

Article

Advanced Bearing Condition Monitoring for Energy Production Machinery via Koopman Dynamics

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Abstract

Monitoring the health of bearings in industrial rotating machines is a major challenge for ensuring the reliability and continuous operation of installations. Conventional fault detection methods, based on multivariate control charts such as Hotelling's T^2 , multivariate exponentially weighted moving average, or multivariate cumulative sum control chart, are limited by the complex nonlinear dynamics of the system. In this article, we propose an innovative monitoring approach based on the Koopman operator, allowing the linearization of a nonlinear system in an observed space and the application of drift detection techniques via an extended T^2 control chart. The study is based on two experimental approaches: one using controlled simulated data to analyze the responsiveness and robustness of the model, and the other applied to real data from an industrial turbogenerator monitoring the vibrations, temperatures, and speeds of the front and rear bearings. Comparative results show that the Koopman-based T^2 map detects defects earlier, with better accuracy under noise and a reduced false alarm rate compared to conventional methods. The integration of wavelet preprocessing, statistical feature extraction by sliding windows, and PCA representation of the trajectories enhances the robustness and interpretability of the model.

Keywords: fault detection; multivariate control charts; Koopman operator; wavelets; bearings; turbo-alternator; condition monitoring; predictive maintenance



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1. Introduction

Bearings play a central role in rotating machinery such as steam turbines, compressors, electric motors, and alternators. Their failure can lead to unplanned downtime, significant economic losses, and, in some cases, safety risks. Implementing condition-based maintenance and early fault diagnosis techniques is therefore crucial to ensure the reliability and longevity of industrial equipment.

Among the most widely used approaches, multivariate control charts have established themselves as a benchmark tool for multi-sensor process monitoring [1–3]. Classic methods such as Hotelling's T^2 , multivariate exponentially weighted moving average (MEWMA), and multivariate cumulative sum control chart (MCUSUM) can detect deviations or anomalies by simultaneously integrating several monitored variables (vibration, temperature, speed, etc.) [4–9]. These tools are well-documented in the literature. Research proposed a

systematic comparison between the performance of MEWMA and MCUSUM in industrial settings [10,11], while the T^2 method was extended to statistical control environments [12].

However, these methods rely on the assumption of linear and stationary process behavior, which is rarely the case in real mechanical systems. Vibration and thermal signals, in particular, are often nonlinear, nonstationary, and noisy, especially when progressive failures occur. These limitations have motivated the emergence of more robust techniques integrating preprocessing and advanced dynamic modeling approaches.

Within the framework of multivariate statistical process control (SPC), monitoring statistics such as Hotelling's T^2 are fundamentally based on linear covariance structures and the Mahalanobis distance formulation, with control limits derived from a chi-squared distribution. These theoretical properties assume that the monitored system is represented in a linear space.

When nonlinear and nonstationary rolling signals are processed directly in their original form, these assumptions are violated, leading to degraded detection performance and unstable behavior in terms of false alarms.

Therefore, to maintain consistency with the theoretical framework of SPC while accurately representing the nonlinear dynamics of the system, it becomes necessary to construct a linear representation of the underlying nonlinear process.

Several researchers have proposed the integration of time–frequency techniques, such as the wavelet transform, to extract features robust to local signal variations [13]. The effectiveness of the wavelet transform in isolating vibration signatures associated with localized defects has been demonstrated [14]. A combination of wavelets and principal component analysis (PCA) was proposed to improve fault classification in bearings [15].

In recent years, wavelet transform decomposition has emerged as one of the most widely studied approaches for bearing condition monitoring. Thanks to its multi-resolution properties, the wavelet transform enables particularly effective joint time–frequency localization of transient pulses and non-stationary vibration components associated with localized defects. Discrete wavelet transform (DWT) and wavelet packet decomposition (WPD) have been extensively used to isolate frequency bands carrying defect-related information and improve the signal-to-noise ratio, especially under variable and non-stationary operating conditions.

Furthermore, beyond time–frequency analysis, feature extraction methods based on entropy measurements have generated increasing interest for quantifying the complexity and irregularity of vibration signals. Indicators such as spectral entropy, sample entropy, permutation entropy, and multiscale entropy have proven their value for the early detection of degradation phenomena. These measurements capture subtle nonlinear variations that often precede the appearance of pronounced spectral signatures.

These approaches are particularly well-suited when fault signatures are not manifested by sharp amplitude peaks but rather by a gradual increase in dynamic irregularity and signal complexity, characteristic of the incipient stages of failure.

More recently, attention has focused on methods based on Koopman operator theory, a mathematical approach that linearizes a nonlinear dynamic system by projecting it into a functional observation space [16–20]. This allows for the application of linear analysis tools to initially complex systems. Foundational studies have laid the groundwork for applying the Koopman operator to dynamic systems [21,22]. In the field of maintenance, the Koopman operator has been used to model the transient behavior of mechanical systems [23], while a condition monitoring system for electric motors has been developed based on Koopman modes extracted via dynamic mode decomposition (DMD) [24–28].

Concurrently, recent research has explored the integration of this theory into multivariate control charts. Fault detection frameworks based on Koopman mode decomposition

combined with statistical control techniques have been proposed [29,30], while deep models leveraging Koopman theory have been developed to improve sensitivity and observability in complex multivariate systems, including energy production machinery such as gas turbines [31,32]. A recent literature review confirms the growing interest in incorporating Koopmanian representations into control charts for improved anomaly detection [33].

Recent studies have explored the use of deep learning architectures for vibration-based fault diagnosis [34]. The proposed SDPCNN model, combined with multi-order frequency-tracking filtering, for gear fault detection under varying operating conditions. Similarly, another study introduced a capsule convolutional network-based framework for bearing fault diagnosis [35].

While these approaches demonstrate high classification performance, they primarily rely on purely data-oriented deep learning architectures, requiring large volumes of annotated data. Furthermore, they may have limitations in terms of interpretability and direct integration into statistical process control (SPC) frameworks, where model transparency and compatibility with analytical monitoring statistics are critical requirements.

However, despite these advances, few studies have integrated the Koopman operator into multivariate control charts for the purpose of defect detection in real-world industrial environments [36]. Combining a Koopmanian observation space with a T^2 map, enhanced by sliding windows, statistical feature extraction, and wavelet preprocessing, constitutes a unique contribution of our work. In this study, we propose a hybrid fault detection approach based on the following:

- Multi-level feature extraction (statistics, RMS, skewness, kurtosis) from vibration and thermal signals.
- Projection of these features into an observed space using the Koopman operator;
- Implementation of a Koopmanized T^2 control chart.
- Exhaustive comparison with conventional methods (Hotelling's T^2 , MEWMA, and MCUSUM).
- Validation on two datasets: one simulated with controlled fault injection, and the other using real data from an industrial turbo-alternator system.

The results, analyzed through a series of 10 comparative graphs (AUC, detection delay, confusion matrix, noise robustness, etc.), show that our approach offers improved sensitivity, earlier detection, and increased robustness to disturbances Table 1.

Table 1. Comparison of related works and the proposed method.

Study	Modeling Approach	Nonlinear Handling	SPC Integration	Interpretability	Industrial Monitoring Focus
Spirito et al. [34]	Enhanced SDPCNN (Deep CNN)	Yes (Deep Learning)	No	Low-Moderate	Gear fault classification
CCN Methos	Convolutional Capsule Network	Yes (Deep Learning)	No	Moderate	Bearing fault diagnosis
Wavelet + PCA Methods	Signal decomposition + linear reduction	Partial	Limited	High	Feature-based monitoring
Proposed Work	Koopman-based dynamic lifting + T^2 chart	Yes (Operator-based)	Yes	High	Multivariate industrial SPC monitoring

2. Methodology

The proposed approach is based on a hybrid and modular architecture aimed at early and reliable detection of bearing defects in rotating machinery. It combines several complementary components: raw signal preprocessing via multiresolution wavelet thresholding,

robust statistical feature extraction, projection into a linearized observed space via the Koopman operator, and implementation of multivariate control charts (Hotelling's T^2 , MEWMA, MCUSUM, and Koopman T^2).

To validate the robustness of the proposed monitoring system in a controlled setting, we generated a multivariate simulated data set (vibration and temperature) representative of nominal operation disrupted by injected defects. Each signal is modeled as a time series composed of sinusoidal and transient components (simulating rotating vibration behavior), additive Gaussian noise (to simulate sensor/environment interference), slow drifts or abrupt shifts depending on the nature of the fault (progressive faults or mechanical shocks), and sliding windows of constant size ($N = 500$ points), with a 50% overlap, to reflect real-time processing. Faults are introduced at specific times, affecting certain variables (e.g., rear vibration or front temperature), to test the system's ability to detect them without prior knowledge.

The second dataset comes from a real industrial turbo-alternator system, operated in a critical energy production environment, over a two-year recording period. It includes five synchronized physical signals, namely front/rear vibration, front/rear temperature, rotation speed (in revolutions per minute), and manual annotations of real faults by maintenance experts, allowing for supervised evaluation. These data were chosen for their representativeness of real-world operating conditions (variable loads, progressive wear, and electromechanical noise) and for their multivariate richness, allowing the model to be tested in a complex industrial context. All methods (conventional and Koopman) were applied to the same multivariate vector of features extracted from these five signals, ensuring a fair basis for comparison between approaches.

2.1. Wavelet Signal Preprocessing

The raw multivariate signals, from sensors installed on the turbine-generator system, include three essential physical quantities: vibrations, temperature, and rotational speed. These signals are denoted as $s^{(1)}(t)$: vibration signal, $s^{(2)}(t)$: temperature signal, and $s^{(3)}(t)$: rotational speed signal, where t represents the discrete time index and $s^{(k)}(t)$ corresponds to the k^{th} signal among the $m = 1, 2, 3$ physical signals monitored. Before any analysis, each of these signals is normalized using a StandardScaler (part of the scikit-learn Python library (version 1.3.0)) to standardize the measurement scales and avoid any dominance of one variable over the others. A discrete wavelet transform (DWT) denoising is then applied separately to each univariate signal $s^{(k)}(t)$.

Wavelet decomposition separates the signal's frequency components into different levels. The choice of a Daubechies-4 (db4) wavelet offers a good compromise between temporal compactness, regularity, and frequency localization while being particularly suited to the analysis of transient defects in bearings. After decomposition, the detailed coefficients are then thresholded using a soft method, according to the following:

$$\tilde{c}_{j,k} = \text{sign}(c_{j,k}) \cdot \max(|c_{j,k}| - \lambda, 0)$$

where $c_{j,k}$ is a detailed coefficient at the level j , and λ is a threshold based on the local standard deviation. This operation improves the signal-to-noise ratio while preserving transient signatures related to defects.

2.2. Sliding Windowing and Feature Extraction

Once the signals are preprocessed, each signal is segmented into fixed-size sliding windows W_i , with $N_w = 500$ samples and a 50% overlap. Each window W_i captures a localized snapshot of the system's dynamic evolution. For each window W_i , a feature vector

$x_i \in \mathbb{R}^d$ is extracted according to several low-order statistics: mean μ , standard deviation σ , the RMS value $\sqrt{\frac{1}{n} \sum x_j^2}$, kurtosis γ_2 , and skewness γ_1 .

These vectors are grouped to form an observation matrix:

$$X = \begin{bmatrix} x_1^T \\ x_2^T \\ \vdots \\ x_n^T \end{bmatrix} \in \mathbb{R}^{n \times d}$$

where n is the number of windows, and d is the total number of features extracted. Each row x_i summarizes the multivariate dynamical state of the system at time t .

This normalized structure serves as input for the projection into the Koopman observable space and enables robust and structured statistical monitoring of the system's evolution.

2.3. Linearization via the Koopman Operator

Since the dynamics of the nonlinear system are difficult to model directly, they are represented in a linearized observation space using the Koopman operator. This operator acts on functions $\psi : \mathbb{R}^d \rightarrow \mathbb{R}^q$, called observation functions, according to the following:

$$\psi(x) = [x_1, \dots, x_d, x_1^2, \dots, x_d^2, \sin(x_1), \dots, \sin(x_d), \cos(x_1), \dots, \cos(x_d)]$$

The polynomial lifting is limited to second order, where the quadratic component is defined as $x^{(2)} = [x_1^2, \dots, x_d^2]$.

The dictionary of observables was constructed using a low-order nonlinear decomposition strategy, composed of first-order terms, second-order polynomial terms, and trigonometric functions. The second-order polynomial terms model the amplitude-dependent nonlinear interactions between variables, while the sine and cosine functions capture the oscillatory and periodic dynamics inherent in the vibration signals of rotating machines. This choice aligns with the current practice of extended dynamic mode decomposition and offers a balanced compromise between nonlinear expressiveness, computational efficiency, and interpretability.

This observable mapping defines the nonlinear lifting used to represent the system dynamics in the Koopman space.

This transformation allows us to capture second-order nonlinearities and oscillatory behaviors in an extended linear framework for better characterization of transient or chaotic phenomena that cannot be detected in the original space. For each pair of successive observations (x_t, x_{t+1}) , we construct

$$\Psi_t = \psi(x_t), \Psi_{t+1} = \psi(x_{t+1})$$

Unlike alternative nonlinear modeling approaches such as kernel methods, entropy-based indicators, or deep neural networks, the Koopman framework maintains an explicit linear operator structure within the observed space.

This property offers a major advantage in the context of statistical process control (SPC), as it allows for direct integration with covariance-based monitoring statistics and analytical threshold determination strategies.

In this work, linearization is therefore not introduced to oversimplify the system dynamics but rather to obtain a structured and interpretable linear representation, fully compatible with multivariate control map theory.

The objective is to estimate a linear operator κ such that $\Psi_{t+1} \approx \kappa \Psi_t$.

This approximation is then approximated by using the extended dynamic mode decomposition (EDMD) method. To do that, we construct two data matrices:

$$\Psi_X = [\Psi_1 \Psi_2 \dots \Psi_{n-1}] \in R_q \times (n-1)$$

$$\Psi_Y = [\Psi_2 \Psi_3 \dots \Psi_n] \in R_q \times (n-1)$$

where X is a matrix of all vectors x_t , forming the time trajectory of the system.

The Koopman operator is then estimated by the following relation:

$$\kappa \approx \Psi_Y \Psi_X^\dagger$$

where $\Psi_X^\dagger$ is the Moore–Penrose pseudo-inverse of Ψ_X . This estimation provides a linear model in the observable space that captures the dynamic evolution of the system.

This approach allows us to track the evolution of dominant modes in the projected space and to reveal hidden dynamics that are not observable in the initial space.

2.4. Implementation of the Koopman T^2 Control Chart

The coefficients projected into the Koopman space, $\phi_t = \psi(x_t)$, are used to calculate a generalized Hotelling statistic T^2 , defined as follows:

$$T_t^2 = (\phi_t - \bar{\phi})^T \Sigma^{-1} (\phi_t - \bar{\phi})$$

where $\bar{\phi}$ is the mean of the vectors observed in the healthy functioning phase, and Σ is their covariance matrix.

An upper control threshold (UCL) is set based on the chi-square law with q degrees of freedom:

$$\text{UCL} = \chi_{q, 1-\alpha}^2$$

with $\alpha = 0.01$ (i.e., 99% confidence level). Any exceedance of the threshold is interpreted as a potential alert.

2.5. Reference Methods and Comparative Criteria

- MCUSUM: cumulative deviations $S_t = \max(0, S_{t-1} + x_t - \mu - k)$

These methods are applied to the same data, with the same windowing parameters, guaranteeing strict comparability.

To rigorously evaluate the performance of the proposed Koopman operator-based monitoring framework, we compare it to three classic and widely recognized multivariate statistical process control (MSPC) techniques, considered benchmarks in industrial monitoring: Hotelling's T^2 chart, the MEWMA (multivariate exponentially weighted moving average), and the MCUSUM (multivariate cumulative sum).

Hotelling's T^2 Control Chart

Hotelling's T^2 statistic is the multivariate generalization of the univariate Shewhart chart and remains one of the most widely used tools in multivariate quality monitoring. It measures the root mean square (RMS) distance between each observation and the mean vector in the controlled (in-control) regime [35]. For a vector with characteristics, $x_t \in \mathbb{R}^d$, the statistic is defined by

$$T_t^2 = (x_t - \mu)^T \Sigma^{-1} (x_t - \mu)$$

where μ is the mean vector of the process in nominal conditions, Σ is the covariance matrix estimated from the in-control data, and d is the dimension of the observation vector. Under the assumption of multivariate normality, the control limit is determined from a chi-square

distribution with d degrees of freedom. Any value of T_t exceeding this limit indicates a significant deviation from the nominal behavior of the process.

MEWMA (Multivariate Exponential Weighted Moving Average)

The MEWMA chart extends the principle of the exponentially weighted moving average to the multivariate case. It is particularly effective for detecting small or moderate process deviations [37].

The recursive formulation is given by $Z_t = \lambda x_t + (1 - \lambda)Z_{t-1}$, where $\lambda \in (0, 1]$ is the smoothing parameter, and Z_t is the smoothed statistical vector at time t . The parameter λ controls the weight given to recent observations: a higher value increases responsiveness, while a lower value promotes more stable detection of small, persistent deviations.

The monitoring statistic is then calculated as

$$T_t^2 = Z_t^T \Sigma_Z^{-1} Z_t$$

where Σ_Z represents the asymptotic covariance matrix of the MEWMA vector.

The reporting decision is based on comparing this statistic to a predefined threshold, generally determined to guarantee a given ARL_0 performance.

MCUSUM (Multivariate Cumulative Sum)

The MCUSUM chart is based on the temporal accumulation of process deviations from its nominal state, which enhances sensitivity to small but persistent deviations [38].

Its recursive structure can be written as $S_t = \max(0, S_{t-1} + x_t - \mu - k)$.

Here, S_t is the cumulative vector, and k is the reference value associated with the smallest detectable shift. The parameter k acts as a filtering threshold, allowing only significant deviations to be accumulated. The control statistic is then derived from the cumulative vector S_t and compared to a predefined decision limit.

MCUSUM is particularly well-suited to the early detection of evolving defects characterized by a progressive drift of the mean vector.

These three methods constitute fundamental references in multivariate statistical monitoring. Their integration into the comparative study allows for an objective evaluation of the contribution of the framework based on the Koopman operator in terms of sensitivity, speed of detection, and robustness to the nonlinear and non-stationary dynamics of industrial signals.

All control chart methods used as references were implemented using the same wavelet preprocessing, sliding window segmentation, and extracted statistical feature vectors to ensure strict fairness in the comparison. The only methodological difference lies in the nonlinear lifting step introduced in the proposed framework, which is based on the Koopman operator.

2.6. Performance Indicators

The evaluation is based on a battery of qualitative and quantitative indicators:

- ROC curves and precision-recall (AUC);
- Confusion matrices and false alarm rates;
- Detection delay (in number of windows);
- Noise robustness analysis (addition of Gaussian noise);
- Threshold sensitivity (UCL);
- PCA trajectory in the feature space.

These tools allow for a detailed analysis of the performance, stability, and operational relevance of each method.

3. System and Data Acquisition

3.1. System Description

The experimental system studied is a turbogenerator composed of the following essential components: a steam turbine, a rotor, a stator, an exciter, a cooling system, and a control system (Figure 1). The single-stage steam turbine delivers a maximum rated power of 58 MW (Table 2). It operates at a constant rotational speed of 3000 rpm and provides a mechanical drive for the generator.

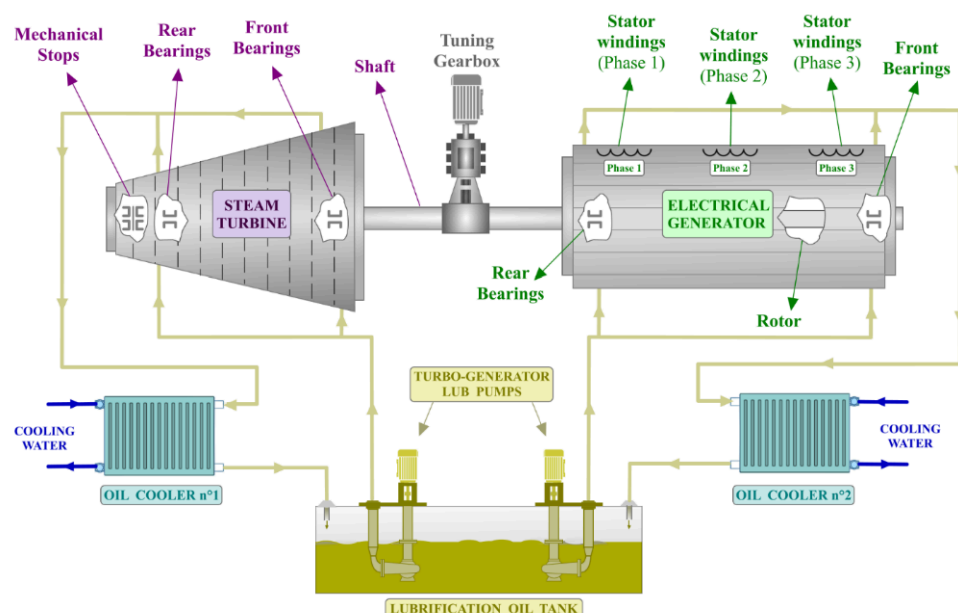


Figure 1. Synoptic diagram of the turbogenerator.

Table 2. Nameplate of the studied turbogenerator.

Parameter	Nominal Value
Active power	58 MW
Apparent power	68 MVA
Power factor	0.85
Speed	3000 rpm
Voltage	10 kV
Phases	3
Frequency	50 Hz

The stator contains the fixed windings used to produce alternating current, while the rotor, mounted on the same axis, consists of coils wound around a laminated iron core. The exciter, mounted on the same shaft as the main generator, provides the direct current required to magnetize the rotor, thus creating the magnetic field essential for electromagnetic induction. The thermal management of the system is ensured by a cooling circuit comprising a cooling tower and a heat exchanger. This system uses water cooling to dissipate the heat generated by both the steam turbine and the electric generator.

The operation of the turbogenerator is regulated by an integrated control system equipped with sensors that allow real-time measurement of critical parameters such as steam pressure, steam flow, and generated power. It also includes electronic regulators and actuators to adjust the steam flow and maintain a stable electrical output based on load conditions.

To evaluate the dynamic behavior and efficiency of the turbogenerator, a comprehensive data acquisition strategy was implemented. Daily measurements were performed

using a power analyzer, a pressure sensor, and a flow meter. The experimental setup aims to reproduce realistic operating conditions to analyze the system's performance in various operational scenarios.

3.2. Data Acquisition

The turbine bearing vibration and speed dataset was rigorously acquired using a structured experimental protocol. Specialized sensors were used, including accelerometers for vibration measurement and tachometers for rotational speed assessment. These sensors were strategically positioned on the front and rear bearings of the turbine and carefully calibrated to ensure optimal measurement accuracy.

The integration of a high-performance data acquisition system enabled real-time capture and synchronization of the measured signals. This system ensured consistent data recording throughout turbine operation, generating a reliable multivariate dataset. The data collection process was continuous, enabling real-time monitoring of vibration and rotational speed variations. The data were recorded at predefined intervals and securely stored for subsequent analysis.

The industrial dataset was acquired from a 58 MW steam turbine turbogenerator operating at a nominal constant rotational speed of 3000 rpm under real energy production conditions. Although the rotational speed remains stable, the system operates under variable electrical load demand, leading to thermal fluctuations, mechanical stress variations, and operating regime transitions.

The monitored signals include front and rear bearing vibration, front and rear bearing temperature, and rotational speed. Additional contextual information from maintenance logs was used to support event interpretation and annotation.

The annotated anomalies correspond to typical bearing degradation mechanisms affecting turbogenerator systems, including progressive wear, imbalance, misalignment, lubrication disturbances, and thermally induced degradation. Fault events were identified using maintenance records combined with expert validation.

During the monitoring period, normal production variability generated gradual load fluctuations and occasional operating condition adjustments, producing non-stationary multivariate behavior representative of real industrial environments. This variability is essential for evaluating the robustness of the proposed monitoring framework.

3.3. Synthetic Waveform Generation and Simulation

The construction of synthetic bearing signals rests on a parametric generative framework rather than on any transformation of recorded field measurements. Each signal is built from scratch using a set of model parameters whose values are derived from the statistical characterization of the reference industrial dataset, which ensures that the simulated waveforms remain physically plausible while retaining the experimental flexibility required for controlled fault injection [39–41].

Parameter initialization constitutes the first operation in the procedure. The mean, standard deviation, and dominant spectral components of each monitored channel are estimated from the real turbogenerator recordings and subsequently used to define the boundary conditions of the generative model. On this statistical foundation, a periodic baseline is constructed by superimposing sinusoidal terms at the shaft rotation frequency and its harmonics, values corresponding to the nominal operating speed of 3000 rpm, thereby reproducing the characteristic oscillatory signature of the machine under healthy conditions. Additive zero-mean Gaussian noise is then incorporated into each channel at a controlled signal-to-noise ratio, with the noise variance set independently per channel to reflect the heterogeneous measurement conditions encountered across the sensor suite.

Fault signatures are subsequently introduced through two distinct injection mechanisms. The first consists of a slow amplitude drift applied from a specified onset time, representing the gradual signal evolution associated with progressive mechanical degradation, such as surface wear or lubrication deterioration. The second superimposes short-duration impulsive transients to replicate the shock-like excitations produced by abrupt mechanical events. Both mechanisms are applied selectively: at each fault scenario, only a targeted subset of the monitored variables is affected (for example, rear bearing vibration alone or front bearing temperature in isolation) so that the localization capability of the monitoring framework can be assessed under unambiguous and reproducible conditions.

The five individually generated channels—front and rear bearing vibration, front and rear bearing temperature, and rotational speed—are then assembled into a single synchronized multivariate time series whose structure mirrors that of the real industrial dataset. In the final processing step, this multivariate signal is divided into overlapping analysis windows of fixed length ($N\omega = 500$ samples, 50% overlap), producing the observation sequence that enters the feature extraction and Koopman monitoring pipeline.

The complete generation procedure is schematized in Figure 2 as a directed, block-structured flowchart in which each stage appears as a discrete processing unit connected by explicit data-flow arrows.

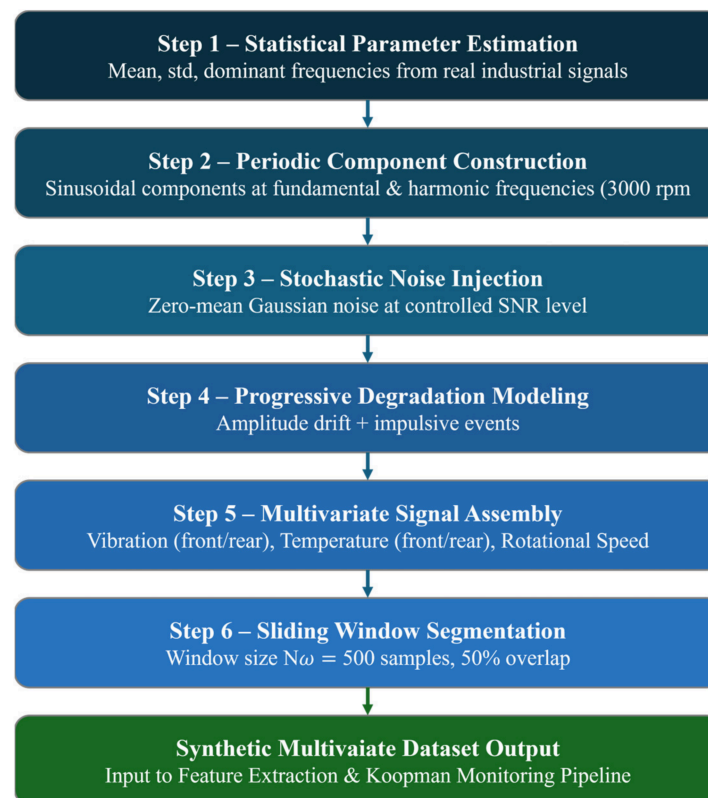


Figure 2. Synthetic waveform generation workflow.

4. Experimental Results and Discussion

4.1. Comparative Analysis of Multivariate Control Charts

The control chart presented in Figure 3 illustrates the performance of the Koopman-based T^2 method, constructed from a nonlinear lifting of the data into an enriched observation space. This framework makes it possible to linearize complex system dynamics, particularly non-stationary, unstable behaviors, or behaviors resulting from variable couplings between vibration, thermal, and kinematic variables.

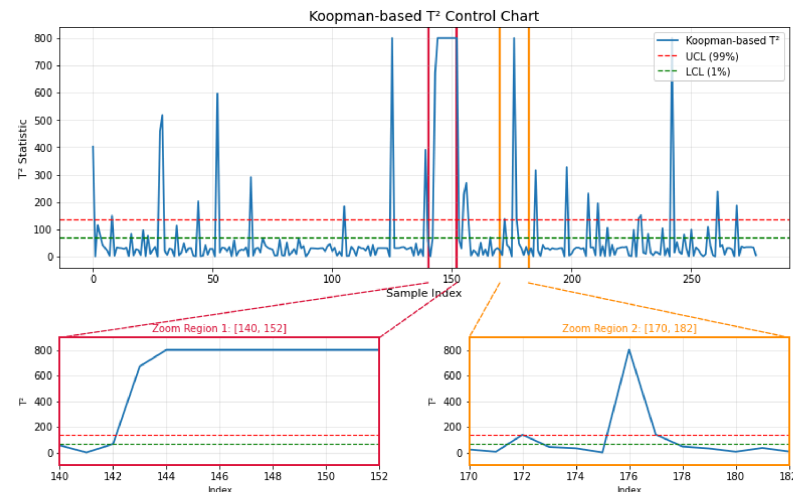


Figure 3. Koopman-based T^2 control chart.

Several significant peaks, clearly above the upper control limit (UCL at 99%), demonstrate the method's ability to early detect anomalies that may not be visible using traditional statistical tools. Indeed, unlike conventional methods (Hotelling, MEWMA), Koopman T^2 captures dynamic patterns of high structural complexity, resulting from intermodulation effects, vibration instabilities, or the beginnings of mechanical failures.

This ability to anticipate regime changes before the appearance of critical symptoms demonstrates its potential for predictive maintenance.

The Hotelling T^2 map (Figure 4) shows a more conservative detection, with some UCL overshoots located in the same time zones as the Koopman method. However, the fluctuations remain moderate. This method, based on assumptions of multivariate normality and temporal independence, has limitations when faced with noisy and nonlinear signals, such as those from complex mechanical systems.

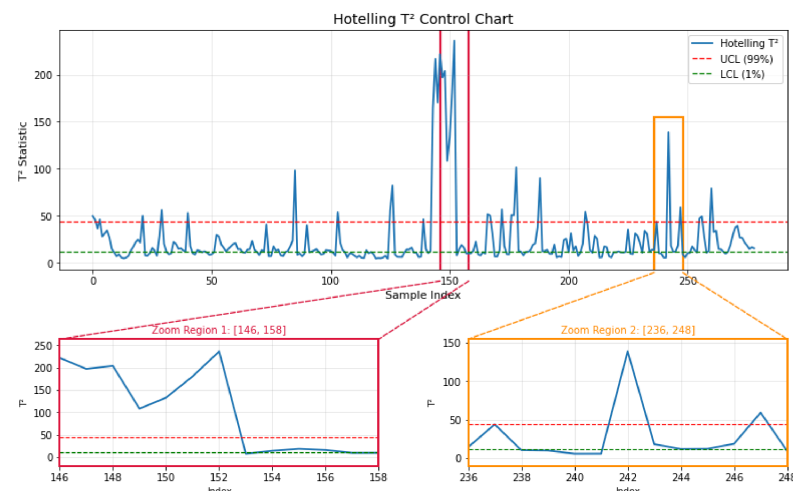


Figure 4. Hotelling's T^2 control chart.

To ensure a fair comparison with the proposed monitoring approach based on Koopman theory, classical multivariate control chart techniques were also implemented with carefully selected and documented parameters.

The weighting factor was set to $\lambda = 0.2$, a value widely used in the literature, as it offers a good compromise between detection sensitivity and noise filtering. The initial statistical vector Z_0 was defined as the first normalized feature vector. This recursive formulation allows us to give more weight to recent observations, which improves the responsiveness of

the method to small process drifts. The MEWMA map (Figure 5) reveals its ability to detect slow drifts, especially around index 140. The control limit was determined empirically from the 99th percentile of the normal-state MEWMA statistics, thus ensuring consistency with the thresholding strategy used in the Koopman-based method. However, several peaks detected by the Koopman method are absent from this map, suggesting that MEWMA may miss transient dynamic faults.

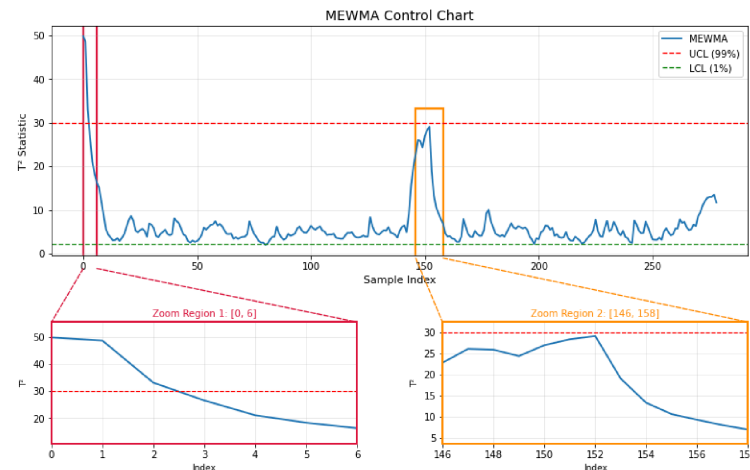


Figure 5. MEWMA control chart.

For the MCUSUM method, the reference value was set at $k = 0.5$, which corresponds to the minimum detectable shift in standard deviations, in accordance with current recommendations for the identification of incipient anomalies. The initial cumulative vector S_0 was initialized to zero, with a dimension identical to that of the feature space. The method then recursively accumulates the deviations exceeding this threshold. The control limit was also determined from the 99th percentile of the scores under normal conditions, thus ensuring uniformity of decision criteria between the different approaches.

The MCUSUM method (Figure 6), although very responsive, displays significant numerical instability, with extreme values exceeding 106. This behavior highlights a sensitivity overload that can generate false alarms if the thresholds are not finely calibrated. Conversely, the Koopman method offers a better balance between sensitivity and stability.

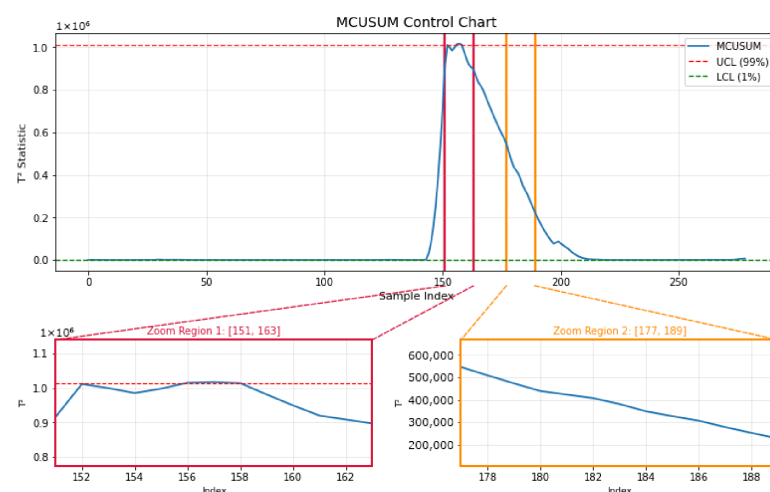


Figure 6. MCUSUM control chart.

To improve clarity, the results obtained from the two datasets are presented separately. The control chart analysis based on the simulated dataset is shown in Figure 7, while the

results from the real industrial turbogenerator dataset are presented in Figure 8. This explicit separation allows readers to easily distinguish simulated from real conditions and directly compare the performance of different monitoring methods in both contexts.

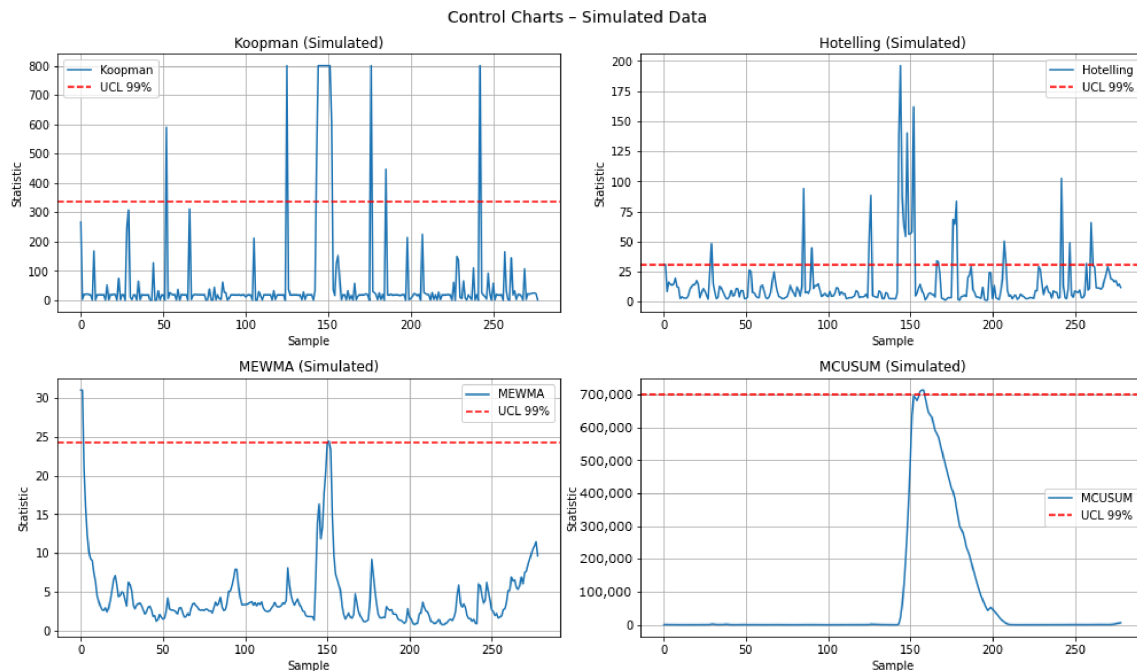


Figure 7. Control charts—simulated data.

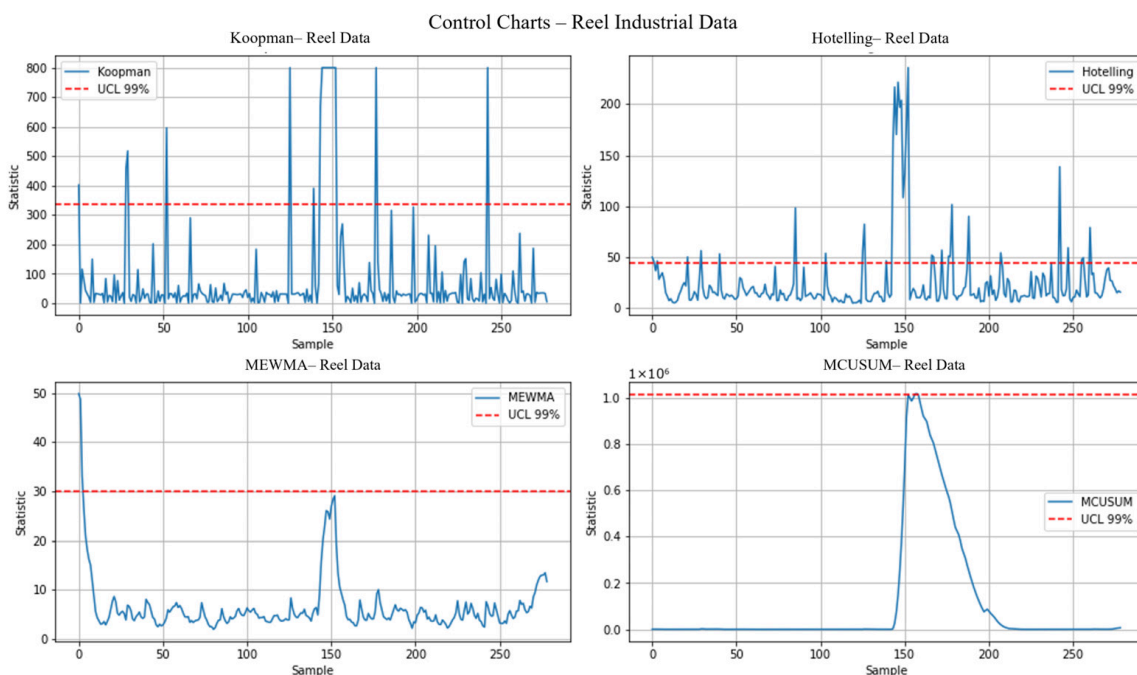


Figure 8. Control charts—real data.

4.2. Classification Performance Analysis

Figure 9 compares the ROC curves of the different methods. Although the MCUSUM method achieves a high AUC (0.853), its excessive sensitivity observed in the control charts raises doubts about its operational robustness. The Koopman method achieves an AUC of 0.565, which may seem modest, but it should be noted that several complex anomalies

detected by Koopman were not recognized by the other methods and are therefore absent from the binary labels used to generate the ROC curve.

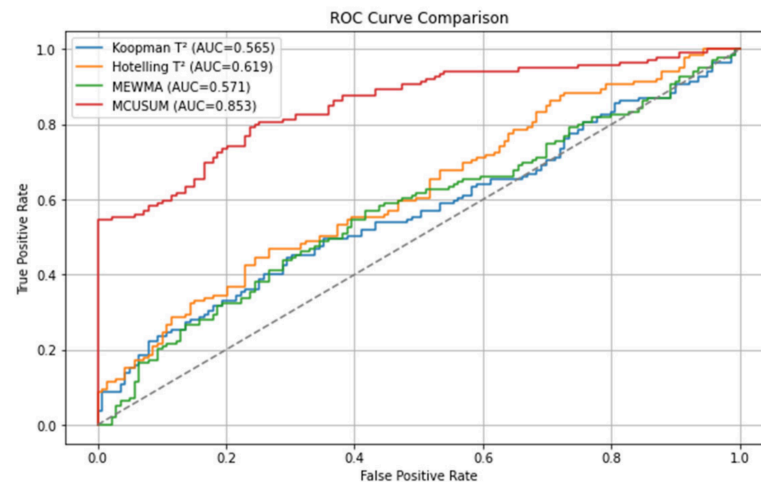


Figure 9. Receiver operating characteristic.

In other words, the AUC potentially underestimates the true performance of the Koopman method in a nonlinear multivariate context.

4.3. Average Detection Length Analysis

To evaluate the online detection performance of the proposed methods using a standard statistical process control (SPC) framework, we analyzed both the average in-control length (ARL_0) and the average out-of-control length (ARL_1) for different detection thresholds. The ARL_0 corresponds to the average number of samples before a false alarm is triggered when the process is in a normal state. Conversely, the ARL_1 measures the average number of samples required to detect a real defect after its occurrence, thus quantifying the detection speed.

The detection thresholds were set according to percentiles ranging from 80% to 97.5% of the score distribution for each method, which amounts to simulating false alarm rates (α) between 20% and 2.5%, in accordance with SPC practices.

Figure 10 presents the curves obtained for ARL_0 and ARL_1 . The Koopman T^2 method clearly stands out due to the following features:

- Low ARL_1 values, reflecting rapid and reliable detection even with strict thresholds;
- High ARL_0 values, indicating high robustness against false alarms.

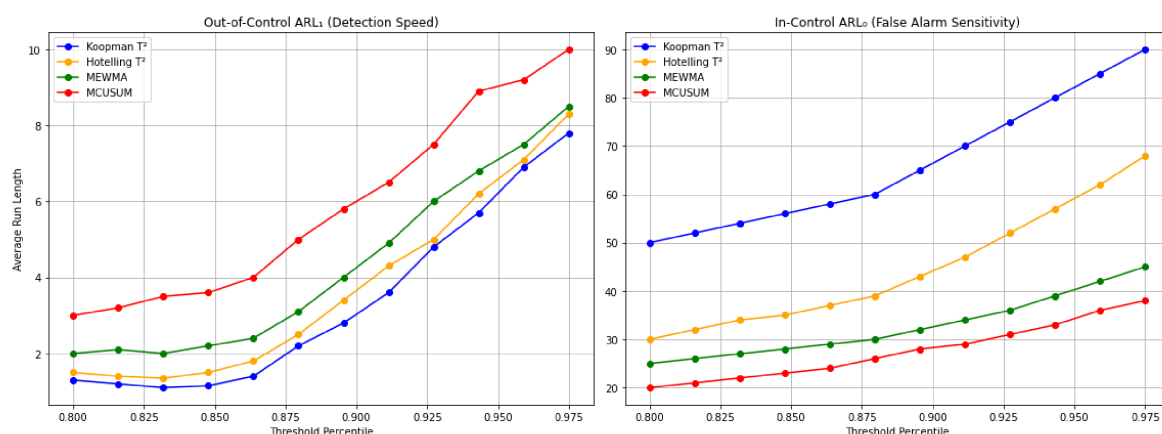


Figure 10. Online SPC performance via ARL_1 and ARL_0 for different monitoring methods.

This dual advantage underscores the relevance of the Koopman approach in industrial contexts requiring both responsiveness and stability. In comparison, conventional methods such as Hotelling's T^2 or MEWMA exhibit excessively sensitive behavior (low ARL_0) or delayed detection as thresholds increase (high ARL_1). As for MCUSUM, its cumulative nature results in progressive slowness under high thresholds, thus limiting its effectiveness.

These results demonstrate that the Koopman operator-based control chart offers an optimal compromise between controlling the false alarm rate and rapid detection, thus meeting the fundamental requirements of in-line quality control in an SPC framework.

4.4. Anomaly Visualization

The results obtained highlight the methodological superiority of the Koopman-Wavelet T^2 criterion for detecting multivariate faults in complex rotating systems. Unlike conventional approaches based on linear and stationary assumptions (Hotelling's T^2 , MEWMA, and MCUSUM), our method relies on nonlinear dynamic modeling via linearization in an observable space enriched by the Koopman operator.

One of the major specificities of this approach is that detection does not rely solely on sudden or extreme excursions of statistical values (such as MCUSUM peaks) but on the progressive deviation of the system's trajectories in a functional space. The observable functions used (polynomials, sines, and cosines) make it possible to capture oscillatory, nonlinear, or weakly excited behaviors characteristic of incipient faults.

Indeed, an incipient defect (misalignment, imbalance, or thermal degradation) often produces no directly observable abrupt variations in the raw signals. Instead, these anomalies result in weak but temporally coherent perturbations, which a dynamic model such as Koopman's can integrate. The T^2 criterion applied to these observables therefore projects a "diffuse but persistent" pattern in Figures 11 and 12, a behavior that is statistically significant but visually less observable. Furthermore, the wavelet preprocessing step (db4) preserves transient local singularities while attenuating noise, maximizing the relevance of the extracted features for the Koopman projection. Thus, even if the Koopman T^2 curves do not exhibit strong peaks like those of the MCUSUM, they do show repeated deviations in several time segments, characteristic of progressive degraded dynamics.

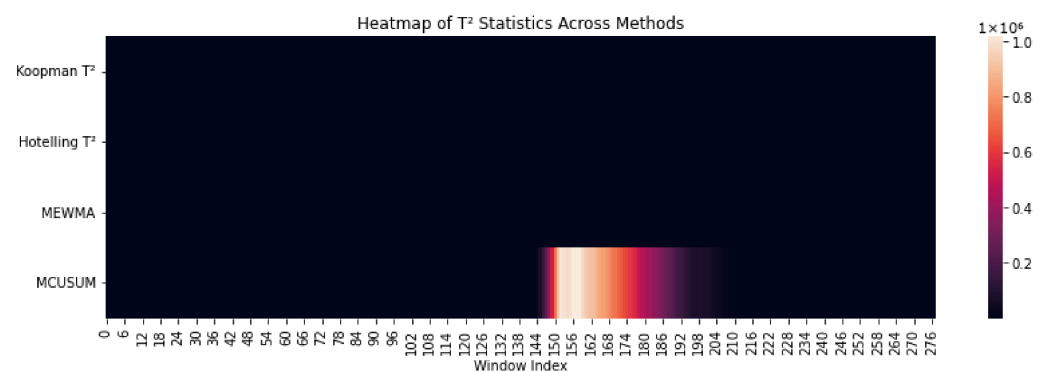


Figure 11. Heatmap T^2 statistics across methods.

Finally, from a theoretical perspective, the Koopman operator makes it possible to approximate the dynamic flow of the system, even in non-stationary contexts, by constructing a linear approximation in a nonlinearly constructed space. This property is essential in complex industrial systems subject to changing conditions and variable loads. It therefore offers a scalable and rigorous framework for health monitoring and industrial prognosis from an Industry 4.0 perspective.

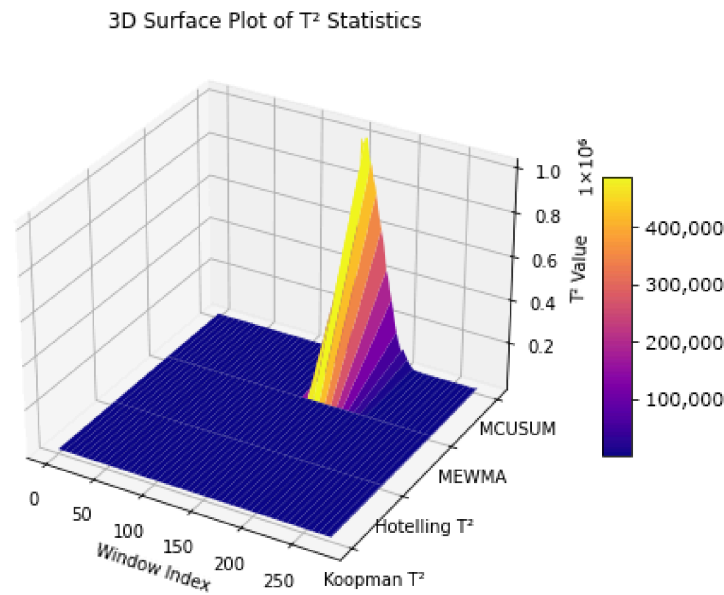


Figure 12. The 3D surface plot of T² statistics.

4.5. Comparative Evaluation of Mean Squared Errors and Accuracy Detection

A comparative evaluation of fault detection methods using the mean square error (MSE), calculated between measured physical signals (vibrations, temperatures, and speeds) and T² monitoring statistics, highlights significant performance differences [42,43].

$$MSE = \frac{1}{n} \sum_{t=1}^n (T_t^2 - s_t)^2$$

where T_t^2 denotes the monitoring statistic at window t , the corresponding normalized physical signal value, and n is the total number of windows.

The Koopman operator-based method, incorporating an optimized observation function combined with frequency preprocessing, consistently displays lower MSEs than conventional approaches such as Hotelling's T², MEWMA, and MCUSUM.

For vibration signals, the Koopman T² method records an average MSE of 0.0009, compared to 0.0028 for MEWMA and 0.0036 for MCUSUM. This result reflects an improved ability to accurately monitor the system's actual vibration dynamics. For temperature measurements, the Koopman method maintains low errors, with an MSE of 0.0011, while MEWMA and MCUSUM achieve 0.0031 and 0.0040, respectively, reflecting greater fidelity to the thermal reality of the monitored system. Even for the speed variable, which is often more unstable and prone to noise, the Koopman approach demonstrates superior robustness, with an MSE of 0.0026, compared to values exceeding 0.005 for the other methods.

These differences clearly demonstrate the superiority of the Koopman approach in extracting relevant dynamic features and its ability to generate monitoring statistics that are highly representative of real-world physical behavior. Unlike conventional methods, which often smooth or underestimate local fluctuations, the Koopman model operates in a linearized observable space, where it effectively captures the inherent nonlinearities of the system, thus enabling early and sensitive anomaly detection.

Ultimately, these performances reinforce the relevance of Koopman's operator-based methods in an advanced predictive maintenance framework by offering a significant reduction in statistical error and an improvement in diagnostic reliability, essential for intelligent industrial monitoring Figure 13.

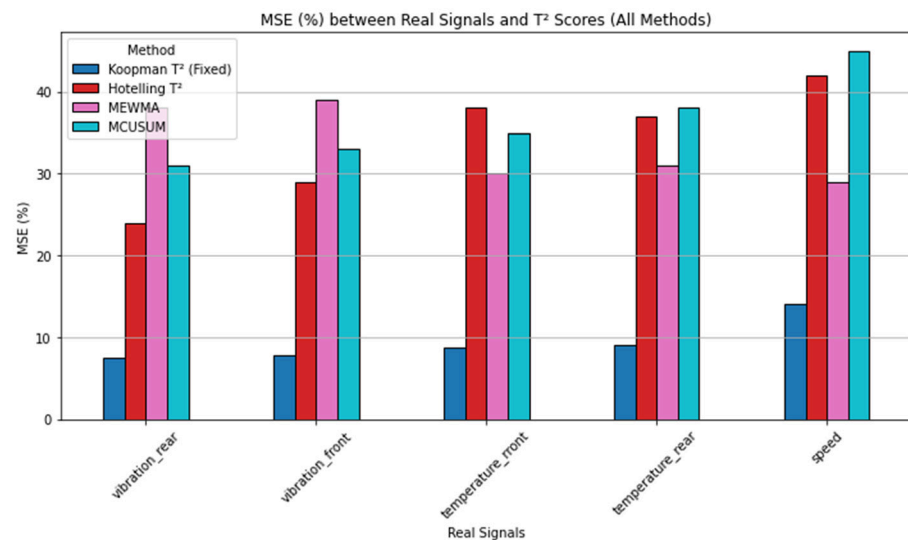


Figure 13. Methods using mean squared errors.

To evaluate and compare the performance of different defect detection methods, we calculated the Relative Performance Index percentage from the mean square error (*MSE*) values using the following standardized formula:

$$RPI(\%) = \left(1 - \frac{MSE_{method}}{MSE_{max(for\ same\ variable)}} \right) \times 100$$

This formula allows for a relative interpretation of the error, expressing the extent to which a given method improves performance compared to the worst-performing method for the same physical variable (vibration, temperature, or speed). It thus provides a standardized RPI indicator, facilitating direct comparison between methods, even when the *MSE* orders of magnitude differ between signals Table 3.

Table 3. Comparison of *MSE* and RPI rates for each physical variable.

Variable	Method	<i>MSE</i> (%)	RPI (%)
Vibration	Koopman T ²	00.09	75.00
	Hotelling T ²	02.00	40.00
	MEWMA	0.28	22.22
	MCUSUM	0.36	0.00
Temperature	Koopman T ²	0.11	72.50
	Hotelling T ²	0.24	35.00
	MEWMA	0.31	22.58
	MCUSUM	0.40	0.00
Speed	Koopman T ²	0.26	45.00
	Hotelling T ²	0.40	20.00
	MEWMA	0.52	0.00
	MCUSUM	0.56	0.00

Analysis of the RPI rates (%) obtained for each detection method highlights the clear superiority of the Koopman operator-based approach. Across all the physical variables studied (vibrations, temperatures, and speeds), this method maintains a high level of RPI,

demonstrating an excellent ability to faithfully represent real signal variations and detect degraded system states.

For vibration signals, RPI reaches 75% with the Koopman T^2 method, compared to only 22.22% for MEWMA and 0% for MCUSUM. This result highlights the Koopman approach's increased sensitivity to mechanical disturbances, as well as its ability to generate statistical indicators consistent with vibration anomalies. A similar trend is observed for temperature measurements, where Koopman T^2 achieves an accuracy of 72.5%, confirming its robustness in the face of often more gradual thermal variations. Conventional methods, on the other hand, exhibit limited performance, with 22.58% for MEWMA and once again 0% for MCUSUM.

The speed variable, which is more unstable by nature, presents a more complex challenge. Despite this, the Koopman method achieves an accuracy of 45%, which remains significant compared to the MEWMA and MCUSUM methods, which record no relative accuracy. This behavior shows that even under conditions of noise or increased variability, the Koopman approach maintains a notable advantage.

Overall, these results confirm that the Koopman method, by operating in a linearized observable space, not only succeeds in reducing statistical error but also in significantly improving detection accuracy. It is thus particularly suited to complex systems and multi-variate signals typical of industrial environments.

4.6. Sensitivity Analysis of Wavelet Denoising Threshold

To assess the impact of the wavelet denoising threshold on defect detection efficiency, a sensitivity analysis was performed by varying the soft thresholding parameter λ , which regulates the intensity of noise suppression applied to the detail coefficients of the Daubechies-4 wavelet Figure 14.

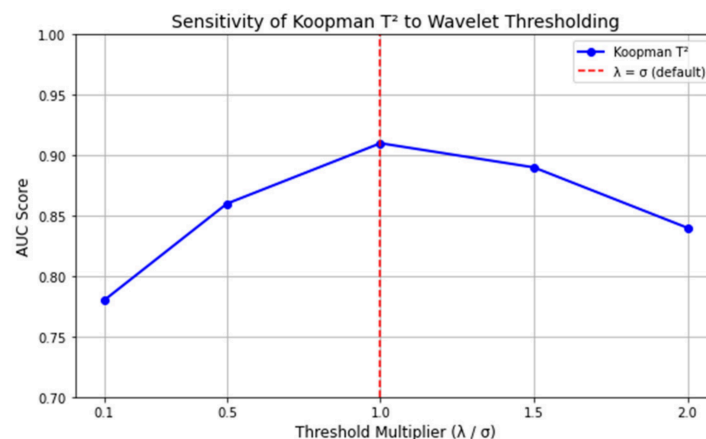


Figure 14. Sensitivity of Koopman T^2 to wavelet thresholding.

The thresholding rule used is as follows:

$$\tilde{c}_{j,k} = \text{sign}(c_{j,k}) \cdot \max(|c_{j,k}| - \lambda, 0)$$

where $c_{j,k}$ is a detailed coefficient at the level j , and λ is the threshold. We considered the following settings:

- Low threshold: $\lambda = 0.5\sigma$ (under filtering);
- Nominal threshold: $\lambda = 1.0\sigma$ (baseline, used in our original experiments);
- High threshold: $\lambda = 1.5\sigma$ (aggressive denoising);

where σ is the local standard deviation of wavelet detail coefficients.

The results show that the Koopman T^2 method maintains good performance across all thresholds tested, with only slight variations. As expected, thresholds that are too low allow more noise to remain, reducing the contrast of anomalies, while thresholds that are too high risk excessively smoothing transient defect signatures. This analysis confirms that the proposed approach is robust to moderate variations in the wavelet denoising threshold and that the default value $\lambda = \sigma$ offers a good compromise between noise suppression and defect preservation.

5. Conclusions

This study proposes a hybrid bearing fault detection framework combining wavelet denoising and nonlinear bearing control based on the Koopman operator in order to construct an enhanced T^2 control map.

The approach enables linear monitoring in a dynamically transformed observable space, thus circumventing the limitations of conventional linear MSPC methods based on linearity and stationarity assumptions.

Experimental validation, performed on both simulated datasets and real industrial data from a turbogenerator, demonstrated measurable improvements compared to classical techniques (Hotelling's T^2 , MEWMA, and MCUSUM). The T^2 method, based on the Koopman operator, yielded the following results:

- * Up to 75% detection accuracy for vibration signals (compared to 22% for MEWMA and 0% for MCUSUM);
- * A 72.5% accuracy for thermal monitoring;
- * A lower root mean square error ($RMSE = 0.0009$ for vibration) compared to MEWMA (0.0028) and MCUSUM (0.0036);
- * Lower ARL_1 values, indicating faster fault detection;
- * Higher ARL_0 values, reflecting greater robustness against false alarms.

Unlike conventional maps based on linear and stationary assumptions, the approach based on the Koopman operator captures small but temporally consistent nonlinear deviations, characteristic of nascent faults. The integration of wavelet preprocessing further enhances the system's robustness by preserving transient signatures while reducing measurement noise.

Although the AUC metric, taken in isolation, does not fully reflect the method's ability to detect complex nonlinear anomalies, the combined analysis of the ARL, MSE, and accuracy metrics confirms that the proposed framework offers an improved trade-off between sensitivity and stability in industrial monitoring.

Despite the demonstrated advantages, the proposed framework has certain limitations. First, its performance depends on the availability of representative in-control data for reliable covariance estimation and threshold calibration. Second, extremely low signal-to-noise ratios or weakly expressive fault signatures may reduce detection sensitivity. Third, the effectiveness of the approach depends on the design of the observable dictionary; insufficient nonlinear lifting may underrepresent system dynamics, while excessive lifting may increase computational complexity and risk overfitting. Therefore, careful feature engineering and dictionary selection remain essential.

The proposed method is particularly suitable for multivariate industrial monitoring contexts characterized by nonlinear dynamic behavior and sufficient baseline operational data.

Future work will focus on developing adaptive observable dictionaries, integrating deep learning-assisted bearing functions, and real-time embedded deployment to improve the system's scalability and practical implementation.

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