

Members in Bending: Annex L of New EN 1999-1-1 [†]

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Abstract

The new version of Eurocode 9 introduces Annex L, which extends the Effective Thickness Method for the assessment of local buckling resistance of aluminium alloy members. This paper presents an overview of the theoretical background and the main procedural steps of Annex L, highlighting its application to thin-walled aluminium profiles susceptible to local instability phenomena. Attention is devoted to the definition of effective thickness, the treatment of plate elements under different stress distributions, and the consistency of the method with the general design framework of Eurocode 9. To evaluate the accuracy and reliability of the proposed approach, a systematic comparison is carried out between analytical predictions obtained using Annex L and numerical results derived from nonlinear finite element analyses. Several representative cross-sections and loading conditions are considered. The comparison allows assessing the level of agreement between the two approaches and identifying the main sources of discrepancy. The results confirm that Annex L provides a rational and generally conservative estimation of local buckling resistance, supporting its adoption in practical design of aluminium structures.

Keywords: local buckling; thin-walled section; Eurocode 9; Annex L

1. Introduction

Thin-walled aluminium profiles under compression or bending are prone to local buckling, which may substantially reduce their load-bearing capacity. Several design methods have been developed over the years to estimate the ultimate strength of members, each rooted in different theoretical assumptions.

In steel design, the Effective Width Method (EWM), pioneered by von Kármán [1] and subsequently refined by Winter [2], remains the dominant approach. It introduces a reduced effective width to account for post-buckling strength and has been codified in Eurocode 3 [3–5]. For aluminium structures, however, the Effective Thickness Method (ETM) is generally preferred. Aluminium alloys exhibit pronounced nonlinearity and strain-hardening, which call for more refined modelling; the ETM accommodates these features by reducing plate thickness rather than width, preserving the cross-section geometry. This proves especially convenient for welded sections, where heat-affected zones induce localised property variations [6–8].

Both methods target Class 4 cross-sections and tend to yield conservative strength estimates. While this is acceptable from a safety perspective, it limits the exploitation of the material's real capacity, a non-trivial drawback given the higher cost of aluminium relative to steel.



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To overcome these limitations, alternative approaches have been proposed. The Direct Strength Method (DSM) [9–12] bypasses effective section calculations by linking member strength directly to elastic buckling loads. The Continuous Strength Method (CSM), developed by Gardner et al. [13,14], exploits strain-hardening through a deformation-based framework, demonstrating improved accuracy for stainless steel [15–17], aluminium [18,19], and cold-formed steel [20,21].

Notwithstanding these developments, Eurocode 9 [6] provisions for aluminium beams remain conservative, as the cross-section classification approach does not fully capture the benefits of strain-hardening and post-elastic stress redistribution.

In this context, the present study investigates the application of the Effective Thickness Method, as formulated in Annex L of Eurocode 9, to box-shaped aluminium beams subjected to non-uniform bending. The analytical procedure is implemented through a dedicated Visual Basic script, enabling the generation of complete normalised moment-curvature responses across a wide parametric space. An extensive comparison is carried out against finite element simulations previously developed by the authors, covering 1008 geometric configurations and four aluminium alloys of the 6000 series (6061-T4/6082-T4, 6063-T5, 6082-T6, 6061-T6) with distinct strain-hardening characteristics. The accuracy of the Annex L predictions is critically assessed through statistical indicators and benchmarked against current EC9 design provisions, demonstrating improved agreement with numerical results and reduced conservatism.

2. Extended Version of ETM: Theoretical Framework

The extended version of the Effective Thickness Method (ETM) is herein illustrated. With reference to a generic cross-section, the elastoplastic buckling stress can be obtained as:

$$\sigma_{cr,p} = \xi \zeta \alpha_{LS} k_{\psi} \frac{\pi^2 E}{12(1 - \nu^2)(b/t)^2} \quad (1)$$

where b and t are, respectively, the width and the thickness of the single plate belonging to the section; E represents the elastic modulus; k_{ψ} is the buckling factor accounting for the edge restraining conditions; ζ is the factor accounting for the interaction of plate elements; α_{LS} is the factor accounting for longitudinal stress gradient in members in bending and, finally, ξ is the plastic coefficient to consider the strain hardening behaviour of aluminium alloys.

In addition, ν is the Poisson's ratio considered variable between the elastic and plastic range according to this relation:

$$\nu = 0.5 - 0.2 \frac{E_t}{E} \quad (2)$$

where E_t represents the tangent modulus.

By introducing the effective thickness t_{eff} to consider the local buckling effects and by considering that:

$$\sigma_{cr,p} = E_s \varepsilon \quad (3)$$

where ε represents the strain corresponding to buckling in non-linear range and E_s is the secant modulus; the relation of the normalised slenderness can be derived:

$$\bar{\lambda}_p = \frac{t_{eff}}{t} = \frac{b}{t} \sqrt{\frac{12}{\pi^2} \frac{1}{\mu} \frac{1}{\zeta \alpha_{LS} k_{\psi}}} \varepsilon \quad (4)$$

with:

$$\mu = \frac{\xi E}{E_s} \frac{1}{1 - \nu^2} \quad (5)$$

According to EN 1999-1-4 [7], the expression of the plate slenderness is equal to:

$$\bar{\lambda}_p \cong \frac{0.06287}{\sqrt{k_0}} \frac{\beta}{\epsilon_0} \quad (6)$$

with $\beta = b/t$ and $\epsilon_0 = \sqrt{250/f_{0.2}}$. the buckling coefficient k_0 is equal to 4 for a plate supported along all four edges and 0.425 for a plate supported along one edge and free along the opposite edge.

By combining Equation (4) with Equation (6), the following relationship is obtained:

$$\frac{\beta}{\epsilon_0} = 17.54 \eta \frac{b}{t} \sqrt{\frac{\epsilon}{\mu \zeta \alpha_{LS}}} \quad (7)$$

with $\eta = \sqrt{k_0/k_{0\psi}}$.

Equation (7) represents the final mathematical relation to evaluate the slenderness parameter of the plate element to be applied with the buckling curves of EN 1999-1-1 [6]. To apply the previous equation, it is useful to note that the non-linear behaviour of aluminium alloys can be described according to the Ramberg–Osgood law:

$$\begin{aligned} \epsilon &= \frac{\sigma}{E} + 0.002 \left(\frac{\sigma}{f_{0.2}} \right)^n \\ n &= \frac{\ln(0.002/\epsilon_{0,u})}{\ln\left(\frac{f_{0.2}}{f_u}\right)} \end{aligned} \quad (8)$$

where n is the strain hardening coefficient, $f_{0.2}$ and f_u are, respectively, the conventional and ultimate stress of aluminium alloy; $\epsilon_{0,u}$ is the residual strain corresponding to the maximum stress.

Regarding the factor for the influence of the longitudinal stress gradient, the following relations can be adopted [22,23]:

- For a flat internal compression cross-section part

$$\alpha_{LS} = 1 + \frac{1}{4} \frac{1.70}{\left(\frac{L_s}{b}\right)^{0.75} - 0.20} \quad (9)$$

- In case of flat outstand compression elements:

$$\alpha_{LS} = 1 + \frac{1}{0.425} \frac{1.70}{\left(\frac{L_s}{b}\right)^{0.95} - 0.60} \quad (10)$$

where L_s represents the shear length and b is the plate width in compression.

Regarding the correction factor ζ for interactive buckling, the main mathematical expressions are given in BS5950-5 [24] for different conditions of the edge constraints.

Finally, regarding the plastic coefficient ζ , the relation presented in the Annex L of new Eurocode 9 [6] can be adopted:

$$\frac{E_s}{E} \left[\frac{n-8}{n} + \frac{8}{n} \sqrt{\frac{E_t}{E_s}} \right] \quad (11)$$

3. Application of ETM

The advanced Effective Thickness Method can be applied to both uniformly compressed members and beams subjected to bending. For the former case, the validity of the procedure has been extensively demonstrated in previous works by the authors [25,26].

The present section focuses specifically on the application of the analytical method to box-shaped aluminium beams under non-uniform bending.

With reference to Figure 1a, a total of 1008 analyses were carried out by varying the following normalised parameters: normalised flange slenderness $\bar{\lambda}_p$, flange-to-web ratio b/h , and shear span-to-flange width ratio L_s/b with L_s equal to the half of the beam. The corresponding ranges of variation are summarised in Table 1. Furthermore, to assess the influence of material properties, four aluminium alloys were considered, whose mechanical characteristics are reported in Table 2.

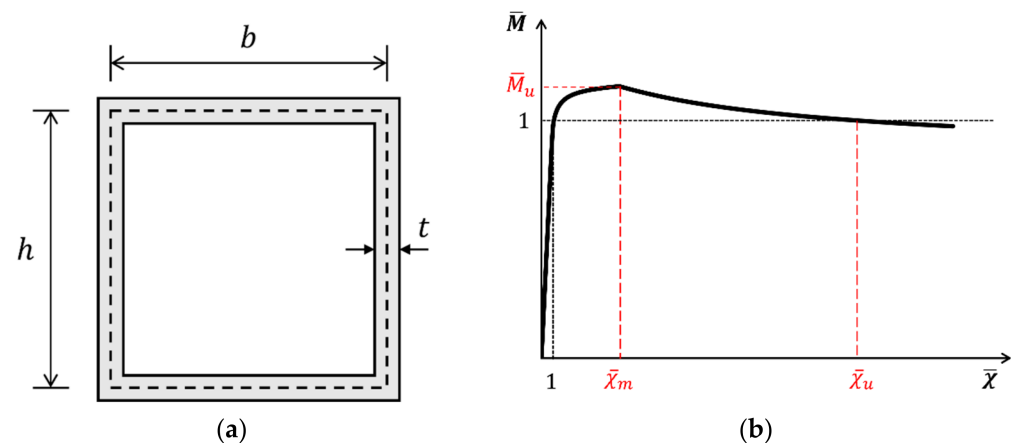


Figure 1. Geometric scheme of the box section (a); scheme of moment–curvature curve (b).

Table 1. Range of non-dimensional parameters.

$\bar{\lambda}_p$:	0.27	0.29	0.33	0.37	0.41	0.44	0.47	0.50	0.55	0.60	0.70	0.80
b/h :	0.50	0.75	1.00	1.25	1.50	1.75	2.00					
L_s/b :	6.25	12.50	18.75									

Table 2. Mechanical properties of aluminium alloys.

Alloy	$f_{0.2}$ [MPa]	f_u [MPa]	n [-]
6061-T4 (or 6082-T4)	110	205	8
6063-T5	130	175	16
6082-T6	260	310	25
6061-T6	240	260	55

It is worth noting that a mid-line cross-section model is adopted, as illustrated in Figure 1a. The moment–curvature response is obtained by incrementally increasing the curvature. At each step, the neutral axis position is determined through translational equilibrium (zero axial force) via an iterative procedure. The resulting strain distribution is then used to evaluate the slenderness of both the compressed flange and the web, accounting for material nonlinearity, plate interaction, and longitudinal stress gradient effects. The effective cross-section is subsequently derived from the reduced plate thicknesses, and the corresponding bending moment is computed from rotational equilibrium. The ultimate flexural resistance is identified as the peak value of the moment–curvature diagram, with the additional check that the maximum strain remains within the limit prescribed by EN 1999-1-1 [6].

With reference to Figure 1b, the results are expressed in terms of the following normalised parameters:

$$\begin{aligned}\bar{M}_u &= \frac{M_u}{M_{0.2}} \\ \bar{\chi}_m &= \frac{\chi_m}{\chi_{0.2}} \\ \bar{\chi}_u &= \frac{\chi_u}{\chi_{0.2}}\end{aligned}\quad (12)$$

with:

$$\begin{aligned}M_{0.2} &= W_{el} f_{0.2} \\ \chi_{0.2} &= \frac{M_{0.2}}{EI} = \frac{2f_{0.2}}{Eh}\end{aligned}\quad (13)$$

where $M_{0.2}$ is the moment at first yielding and $\chi_{0.2}$ is the corresponding curvature. The quantity $\bar{M}_u$ denotes the normalised ultimate moment, $\bar{\chi}_u$ the normalised curvature at which $\bar{M} = \bar{M}_u$ along the softening branch, and W_{el} the elastic section modulus under the mid-line assumption.

The numerical analyses were carried out by means of a dedicated Visual Basic script [27]. As an example, Figure 2 shows the normalised moment–curvature curves obtained for the four investigated alloys across the full range of considered beam geometries for $b/h = 1.00$ and $L_s/b = 6.25$.

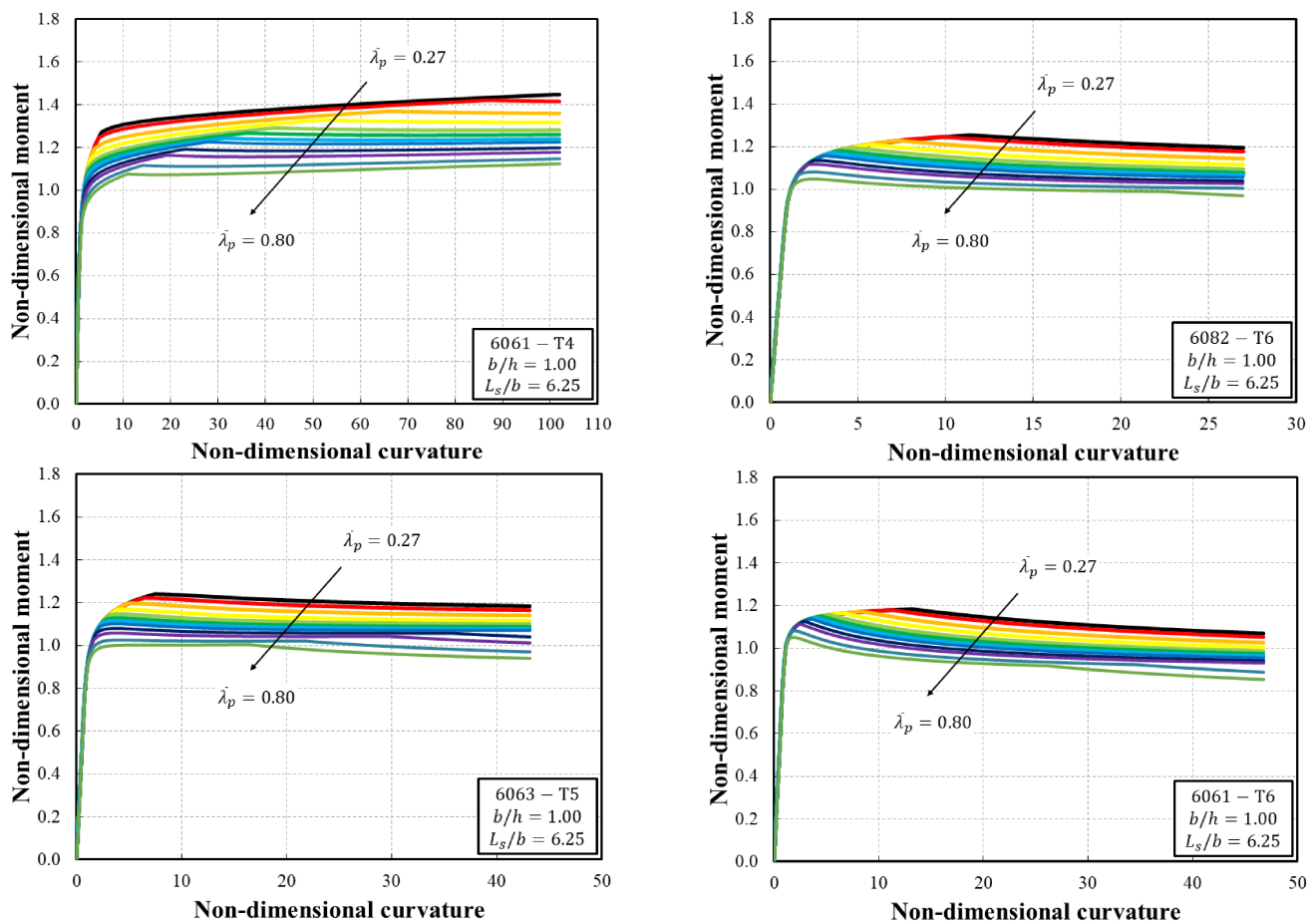


Figure 2. Non dimensional moment–curvature curves obtained by extended version of ETM.

4. Accuracy of the ETM Against Numerical Results

To assess the accuracy of the procedure outlined in Annex L of EC9, the normalised moment values predicted by the analytical method were compared against those obtained from finite element simulations previously developed by the authors [28,29]. The numerical analyses were performed using Abaqus [30], employing a finite element model calibrated against experimental data available in the literature [31].

Figure 3 shows the comparison for all 1008 analysed cases. It can be readily observed that the results are almost entirely on the safe side, with a degree of accuracy that varies depending on the alloy and, consequently, on the extent of strain-hardening. In particular, the best agreement is achieved for the 6061-T4 alloy, while the prediction accuracy progressively deteriorates as the Ramberg–Osgood exponent increases. Table 3 summarises the comparison for each alloy in terms of the main statistical parameters: mean value, standard deviation, coefficient of variation, 5th and 95th percentile fractiles, and maximum absolute error.

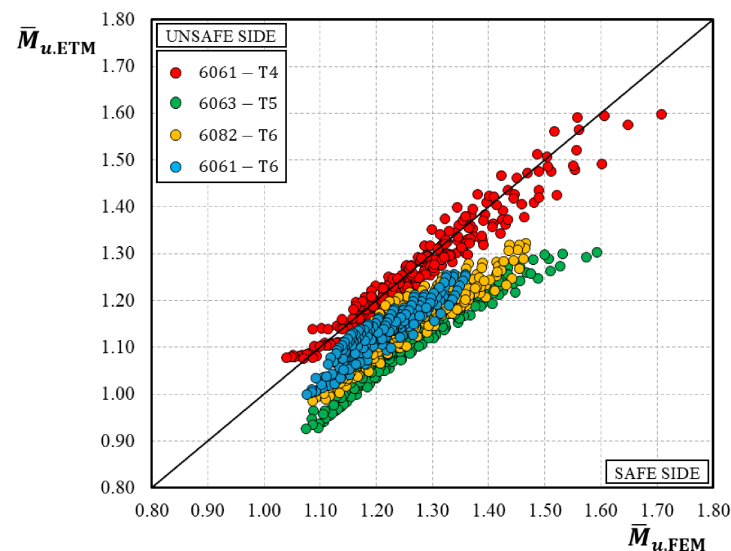


Figure 3. Comparison of numerical (FEM) and analytical (ETM) results for box sections.

Table 3. Summary of the comparison between FE and ETM results for box sections.

	Aluminium Alloys			
	6061 – T4	6063 – T5	6082 – T6	6061 – T6
Meanvalue [μ]	0.99	0.88	0.92	0.93
5% fractile	0.96	0.85	0.88	0.91
95% fractile	1.03	0.92	0.95	0.96
St.dev.[σ]	0.022	0.022	0.023	0.017
Coef.of var.[cov]	0.022	0.025	0.025	0.018
MAE [%]	6.05	18.36	13.64	10.92

In addition, Table 4 presents a statistical comparison between the flexural resistance predictions obtained from the current EC9 design rules and those derived from the Annex L procedure, i.e., through the extended Effective Thickness Method. It is worth noting that the EC9 provisions were applied assuming a unit partial safety factor. It is immediately apparent that EC9 yields considerably conservative estimates, with a mean ratio of 0.86 and a relatively low standard deviation of 0.044. In contrast, Annex L provides predictions in much closer agreement with the Abaqus results, with a mean of 0.93 and a standard deviation of 0.049. From the perspective of fully exploiting the structural capacity of aluminium while avoiding over-design and material waste, Annex L represents a well-balanced and effective design alternative.

Table 4. Comparison between EN 1999-1-1 and Annex L.

	$\frac{\overline{M}_{u,EC9}}{\overline{M}_{u,FEM}}$	$\frac{\overline{M}_{u,ETM}}{\overline{M}_{u,FEM}}$
Meanvalue [μ]	0.86	0.93
5% fractile	0.79	0.85
95% fractile	0.93	1.00
St.dev.[σ]	0.044	0.045
Coef.of var.[cov]	0.051	0.049
MAE [%]	58.90	18.44

5. Conclusions

This study investigated the application of the advanced Effective Thickness Method to box-shaped aluminium beams subjected to non-uniform bending. A total of 1008 parametric analyses were carried out by varying the normalised flange slenderness, the flange-to-height ratio, and the shear span-to-flange width ratio, considering four aluminium alloys of the 6000 series with different strain-hardening characteristics.

The comparison between the analytical predictions and the finite element results demonstrates that the Annex L procedure led to estimate in close agreement with the numerical simulations, with a mean ratio of 0.93 and a standard deviation of 0.049, outperforming the current EC9 design rules, which tend to be considerably conservative (mean of 0.86). The results also highlight that prediction accuracy is influenced by the degree of strain-hardening of the alloy, with better performance observed for alloys characterised by lower values of the Ramberg–Osgood exponent.

Overall, the Annex L approach represents a more rational and material-efficient design alternative to the conventional EC9 provisions, enabling a better exploitation of the actual structural capacity of aluminium members while maintaining results predominantly on the safe side.

Future work will extend the proposed procedure to H-shaped sections, broadening the applicability of the method to a wider range of structural members. In addition, the development of advanced formulations for the plastic reduction factor will be pursued, with the aim of more explicitly accounting for the influence of strain-hardening on the flexural response of aluminium alloys, thereby further improving prediction accuracy across the full spectrum of materials considered in structural applications.

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