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Ranking DMUs by Comparing DEA Cross-Efficiency Intervals Using Entropy Measures

Tim Lu [1] and Shiang-Tai Liu [2,*]

1 Department of Marketing and Logistics, Vanung University, Chung-Li, Tao-Yuan 320, Taiwan; timlu@vnu.edu.tw

2 Graduate School of Business and Management, Vanung University, Chung-Li, Tao-Yuan 320, Taiwan

* Correspondence: stliu@vnu.edu.tw; Tel.: +886-3-443-4460

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Abstract: Cross-efficiency evaluation, an extension of data envelopment analysis (DEA), can eliminate unrealistic weighing schemes and provide a ranking for decision making units (DMUs). In the literature, the determination of input and output weights uniquely receives more attentions. However, the problem of choosing the aggressive (minimal) or benevolent (maximal) formulation for decision-making might still remain. In this paper, we develop a procedure to perform cross-efficiency evaluation without the need to make any specific choice of DEA weights. The proposed procedure takes into account the aggressive and benevolent formulations at the same time, and the choice of DEA weights can then be avoided. Consequently, a number of cross-efficiency intervals is obtained for each DMU. The entropy, which is based on information theory, is an effective tool to measure the uncertainty. We then utilize the entropy to construct a numerical index for DMUs with cross-efficiency intervals. A mathematical program is proposed to find the optimal entropy values of DMUs for comparison. With the derived entropy value, we can rank DMUs accordingly. Two examples are illustrated to show the effectiveness of the idea proposed in this paper.

Keywords: data envelopment analysis; cross-efficiency; entropy; ranking

1. Introduction

Data Envelopment Analysis (DEA)—originally developed by Charnes, Cooper, and Rhodes (CCR) [1]—is an effective method for evaluating the efficiencies of decision making units (DMUs) with the same inputs and outputs. The idea of DEA models is to generate a set of optimal weights for each DMU to maximize the ratio of its sum of weighted outputs to its sum of weighted inputs while keeping all the DMU ratios at most the unity. Although DEA has been widely used as an effective approach in finding the frontiers, its flexibility in weighting multiple inputs and outputs and its nature of self-evaluation may lead to the situation that many DMUs are evaluated as efficient, and the DEA efficient units cannot be further discriminated. Rating too many units as efficient is a commonly recognized problem of DEA.

As an extension of DEA, cross-efficiency evaluation is to provide a ranking for CCR-efficient units [2,3]. The purpose of this method is to employ DEA to do peer-evaluation, rather than to have it perform in a self-evaluation mode. There are mainly two advantages of the cross-efficiency evaluation method. It provides an ordering among DMUs, and it eliminates unrealistic weight schemes without requiring the elicitation of weight restrictions from experts [4]. These merits let the method be widely used for ranking performance of DMUs, for example: advanced manufacturing technology selection [5], economic-environmental performance [6], measuring the performance of the nations participating in Olympic Games [7], supply chain management [8], public resource management [9], fixed cost and resource allocation [10], portfolio selection [11], premium allocation for academic faculty [12], and baseball player ranking [13].

However, the multiple optimum solutions for DEA weights might reduce the effectiveness of the cross-efficiency. Specifically, cross-efficiency scores obtained from the original DEA methodology are generally not unique [3]. It may be possible to improve a DMU's (cross-efficiency) performance rating, but generally only by worsening the ratings of others [14]. In this regard, the methods of Sexton et al. [2] and Doyle and Green [3] use a secondary-goal methodology to deal with the multiple DEA solutions. They develop aggressive (minimal) and benevolent (maximal) formulations to identify optimal weights that not only maximize the efficiency of a particular DMU under evaluation, but also minimize the average efficiency of other DMUs. In addition to the well-known aggressive (minimal) and benevolent (maximal) formulations, other secondary-goal techniques are proposed and investigated [14–28]. Since it is possible that these two different formulations produce two different ranking results, decision makers may need to make a choice between these two formulations.

There is another direction of studies, where all the possible weights are considered in the proposed approaches, and a cross-efficiency interval is derived for a DMU being evaluated. Yang et al. [29] calculate both the minimal and maximal game cross-efficiency scores for each given DMU according to the idea of Liang et al. [14]. The holistic acceptability index (HAI) provides a measure of the overall acceptability of the obtained cross-efficiency scores for ranking DMUs. Alcaraz et al. [30] take into consideration all the possible choices of weights that all DMUs can make, and yields for each DMU a range for its possible ranking rather than a single ranking. Ramón et al. [31] develop a pair of models that allow for all possible weights for all DMUs, and a cross-efficiency interval is obtained for each DMU. Existing order relations for interval numbers are used to identify dominance relations among DMUs and a ranking result of DMUs is derived. These approaches perform the cross-efficiency evaluation without choosing the DEA weights.

Information entropy is an effective tool to measure the uncertainty. According to the idea of entropy, the amount or quality of information is one of the determinants for making decisions accurately [32]. For this reason, it has been widely applied to different cases of assessments, such as physics, social sciences, and so on [33–36]. There are several studies that integrate entropy and DEA models. Soleimani-Damaneh et al. [37] integrate a series of efficiency scores of a DMU, which are calculated from different DEA models, into a comprehensive efficiency score via using Shannon entropy to calculate the degree of importance of each model. Hsiao et al. [38] propose an entropy-based approach to deal with the problem of the distorted efficiency measurement in the non-proportional radial measure. Bian and Yang [39] extend the Shannon-DEA procedure to establish a comprehensive efficiency measure for appraising DMUs' resource and environment efficiencies. Xie et al. [40] employ Shannon entropy theory to calculate the degree of the importance of each DMU. Then they combined the obtained efficiencies and the degrees of importance to improve the discrimination of traditional DEA models. Qi and Guo [41] propose a modified weight restricted DEA model for calculating non-zero optimal weights, and the non-zero optimal weights are aggregated to be the common weights using Shannon entropy. Storto [42] investigates an index that calculates the ecological efficiency of a city through combining the Shannon's entropy and the cross-efficiency model. Wang et al. [43] use the DEA entropy model to calculate the intervals of all cross-efficiency values with imprecise inputs and outputs, and all DMUs are evaluated and ranked based upon the distance to ideal positive cross efficiency.

The current approaches for cross-efficiency evaluation are often averaging the entries of the cross-efficiency matrix column-wise for comparison of DEA efficient units, or concentrate on how to determine DEA weights uniquely. In these cases, however, the problem of choosing the aggressive (minimal) or benevolent (maximal) formulation for decision-making might still remain. In this paper, we treat the cross-efficiency of a DMU as an interval, where the lower bound and upper bound are derived by minimal and maximal formulations, respectively. That is, the cross-efficiency interval takes the minimal and maximal formulations into account at the same time, and the choice of DEA weights can then be avoided. To rank DMUs with their cross-efficiency intervals, a numerical index is required for comparison. The entropy, which is based on information theory, is an effective tool to measure the uncertainty. We utilize the concept of entropy to construct a numerical index for ranking DMUs with cross-efficiency intervals. Following the idea of Yang et al. [29], a number of cross-efficiency intervals are obtained for DMUs in the cross-efficiency evaluation. The entropy values are then calculated for the DMUs with cross-efficiency intervals. A nonlinear fractional program with bound constraints is formulated to find the optimal value of entropy among cross-efficiency intervals. By variable substitution, this nonlinear fractional program is transformed into a convex optimization problem for deriving the global optimum solution. With the obtained entropy values, the DMUs are ranked accordingly.

In the sections that follow, we first introduce the aggressive and benevolent formulations in the cross-efficiency evaluation method. Next, the concept of entropy is introduced, and a nonlinear fractional program with cross-efficiency intervals is formulated. Then we develop the solution procedure to find the optimal entropy value for comparison of DMUs. Two numerical examples are employed to illustrate the ideal proposed in this study. Finally, some conclusions of this study are presented.

2. Solution Procedure

2.1. Cross-Efficiency Intervals

Let X_{ij} and Y_{rj} denote the i-th input, $i = 1, \ldots, m$, and r-th output, $r = 1, \ldots, s$, respectively, of the j-th DMU, $j = 1, \ldots, n$. The DEA model proposed by Charnes et al. [1] for calculating the efficiency of DMU d under the assumption of constant returns-to-scale, referred to as the CCR model, is:

$$E_{dd} = \max \sum_{r=1}^{s} u_{rd} Y_{rd} \tag{1}$$

$$\text{s.t.} \sum_{i=1}^{m} v_{id} X_{id} = 1$$

$$\sum_{r=1}^{s} u_{rd} Y_{rj} - \sum_{i=1}^{m} v_{id} X_{ij} \leq 0, j = 1, \ldots, n$$

$$u_{rd},\ v_{id} \geq 0,\ r = 1, \ldots, s,\ i = 1, \ldots, m$$

where u_{rd} and v_{id} are the weights assigned to the s outputs and m inputs, respectively.

In the cross-efficiency evaluation we use the optimal solutions of (1) to calculate the cross-efficiencies. To be specific, if v^*_{id} ($i = 1, \ldots, m$) and u^*_{rd} ($r = 1, \ldots, s$) is an optimal solution of (1) for a given DMU d, then the cross-efficiency of DMU j ($j = 1, \ldots, n$, $j \neq d$) peer-evaluated by DMU d is given by

$$E_{dj} = \frac{\sum_{r=1}^{s} u^*_{rd} Y_{rj}}{\sum_{i=1}^{m} v^*_{id} X_{ij}}, d, j = 1, \ldots, n \tag{2}$$

The cross-efficiency score of DMU j, $j = 1, \ldots, n$, is usually defined as the average of its cross-efficiencies obtained with the weights of all the DMUs. That is, the cross-efficiency of DMU j is defined as

$$\overline{E}_j = \frac{1}{n} \sum_{d=1}^{n} E_{dj},\ j = 1, \ldots, n \tag{3}$$

Generally, the cross-efficiency scores, which are calculated from DEA models, are not unique due to the multiple optimum solutions for DEA weights, and one may obtain different efficiency scores with different optimum solutions of the DEA weights. One remedy to eliminate the non-uniqueness, as suggested by Sexton et al. [2], is to use secondary goals to choose the weights among the optimal solutions. The most commonly used secondary goals are proposed by Doyle and Green [3], and they present aggressive and benevolent formulations. In the case of the benevolent formulation, for example, the idea is to identify optimal weights that maximize not only the efficiency of a particular DMU under evaluation but also the average efficiency of other DMUs. In the aggressive formulation, however, one seeks weights that minimize the average efficiency of those other units. Each DMU in Liang et al. [14] is viewed as a player that seeks to maximize its own efficiency, under the condition that the cross-efficiency of each of the other DMUs does not deteriorate. The cross efficiency derived by using the model of [14] is called the game cross-efficiency. Consider two DMUs, that is, DMU d and DMU j. Based on the idea of [14], the aggressive and benevolent game cross-efficiencies are calculated, respectively, by the following formulations [29].

$$E_{dj}^{L} = \min \sum_{r=1}^{s} u_{rd} Y_{rj} \tag{4}$$

$$\text{s.t.} \sum_{i=1}^{m} v_{id} X_{ij} = 1$$

$$\sum_{r=1}^{s} u_{rd} Y_{rd} - E_{dd} \sum_{i=1}^{m} v_{id} X_{id} = 0$$

$$\sum_{r=1}^{s} u_{rd} Y_{rj} - \sum_{i=1}^{m} v_{id} X_{ij} \leq 0, j = 1, \ldots, n, j \neq d$$

$$u_{rd},\ v_{id} \geq 0,\ r = 1, \ldots, s,\ i = 1, \ldots, m$$

$$E_{dj}^{U} = \max \sum_{r=1}^{s} u_{rd} Y_{rj} \tag{5}$$

$$\text{s.t.} \sum_{i=1}^{m} v_{id} X_{ij} = 1$$

$$\sum_{r=1}^{s} u_{rd} Y_{rd} - E_{dd} \sum_{i=1}^{m} v_{id} X_{id} = 0$$

$$\sum_{r=1}^{s} u_{rd} Y_{rj} - \sum_{i=1}^{m} v_{id} X_{ij} \leq 0, j = 1, \ldots, n, j \neq d$$

$$u_{rd},\ v_{id} \geq 0,\ r = 1, \ldots, s,\ i = 1, \ldots, m$$

Under maintaining the efficiency score E_{dd} of DMU d unchanged, Models (4) and (5), respectively, search for the minimization and maximization of the cross-efficiency scores that DMU j would reach.

From viewpoints of the cross-efficiency, the values of E_{dj}^{L} and E_{dj}^{U} solved from (4) and (5), respectively, are the lower bound and upper bound of the cross-efficiency interval between DMU d and DMU j. In other words, for DMUs d and j, their cross-efficiency lies in the range of $[E_{dj}^{L}, E_{dj}^{U}]$. The conventional single-valued data can be regarded as degenerate interval data with only one value in that interval. Table 1 shows a generalized cross-efficiency matrix, where all the cross-efficiency scores are interval-valued rather than a single value, for DMUs. Note that the elements in the diagonal are $E_j = E_{jj}^{L} = E_{jj}^{U}, \forall j$.

To rank all DMUs with cross-efficiency intervals, a numerical index for each DMU is required for easy comparison. In this study we use the concept of Gibbs' entropy for ranking DMUs with cross-efficiency intervals. The major difficulty is how to deal with the lying ranges of the cross-efficiency intervals in calculating the entropy. In the next section, a methodology is proposed to find the optimal entropy among the cross-efficiency intervals for discrimination of DMUs.

Table 1. Cross-efficiency interval matrix.

DMU	1	2	...	*n*
1	$[E_{11}^L, E_{11}^U]$	$[E_{12}^L, E_{12}^U]$	...	$[E_{1n}^L, E_{1n}^U]$
2	$[E_{21}^L, E_{21}^U]$	$[E_{22}^L, E_{22}^U]$	...	$[E_{2n}^L, E_{2n}^U]$
...	...	...	...	...
n	$[E_{n1}^L, E_{n1}^U]$	$[E_{n2}^L, E_{n2}^U]$	...	$[E_{nn}^L, E_{nn}^U]$
Average	$[\overline{E}_1^L, \overline{E}_1^U]$	$[\overline{E}_2^L, \overline{E}_2^U]$	...	$[\overline{E}_n^L, \overline{E}_n^U]$

2.2. Entropy of Cross-Efficiency Intervals

In this section, we adopt Gibbs' entropy to consider all the cross-efficiency intervals derived in Models (4) and (5) for determining the rankings of DMUs. Information entropy is a measurement of uncertainty of the system state, when the system is in limited states with the probability P_i (i = 1, 2, ..., n) of each state, then the entropy of the system is [32]

$$h = -K\sum_{i=1}^{n} P_i \ln P_i \tag{6}$$

where K is a constant and $0 \leq P_i \leq 1,\ \sum_{i=1}^{n} P_i = 1$.

Without loss of generality, we discuss the case where all the cross-efficiency scores are of interval-valued type because the single-valued cross-efficiency can be treated as the degenerated interval with only one value in the interval. When the cross-efficiency is a constant value, the entropy of DMU j can be defined as:

$$H_j = -K_j\sum_{d=1}^{n} G_{dj} \ln G_{dj} = -K_j\sum_{d=1}^{n}\left(\frac{E_{dj}}{\sum\limits_{d=1}^{n} E_{dj}} \ln \frac{E_{dj}}{\sum\limits_{d=1}^{n} E_{dj}}\right) \tag{7}$$

where $G_{dj} = \frac{E_{dj}}{\sum_{d=1}^{n} E_{dj}}$ and $K_j = \frac{(\overline{E}_j^L + \overline{E}_j^U)}{2}$ is a constant value. Note that $\overline{E}_j^L$ and $\overline{E}_j^U$ are, respectively, aggressive and benevolent efficiencies that are defined in (3).

Since E_{dj}^L and E_{dj}^U are calculated, respectively, from (4) and (5), the cross-efficiency is an interval rather than a constant value. Then Equation (7) becomes the following formulation with cross-efficiency intervals

$$\hat{H}_j = -K_j\sum_{d=1}^{n}\left(\frac{\hat{E}_{dj}}{\sum\limits_{d=1}^{n} \hat{E}_{dj}} \ln \frac{\hat{E}_{dj}}{\sum\limits_{d=1}^{n} \hat{E}_{dj}}\right) \tag{8}$$

where $\hat{E}_{dj} \in \left[E_{dj}^L,\ E_{dj}^U\right]$.

To find the smallest uncertainty of a DMU with cross-efficiency intervals, we need to find the minimum value of entropy in (8). If every evaluated DMU has its own smallest entropy (uncertainty), then we can use this obtained entropy for comparison of DMUs. The minimization of $\hat{H}_j$ is equal to $\min\{\hat{H}_j \mid E_{dj}^L \leq \hat{E}_{dj} \leq E_{dj}^U, \forall d\}$. In symbols, it can be expressed as:

$$\hat{H}_j = \min_{\substack{E_{dj}^L \leq \hat{E}_{dj} \leq E_{dj}^U \\ \forall d}} = -K_j\sum_{d=1}^{n}\left(\frac{\hat{E}_{dj}}{\sum\limits_{d=1}^{n} \hat{E}_{dj}} \ln \frac{\hat{E}_{dj}}{\sum\limits_{d=1}^{n} \hat{E}_{dj}}\right) \tag{9}$$

Model (9) can be reduced to the following formulations:

$$\hat{H}_j = \min(-K_j \sum_{d=1}^{n} \left(\frac{\hat{E}_{dj}}{\sum_{d=1}^{n} \hat{E}_{dj}} \ln \frac{\hat{E}_{dj}}{\sum_{d=1}^{n} \hat{E}_{dj}} \right)) \tag{10}$$

$$\text{s.t. } E_{dj}^{L} \leq \hat{E}_{dj} \leq E_{dj}^{U},\ d = 1, \ldots, n. \tag{10a}$$

Model (10) is a nonlinear fractional program with bound constraints, where there is no guarantee to have stationary points. In (10), we want to find a set of $\hat{E}_{dj}$, $d = 1, \ldots, n$, that produces the smallest objective value. Following the variable substitution of Charnes and Cooper [44], we let $t = 1/\sum_{d=1}^{n} \hat{E}_{dj}$ and $w_{dj} = t\hat{E}_{dj}$. Because the value of $t > 0$, one can multiply constraint (10a) by t and transform (10) into the following mathematical program:

$$\hat{H}_j = \min(-K_j \sum_{d=1}^{n} w_{dj} \ln\ w_{dj}) \tag{11}$$

$$\text{s.t. } \sum_{d=1}^{n} w_{dj} = 1$$

$$E_{dj}^{L} t \leq w_{dj} \leq E_{dj}^{U} t,\ d = 1, \ldots, n.\ \ t > 0.$$

Since $\ln\ w_{dj}$ and $w_{dj} \ln\ w_{dj}$ are both increasing functions and K_j is a constant value, the objective function is a concave function. The values of E_{dj}^{L} and E_{dj}^{U} are the lower bound and upper bound of $\hat{E}_{dj}$, $d = 1, \ldots, n$, respectively, and they are constant values. Therefore, $E_{dj}^{L} t \leq w_{dj} \leq E_{dj}^{U} t$, $d = 1, \ldots, n$, are linear constraints. Model (11) is minimizing a concave function subject to linear constraints, and we can derive a stationary point—the global optimum solution for (11). Moreover, because the objective function is concave upward and the constraints are linear and boxed in (11), the optimal solution should occur at extreme points [45].

With the derived value of $\hat{H}_j^*$, we are able to rank all DMUs accordingly. The larger the value of $\hat{H}_j^*$ the better the DMU is since the minimum uncertainty is assured in the solution processes.

3. Examples

3.1. Academic Departments in a University

To illustrate the methodology proposed in this paper, we first use an example, which is taken from Wong and Beasley [46] that is an evaluation of seven academic departments in a university. This example is also used for the illustrations by related studies [20,23,29]. Wong and Beasley [46] evaluate seven academic departments in a university in terms of three inputs and three outputs. The number of academic staff (X_1), academic staff salaries in thousands of pounds (X_2), and support staff salaries in thousands of pounds (X_3) are used as input items, and number of undergraduate students (Y_1), number of postgraduate students (Y_2), number of research papers (Y_3) are selected as output items. Table 2 shows their input and output data together with their calculated CCR efficiency scores.

Clearly, there are six of seven departments being evaluated as CCR-efficient in Table 2, and it is hard to discriminate among them. Table 3 reports the lower bound and upper bound cross-efficiencies, E_{dj}^{L} and E_{dj}^{U}, $d, j = 1, \ldots, 7$, of the seven departments, which are calculated from Models (4) and (5), respectively. The last row of Table 3 lists the average cross-efficiency scores, which stand for the aggressive and benevolent cross-efficiencies, respectively, for all DMUs.

Table 2. Input and output data for seven departments.

Department	Inputs			Outputs			CCR Efficiency
	x_1	x_2	x_3	y_1	y_2	y_3	
1	12	400	20	60	35	17	1
2	19	750	70	139	41	40	1
3	42	1500	70	225	68	75	1
4	15	600	100	90	12	17	0.8197
5	45	2000	250	253	145	130	1
6	19	730	50	132	45	45	1
7	41	2350	600	305	159	97	1

To find the optimal objective of entropy, $\hat{H}_1$, for Department 1, we need to put the data listed in the first two and three columns of Table 3 into (10), and the mathematical formulation for $\hat{H}_1$ is as follows:

$$\hat{H}_1 = \min(-\frac{(0.7050 + 0.9462)}{2}\sum_{d=1}^{7}\left(\frac{\hat{E}_{d1}}{\sum_{d=1}^{7}\hat{E}_{d1}}\ln\frac{\hat{E}_{d1}}{\sum_{d=1}^{n}\hat{E}_{d1}}\right)) \quad (12)$$

$$\text{s.t. } \hat{E}_{11} = 1.0,\ 0.6856 \le \hat{E}_{21} \le 0.9361,\ 0.7933 \le \hat{E}_{31} \le 1.0,\ \hat{E}_{41} = 0.6874,$$
$$0.4904 \le \hat{E}_{51} \le 1.0,\ 0.6499 \le \hat{E}_{61} \le 1.0,\ 0.6336 \le \hat{E}_{71} \le 1.0.$$

Based on (11), Model (12) can be rewritten to the following mathematical form:

$$\hat{H}_1 = \min(-0.8256\sum_{d=1}^{7} w_{d1}\ln\ w_{d1}) \quad (13)$$

$$\text{s.t.} \sum_{d=1}^{7} w_{d1} = 1$$

$$w_{11} = t,\ 0.6856t \le w_{21} \le 0.9361t,\ 0.7933t \le w_{31} \le t,$$

$$w_{41} = 0.6847t,\ 0.4904t \le w_{51} \le t,\ 0.6499t \le w_{61} \le t,$$

$$0.6336t \le w_{71} \le t,\ t > 0.$$

The objective value of $\hat{H}_1$ is solved as 1.5805 occurring at $t* = 0.1819$, $w^*_{11} = 0.1819$, $w^*_{21} = 0.1247$, $w^*_{31} = 0.1819$, $w^*_{41} = 0.1251$, $w^*_{51} = 0.0892$, $w^*_{61} = 0.1819$, and $w^*_{71} = 0.1153$. After transformation, we have $\hat{E}^*_{11} = 1.0$, $\hat{E}^*_{21} = 0.6856$, $\hat{E}^*_{31} = 1.0$, $\hat{E}^*_{41} = 0.6874$, $\hat{E}^*_{51} = 0.4904$, $\hat{E}^*_{61} = 1.0$, and $\hat{E}^*_{71} = 0.6336$. The solutions of $\hat{E}^*_{21}$, $\hat{E}^*_{51}$, and $\hat{E}^*_{71}$ are at their lower bounds. On the contrary, the solutions of $\hat{E}^*_{31}$ and $\hat{E}^*_{61}$ are at their upper bounds. This verifies that the optimal solution occurs at extreme point, which is discussed in the previous section. One can calculate the optimal entropies for the other departments with the same solution procedure, and we have $\hat{H}_2 = 1.5632$, $\hat{H}_3 = 1.4771$, $\hat{H}_4 = 0.8701$, $\hat{H}_5 = 1.4171$, $\hat{H}_6 = 1.7232$, and $\hat{H}_7 = 1.2197$.

Table 4 lists the aggressive and benevolent game cross-efficiencies measured by [14], holistic acceptability indices (HAI) calculated by [29], and the entropy calculated in this paper and their rankings (in parentheses) by different formulations, from which it is observed that the aggressive and benevolent formulations lead to different cross-efficiency rankings for the seven academic departments. Interestingly, the ranking result of this paper is the same as that of the aggressive formulation suggested by [14].

Table 3. Lower bound and upper bound cross efficiencies of seven departments.

Dep.	Dep. 1		Dep. 2		Dep. 3		Dep. 4		Dep. 5		Dep. 6		Dep. 7	
	L	**U**	**L**	**U**	**L**	**U**	**L**	**U**	**L**	**U**	**L**	**U**	**L**	**U**
1	1.0000	1.0000	0.3347	0.9850	0.5181	1.000	0.0686	0.6836	0.3314	1.0000	0.5143	1.0000	0.1514	1.0000
2	0.6856	0.9361	1.0000	1.0000	0.7346	0.8481	0.6868	0.8197	0.6620	0.9208	0.9506	1.0000	0.6044	1.0000
3	0.7933	1.0000	0.5533	0.8584	1.0000	1.0000	0.1515	0.4695	0.3148	0.7081	0.8213	1.0000	0.1509	0.2941
4	0.6874	0.6874	1.0000	1.0000	0.7349	0.7349	0.8197	0.8197	0.7649	0.7649	0.9506	0.9506	1.0000	1.0000
5	0.4904	1.0000	0.6990	0.9703	0.5505	0.8285	0.2417	0.6721	1.0000	1.0000	0.7799	1.0000	0.5252	1.0000
6	0.6449	1.0000	0.6954	1.0000	0.7488	1.0000	0.2136	0.7718	0.4778	1.0000	1.0000	1.0000	0.2460	1.0000
7	0.6336	1.0000	0.5564	1.0000	0.4175	0.7719	0.2063	0.8197	0.7558	1.0000	0.6107	1.0000	1.0000	1.0000
Ave.	0.7050	0.9462	0.6884	0.9734	0.6720	0.8834	0.3412	0.7223	0.6153	0.9134	0.8039	0.9929	0.5254	0.8992

L: lower bound, U: upper bound.

Table 4. Cross efficiencies, HAI, entropies, and ranks of the seven departments.

Department	Aggressive	Benevolent	HAI	Entropy
1	0.7050 (2)	0.9462 (3)	35.06 (3)	1.5805 (2)
2	0.6884 (3)	0.9734 (2)	60.95 (2)	1.5632 (3)
3	0.6720 (4)	0.8834 (6)	25.33 (6)	1.4771 (4)
4	0.3412 (7)	0.7223 (7)	6.21 (7)	0.8701 (7)
5	0.6153 (5)	0.9134 (4)	27.01 (5)	1.4171 (5)
6	0.8039 (1)	0.9929 (1)	86.06 (1)	1.7232 (1)
7	0.5254 (6)	0.8992 (5)	29.36 (4)	1.2197 (6)

3.2. Chinese City

In this example, two inputs and three outputs are chosen to characterize the technology of 18 Chinese cities, and the original data of this example can be found in [47]. The investment in fixed assets by stated-owned enterprises (X_1) (10,000 RMB), where RMB is the Chinese monetary unit, and foreign funds actually used (X_2) (10,000 USD) are treated as inputs; total industry output value (Y_1) (10,000 RMB), total value of retail sales (Y_2) (10,000 RMB), and handling capacity of coastal ports (Y_3) (10,000 tons) are chosen as outputs. Table 5 records the input and output data.

Table 5. Input and output data of 18 Chinese cities.

DMU	x_1	x_2	y_1	y_2	y_3
1	2874.8	16,738	160.89	80,800	5092
2	946.3	691	21.14	18,172	6563
3	6854.0	43,024	375.25	144,530	2437
4	2305.1	10,815	176.68	70,318	3145
5	1010.3	2099	102.12	55,419	1225
6	282.3	757	59.17	27,422	246
7	17,478.3	116,900	1029.09	351,390	14,604
8	661.8	2024	30.07	23,550	1126
9	1544.2	3218	160.58	59,406	2230
10	428.4	574	53.69	47,504	430
11	6228.1	29,842	258.09	151,356	4649
12	697.7	3394	38.02	45,336	1555
13	106.4	367	7.07	8236	121
14	4539.3	45,809	116.46	56,135	956
15	957.8	16,947	29.20	17,554	231
16	1209.2	15,741	65.36	62,341	618
17	972.4	23,822	54.52	25,203	513
18	2192.0	10,943	25.24	40,627	895

We first employ Models (4) and (5) to calculate the lower bound and upper bound of the cross-efficiency intervals, E_{dj}^L and E_{dj}^U, respectively, for each Chinese city, and the obtained results are presented in Table 6. Similar to Example 1, we put the cross-efficiency scores contained in Table 6 into (11) to derive the corresponding entropy for each Chinese city. Table 7 reports the CCR efficiencies, aggressive and benevolent cross-efficiencies measured by [14], and the entropies of the 18 Chinese city calculated in this paper. The numbers in the parentheses are their associated ranks of these methods.

Three cities—DMUs 2, 6, and 10—are rated as CCR-efficient. This paper determines the order of the 18 Chinese cities by their calculated entropies, and the top five places of the Chinese cities—namely, DMU 1, DMU 6, DMU 10, DMU 12, and DMU 13—are exactly the same top five places evaluated by CCR model. This shows that the methodology proposed in this paper works well in a complex problem for discriminating among efficient DMUs. There is no wonder that the aggressive and benevolent formulations measured by [14] result in different cross-efficiency rankings for the 18 Chinese cities. A numerical index like entropy is helpful for ranking DMUs with cross-efficiency intervals.

Table 6. Lower bound and upper bound cross-efficiency of 18 Chinese cities.

DMU	1		2		3		4		5		6		7		8		9	
	L	U	L	U	L	U	L	U	L	U	L	U	L	U	L	U	L	U
1	0.469	0.469	1.000	1.000	0.249	0.251	0.461	0.463	0.629	0.631	1.000	1.000	0.306	0.309	0.491	0.496	0.556	0.561
2	0.032	0.469	1.000	1.000	0.006	0.278	0.031	0.502	0.061	0.631	0.034	1.000	0.013	0.358	0.059	0.496	0.073	0.658
3	0.468	0.468	0.999	1.000	0.278	0.278	0.502	0.502	0.586	0.586	1.000	1.000	0.358	0.358	0.414	0.414	0.628	0.628
4	0.468	0.468	1.000	1.000	0.278	0.278	0.502	0.502	0.586	0.586	1.000	1.000	0.358	0.358	0.414	0.414	0.628	0.628
5	0.446	0.469	0.999	1.000	0.239	0.249	0.447	0.461	0.631	0.631	1.000	1.000	0.292	0.306	0.484	0.496	0.556	0.560
6	0.138	0.469	0.107	1.000	0.126	0.278	0.228	0.502	0.482	0.631	1.000	1.000	0.128	0.358	0.195	0.496	0.422	0.658
7	0.468	0.468	1.000	1.000	0.278	0.278	0.502	0.502	0.586	0.586	1.000	1.000	0.358	0.358	0.414	0.414	0.628	0.628
8	0.469	0.469	1.000	1.000	0.249	0.249	0.460	0.461	0.631	0.631	0.999	1.000	0.306	0.306	0.496	0.496	0.556	0.557
9	0.183	0.183	0.999	1.000	0.133	0.133	0.267	0.267	0.629	0.629	1.000	1.000	0.146	0.146	0.265	0.265	0.658	0.658
10	0.058	0.469	0.173	1.000	0.041	0.249	0.079	0.461	0.319	0.631	0.437	1.000	0.036	0.306	0.141	0.496	0.223	0.658
11	0.469	0.469	0.999	1.000	0.249	0.249	0.461	0.461	0.631	0.631	0.999	1.000	0.306	0.307	0.496	0.496	0.556	0.557
12	0.439	0.439	1.000	1.000	0.210	0.210	0.408	0.408	0.582	0.582	0.875	0.875	0.261	0.261	0.490	0.490	0.481	0.481
13	0.439	0.439	0.999	1.000	0.210	0.210	0.408	0.408	0.582	0.582	0.875	0.875	0.261	0.261	0.489	0.490	0.481	0.481
14	0.469	0.469	1.000	1.000	0.249	0.249	0.461	0.461	0.631	0.631	1.000	1.000	0.306	0.307	0.496	0.496	0.556	0.557
15	0.469	0.469	0.999	1.000	0.249	0.249	0.461	0.461	0.631	0.631	1.000	1.000	0.306	0.306	0.496	0.496	0.556	0.557
16	0.437	0.439	0.990	1.000	0.210	0.211	0.407	0.409	0.581	0.583	0.875	0.877	0.260	0.262	0.487	0.490	0.479	0.482
17	0.468	0.468	0.999	1.000	0.277	0.278	0.502	0.502	0.586	0.586	1.000	1.000	0.357	0.358	0.414	0.416	0.627	0.628
18	0.430	0.442	0.962	1.000	0.208	0.215	0.402	0.414	0.578	0.588	0.868	0.889	0.258	0.266	0.482	0.490	0.475	0.490
Ave.	0.379	0.446	0.901	1.000	0.208	0.244	0.388	0.453	0.552	0.610	0.887	0.973	0.257	0.305	0.401	0.464	0.508	0.579

DMU	10		11		12		13		14		15		16		17		18	
	L	U	L	U	L	U	L	U	L	U	L	U	L	U	L	U	L	U
1	0.981	1.000	0.299	0.301	0.749	0.763	0.708	0.725	0.662	0.666	0.670	0.674	0.678	0.682	0.686	0.690	0.693	0.697
2	0.079	1.000	0.016	0.301	0.048	0.787	0.035	0.751	0.354	0.353	0.351	0.350	0.348	0.347	0.345	0.344	0.343	0.341
3	0.661	0.661	0.273	0.273	0.521	0.522	0.429	0.430	0.449	0.442	0.434	0.427	0.420	0.413	0.405	0.398	0.391	0.383
4	0.661	0.661	0.273	0.273	0.521	0.522	0.429	0.430	0.449	0.442	0.435	0.427	0.420	0.413	0.405	0.398	0.391	0.383
5	0.999	1.000	0.290	0.301	0.724	0.763	0.702	0.725	0.661	0.665	0.669	0.673	0.677	0.681	0.685	0.689	0.693	0.697
6	0.598	1.000	0.121	0.301	0.157	0.763	0.258	0.725	0.535	0.540	0.545	0.550	0.555	0.560	0.565	0.570	0.575	0.580
7	0.661	0.661	0.273	0.273	0.521	0.522	0.429	0.430	0.449	0.442	0.435	0.427	0.420	0.413	0.405	0.398	0.391	0.384
8	1.000	1.000	0.301	0.301	0.763	0.763	0.725	0.725	0.668	0.672	0.677	0.681	0.685	0.689	0.693	0.697	0.702	0.706
9	0.999	1.000	0.142	0.142	0.223	0.224	0.296	0.296	0.396	0.392	0.387	0.383	0.378	0.374	0.369	0.365	0.360	0.356
10	1.000	1.000	0.061	0.301	0.120	0.787	0.206	0.751	0.015	0.138	0.013	0.187	0.044	0.470	0.013	0.303	0.025	0.197
11	0.999	1.000	0.301	0.301	0.762	0.763	0.724	0.725	0.138	0.138	0.186	0.187	0.465	0.465	0.303	0.303	0.185	0.185
12	1.000	1.000	0.284	0.284	0.787	0.787	0.751	0.751	0.124	0.124	0.174	0.174	0.470	0.470	0.270	0.270	0.197	0.197
13	1.000	1.000	0.283	0.284	0.786	0.787	0.751	0.751	0.124	0.124	0.174	0.175	0.470	0.470	0.269	0.270	0.197	0.197
14	0.999	1.000	0.301	0.301	0.762	0.763	0.724	0.725	0.138	0.138	0.187	0.187	0.465	0.465	0.303	0.303	0.184	0.185
15	1.000	1.000	0.301	0.301	0.762	0.763	0.725	0.725	0.138	0.138	0.187	0.187	0.465	0.465	0.303	0.303	0.185	0.185
16	1.000	1.000	0.283	0.284	0.784	0.787	0.751	0.751	0.123	0.124	0.174	0.175	0.470	0.470	0.269	0.270	0.196	0.197
17	0.661	0.666	0.273	0.273	0.521	0.525	0.429	0.434	0.136	0.136	0.160	0.161	0.295	0.298	0.306	0.306	0.102	0.103
18	0.992	1.000	0.281	0.286	0.777	0.784	0.746	0.749	0.122	0.125	0.169	0.176	0.460	0.470	0.258	0.273	0.195	0.195
Ave.	0.849	0.925	0.242	0.282	0.572	0.687	0.546	0.645	0.316	0.322	0.335	0.344	0.455	0.479	0.381	0.397	0.333	0.343

L: lower bound, U: upper bound.

Table 7. CCR-efficiency, cross-efficiencies, entropies, and ranks (in parentheses) of the 18 Chinese cities.

DMU	CCR	Aggressive	Benevolent	Entropy
1	0.469 (11)	0.379 (12)	0.446 (11)	1.147 (11)
2	1.000 (1)	0.901 (1)	1.000 (1)	2.684 (1)
3	0.278 (15)	0.208 (18)	0.244 (18)	0.630 (18)
4	0.502 (8)	0.388 (10)	0.453 (10)	1.178 (10)
5	0.631 (7)	0.552 (5)	0.610 (6)	1.653 (6)
6	1.000 (1)	0.887 (2)	0.973 (2)	2.632 (2)
7	0.358 (12)	0.257 (16)	0.305 (16)	0.780 (16)
8	0.496 (9)	0.401 (9)	0.464 (9)	1.218 (9)
9	0.658 (6)	0.508 (7)	0.579 (7)	1.542 (7)
10	1.000 (1)	0.849 (3)	0.925 (3)	2.514 (3)
11	0.301 (14)	0.242 (17)	0.282 (17)	0.733 (17)
12	0.787 (4)	0.572 (4)	0.687 (4)	1.738 (4)
13	0.751 (5)	0.546 (6)	0.645 (5)	1.655 (5)
14	0.138 (18)	0.316 (15)	0.322 (15)	0.880 (15)
15	0.187 (17)	0.335 (13)	0.344 (13)	0.924 (13)
16	0.470 (10)	0.455 (8)	0.479 (8)	1.266 (8)
17	0.306 (13)	0.381 (11)	0.397 (12)	1.053 (12)
18	0.195 (16)	0.333 (14)	0.343 (14)	0.923 (14)

A few studies have investigated methods for discriminating CCR-efficient units. One of the well-known methods is the super-efficiency DEA model [48]. Nevertheless, under certain conditions this approach suffers the infeasibility in deriving the efficiency score. In this case the application of this approach is limited. There are other methods for investigating the discrimination power of DEA, such as common weights [49] and the context-dependent DEA method [50]. However, the most popular one in discriminating efficient units perhaps is the cross-efficiency method [28]. The advantages of the approach proposed in this paper are that it is always feasible for calculating cross-efficiency, the multiple optimum solutions for DEA weights can be ignored, and the uncertainty of cross-efficiency intervals are considered in discrimination of DMUs.

4. Conclusions

It is possible that the cross-efficiency evaluation has multiple optimum solutions for the DEA weights that result in different cross-efficiency scores, and consequently to different ranking results of DMUs. The traditional approaches in the literature may make a choice of weights according to their alternative secondary goals in performing the cross-efficiency evaluation. However, decision-makers need to make a choice between the aggressive and benevolent formulations, and the issue of multiple solutions of weights still exists.

Different from previous approaches in the literature, this paper considers not only the cross-efficiency intervals but also their entropy values for ranking DMUs. The merits of the proposed approach are that the determination of the weights can be ignored and the uncertainty of the cross-efficiency intervals is considered as a ranking factor in comparison of DMUs. Since the aggressive and benevolent formulations are considered simultaneously, a number of cross-efficiency intervals are obtained for a specific DMU in the evaluation process. To find the optimal value of the entropy among cross-efficiency intervals, a nonlinear fractional programs with bound constraints is formulated. By variable substitutions, this nonlinear fractional program is transformed into a convex optimization problem to solve. With the derived entropy values, we are able to rank DMUs accordingly. Two examples are used to illustrate the approach proposed in this paper, and the derived results show that this research is indeed able to ranking the CCR-efficient units effectively.

There are different approaches proposed for enhancing and extending the cross-efficiency evaluation. In this study, the input and output data are measured by exact values. However, in some cases, the input and output items of DMUs could be imprecise data or fuzzy data. How to deal with the imprecise and fuzzy data is a possible direction of future research and an extension of the approach proposed in this study.

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