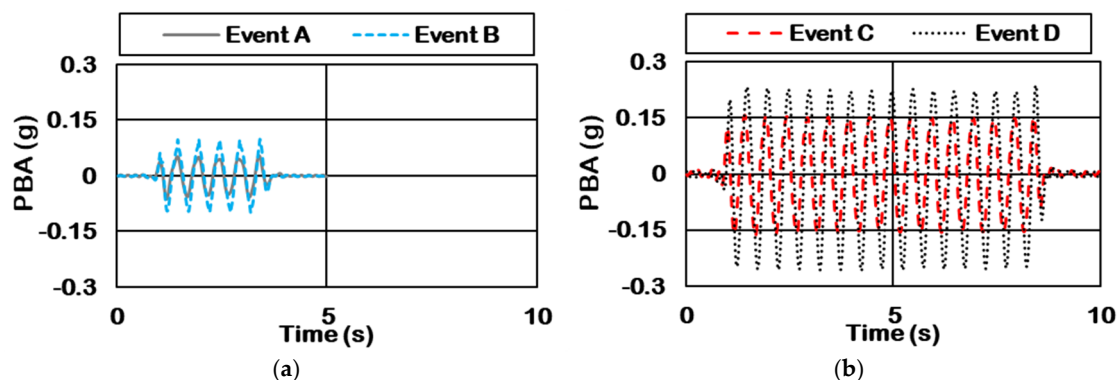


Figure 2. Input motion used in this study: (a) weak shakings and (b) strong shakings.

The dataset includes results from nine experiments conducted under varying seismic histories, encompassing a total of 105 shaking events, as presented in Table 1. The initial relative density used in this study ranges from 30% to 52%. The primary objective of this research is to establish shear modulus degradation curves based on a comprehensive dataset comprising both virgin and pre-shaken samples. Measurements were collected at four different depths, yielding an overall dataset of 420 observations utilized for the present analysis.

Table 1. Seismic events sequence for Tests 1–9.

Shaking #	S1	S2	S3	S4	S5	S6	S7	S8	S9	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20	S21	S22	S23	S24	S25
Test 1 (Dr = 30%)	A	A	A	A	A	A	A	A	A	A	C	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Test 2 (Dr = 32%)	C	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Test 3 (Dr = 30%)	B	B	B	B	B	B	B	B	B	B	C	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Test 4 (Dr = 53%)	C	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Test 5 (Dr = 30%)	C	C	C	C	C	C	C	C	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Test 6 (Dr = 30%)	D	D	D	D	D	D	D	D	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
Test 7 (Dr = 30%)	A	A	A	C	A	A	A	C	A	A	A	C	A	A	A	C	A	A	A	-	-	-	-	-	-
Test 8 (Dr = 50%)	C	C	C	A	A	A	C	C	C	A	A	A	C	C	C	A	A	A	C	C	C	-	-	-	-
Test 9 (Dr = 50%)	B	B	B	D	B	B	B	D	B	B	B	D	B	B	B	D	B	B	B	C	A	A	C	C	A

Note: # : Number and Dr : Relative Density.

3. Results and Discussion

As illustrated in Figure 1, a total of five accelerometers were installed to capture acceleration time histories at depths of 7.5, 22.5, 37.5, 52.5, and 67.5 cm during each shaking event. For conditions involving vertically propagating shear horizontal (SH) waves within a horizontally stratified soil deposit, the system can be idealized using one-dimensional shear beam theory. Following the approach introduced by Elgamal et al. [20], shear stress–strain time histories can be derived from recorded acceleration (a , in m/s^2) responses within the soil profile, enabling the evaluation of its mechanical characteristics. In this framework, the soil deposit is modeled as a one-dimensional shear beam and discretized

into multiple layers for analysis. The shear stress (τ , in kPa) at a specified depth (z , in m) is determined using Equation (1). The soil density (ρ , in kN/m³) was determined from the average relative density derived using measurements of surface settlement, assuming uniform vertical strain within the specimen, based on the methods proposed by [17,21].

This equation was originally formulated for field applications, where surface acceleration records are typically available as part of site observations. In contrast, measuring surface acceleration in physical model tests is difficult, as sensors must be embedded within the soil to ensure proper coupling. To address this limitation, a linear interpolation approach between adjacent instrument pairs is commonly adopted, allowing extrapolation from the uppermost sensor pair to estimate the surface response. This technique has been widely implemented in physical modeling studies [4,22]. Shear strain is determined by first computing displacement (u , in m) by integrating the recorded acceleration time histories twice. Prior to each integration step, the acceleration signals are processed using band-pass filtering to reduce noise, followed by baseline correction after each stage [20]. The resulting shear strain is then evaluated at the midpoint between each accelerometer pair (z , in m), which coincides with the locations of the pore pressure transducers, as expressed in Equation (2). Consequently, the stress–strain time histories can be represented in the form of hysteresis loops. The shear modulus (G , in kPa) is then evaluated as the secant stiffness, defined by the slope of a line connecting two points on the hysteresis loop, as expressed in Equation (3). The aforementioned process is demonstrated in Figure 3.

$$\tau = \int_0^z \rho(z) a(z) dz \quad (1)$$

$$\gamma(t) = \frac{u_i - u_{i-1}}{z_i - z_{i-1}} \quad (2)$$

$$G = \frac{\tau_{\max} - \tau_{\min}}{\gamma_{\max} - \gamma_{\min}} \quad (3)$$

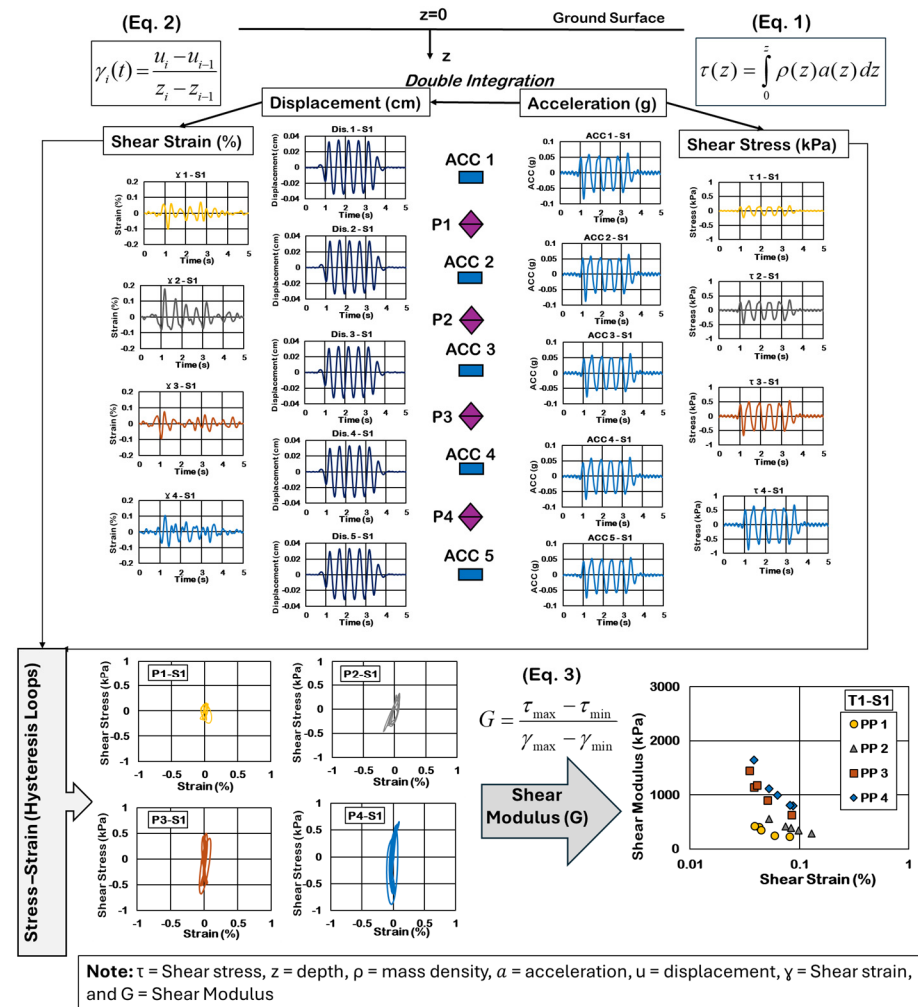


Figure 3. Procedure for generating hysteresis loops and calculating shear modulus for Test 1 at S1.

A best-fit trendline was developed for each depth to estimate the shear modulus, owing to the proximity of the shear strain measurements to 0.01% [1]. Figure 4 demonstrates the trendline drawn from shakings during Test 1 as an example. Similarly, the shear modulus at $\gamma = 0.01\%$ was computed using this method for each test.

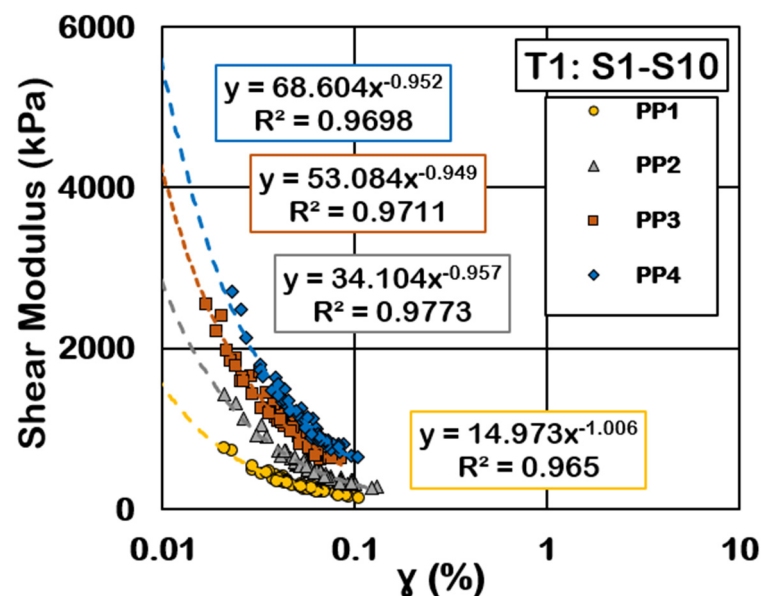


Figure 4. Shear modulus determined at $\gamma = 0.01\%$ using trendline in Test 1.

Figure 5 presents a comparison between the normalized shear modulus degradation curves obtained in this research and those reported by Ko et al. [11] for saturated sand from shaking table tests, as well as the widely used model of Seed and Idriss [23], which was obtained from small-scale dynamic testing on virgin sand under relatively high confining pressures. Overall, the results of the present study show good agreement with those of Ko et al. [11]. Nevertheless, due to the low overburden pressures inherent in shaking table experiments, greater shear strain development occurs in conjunction with excess pore water pressure (EPWP) generation compared to other dynamic testing methods. In general, the shear modulus degradation curves obtained from the shaking table tests tend to be lower than those predicted by the Seed and Idriss model. In Ko et al. [11], the reported upper and lower bounds for virgin sand correspond to higher and lower overburden pressures (or depths), respectively. Since this experimental research was conducted under comparatively lower confining stresses, data points for Events A and B representing soil behavior within the elastic range show good agreement with the lower bound reported by Ko et al. [11]. In contrast, the influence of pre-shaking is evident during Events C and D, where an increase in soil stiffness is observed, particularly at greater depths. Thus, the degradation curves associated with large shaking events (Events C and D) due to the considerable increase in the relative densities caused by pre-shakings exceed the upper bound reported by Ko et al. [11], which was established for virgin sand conditions. The results obtained using this method consistently remained within $\pm$ one standard deviation (± 1 SD), demonstrating relatively low variability.

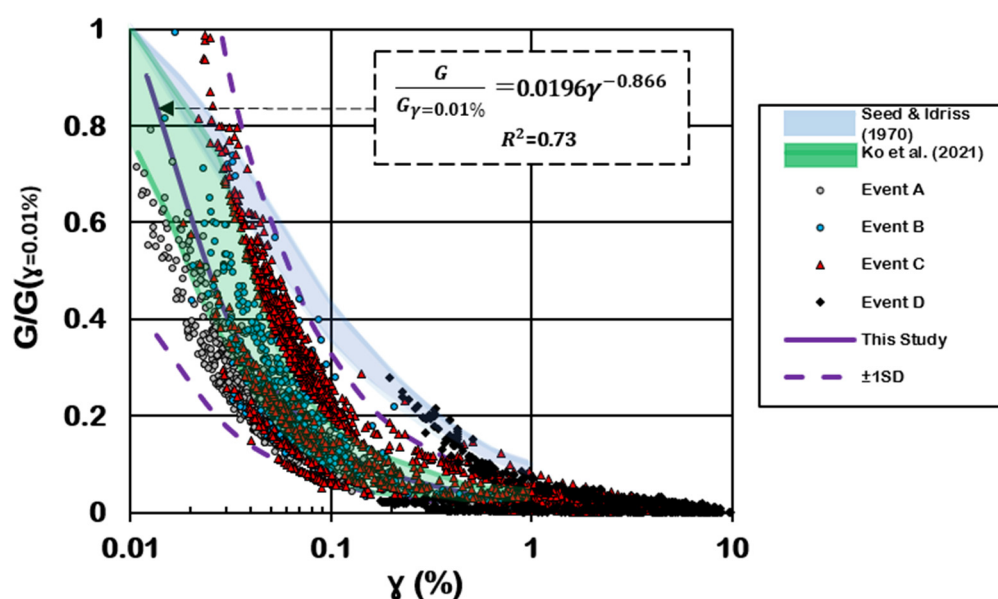


Figure 5. Comparison of shear modulus normalized by $G_{\gamma=0.01\%}$ between the findings of this study and other studies [11,23].

Traditionally, the maximum shear modulus ($G_{\max}$), corresponding to a shear strain of 0.0001%, is taken as the reference value for normalization. In this study, ($G_{\max}$) was evaluated using the empirical relationship developed by Hardin and Black [24] (Equation (4)):

$$G_{\max} = 707 \times \frac{(2.973 - e)^2}{1 + e} \times \sigma'_m{}^{0.5} \quad (4)$$

where e denotes the void ratio and σ'_m represents the mean effective confining stress (kPa). The void ratio of the soil was estimated from surface settlement measurements by assum-

ing a uniform vertical strain distribution within the specimen, following the procedures outlined by [17,21]. The post-shaking void ratio from each test was adopted as the initial void ratio for the subsequent shaking event. The mean effective stress was calculated in the middle of accelerometer pairs (P1–P4 shown in Figure 3) before starting the test [1].

Despite the previous approach, the use of empirically estimated $G_{\max}$ resulted in greater scatter, with data predominantly distributed within ± 2 SD. The comparison of shear modulus degradation curves normalized at $G_{\max}$ (at $\gamma = 0.0001\%$) indicates that a subset of the data associated with small shaking events and limited pre-shaking history falls below the boundary suggested by Seed and Idriss [23], as shown in Figure 6. This behavior is attributed to a higher rate of excess pore water pressure generation, which results in greater shear modulus degradation. However, with increasing pre-shaking, the observed trend progressively converges toward the Seed and Idriss [23] boundary, which was obtained from small-scale tests conducted under higher confining pressures. This observation is consistent with previous shaking table studies [1], which reported that pre-shaking can significantly increase the shear modulus of sandy deposits due to improvement in soil fabric and densification. In addition, since the empirical correlation was established based on virgin soil conditions, the impact of pre-shaking history on the stiffness characteristics of sand was not considered. In contrast, the best-fit trendline approach, derived directly from experimental measurements, provides a greater capability to capture the effects of prior seismic loading on soil behavior. Previous studies have shown that shear modulus degradation is more pronounced under lower confining pressures and in saturated sands [4,25]. Given that the shaking table tests in this study were performed on saturated sand at relatively low confining pressures, this condition likely explains the higher rate of shear modulus degradation observed herein compared to the findings of Seed and Idriss [11] and Ko and Chen [26]. The proposed correlation is recommended for saturated poorly graded sand (SP) under conditions represented by the present shaking table tests, including effective confining stresses less than 6 kPa, relative densities ranging from approximately 30% to 90%, peak base accelerations between 0.06 g and 0.25 g, and shear strain levels greater than 0.01%.

The underlying assumption of normalizing the shear modulus at $\gamma = 0.01\%$ is that the variation in shear modulus between $\gamma = 0.0001\%$ and $\gamma = 0.01\%$ is minimal. Under this assumption, normalization at $\gamma = 0.01\%$ is considered reliable. According to Figure 6, the comparison indicates that normalization by $G_{\max}$ yields slightly higher values than normalization at $\gamma = 0.01\%$. This difference can be attributed to the method used to determine $G_{\max}$. Due to instrumentation limitations in shaking table tests, $G_{\max}$ is not measured directly but is estimated using empirical correlations, such as those proposed by Hardin and Black [24]. Under the specific test conditions characterized by low confining pressure, pre-shaking history, and potential soil structure change, this correlation may underestimate the actual small-strain stiffness. Consequently, the denominator in $G/G_{\max}$ becomes smaller than the actual stiffness, leading to an overestimation of the normalized values and, hence, higher curves. In contrast, $G_{\gamma=0.01\%}$ is obtained directly from measured stress–strain data or derived from best-fit trendlines within a reliable strain range. As such, it provides a more representative and consistent reference, minimizing uncertainty. Therefore, normalization at $\gamma = 0.01\%$ results in smoother and comparatively lower degradation curves. The results indicate a reasonable agreement between shear modulus normalization at $\gamma = 0.01\%$ and $G_{\max}$, supporting the validity of this approach. Accordingly, normalization at $\gamma = 0.01\%$ is deemed appropriate for the interpretation of shaking table test results in the absence of direct measurement of $G_{\max}$.

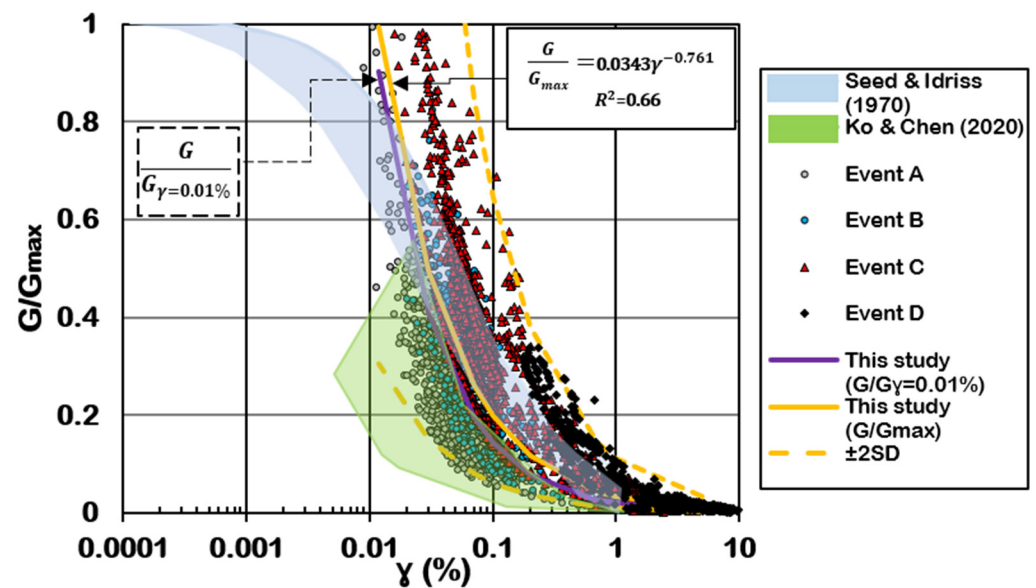


Figure 6. Shear modulus normalized by G_{max} comparison among the results of this study and other studies [23,26].

According to the statistical comparison of the two normalization approaches shown in Figure 7, the mean value of the data normalized using $G_{\gamma=0.01\%}$ was about 19% lower than that obtained using G_{max} . The standard error (SE) for both normalization methods was small and comparable, indicating consistent mean estimates with limited statistical uncertainty.

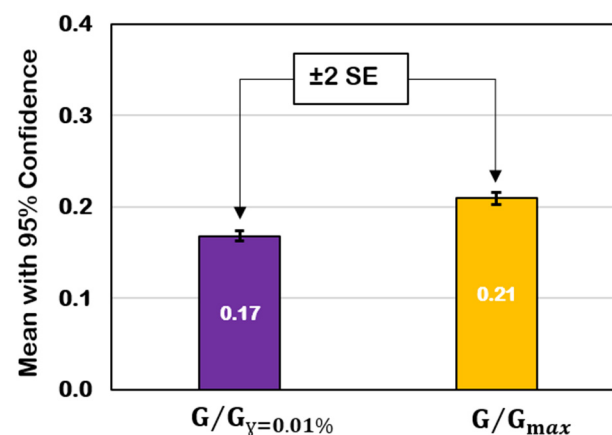


Figure 7. Mean shear modulus normalized using both approaches with 95% confidence intervals.

4. Conclusions

This experimental research studies the influence of normalization approaches on the shear modulus degradation behavior of saturated sand using 1 g shaking table tests. Acceleration time-history data were employed to derive stress–strain hysteresis loops, from which the shear modulus was evaluated for each loading cycle. The normalized shear modulus obtained using empirically estimated G_{max} was subsequently compared with that normalized by the reference shear modulus at $\gamma = 0.01\%$, directly determined from shaking table measurements. Based on the experimental results and analyses, the following conclusions can be drawn:

- The comparison of the two normalization approaches demonstrated that the choice of normalization method has a minimal influence on the resulting shear modulus degradation curves.
- The normalization using $G_{\gamma=0.01\%}$ exhibited reduced scatter, with the majority of data falling within ± 1 standard deviation, whereas normalization using empirically estimated $G_{\max}$ produced greater variability, with most observations distributed within ± 2 standard deviations.
- Both normalization approaches produced generally consistent trends, indicating that normalization at $\gamma = 0.01\%$ provides a reliable alternative when direct measurements of $G_{\max}$ are unavailable. This approach better captures the influence of previous shaking events, as empirical $G_{\max}$ correlations were developed for virgin soils under higher confining pressures and do not consider the effects of pre-shaking history.
- Empirical shear modulus degradation correlations were developed for saturated sandy soils under low confining stress and shear strains greater than 0.01%. These correlations are applicable to the range of conditions investigated in this study and are particularly suitable for shallow saturated sandy deposits. Future studies should focus on expanding the experimental database to include a broader range of soil types, relative densities, confining pressures, and recorded earthquake motions in order to improve the generalizability and engineering applicability of the proposed correlation.

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