Proceeding Paper

Studying the Dynamics of a Vibratory Finishing Machine Providing the Single-Sided Lapping and Polishing of Flat Surfaces †

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Abstract: The improved design of a vibratory lapping machine is developed in the SolidWorks software on the basis of a suspended double-mass oscillatory system. The system is set into motion by three pairs of electromagnets generating periodic excitation forces applied between the upper and lower laps. By adopting the same forced frequencies and the specific phase shifts of the excitation forces, we aim to produce antiphase translational (circular) oscillations of the laps. In such a case, the best accuracy and operational efficiency of the lapping (polishing) process can be reached. The present research is aimed at analyzing the dynamic behavior of the lapping machine’s oscillatory system. In particular, the motion trajectories of the laps, as well as their kinematic characteristics (displacements, velocities, and accelerations) are considered. The mathematical model of the oscillatory system is developed using the Euler–Lagrange equations. The numerical modelling of the system motion is performed in the Mathematica software using the Runge–Kutta methods. The computer simulation of the laps oscillations is conducted in the SolidWorks software under different friction conditions. The experimental prototype of the vibratory lapping machine was tested in the Vibroengineering Laboratory of Lviv Polytechnic National University. The possibility of generating controllable translational (circular) oscillations of the laps is theoretically studied and experimentally confirmed. Further investigations on the subject of the present paper could focus on the physical-mechanical and technological parameters (surface flatness, roughness, hardness, wear resistance, etc.) obtained through the lapping and polishing processes using the proposed vibratory finishing machine.

Keywords: lapping machine; double-mass oscillatory system; circular oscillations; dynamic behavior; kinematic characteristics; numerical modelling; computer simulation; experimental prototype

1. Introduction

Polishing and lapping processes are among the most widespread of processes for the finishing treatment of flat surfaces. Numerous scientific papers focus on the technological efficiency of these operations. Various mathematical models describing the material removing processes during double-sided polishing are compared in [1], and the analysis of the laps’ kinematics and the problems of optimizing the treatment parameters are considered in [2]. The paper [3] is dedicated to improving the methods of defining the friction coefficients between the movable parts of the double-sided lapping machine, and in [4], the authors developed mathematical and simulation models to optimize the lapping operations. Models describing the tool wear processes taking place during the single-sided lapping are analyzed in [5]. A novel design of the driving system of the single-sided planetary-type lapping machine is proposed in [6]. The paper [7] is focused on the theoretical and experimental investigations of the kinematic parameters characterizing the double-sided cylindrical lapping process.
In contrast to the traditional methods of the finishing treatment, the vibratory methods are currently receiving increased interest. A general review of the existent vibratory finishing technologies is presented in [8]. Simplified mathematical models describing the vibratory finishing treatment processes are considered in [9,10]. The paper [11] is dedicated to theoretical and experimental investigations on the kinematics of the particles during the vibratory finishing process. A similar study on the dynamic behavior of the vibratory finishing machine with fixed parts being treated in the rotary devices is presented in [12]. The present paper is based on our previous investigations presented in [13,14]. The initial idea of developing the vibratory lapping machine equipped with a complex electromagnetic exciter was proposed and theoretically investigated in [13]. A 3D-model of the machine and some results of its experimental testing are presented in [14]. The major purpose of the present research is to analyze the dynamic behavior of the lapping machine’s double-mass oscillatory system driven by six pairs of electromagnets. Based on the obtained results, the corresponding recommendations for designers and researchers of similar equipment can be drawn.

2. Materials and Methods

2.1. General Design and Simplified Kinematic Diagram of the Vibratory Lapping Machine

The improved design of the vibratory machine for single-sided lapping of flat surfaces is presented in Figure 1a,b. The corresponding kinematic diagram of the machine’s oscillatory system is shown in Figure 1c. The lower lap 2 is suspended from the stationary body 5 by the metal ropes 4. The parts 6 being treated by the upper lap 1 are fixed on the lower lap 2. The laps are connected with one another by the system of six coil springs 3. The vibration exciter is formed by six electromagnets 7 hinge mounted on the lower lap 2 with the help of the bearing units 8. The electromagnets’ armatures (retractable (sliding) rods) 9 are connected with the upper lap 1 by the hinges 10.

![Figure 1. Vibratory lapping machine: (a) 3D-design; (b) electromagnets connection scheme; (c) kinematic diagram.](image)

The system is set into motion by three pairs of electromagnets generating periodic disturbing forces applied between the upper and lower laps. By applying the same forced frequencies and the specific phase shifts of the excitation forces, the antiphase translational (circular) oscillations of the laps can be generated. The major difference between the improved machine and the previous one considered in [14] is that new electromagnets with retractable rods (sliding armatures) are used. They are hinge-joined with the lower lap with the possibility of turning. The previous machine was equipped with principally different electromagnets with an air gap between the electromagnet and the armature. The previous electromagnets were fixed (rigidly connected) to the machine’s lower platform without
any possibility of turning (rotating). This allows for increasing the energy efficiency of the machine’s drive and the accuracy of the surface treatment.

2.2. Mathematical Model of the System Motion

In order to study the laps oscillatory motion, the inertial coordinate system \( xOy \) and the corresponding generalized coordinates \( x_1, x_2, y_1, y_2 \) are applied (see Figure 1c). The latter describe the displacements of the upper lap and the lower lap relative to the adopted coordinate system. The origin \( O \) is placed at the upper lap’s mass center in its equilibrium position (state of rest). The masses of the upper lap, lower lap, and the parts being treated are denoted as \( m_1, m_2, m_3 \), respectively. The spring elements are characterized by the stiffness coefficients \( c_1 \). The energy dissipation during the lap sliding over the parts being treated is taken into account by the viscous friction coefficient \( \mu \), which depends on numerous factors: physical and mechanical properties of the contacting materials, specific features of the abrasive medium, lapping (polishing) conditions, etc. Therefore, the coefficient \( \mu \) is usually determined experimentally.

Using the Euler–Lagrange equations, the simplified mathematical model describing the machine’s oscillatory system motion can be written as follows:

$$
\begin{align}
\dot{x}_1(t) + \mu \cdot (x_1(t) - \dot{x}_2(t)) + c_x \cdot (x_1(t) - x_2(t)) &= F_1(t) \cdot \cos 0 + F_2(t) \cdot \cos(\pi/3) + F_3(t) \cdot \cos(5\pi/3), \\
\dot{x}_2(t) + \mu \cdot (x_2(t) - \dot{x}_1(t)) + c_x \cdot (x_2(t) - x_1(t)) &= -F_1(t) \cdot \cos 0 - F_2(t) \cdot \cos(\pi/3) - F_3(t) \cdot \cos(5\pi/3), \\
\dot{y}_1(t) + \mu \cdot (y_1(t) - \dot{y}_2(t)) + c_y \cdot (y_1(t) - y_2(t)) &= F_1(t) \cdot \sin 0 + F_2(t) \cdot \sin(\pi/3) - F_3(t) \cdot \sin(5\pi/3), \\
\dot{y}_2(t) + \mu \cdot (y_2(t) - \dot{y}_1(t)) + c_y \cdot (y_2(t) - y_1(t)) &= -F_1(t) \cdot \sin 0 - F_2(t) \cdot \sin(\pi/3) + F_3(t) \cdot \sin(5\pi/3),
\end{align}
$$

where \( F_1(t) = F \cdot \sin(\omega t), F_2(t) = F \cdot \sin(\omega t + \pi/3), F_3(t) = F \cdot \sin(\omega t + 2\pi/3) \) are the excitation (disturbing) forces; \( F \) is the maximal (amplitude) value of the excitation (disturbing) force; and \( \omega \) is the forced frequency. The projections of the reduced (equivalent) stiffness coefficients on the \( Ox \) and \( Oy \) axes can be expressed as follows:

$$
\begin{align}
c_x &= c_1 \cdot \cos 0 + c_1 \cdot \cos(\pi/3) + c_1 \cdot \cos(5\pi/3) \approx 2c_1, \\
c_y &= c_1 \cdot \sin 0 + c_1 \cdot \sin(\pi/3) - c_1 \cdot \sin(5\pi/3) \approx 1.732c_1.
\end{align}
$$

The numerical modeling is carried out by solving the derived system of differential equations with the help of the Runge–Kutta methods in the Mathematica software.

2.3. Experimental Prototype of the Vibratory Lapping Machine

Based on the proposed 3D-design of the vibratory lapping machine (see Figure 1a), its experimental prototype was developed at the Vibroengineering Laboratory of Lviv Polytechnic National University (see Figure 2a). The machine’s frame (stationary body) 1 was welded using square-shape tubes. The lower lap 2 is suspended from the frame 1 by the metal ropes 3. The cylindrical part (disc) 4 being treated is fixed to the lower lap 2 and is made of mild (soft) AISI 1018 steel. The electromagnets (push-pull-type linear solenoids ZUIDID KK-1564B, China) 5 are fixed on the lower lap 2. The electromagnets’ retractable rods (sliding armatures) are spring-loaded and hinge-joined with the upper lap 6 made of the synthetic-resin bonded (SRB) paper laminate. The fine-grained abrasive medium Abro GP-201 (Savannah, GA, USA) is applied between the contacting surfaces of the part (disc) 4 being treated and the upper lap 6.
The experimental tests focused on studying the machine’s free damped oscillations with the help of the WitMotion BWT901CL (Shenzhen, China) accelerometer 7. The processing of the experimental data was carried out using the corresponding WitMotion software (see Figure 2b). The instantaneous value of the upper lap acceleration during the machine stopping conditions was registered by the accelerometer 7. Based on the obtained experimental data, the approximate value of the reduced (equivalent) damping coefficient \( \mu \) was determined according to the technique described in [14].

3. Results and Discussion
3.1. Results of Numerical Modeling of the Oscillatory System Motion

The numerical modeling was performed in the Mathematica software, and the following input parameters were applied: \( m_1 = 0.42 \text{ kg}, m_2 = 0.5 \text{ kg}, m_3 = 0.75 \text{ kg}, \omega = 100.5 \text{ rad/s} \) (16 Hz), \( F = 4 \text{ N}, c_1 = 3700 \text{ N/m}, \) and \( \mu = 20 \text{ N}\cdot\text{s/m}. \) The time dependencies of the instantaneous displacements of the upper lap \((x_1, y_1)\) are presented in Figure 3a, and the upper lap motion trajectory is shown in Figure 3b.

Considering the steady-state operational conditions, the lap’s maximal horizontal displacement is equal to the vertical displacement, and takes the value of 0.0008 m (0.8 mm). The transient mode duration is about 0.2 s. Analyzing the modeled trajectory of the lap motion, it can be concluded that the initial idea of the proposed machine design is satisfied: the lap performs the translational (circular) oscillations, which are characterized by the
uniform speed of each point of the lapping plate. In such a case, the best accuracy and operational efficiency of the lapping (polishing) process can be reached.

3.2. Results of Computer Simulation (Virtual Experiments)

The computer simulation (virtual experiment) was carried out in the SolidWorks Motion software (Waltham, MA, USA) using the developed 3D-model of the vibratory lapping machine. The corresponding results are presented in Figure 4. All the input parameters correspond to the those mentioned above. In general, the simulation results satisfactorily agree with the those obtained by numerical modeling. After the transient conditions lasting about 0.8 s, the lap describes the circular trajectory (path) of the radius equal to 0.8 mm. The only difference is that the center of this circle moved upwards and to the right. This fact can be described by the complex friction conditions present during the lapping process. The simplified mathematical model has not considered this phenomenon.

Considering the steady-state operational conditions, the lap’s maximal horizontal displacement is equal to the vertical displacement, and takes the value of 0.8 mm. The results of the numerical modeling and computer simulation indicate that the initial idea
of the proposed machine design is satisfied: the lap performs the translational (circular) oscillations, which are characterized by the uniform speed of each point of the lapping plate. In such a case, the best accuracy and operational efficiency of the lapping (polishing) process can be reached. The obtained results can be used by designers and researchers of similar technological equipment, as well as by technologists implementing vibratory finishing operations. Further investigations on the subject of this paper could focus on analyzing the machine’s drive power consumption under different operational conditions and determining optimization criteria for minimizing the power consumption and maximizing the machine’s technological efficiency.

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