

Feedback Control of the Inverted Pendulum [†]

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Abstract

This paper considers the control problem for the cart–pendulum system. The equations of system dynamics are derived and its parameters are estimated by applying simple identification procedures. The design goals include stabilization of the pendulum in upper equilibrium position and disturbance rejection for plant input disturbances. For satisfying the design goals, the conditional feedback control structure is introduced and its properties are discussed. The first objective is to stabilize the unstable pendulum in an upright position. Since the pendulum cannot be stabilized by a simple gain, a state feedback is designed in the local feedback contour. Based on the conditional feedback, the design problem for the forward controller is solved by using desired closed-loop system complementary sensitivity function and achieving low sensitivity with respect to disturbances at the plant input. The performance of the obtained designs are tested by a numerical example.

Keywords: feedback control; inverted pendulum; stabilization; conditional feedback

1. Introduction

There exist some special features and requirements for the education of specialists in the area of technical sciences, where two basic approaches of technical education prevail. The first approach is related to the physics of the explored phenomenon and is practically oriented, relying on the particular technical application. The other approach is mathematically oriented and uses theoretical concepts and methods to extract the properties of the explored phenomenon. Depending on the particular realization, these two approaches sometimes cause the undesired effect of separation between theory and practice, which is the most common source of argument and mutual accusations between practitioners and theoretical engineers. Nevertheless, both approaches serve their own well-deserved part in the process of student education. One particular way of bringing these two approaches closer is the development of university laboratories and laboratory equipment. The usage of more advanced and complicated machinery, materials and provisions demonstrates the necessity of combining theoretical and practical knowledge in contemporary engineering education; see [1,2]. The present paper serves the general goal to improve the laboratory environment for modern control systems analysis and design.

The main part of the developed laboratory models is built to represent certain characteristics and properties of existing industrial processes and systems. However, there exists a large group of models that do not have a direct link to real systems, but serve as a test bed for a variety of control algorithms. The typical example of such a device is the physical model of the inverted pendulum. The inverted pendulum laboratory model is one of the most popular laboratory devices, which is widely distributed for practical realization of



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complex control systems. Frequently, it is regarded as a benchmark example for the application of complicated control methods for analysis and design of classical electromechanical devices. One of the main purposes of the inverted pendulum is to represent the difficulties in controlling an inherently unstable control plant, containing many nonlinearities, having more than one equilibrium positions and possessing the inherent structural limitations of using negative feedback structures. The inverted pendulum can be used as a simplified model for controlling a load crane at shipping ports or as a meaningful demonstration tool for missile stabilization problems. The inverted pendulum has been used to illustrate task-oriented control problems such as swinging up the pendulum in an upright position and its stabilization.

The laboratory model of the inverted pendulum is shown in Figure 1. The physical system consists of a cart driven by a DC motor and a two-pole pendulum attached to the cart; see [1]. The cart can move along a horizontal rail and the pendulum is able to rotate freely in a vertical plane parallel to the rail. In order to swing up or balance the pendulum around equilibrium points, the cart is moved back and forth on the rail. Both the position of the cart on the rail and the angle of the pendulum can be measured by using two optical encoders. The control objective is to move the cart from one position to another along the rail with the pendulum stabilized in an upward position. This motion can be achieved by forcing several oscillations without letting the angle and velocity become too large. In close proximity to the pendulum upward position, the pendulum is stabilized by using its linearized model and implementing certain linear control algorithms.



Figure 1. The inverted pendulum laboratory model.

The goal of this paper is to develop a control algorithm for the stabilization of the inverted pendulum around its upper unstable equilibrium position. At the first stage of algorithm development, the model of the inverted pendulum is derived. The pendulum model is obtained by using the Newton–Euler equations of dynamics and then applying certain identification procedures for estimating the model parameters. At the second stage of control, the conditional feedback structure is introduced, which allows us to overcome the error of linear approximation of the nonlinear pendulum model around the upward equilibrium position. Several experiments are performed, demonstrating the stabilizing properties of the suggested control algorithm.

2. Equations of Dynamics of the Cart–Pendulum System

The equations of motion are derived from the formulation of Newton–Euler, which is based on the second law of Newton [3]. We consider the free body diagram of the cart–pendulum system, which is shown in Figure 2.

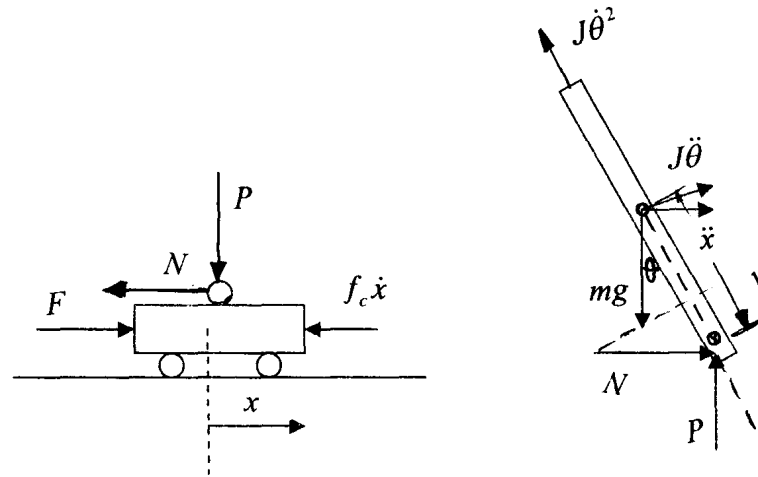


Figure 2. Free body diagram of the cart–pendulum system.

The cart–pendulum system can be described as a fourth order dynamical system with state variables defined as follows: cart position x , cart velocity $\dot{x}$, angular deviation of the pendulum from its vertical upward position θ and the pendulum angular velocity $\dot{\theta}$, both measured in clockwise direction. The input signal is the horizontal force applied to the cart F and g is the gravitational constant. N is the reaction force of the pendulum in horizontal direction and P is the reaction force in vertical direction. f_c is the coefficient of viscous friction of the cart and f_p is the coefficient of viscous friction of the pendulum. We denote the cart mass as M , the pendulum mass as m , the length from the axis of rotation to the pendulum center of mass as l and the pendulum moment of inertia with respect to the axis of rotation as J . We first write the equations of Newton in horizontal direction:

$$M\ddot{x} = F - N - f_c \dot{x} \quad (1)$$

where the horizontal force exerted by the cart on the pendulum is obtained as follows:

$$N = m \frac{d^2}{dt^2} (x + l \sin \theta) = m\ddot{x} + ml \cos \theta \ddot{\theta} - ml \sin \theta \dot{\theta}^2 \quad (2)$$

After substitution of N , we obtain the following equation in horizontal direction for the cart–pendulum system:

$$(M + m)\ddot{x} + ml \cos \theta \ddot{\theta} - ml \sin \theta \dot{\theta}^2 + f_c \dot{x} = F \quad (3)$$

For the rotational motion, we determine the forces that are perpendicular to the pendulum body, see Figure 2. Based on their contribution to the rotational motion of the pendulum, we can write the following equation:

$$P \sin \theta + mg \sin \theta = ml \ddot{\theta} + N \cos \theta + m \cos \theta \ddot{x} \quad (4)$$

The equation of the rotational motion around the axis of rotation through the pendulum center of mass is obtained as follows:

$$Nl\cos\theta - Pl\sin\theta = j\ddot{\theta} \quad (5)$$

After substitution of (5) into (4), we obtain the equation of rotational motion of the cart-pendulum system:

$$(J + ml^2)\ddot{\theta} - mgl\sin\theta + ml\cos\theta\ddot{x} + f_p\dot{\theta} = 0 \quad (6)$$

where by $f_p\dot{\theta}$ we have denoted the effect of the viscous friction on the rotational motion of the pendulum and g is the gravitational constant. Equations (3) and (6) constitute the equations of dynamics of the cart-pendulum system. Equation (3) describes the translational motion of the system and Equation (6) determines its rotational motion.

3. Parametric Identification of the Cart–Pendulum System

The next step in determining the cart–pendulum system model is the estimation of the model parameters. Some parameters are determined by measurement and some of them are obtained after conducting an experiment. For example, the masses and lengths are constants and can be directly measured. Other parameters, such as moments of inertia, can be evaluated by using certain computations. The third group of parameters, like the viscous friction coefficients, can be evaluated by performing statistical experiments and using the methods of identification, see [1]. A dense cylindrical body with mass m , length l and radius r has the following moment of inertia with respect to axis of rotation, passing through the center of mass; see Figure 3:

$$J_C = \frac{1}{12}ml^2 + \frac{1}{4}mr^2 \quad (7)$$

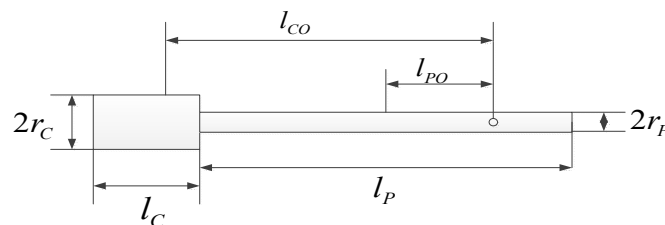


Figure 3. The pendulum body with all measures.

The moments of inertia with respect to axis of rotation, which is located at a distance d from the center of mass, is determined by applying the theorem of Steiner [4]:

$$J_P = J_C + md^2 \quad (8)$$

Using expressions (7) and (8) and employing the fact that the pendulum body consists of two parts, the pendulum pole and the pendulum bar, we can compute the pendulum moment of inertia by the following expression:

$$J = \frac{1}{12}m_L l_C^2 + \frac{1}{4}m_L r_C^2 + m_L l_{CO}^2 + \frac{1}{12}m_P l_P^2 + \frac{1}{4}m_P r_P^2 + m_P l_{PO}^2 \quad (9)$$

where m_L is the load mass, m_P is the pole mass, l_C is the load length, r_C is the load radius, l_P is the pole length, r_P is the pole radius, l_{CO} is the distance from the load center of mass to

the axis of rotation and l_{PO} is the distance from the pole center of mass to the axis of rotation. Using the data from catalog [1] and certain measurements, we obtain the following values:

$$m_L = 0.014 \text{ kg}, m_P = 0.038 \text{ kg}, l_C = 0.03 \text{ m}, r_C = 0.0055 \text{ m}, l_P = 0.5 \text{ m}, r_P = 0.003 \text{ m}, \\ l_{CO} = 0.354 \text{ m}, l_{PO} = 0.107 \text{ m}, \text{ the gravitational constant is } g = 9.81 \text{ m/s}^2.$$

From (9), we obtain the pendulum moment of inertia $J_p = 2.98 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$ and since the cart–pendulum system consists of two pendulum poles and loads, the total pendulum moment of inertia is: $J_P = 2J_p = 5.96 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$. Similarly, we compute the pendulum mass $m = 2m_P + 2m_L = 0.104 \text{ kg}$. The cart mass is obtained from the catalog data and is equal to $M = 0.768 \text{ kg}$. The distance from the center of mass to the axis of rotation is determined as $l = \frac{m_L l_{CO} + m_P l_{PO}}{m_L + m_P} = 0.1735 \text{ m}$. After applying the theorem of Steiner, we obtain the moment of inertia with respect to the axis of rotation at the center of mass as $J = J_P - ml^2 = 2.829 \cdot 10^{-3} \text{ kg} \cdot \text{m}^2$.

In order to find the pendulum viscous friction coefficient for rotational motion, we have to determine pendulum damping. Pendulum damping can be determined from the following experiment. We fix the cart at the end of the rail by using a spring holder and then deviate the pendulum around the equilibrium at an angle $\theta \approx 20^\circ$. We let the pendulum swing freely around the equilibrium position and gather data from the oscillating motion. The collected data contains the amplitudes of oscillation as they vary with time. Since the amplitudes are decreasing due to torque friction, we can determine the damping coefficient as follows:

$$\zeta = \frac{2}{T(2n-1)} \lg \frac{A_1}{A_n} \quad (10)$$

where T is the period of oscillations, n is the number of oscillations, and A_1 and A_n are the first and n -th amplitudes. Then, the viscous friction coefficient is obtained as follows:

$$f_P = \zeta \cdot J \quad (11)$$

where J is the pendulum moment of inertia. After averaging the experimental data, we obtain the following value of the viscous friction coefficient: $f_P = 0.1035 \cdot 10^{-3} \frac{\text{N} \cdot \text{ms}}{\text{rad}}$. The cart viscous friction coefficient is obtained from catalog data as $f_C = 0.5 \frac{\text{N} \cdot \text{s}}{\text{m}}$, see [1].

The next step in model building is to linearize Equations (3) and (6) around the upper equilibrium position. For small values of θ , we use the small argument expressions for the harmonic functions, more precisely $\sin \theta \approx \theta$ and $\cos \theta \approx 1$. Then, the linearized Equations (3) and (6) around $\theta = 0$ are obtained in the following form:

$$(M+m)\ddot{x} + ml\ddot{\theta} + f_c \dot{x} = F \quad (12)$$

$$(J + ml^2)\ddot{\theta} - mgl\theta + ml\ddot{x} + f_p \dot{\theta} = 0 \quad (13)$$

Since the rotational friction coefficient and the pendulum moment of inertia are small in comparison to the other model parameters, they can be neglected, i.e., $f_P \approx 0$ and $J \approx 0$. Based on these assumptions, we obtain the equations of dynamics as follows:

$$(M+m)\ddot{x} + ml\ddot{\theta} + f_c \dot{x} = F \quad (14)$$

$$ml^2\ddot{\theta} - mgl\theta + ml\ddot{x} = 0 \quad (15)$$

Using Equations (14) and (15), we obtain the transfer function of the cart-pendulum system with an input signal, the external translational force applied to the cart F and output signal, the pendulum deviation around the upper equilibrium position θ as follows:

$$G(s) = \frac{\Theta(s)}{F(s)} = \frac{-s}{Mls^3 + f_Cls^2 - (M+m)gs - f_Cg} \quad (16)$$

Similarly:

$$G(s) = \frac{\Theta(s)}{F(s)} = \frac{-(Ml)^{-1}s}{s^3 + (f_C/M)s^2 - [(M+m)g/Ml]s - (f_Cg/Ml)} \quad (17)$$

The negative sign of the transfer function (17) is due to the assumption that the angle changes in a clockwise direction. Substituting the calculated parameter values in (17) and assuming counter-clockwise change in angle, we obtain the following transfer function between the translational force and the pendulum angle of rotation:

$$G(s) = \frac{\Theta(s)}{F(s)} = \frac{7.5048s}{s^3 + 0.6509s^2 - 64.2215s - 36.8243} \quad (18)$$

As can be clearly seen from (18), the necessary condition for stability is violated and the obtained transfer function is unstable. This fact is a direct consequence from the unstable character of the pendulum upper equilibrium position. Therefore, we need a stabilizing controller. We also set some performance requirements, like a small overshoot and fast response. In order to achieve these design goals, we introduce a feedback structure called the conditional feedback control structure.

4. Feedback Control of the Cart–Pendulum System

The conditional feedback control structure is shown in Figure 4. This structure consists of two parts connected in cascade. The first part is a parallel connection of two elements with transfer functions $C(s)$ and $F(s)$. The second part is a feedback connection of the plant element $G(s)$ and the feedback element with a transfer function $K(s)$. The special properties of the conditional feedback control structure are based on the feedback connection. These properties include low sensitivity with respect to changes in the parameter values of the plant, disturbance rejection at the input of the plant, linearization properties of feedback and others. The special form of the element $F(s)$ explains the reason for the name of this control structure.

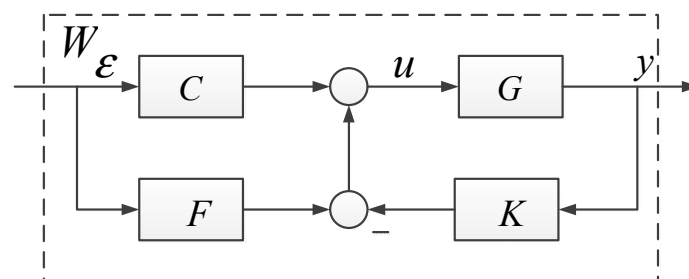


Figure 4. The conditional feedback control structure.

We assume that the plant contains an uncertain parameter λ , i.e., $G(s) = G(s, \lambda)$. In order to present the special properties of the conditional feedback structure, we introduce

the concept of logarithmic sensitivity. The logarithmic sensitivity of a transfer function $G(s, \lambda)$ with respect to an uncertain parameter λ is defined by the following expression:

$$LS_{\lambda}^G(s) = \frac{1}{G(s, \lambda_n)} \frac{\partial G}{\partial \lambda} \Big|_{\lambda=\lambda_n} \quad (19)$$

There exist two basic properties of the conditional feedback structure utilizing the concept of logarithmic sensitivity. The first property is related to computing the logarithmic sensitivity of a cascade connection of elements. The logarithmic sensitivity of a cascade connection of several elements with respect to the uncertain parameter is the logarithmic sensitivity of the element, containing the uncertain parameter. For example, if we have $G(s, \lambda) = G_1(s)G_2(s, \lambda)G_3(s)$, then $LS_{\lambda}^G(s) = \frac{1}{G(s, \lambda_n)} \frac{\partial G}{\partial \lambda} \Big|_{\lambda=\lambda_n} = \frac{1}{G_2(s, \lambda_n)} \frac{\partial G_2}{\partial \lambda} \Big|_{\lambda=\lambda_n} = LS_{\lambda}^{G_2}$. The second property is concerned with the feedback connection. The logarithmic sensitivity of the closed-loop transfer function with respect to the uncertain parameter λ in the plant can be computed as $LS_{\lambda}^H = \frac{1}{H(s, \lambda_n)} \frac{\partial H}{\partial \lambda} \Big|_{\lambda=\lambda_n} = \frac{1}{1+K(s)G(s, \lambda_n)} \frac{1}{G(s, \lambda_n)} \frac{\partial G}{\partial \lambda} \Big|_{\lambda=\lambda_n} = \frac{1}{1+K(s)G(s, \lambda_n)} LS_{\lambda}^G$. If the condition $|1 + K(j\omega)G(j\omega, \lambda_n)| > 1$ is satisfied over a frequency band of interest $\omega \in \Omega$, then the sensitivity is $LS_{\lambda}^H(j\omega) < LS_{\lambda}^G(j\omega)$ with respect to parameter change in the plant model. As we consider the conditional feedback structure as a cascade connection between parallel and feedback components, the above properties apply and the conditional feedback possesses reduced sensitivity with respect to uncertainties in the plant model and disturbances at the plant input. In addition to these properties, we can include the stabilizing effect of the controller in the global feedback control structure, which is presented in Figure 5.

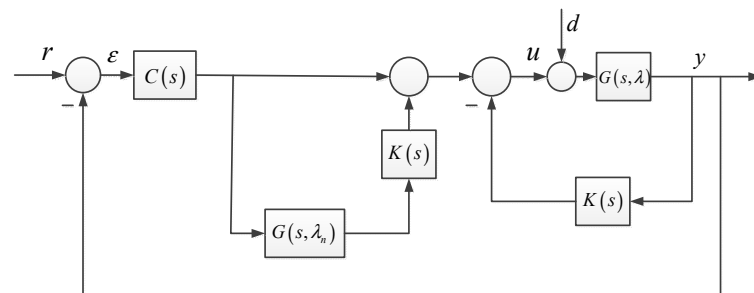


Figure 5. The conditional feedback structure as part of the closed-loop system.

The closed-loop system shown in Figure 5 contains the transformed version of the conditional feedback as part of the control structure. The plant presented by the transfer function $G(s)$ is the cart-pendulum system, with the pendulum raised around the upper equilibrium position. The goal is to stabilize the pendulum in the unstable equilibrium point, to reduce the sensitivity with respect to the unmodeled dynamics, including the nonlinearities acting on the system, and to reduce the influence of disturbance at the plant input. In order to simplify the design procedure, we assume that, in the parallel branch of the forward loop, the $K(s)$ element is a gain with a constant transfer function, i.e., $K(s) = K_F$.

However, if we look at expression (18), it is clear that the transfer function of the cart-pendulum system cannot be stabilized by simple gain in the feedback. Therefore, we need dynamic feedback in order to stabilize the pendulum in upright unstable equilibrium position. We suggest the following approach. The local feedback structure has the goal to stabilize the unstable plant by means of a dynamic feedback, with the only purpose being the stabilization of the pendulum. The regulator in the forward path $C(s)$ is designed to achieve the desired closed-loop system performance.

One approach is to use state feedback that is designed by using the pole placement method and the controller in the forward path, which includes an integrator in order to fix the pendulum in an upright position. For this purpose, we develop the state space model of the cart-pendulum system. Based on Equations (14) and (15), we develop the expressions for the second-order derivatives of the linear translational motion x and the angular deviation of the pendulum θ :

$$\begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} = \frac{1}{Mml^2} \begin{bmatrix} ml^2 & -ml \\ -ml & M+m \end{bmatrix} \left\{ \begin{bmatrix} -f_C \dot{x} \\ mgl\theta \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} F \right\} \quad (20)$$

The state space model of the cart-pendulum system has the form:

$$\dot{z} = Az + Bu \quad (21)$$

$$y = Cz \quad (22)$$

The state space model for (17) is of order four, with state variables defined as follows: $z_1 = x, z_2 = \dot{x}, z_3 = \theta, z_4 = \dot{\theta}$, the input variable is $u = F$ and the output variable is y . By using (14) and (15), we obtain the system matrices as follows:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -f_C/M & -mg/M & 0 \\ 0 & 0 & 0 & 1 \\ 0 & f_C/Ml & (M+m)g/Ml & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1/M \\ 1/Ml \end{bmatrix}, C = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}^T \quad (23)$$

After substitution with the parameter values, we obtain:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -0.65 & -1.33 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 3.75 & 64.2 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1.3 \\ 0 \\ -7.5 \end{bmatrix}, C = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}^T \quad (24)$$

Comparing (24) with (18), we can easily conclude that the realization (24) is not minimal. It is controllable but not observable. Obviously, the order of the minimal realization is three. In order to simplify the design procedure, we consider the transfer function (18) and obtain the controllable canonical for realization as follows:

$$A_c = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 36.82 & 64.22 & -0.651 \end{bmatrix}, B_c = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, C_c = \begin{bmatrix} 0 & 7.5 & 0 \end{bmatrix} \quad (25)$$

We use the controllable canonical form of the cart-pendulum system to design state feedback, which stabilizes the inverted pendulum in the upper unstable equilibrium position, and then design the forward controller $C(s)$ achieving desired performance. This closed-loop system control structure is shown in Figure 6.

The stabilizing state feedback is designed by applying the pole placement method. If we select the desired closed-loop system poles as:

$$p(s) = \{-1 + j3 \quad -1 - j3 \quad -5\}$$

the state feedback matrix of the closed-loop system is obtained as follows:

$$K_c = \begin{bmatrix} 86.82 & 84.22 & 6.35 \end{bmatrix}$$

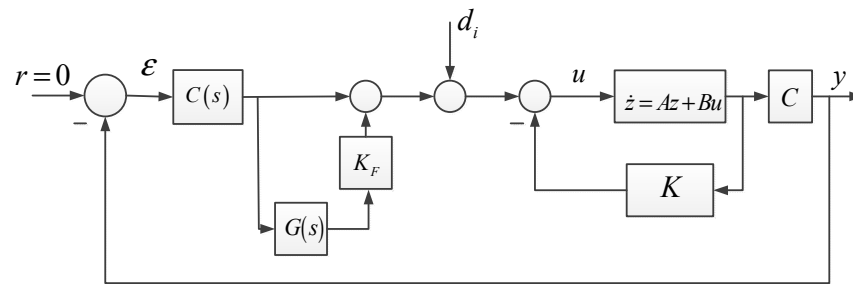


Figure 6. The closed-loop system with stabilizing state feedback.

The step response of the pendulum at the upper equilibrium position is shown in Figure 7.

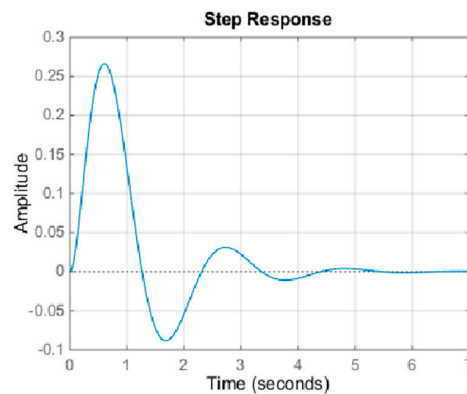


Figure 7. Step response of the 3-rd order cart–pendulum system at upright equilibrium position.

Similarly, if we assume the cart–pendulum description (24) and select the closed-loop system poles at the following location:

$$p(s) = \left\{ -1 + j3 \quad -1 - j3 \quad -5 \quad -3 \right\}$$

the state feedback matrix of the system is obtained as follows:

$$K_c = \begin{bmatrix} -2.04 & -2.0 & -14.38 & -1.59 \end{bmatrix}$$

The step response of the pendulum at the upper equilibrium position is shown in Figure 8. Although the fourth-order realization is not minimal, it is controllable and thus allows the usage of state feedback to place the closed-loop system poles at desired locations.

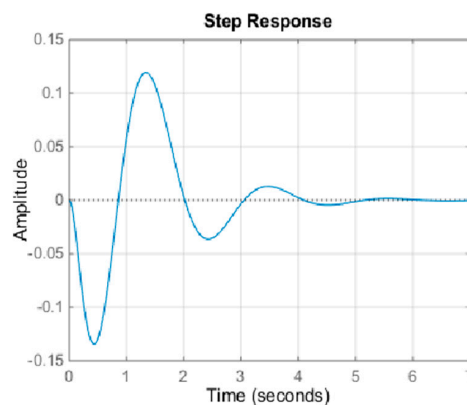


Figure 8. Step response of the 4-th order cart–pendulum system at upright equilibrium position.

The step response of the pendulum at the upper equilibrium position is shown in Figure 8. Although the fourth-order realization is not minimal, it is controllable, and thus allows the usage of state feedback to place the closed-loop system poles at desired locations. The advantage of using the fourth-order realization is that all the system states are measurable and the state feedback can be practically implemented. In this sense, the goal of the local feedback structure is achieved, namely stabilization of the pendulum in an upper equilibrium position. The transfer function of the closed-loop system in the local feedback is obtained as follows: $H(s) = \frac{7.5s}{s^3 + 7s^2 + 20s + 50}$. The fourth-order model is obtained as follows:

$$H(s) = \frac{7.5s^2}{s^4 + 10s^3 + 41s^2 + 110s + 150} \quad (26)$$

The next step of the design procedure is to determine the controller $C(s)$ description in order to achieve desired performance. A standard procedure is to select the desired closed-loop transfer function in low-pass filter form as follows $f(s) = 1/(\tau s + 1)^n$, see [5]. The design procedure is based on equality [5]:

$$T(s) = \frac{C(s)H(s)}{1 + C(s)H(s)} = f(s) \quad (27)$$

where $H(s) = C(sI - A + BK)^{-1}B$ is the stable closed-loop system in the local feedback, which is obtained as a result from the state feedback design procedure, $f(s) = 1/(\tau s + 1)^2$, since the relative degree of the cart-pendulum system is two, and we have set $K_F = 0$. Based on (27), we obtain the transfer function of the controller $C(s)$ as follows:

$$C(s) = \frac{1}{H(s)} \frac{1}{f(s)^{-1} - 1} = \frac{s^3 + 7s^2 + 20s + 50}{7.5s} \frac{1}{\tau^2 s^2 + 2\tau s + 1 - 1} = \frac{s^3 + 7s^2 + 20s + 50}{7.5s} \frac{1}{\tau^2 s^2 + 2\tau s} \quad (28)$$

Similarly, for the fourth order model, we obtain:

$$C(s) = \frac{1}{H(s)} \frac{1}{f(s)^{-1} - 1} = \frac{s^4 + 10s^3 + 41s^2 + 110s + 150}{7.5s^2} \frac{1}{\tau^2 s^2 + 2\tau s + 1 - 1} = \frac{s^4 + 10s^3 + 41s^2 + 110s + 150}{7.5s^2} \frac{1}{\tau^2 s^2 + 2\tau s} \quad (29)$$

We select the filter time constant as $\tau = 0.2$ s. Finally, we obtain the impulsive disturbance response of the stabilized pendulum in upright equilibrium position for the 3-rd order cart-pendulum, shown in Figure 9, and similarly, the impulsive disturbance response of the stabilized pendulum in an upright position for the 4-th order cart-pendulum system, shown in Figure 10.

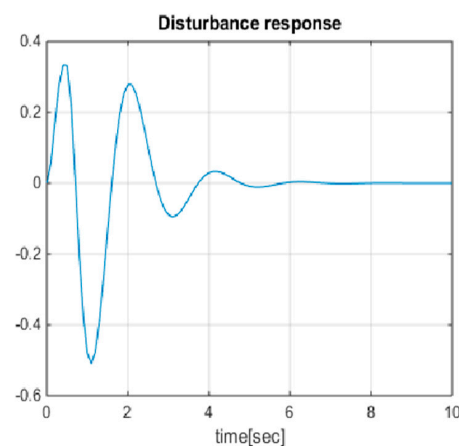


Figure 9. Impulsive disturbance response of the closed-loop 3-rd order cart-pendulum system.

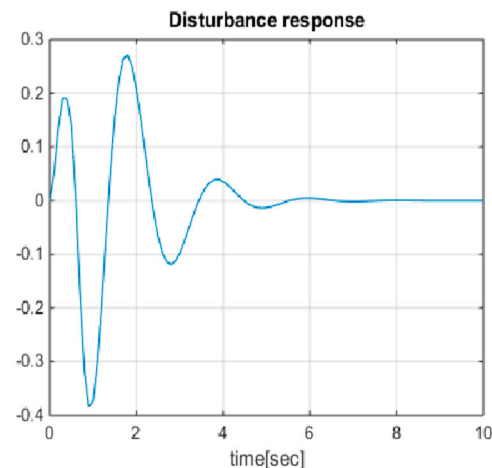


Figure 10. Impulsive disturbance response of the closed-loop 4-th order cart–pendulum system.

From the figures it is clearly seen that the control system stabilizes the pendulum in an upright position in presence of impulsive disturbance.

After small oscillation, when the impulsive disturbance is applied, the pendulum returns back to the upper unstable equilibrium position. The higher the controller order, the lower the maximum amplitude of the response oscillation. In both cases, the time response settles in almost the same settling time. Therefore, the conditional feedback control structure can be successfully used in the case of unstable plants with disturbance perturbations.

5. Conclusions

This paper considers the problem of the inverted pendulum stabilization in an upright unstable equilibrium position in presence of disturbances. The equations of system dynamics are derived and the parameters of the cart-pendulum system are estimated. The conditional feedback control structure is introduced and some of its properties are discussed. Since the upright equilibrium position of the pendulum is unstable, it is shown that the pendulum cannot be directly stabilized by using a simple gain in the local feedback. Therefore, a dynamic feedback has to be used, which is implemented in terms of a state feedback. After stabilization of the pendulum in the upper equilibrium position, the controller in the forward loop is designed by using the desired complementary sensitivity function, thus improving the system performance. The performance of the obtained design is tested in presence of impulsive disturbance at the plant input.

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