

Time-Domain Analysis of SH-Wave Scattering by a Near-Source Loess Yuan [†]

Baitao Sun ¹, Jing Guo ^{1,2,*} and Guixin Zhang ¹

¹ Institute of Engineering Mechanics, China Earthquake Administration, Harbin 150001, China; sunbt@iem.ac.cn (B.S.); zgx@iem.cn (G.Z.)

² College of Aerospace Engineering, Harbin Engineering University, Harbin 150001, China

* Correspondence: gj3041@126.com

[†] Presented at the 7th International Conference on Civil, Architecture and Disaster Prevention and Control, Dali, China, 30 January–1 February 2026.

Abstract

Local topography critically influences seismic hazards by amplifying ground motions and altering their spectral content. This study presents a novel semi-analytical solution for modeling the time-domain scattering of SH-waves by a near-source loess yuan, idealized as an asymmetric trapezoidal ridge. To accurately represent near-source conditions, cylindrical wave incidence is incorporated. The frequency-domain solution is derived using a wave function expansion method within a multi-region framework, employing the wave field mirror method. The transient response is efficiently synthesized via the inverse Fourier transform using a Ricker wavelet source. The results reveal that the asymmetric topography induces significant, incidence-dependent amplification due to wave focusing and prolonged shaking duration caused by multiple internal reflections and scattering within the topographic feature. A key finding is that while a steeper incident slope provides surface shielding, it can generate pronounced subsurface amplification. The solution is rigorously validated against independent finite-element simulations, confirming its accuracy. Furthermore, the proposed method demonstrates a substantial computational advantage. This efficient and accurate framework provides a valuable tool for parametric analysis in site-specific seismic hazard assessment.

Keywords: near-source topography; time-domain; seismic amplification

1. Introduction

Local topography is a critical factor influencing seismic hazard, capable of significantly altering the amplitude, acceleration, and spectral characteristics of ground motion in its vicinity [1,2]. These modifications directly impact the seismic performance and safety of structures [3,4]. With accelerating global urbanisation, populations and critical infrastructure are increasingly concentrated in seismically active regions. Consequently, a major earthquake in such areas carries the potential for catastrophic human and economic losses. It is therefore imperative to deepen our understanding of seismic wave scattering phenomena in complex near-source geological settings.

The loess yuan—a vast, table-like landform characteristic of loess plateau regions—is both a region of strategic socio-economic importance [5] and one prone to strong seismic activity [6,7]. This study investigates the scattering of SH-waves by such a prototypical near-source topography, which is idealized as an asymmetric trapezoidal ridge. We develop a semi-analytical solution based on an extended wave function expansion framework,



Academic Editor: Mohamed Abdelkader Ismail

Published: 8 July 2026

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integrating several key theoretical techniques including the wave field mirror method, multi-region matching, and complex function theory [8–10].

The principal advancements of this work are threefold: (i) the incorporation of cylindrical wavefront incidence to model near-source effects; (ii) the development of a multi-region formulation to handle the asymmetric trapezoidal geometry; and (iii) the derivation of an efficient semi-analytical solution that is synthesized into the time domain analysis. The proposed method is rigorously validated and demonstrates a significant computational advantage over conventional numerical approaches.

2. Geometric Model and Analytical Formulation

The physical problem is conceptualized within a two-dimensional, infinite, elastic half-space. The loess yuan topography is idealized as a homogeneous, isotropic, linearly elastic trapezoidal protrusion with dissimilar flank slopes, representing a canonical local topographic feature. The incident seismic excitation is modelled as a cylindrical SH-wave emanating from a line source located at coordinates (x_0, y_0) , simulating near-source conditions. The geometric configuration is depicted in Figure 1. The trapezoid is defined by a top width a , a base width $2a$, and left- and right-side slopes of $1 : n_1$ and $1 : n_2$, respectively.

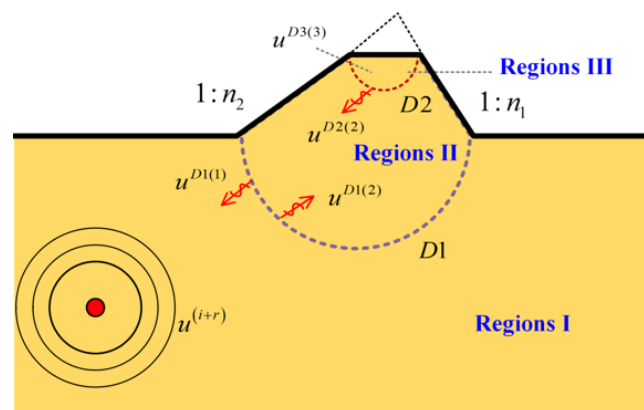


Figure 1. Model and Regional Partitioning.

The medium is characterized by a constant shear modulus μ and shear wave velocity c (with wavenumber $k = \omega/c$). To facilitate the application of the wave function expansion method, the total domain is partitioned into three sub-regions (Regions I, II, III) by introducing two virtual, circular auxiliary boundaries, D1 and D2 (Figure 1).

The total wavefield in Region I (the semi-infinite domain exterior to D1) comprises the incident-reflected field, $u^{(i+r)}$, and the scattered field $u^{D1(1)}$ radiating outward from boundary D1:

$$u^{(i+r)} = \frac{i}{4\mu} \left[H_0^{(1)}(k|z|) + H_0^{(1)}(k|z'|) \right] \quad (1)$$

$$u^{D1(1)}(z_1, \bar{z}_1) = \sum_{m=-\infty}^{+\infty} A_m H_m^{(1)}(k|z_1|) \left[\left(\frac{z_1}{|z_1|} \right)^m + (-1)^m \left(\frac{z_1}{|z_1|} \right)^{-m} \right] \quad (2)$$

where $H_m^{(1)}(\cdot)$ is the m -th order Hankel function of the first kind, z_1 is the complex coordinate in the local system, and z' denotes the image of the source about the flat free surface. The form of the series ensures automatic satisfaction of the stress-free condition on the horizontal ground surface.

Region II (the annular domain between D1 and D2) hosts a standing wavefields, constructed using Bessel $J_m(\cdot)$ and Hankel $H_m^{(1)}(\cdot)$ functions, respectively:

$$\begin{aligned} u^{D1(2)}(z_3, \bar{z}_3) &= \sum_{m=-\infty}^{+\infty} B_m H_{mp_0}^{(1)}(k|z_3 e^{qi}|) \left[\left(\frac{z_3 e^{qi}}{|z_3 e^{qi}|} \right)^{mp_0} + (-1)^m \left(\frac{z_3 e^{qi}}{|z_3 e^{qi}|} \right)^{-mp_0} \right] \\ u^{D2(2)}(z_3, \bar{z}_3) &= \sum_{m=-\infty}^{+\infty} C_m J_{mp_0}(k|z_3 e^{qi}|) \left[\left(\frac{z_3 e^{qi}}{|z_3 e^{qi}|} \right)^{mp_0} + (-1)^m \left(\frac{z_3 e^{qi}}{|z_3 e^{qi}|} \right)^{-mp_0} \right] \end{aligned} \quad (3)$$

Region III (the interior domain bounded by D2) contains a standing wavefield:

$$u^{D3(3)}(z_2, \bar{z}_2) = \sum_{m=-\infty}^{+\infty} D_m J_m(k|z_2|) \left[\left(\frac{z_2}{|z_2|} \right)^m + (-1)^m \left(\frac{z_2}{|z_2|} \right)^{-m} \right] \quad (4)$$

The unknown complex expansion coefficients $\mathbf{X} = \{A_m, B_m, C_m, D_m\}^T$ are determined by enforcing continuity of displacement and shear stress across the virtual interfaces D1 and D2. This yields the following system of coupled equations:

$$\begin{aligned} -u^{D1(1)} + u^{D1(2)} + u^{D2(2)} &= u^{(i+r)} \text{ on } |Z_1| = a, \\ -\tau_{rz}^{D1(1)} + \tau_{rz}^{D1(2)} + \tau_{rz}^{D2(2)} &= \tau_{rz}^{(i+r)} \text{ on } |Z_1| = a, \\ u^{D1(2)} + u^{D2(2)} - u^{D3(3)} &= 0 \text{ on } |Z_2| = a/2, \\ \tau_{rz}^{D1(2)} + \tau_{rz}^{D2(2)} - \tau_{rz}^{D3(3)} &= 0 \text{ on } |Z_2| = a/2. \end{aligned} \quad (5)$$

Shear stress components τ_{rz} are derived from the constitutive relation $\tau = \mu \partial u / \partial r$. The problem is reduced to a system of linear algebraic equations via the collocation and truncation method. The unknown coefficients are subsequently determined through standard matrix operations, yielding the semi-analytical solution.

3. Time-Domain Analysis and Numerical Validation

The semi-analytical solution derived in the frequency domain is transformed into the time domain to elucidate the transient response characteristics of the near-source yuan topography. This is achieved through the application of the Inverse Fourier Transform (IFT). To ensure broad relevance and practical utility for seismic engineering, a Ricker wavelet is selected as the excitation signal.

A total of 100 points from 0.06π Hz to 8π Hz are taken to ensure adequate frequency sampling. The model parameters are set as follows: shear wave velocity $c = 1$ m/s, central frequency $f_c = 2$ Hz, and shear modulus $\mu = 1$. The trapezoidal topography has slopes $n_1 = 0.5$ (gentle) and $n_2 = 1.5$ (steep). Four distinct source locations are analyzed: $(0, -4a)$, $(0, 4a)$, $(4a, -4a)$, and $(4a, 4a)$, representing incidence from the gentle-slope side and the steep-slope side, respectively. Snapshots of the resultant out-of-plane displacement field at $t = 1, 4, 6$, and 8 s are presented in Figure 2.

The wave propagation exhibits clear cylindrical spreading from the line source, with amplitude decay over distance consistent with geometric spreading. The sequence of snapshots reveals the evolution from a localized disturbance near the source ($t = 1$ s) to a wavefield encompassing the entire domain ($t = 6, 8$ s).

The interaction with the trapezoidal topography induces complex scattering phenomena. At $t = 4$ s, as the wavefront impinges on the protrusion, a pronounced focusing effect is observed, leading to significant amplification of displacement amplitudes within and atop the yuan. Crucially, this amplification persists at later times ($t = 6, 8$ s), indicating that the topographic feature acts as a sustained secondary source due to multiple internal reflections and scattering, prolonging the duration of shaking.

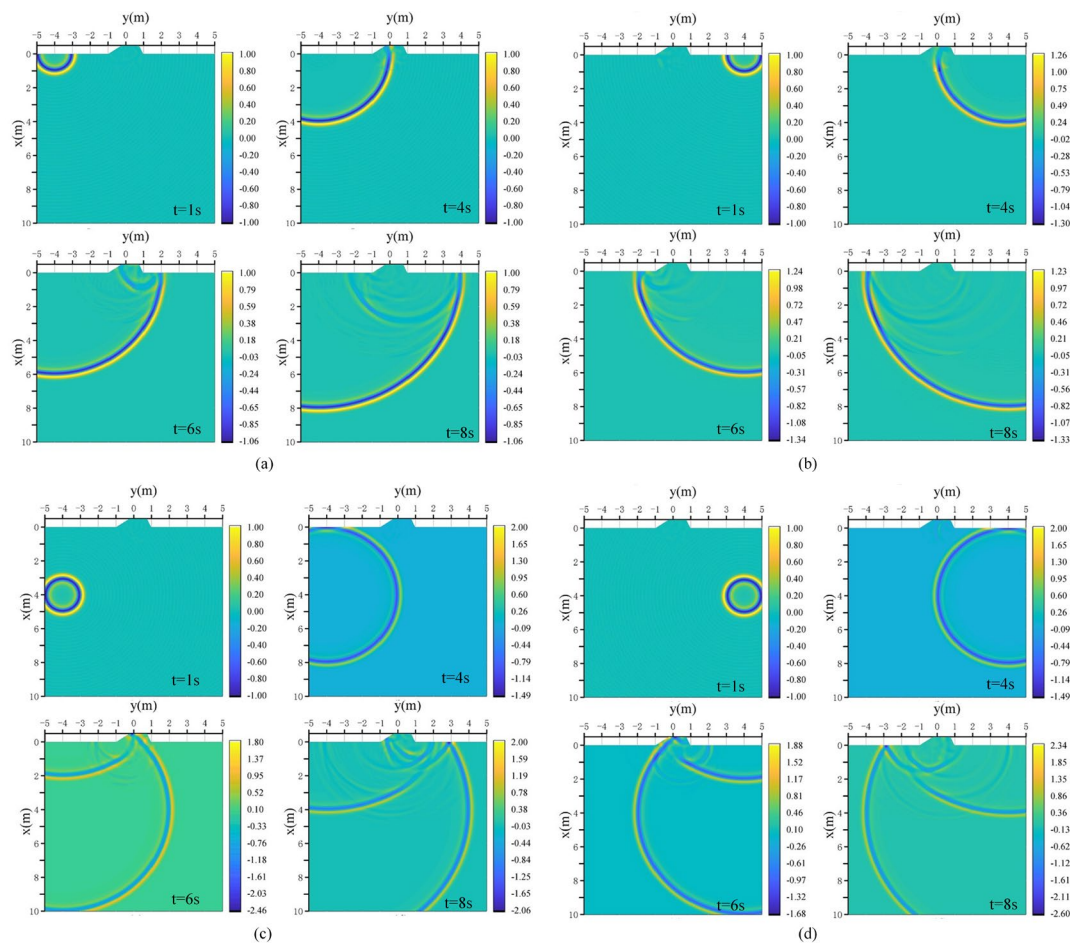


Figure 2. The propagation process of the wave within the domain: (a) excited at point $(0, -4a)$; (b) excited at point $(0, 4a)$; (c) excited at point $(4a, -4a)$; (d) excited at point $(4a, 4a)$.

The asymmetric slope geometry ($n_1 \neq n_2$) leads to direction-dependent responses. Comparison between gentle-slope incidence (Figure 2a, source at $(0, -4a)$) and steep-slope incidence (Figure 2b, source at $(0, 4a)$) shows broadly similar wavefront evolution but distinct amplification patterns. Incidence against the steeper flank results in larger positive/negative displacement peaks in the subsurface region directly beneath the slope, highlighting a significant amplification hazard for embedded structures despite the surface screening effect of the steep face. Subsurface sources (Figure 2c,d) exhibit roughly doubled amplitudes compared to surface sources (Figure 2a,b), consistent with the constructive interference between incident and surface-reflected waves prior to topographic interaction.

The semi-analytical time-domain solution is validated through finite element (FEM) simulations in COMSOL Multiphysics® version 6.2, configured with identical parameters and absorbing boundaries. The wavefield patterns and amplification factors show excellent agreement with the semi-analytical results (cf. Figures 2c and 3), confirming the method's accuracy. Minor discrepancies are attributable to numerical approximations in the FEM model versus the theoretical precision of the 2D semi-analytical solution.

A critical advantage of the semi-analytical approach is its computational efficiency. A direct comparison was performed on an identical computing platform (Intel Core i7-12700F, 32GB RAM). For the $n_1 = 0.5$, $n_2 = 1.5$ model, the time required to compute the wavefield snapshot at $t = 8$ s was 53.4 s using the semi-analytical/IFFT method, compared to 308 s for the FEM simulation—an acceleration of $\sim 5.8\times$. Even for an earlier snapshot ($t = 1$ s), the semi-analytic method was 5.2 times faster (45.3 s vs. 234 s). This efficiency gain, which becomes more pronounced for later times in the simulation.

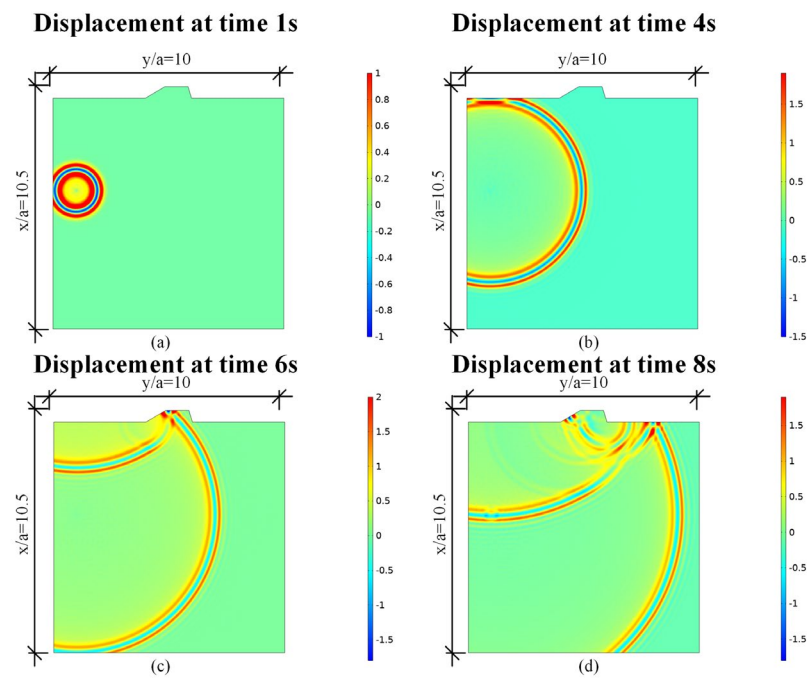


Figure 3. Finite element results of the displacement amplitude when $(4a, -4a)$ is excited: (a) $t = 1$ s; (b) $t = 4$ s; (c) $t = 6$ s; (d) $t = 8$ s.

This gain in efficiency becomes increasingly pronounced during the later stages of the simulation, a direct consequence of the fundamental methodological differences between the two approaches. The Finite Element Method (FEM) requires a spatially fine mesh to accurately resolve the wavefield across the entire frequency spectrum excited by the Ricker wavelet, particularly at higher frequencies. This necessity results in a large system of equations that must be solved at each time step or frequency point. In contrast, the semi-analytical solution, rooted in the wave-function expansion method, is inherently a mesh-free boundary-value technique. Its convergence is primarily governed by the truncation order N of the expansion series. Once an adequate N is determined for the frequency range of interest—which is typically much smaller than the number of degrees of freedom in an FEM model—the solution is obtained by solving a relatively small, dense system of linear equations for the expansion coefficients at each frequency. Subsequent synthesis into the time domain via the Inverse Fast Fourier Transform (IFFT) is computationally inexpensive. This inherent efficiency renders the semi-analytical method particularly well-suited for parametric studies.

4. Conclusions

A novel semi-analytical model is developed for the time-domain scattering of near-source SH-waves by an asymmetric trapezoidal loess yuan. The solution, derived via wave function expansion and synthesized into the time domain, reveals that the topography induces significant, incidence-dependent amplification due to wave focusing and multiple scattering. Notably, steeper slopes, while shielding the surface, can amplify motions in the critical subsurface region. The results are rigorously validated against finite-element simulations, and the method demonstrates superior computational efficiency. This work provides an accurate and efficient tool for assessing topographic effects in seismic hazard analysis.

Author Contributions: Conceptualization, B.S. and J.G.; methodology, B.S.; software, G.Z.; validation, B.S., J.G. and G.Z.; formal analysis, B.S.; investigation, B.S. and G.Z.; resources, J.G.; data curation, B.S.; writing—original draft preparation, J.G.; writing—review and editing, J.G.; visualization, G.Z.; supervision, J.G.; project administration, J.G.; funding acquisition, J.G. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by [the National Key Research and Development Program of China] grant number [2022YFC3003605]. And the APC was funded by [2022YFC3003605].

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The data presented in this study are available within the article.

Conflicts of Interest: The authors declare no conflict of interest.

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