An MI-SOCP Model for the Economic Dispatch Problem in BESS Distribution Using Optimal Placement †

Muhammad Hussain, Raja Masood Larik * and Kamran Ahmed

Department of Electrical Engineering, NED University of Engineering & Technology, Karachi 75270, Pakistan
* Correspondence: rmlarik@neduet.edu.pk
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Abstract: As the demand for electricity increases day by day, the integration of battery energy storage systems (BESS) has become highly influential when considering power systems. Battery energy storage systems have the advantage of improving the economy of power with environmental benefits. For the successful integration of battery energy storage systems, the optimal placement and economic dispatching of energy is required to avoid problems. A BESS will provide power whenever the DGs are unable to provide power to the system. In the case of solar power, the sun cannot provide light to solar panels all day and wind turbines encounter variations in wind supply. This research proposes optimal placement within a BESS and a reduction in the cost of energy usage from conventional sources by considering BESSs with active power capability via an IEEE-33 bus system using an algorithm called mixed-integer second-order cone programming (MI-SOCP), modeled in MATLAB using MOSEK solver to reduce the cost of energy as an objective function. The MI aspect of the algorithm is used to find out the optimal placement of the BESS and the SOCP part is used to reformulate the economic dispatch problem in an AC-distributed BESS.

Keywords: BESS; MI-SOCP; economic dispatch problem; Convex Optimization; hyperbolic relaxation

1. Introduction

As the demand for electricity is increasing, the integration of battery energy storage systems has become important in power systems; yet, if not optimally placed, the cost of the purchased energy cannot be reduced. A BESS will provide power whenever DGs are unable to provide power to the system. In the case of solar power, the sun cannot provide light to solar panels all day, and wind (for wind turbines) also variates. Therefore, the purpose of this research is to propose the optimal placement of BESS and reformulate the economic dispatch problem in BESSs, reducing the cost of purchased energy as an objective function. In [1], the optimal dispatch problem is solved using SOCP but without considering overall optimal placement. In [2], the optimal placement and sizing of DGs were calculated using the MI-SOCP algorithm. In [3], optimal placement and sizing of BESS were found out in order to compare with the heuristic reinforcement strategy. In [4], addresses the problem about finding the sizing and placement of distributed energy resources in the distribution network by using GAMS. In [5], works on the real scenario by calculating the optimal operation of BESS in order to reduce the electricity cost. The operation of BESS consists with two modes, first one was for optimal scheduling and the second one was for the economic dispatch. In [6], used the mixed-integer conic formulation for optimal placement and dimensioning of DGs on 21 and 69 bus system in dc distribution system and the results in terms of efficiency was compared with the MINLP method available in GAMS. The MI-SOCP model enables the finding of the global optimal solution. In [7], the optimal placement and sizing of BESS is found out through the injection of P and Q into the distribution system. The examination is carried out on a solar and wind medium voltage IEEE-33 bus distribution system. In [8], a meta-heuristic algorithm which is known as...
whale optimization algorithm is used to find out the optimal allocation and sizing of battery energy storage system. In [9], the optimal placement and sizing of multiple distributed generators in distribution system was found out on efficient way in this work a chaos map theory and sine cosine algorithm was used for optimization. In [10], the optimal placement and sizing of distributed generators was found out to reduce the losses, voltage profile improvement, and the rises of voltage stability in radial distribution system. In [11], propose the optimal placement and sizing of distributed generators by using the hybrid technique to decrease the losses in distributed system. the technique of hybrid is joined execution of cuckoo search and grasshopper optimization algorithm. In [12], a new approach is adopted to find out the optimal placement and sizing of BESS. The type of BESS in the proposed work is lithium-ion battery. The purpose of this work is to decrease losses in the distribution network. In [13], works on the efficiency comparison between AC and DC distribution system by integrating the solar, wind and BESS in order to reduce the energy losses and carbon emission. In [14], IEOA is used with the combination of recycling strategy for configuring the distribution system with optimal allocation of multiple DG’S. In [15], MRFO (Manta Ray Foraging Optimization Algorithm) was used to find out the optimal placement and sizing of distributed generators in radial distribution network. The purpose of the proposed work was to minimize the power losses in the radial distribution network. In this work the proposed algorithm was applied on three different bus systems, IEEE 33, 69 and 85 bus networks. In [16], works on the optimal placement and sizing of distributed energy resources in dc distribution system by using the mathematical programming known as semi-definite programming in order to ensure the global optimal solutions. In [17], wind energy resources are integrated into IEEE-33 bus system in order to reduce power losses as well as to improve voltage profile by using particle swarm optimization. In [18], proposes an MMF (modified moth flame optimization) method. There are two proposed modifications in the modified moth flame algorithm for the purpose of exploitation and exploration balance and also for the purpose of original MFO shortcoming overcome. The objective of using the proposed algorithm is to find out the optimal allocation and sizing of distributed generating units in distribution network. The DER’s are based on the renewable sources. In [19], proposed a method for the photovoltaic generation and ESS by the consideration of the MV distribution network to be deployed country wide to derive cost-optimal plans. In [20], proposes a new method of optimization. The sine cosine algorithm is proposed with the multi-objective function to find out the optimal location and sizing of distributed generators in the radial distribution network. The aim of this work is to minimize the active power loss, increase the stability of voltage, reduction in annual energy loss costs and also the emission of pollutant gas without crossing the limits of DG operating constraints. In [21], the work proposes the optimal integration of DG units in distribution network, by observing the load of plug-in electric vehicles with behavior of their charging under the daily load pattern with the consideration of 24 h improvement in the voltage and losses in power as an objective for the betterment of performance of the system. To minimize all these objectives, two algorithms are selected which are butterfly optimization and also particle swarm optimization. In [22], paper proposes the technique of FLC (fuzzy logic controller) and PSO (particle swarm optimization) with ALOA (Ant-Lion Optimization Algorithm). The main purpose of Ant-Lion optimization algorithm is to imitates the mechanism of hunting of ant lions in nature and the purpose of particle swarm optimization is to better the ant-lion performance by updating elitism phase of ant-lion optimization and, the purpose of using Fuzzy logic controller is to create the parameters with optimal allocation on the variation of parameters of radial distribution system. the PV (photovoltaic) and WT (wind turbine) are considered as DG units in this hybrid approach algorithms. The minimization of power loss voltage stability index improvement and voltage deviation are taken as multi-objective function. The proposed work is implemented on IEEE-33 bus system and the analyzation of DG’s allocation at different location and the optimal flow of load at different cases are discussed using hybrid approach. In [23], the author’s proposed the method of finding the optimum location, selection and also
the BESS (battery energy storage system) and capacitor bank operation in the distribution network. The MINLP algorithm for the proposed work. The objective function of this work is the energy loss minimization in the distribution system. due to the presence of constraints, the regulation of the voltage, balance in the active as well as reactive energy, therefore, in this research work, an MI-SOCP algorithm was used to reduce the cost of purchased energy. The purpose of using MI-SOCP is to convert the non-linear power flow equation into a convex structure, which ensures the global optimal point of the objective function.

The summarized points of this research are as follows:

To convert the non-linear power flow equations into a convex structure called MI-SOCP using hyperbolic relaxation to find out “optimal placement”, which, in turn, will also address the economic dispatch problem in BESSs with only active power capability to reduce the cost of purchased energy from conventional sources.

2. Nonlinear Formulation of the BESS System

For the optimal operation of BESSs, distribution corresponds to the nonlinear approach of optimization regarding variables such as voltage, power generation, and states of charge. Here, the objective function of the problem is to minimize the cost of the purchased energy from conventional sources such as substations and diesel generators. The mathematical non-linear formulation of the optimal operation of BESSs in distribution systems will be as follows:

2.1. Objective Function

The objective function of this problem is to reduce the cost of the energy purchase from conventional sources:

\[
\text{min } E_{\text{cost}} = \sum_{i=1}^{N} \sum_{\gamma \in N} c_{i,\gamma}^{gc} \gamma_{i,\gamma} \Delta T
\]

where \( E_{\text{cost}} \) is the objective function, \( c_{i,\gamma}^{gc} \) is the power generation cost from a conventional source connected at node \( i \) for the duration of time, \( t \), and \( \gamma_{i,\gamma} \) is the power output from the conventional sources at node \( i \) for the duration of \( t \). The fraction of time is denoted by \( \Delta T \) for the economic dispatch analysis of the distribution system, which can be 0.25 h, 0.50 h, and 1.00 h, yet this time duration depends on the information related to the consumption of power. \( T \) and \( T_{gc} \) are the total considered periods in the days ahead of the economic dispatch and the number of conventional sources present in the distribution system, respectively.

2.2. Set of the Constraints

The constraints consist of the power flow equations and voltage regulation constraints, with the operative limit in the distribution system.

\[
P_{i,t}^{gc} + P_{i,t}^{b} - P_{i,t}^{d} = V_{i,t} \sum_{j=1}^{N} \gamma_{ij} V_{j,t} \cos(\theta_{i,t} - \theta_{j,t} + \delta_{i,j}), \quad \{ t \in T, \ i \in N \}
\]

\[
q_{i,t}^{gc} - q_{i,t}^{d} = V_{i,t} \sum_{j=1}^{N} \gamma_{ij} V_{j,t} \sin(\theta_{i,t} - \theta_{j,t} + \delta_{i,j}), \quad \{ t \in T, \ i \in N \}
\]

\[
V_{i,t}^{\min} \leq V_{i,t} \leq V_{i,t}^{\max}, \quad \{ t \in T, \ i \in N \}
\]

where the parameters \( P_{i,t}^{gc} \) and \( q_{i,t}^{gc} \) are the output power (active and reactive) from the conventional source at node \( i \) for the period of time, \( t \), \( P_{i,t}^{b} \) is the active power of the BESS connected at node \( i \) for the period of time, \( t \), and \( P_{i,t}^{d} \), \( q_{i,t}^{d} \) are the active and reactive power demanded at node \( i \) for the time, \( t \), respectively. The magnitude of the voltage and angle at node \( i \) at time \( t \) are represented by the \( V_{i,t} \) and \( \theta_{i,t} \), respectively. The magnitude of the admittance and the angle at node \( i \) and \( j \) are represented by \( \gamma_{ij} \) and \( \delta_{i,j} \), respectively. The \( V_{i,t}^{\min} \) shows the minimum limit of the voltage at each node and each time period, and \( V_{i,t}^{\max} \) shows the maximum limit of the voltage at each node and each time period.
For the mathematical model, the constraints of the BESS with the economic dispatch problem must be added:

\[ soc_{i,t}^b = soc_{i,t-1}^b - \eta_0 P_{i,t}^b \Delta T, \quad \{ t \in T, i \in N \} \quad (5) \]

\[ \left( P_{i,t}^b \right)^2 \leq \left( P_{i,t}^{b,\text{max}} \right)^2, \quad \{ t \in T, i \in N \} \quad (6) \]

\[ P_{i,t}^{b,\text{min}} \leq P_{i,t}^b \leq P_{i,t}^{b,\text{max}}, \quad \{ t \in T, i \in N \} \quad (7) \]

\[ soc_{i,t}^{b,\text{min}} \leq soc_{i,t}^b \leq soc_{i,t}^{b,\text{max}}, \quad \{ t \in T, i \in N \}, \quad (8) \]

where the state of charge of the BESS at node \( i \) at time \( t \) is denoted by \( soc_{i,t}^b \) and the efficiency of BESS at node \( i \) is represented by \( \eta_0 \), whereas the minimum and maximum state of the charges are denoted by \( soc_{i,t}^{b,\text{min}} \) and \( soc_{i,t}^{b,\text{max}} \), respectively. The minimum and the maximum active power bounds of the BESS are denoted by \( P_{i,t}^{b,\text{min}} \) and \( P_{i,t}^{b,\text{max}} \), respectively. The maximum apparent power of the BESS at node \( i \) is represented by \( s_{i,t}^{b,\text{max}} \).

\[ Z_i \in \{0,1\}, \quad \forall i \in N Z \in B, \quad \forall i \in N \quad (9) \]

Equation (9) shows the binary variable; if the value of the variable is 1, then the BESS is installed at \( i \), and if the value of the variable is 0, then it shows that the BESS is not installed at \( i \).

The above mathematical model is non-linear and non-convex because of Equations (1)–(8). The reasons for this are the constraints of power (active and reactive), which are explained by Equations (2) and (3). This non-linear/non-convex system complicates the finding of a global optimum point of the solution. Therefore, the model must be converted into a convex structure via the MI-SOCP approach. Equation (6) is non-linear but convex because the border, as well as the inside of the circle of powers, is free from holes.

### 2.3. MI-SOCP Reformulation Approach

The main reason behind the conversion of the non-linear formulation into a second-order cone is to rewrite Equations (2) and (3) into a linear equivalent via hyperbolic relaxation. For this purpose, the power equations will be rewritten in the complex form:

\[ \left( S_{i,t}^c + S_{i,t}^b - S_{i,t}^d \right)^* = V_{i,t}^* \sum_{j=1}^n Y_{ij} V_{j,t}, \quad \{ t \in T, i \in N \} \quad (10) \]

where \( S_{i,t}^c = P_{i,t}^c + j q_{i,t}^c \) is the apparent power from a conventional source, \( S_{i,t}^b = P_{i,t}^b \) is the apparent power of the batteries, and \( S_{i,t}^d = P_{i,t}^d + j q_{i,t}^d \) is the apparent power demanded by the load. \( V_{i,t}^* \) represents the voltage profile, where \((\cdot)^*\) denotes the complex conjugate. The complex component of the admittance between node \( i \) and \( j \) is represented by \( Y_{ij} \).

Now, to convert Equation (10) into the linear convex form, we have to define the auxiliary variables:

\[ U_{i,j,t} = V_{i,t}^* V_{j,t} \quad (11) \]

Let us consider that \( U_{i,t} = \| V_{i,t} \|^2 \) and \( U_{j,t} = \| V_{j,t} \|^2 \), which implies that, by taking the square at both sides of Equation (11):

\[ \| U_{i,t} \|^2 = \| V_{i,t} \|^2 \| V_{j,t} \|^2 = U_{i,t} U_{j,t} \quad (12) \]

the right-hand side of Equation (12) can be rewritten through hyperbolic representation, as follows:

\[ \| U_{i,j,t} \|^2 = U_{i,t} U_{j,t} = \frac{1}{4} \left( U_{i,t} + U_{j,t} \right)^2 - \frac{1}{4} \left( U_{i,t} - U_{j,t} \right)^2 \]

\[ 2 U_{i,j,t} \| U_{i,j,t} \|^2 = \left( U_{i,t} + U_{j,t} \right)^2 \]

\[ 2 U_{i,j,t} + U_{i,j,t} \| U_{i,j,t} \|^2 = \left( U_{i,t} + U_{j,t} \right)^2 \]
\[ \| \frac{2U_{i,j}^b}{U_{i,t} + U_{j,t}} \| = U_{i,t} + U_{j,t} \]  

Equation (13) is still non-convex due to the equality symbol, but this can be replaced by a lower equal one, which will convert the above equation into a conic-convex constraint:

\[ \| \frac{2U_{i,j}^b}{U_{i,t} - U_{j,t}} \| \leq U_{i,t} + U_{j,t} \]  

To complete the SOCP reformulation, substitute Equation (11) into Equation (10), and we find:

\[ (S_{i,j}^g + S_{i,j}^b - S_{i,j}^d)^t = \sum_{j=1}^n Y_{ij}U_{ij,t}, \quad \{ t \in \mathcal{T}, i \in \mathcal{N} \} \]  

\[ U_{ij,t} \text{ and } Y_{ij} \text{ can be seperated into real and imaginary parts, } \left( \mathcal{U}_{ij,t} = \mathcal{U}_{ij,t}^r + j\mathcal{U}_{ij,t}^i \right) \]  

and \( Y_{ij} = G_{ij} + jB_{ij}. \)

\[ P_{i,t}^b + P_{i,t}^d - P_{i,t}^a = \sum_{j=1}^n \left( G_{ij}\mathcal{U}_{ij,t}^r - B_{ij}\mathcal{U}_{ij,t}^i \right), \quad \{ t \in \mathcal{T}, i \in \mathcal{N} \} \]  

\[ q_{i,t}^g - q_{i,t}^d = -\sum_{j=1}^n \left( B_{ij}\mathcal{U}_{ij,t}^i - G_{ij}\mathcal{U}_{ij,t}^r \right), \quad \{ t \in \mathcal{T}, i \in \mathcal{N} \} \]  

The voltage regulation constraints in Equation (4) can be redefined as:

\[ (\gamma_{i,t}^{\text{min}})^2 \leq \mathcal{U}_{i,t}^r \leq (\gamma_{i,t}^{\text{max}})^2, \quad \{ t \in \mathcal{T}, i \in \mathcal{N} \} \]  

and the inequality constraint in Equation (6) can also be rewritten by using the conic form:

\[ \| P_{i,t}^b \| \leq \mathcal{S}_{i,t}^{b,\text{max}}, \quad \{ t \in \mathcal{T}, i \in \mathcal{N} \} \]  

\[ \mathcal{Z}_t \in \mathcal{B}, \quad \forall i \in \mathcal{N} \]  

Equation (20) leads the problem into the MI-SOCP formulation of BESS.

3. Results
3.1. IEEE 33-Bus Distribution System

The 33-bus distribution system consists of 33 nodes and 32 lines with a voltage profile of 12.66 kv. The total demand of the system is 3715 KW and 2300 Kvar. Figure 1 below shows the interconnection of the nodes and Table 1 shows the information about the sending and receiving ends of the nodes, the line resistance and reactance, as well as the powers (active and reactive) consumed at the receiving end.

![Figure 1. IEEE 33-Bus System.](image-url)
Table 1. IEEE 33-Bus distribution Network parameters.

<table>
<thead>
<tr>
<th>(i)</th>
<th>(j)</th>
<th>(R_{ij})</th>
<th>(X_{ij})</th>
<th>(P_j)</th>
<th>(Q_j)</th>
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<td>0.5302</td>
<td>60</td>
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</tr>
</tbody>
</table>

3.2. Parameters of IEEE 33-Bus System

The Table 1, shows the parameters of IEEE 33-bus distribution network in terms of resistance, reactance, active and reactive powers.

3.3. Information about the Battery

The utility company installed three lithium-ion batteries, as reported in [1], and information about these batteries is given in Table 2 below.
Table 2. Information of Batteries installed by utility company.

<table>
<thead>
<tr>
<th>Type</th>
<th>Nominal Energy (kWh)</th>
<th>Charge/Discharge Time (h)</th>
<th>Nominal Power (kW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1000</td>
<td>4</td>
<td>250</td>
</tr>
<tr>
<td>B</td>
<td>1500</td>
<td>4</td>
<td>375</td>
</tr>
<tr>
<td>C</td>
<td>2000</td>
<td>5</td>
<td>400</td>
</tr>
</tbody>
</table>

In the previous work reported [1], information regarding the distribution of the batteries in the IEEE-33 bus system are as follows:
A-type at bus 14, B-type at bus 31, and C-type at bus 6.

3.4. Results of MI-SOCP Algorithm for the BESS

In the previous work [1], four cases were considered for the optimal operation of the BESS via the distribution network by using a solver called GAMS software. These cases are as follows:
Case 1: in the first case only, DGs were considered as activated when evaluating the operating costs of the distribution system.
Case 2: in this case, DGs, as well as the BESS (with only injecting reactive power), were considered.
Case 3: in this case, DGs, as well as the BESS (at unity power factor), were considered.
Case 4: in the last case, DGs, as well as the BESS (with apparent power capability), were considered.

As we know, in the above cases (2–4), both the DGs and the BESS were both considered; however, in this work, the authors were unable to use the binary variable to find out the optimal placement of the BESS in the distribution network.

So, in this research work, adding the binary variable for the optimal placement of the battery energy storage system was implemented to find the optimal operation of the BESS in the distribution network. In this proposed work, a new case is considered with the proposed optimal placement of the BESS considered as:
Proposed case: the operation cost of the distribution system is considered via the BESS with an active power capability and with an optimal placement of the BESS.

After considering the above proposed case, the results are shown in Tables 3 and 4, respectively.

Table 3. BESS without and with optimal placement in distribution system.

<table>
<thead>
<tr>
<th>From [1]: placement of the BESS in distribution network</th>
<th>Proposed optimal placement of the BESS in the distribution network</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bus {6,14,31}</td>
<td>Bus {10,16,32}</td>
</tr>
</tbody>
</table>

Table 4. Daily energy purchasing costs without and with optimal placement in distribution system.

<table>
<thead>
<tr>
<th>Daily energy purchasing costs without optimal placement</th>
<th>Daily energy purchasing costs with optimal placement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8.44099/kWh</td>
<td>$7.24916/kWh</td>
</tr>
</tbody>
</table>

4. Discussion

In [1], the optimal placement of battery energy storage was not found. In our research work, the optimal placement of battery energy storage was found by adding the binary variable, which changed the algorithm from SOCP to MI-SOCP. The proposed research worked to find the optimal placement, as well as to solve the economic dispatch problem of
BESSs in a distribution system. The optimal placement of the BESS in a distribution system helps to reduce the purchased energy cost from conventional sources.

5. Conclusions and Future Works

In this research work, an MI-SOCP algorithm was applied to reduce the cost of purchased energy from conventional sources by considering a battery energy storage system with only an active power capability. The uniqueness of the proposed algorithm is that it gives a guarantee of global optimal points by converting non-linear power flow equations into a convex structure. Therefore, in this work, the optimal placement of a BESS with an economic dispatch problem is solved. The main objective was to reduce the cost of the purchased energy from a conventional source by considering the optimal placement of the BESS in a distribution system.

In future work, we recommend that this work can be extended to combine MI-SOCP with optimization techniques, such as metaheuristics and heuristics, to plan AC distribution using DGs and BESSs in a rural area to reduce fossil fuel dependency.

The MI-SOCP algorithm can be used to find out the optimal placement of fixed-capacitor banks with discrete as well as continuous capacities.

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