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Finite-Time H_∞ Controllers Design for Stochastic Time-Delay Markovian Jump Systems with Partly Unknown Transition Probabilities

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Abstract: This paper concentrates on the finite-time H_∞ control problem for a type of stochastic discrete-time Markovian jump systems, characterized by time-delay and partly unknown transition probabilities. Initially, a stochastic finite-time (SFT) H_∞ state feedback controller and an SFT H_∞ observer-based state feedback controller are constructed to realize the closed-loop control of systems. Then, based on the Lyapunov–Krasovskii functional (LKF) method, some sufficient conditions are established to guarantee that closed-loop systems (CLSs) satisfy SFT boundedness and SFT H_∞ boundedness. Furthermore, the controller gains are obtained with the use of the linear matrix inequality (LMI) approach. In the end, numerical examples reveal the reasonableness and effectiveness of the proposed designing schemes.

Keywords: Markovian jump systems; discrete-time systems; finite-time control; H_∞ control; partly unknown transition probabilities

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1. Introduction

The structure or parameters of various practical systems often undergoes changes due to environmental mutations, component failures, and other factors, resulting in a decrease in system performance and potential instability [1]. How to ensure the stability of mutation systems has been one of the hot topics for scholars. Markovian jump systems (MJSs), a type of hybrid systems consisting of several subsystems, can be used to model dynamical systems with structural mutations and have been extensively researched in both the practical and theoretical domains [2,3]. An adaptive neural network-based control approach was devised in [4] to address the problem of fault-tolerant control for nonlinear MJSs. In [5], the asynchronous filtering problem of MJSs affected by time-varying and infinite distributed delays was studied by using the homogeneous polynomial method. For stochastic T-S fuzzy singular MJSs, the robust H_∞ sliding mode control problem was studied in [6,7]. The authors of [8] studied the fault-detection filter design problem of uncertain singular MJSs by means of the LKF and convex polyhedron techniques. In addition, for the achievements regarding the stability and stabilization of MJSs, readers may see [9,10] and references therein.

It should be emphasized that there is a qualification in references [4–10], that is, the transition probability (TP) information of MJSs must be exactly and completely known. However, due to the limitations of measurement costs and measuring instruments, this condition is difficult to meet in the actual system modeling. As a result, it is essential and significant to investigate MJSs with partly unknown TPs [11]. For networked MJSs with partly unknown TPs, the event-triggered dynamic output feedback control problem and

sliding mode control problem were solved in [12,13], respectively. For a type of singular MJSs with partly unknown TPs, the H_∞ filtering problem was studied in [14,15]. The authors of [16] achieved the event-triggered guaranteed cost control for time-delay MJSs with partly unknown TPs, and some sufficient conditions were established to guarantee the presence of guaranteed cost controllers. In [17], a state feedback controller was constructed to ensure that MJSs with partly unknown TPs were stochastic stable. A sliding mode controller based on an adaptive neural network was proposed in [18], and the reliable control problem of uncertain MJSs with partially unknown TPs was studied.

Notably, most of the above research findings mainly concentrate on the asymptotic behavior of systems in an infinite-time interval, namely, as in the Lyapunov stability theory. However, in many practical systems, such as vehicle emergency braking systems [19], aircraft-tracking systems [20], and ship-maneuvering systems [21], it is required that the systems respond ideally to work in a finite time interval. To realize this practical need, in 1961, Dorato proposed finite-time stability [22]. Since finite-time stability has a better transient performance, a faster response speed, and a higher tracking accuracy, it has been applied to MJSs [23–26], T-S fuzzy systems [27–29], nonlinear pulse systems [30,31], mean-field systems [32–34], and so on.

In addition to requiring better transient performance, modern industries increasingly emphasize the anti-interference performance of control systems. Both external disturbances and imprecise modeling can adversely affect the performance of control systems. To weaken the effect of external disturbances, H_∞ control has emerged. Recently, many scholars have carried out plenty of research on finite-time H_∞ control [35–41]. Specifically, ref. [35] introduced a new switching dynamic event-triggering mechanism, and discussed the finite-time H_∞ control problem for switching fuzzy systems. In [36], the finite-time H_∞ control problem of nonlinear pulse switching systems was studied to guarantee that the CLS was bounded. On the other hand, due to the constraints of measurement technology and measurement costs, the system state information is frequently challenging to measure directly. In tackling this challenge, many meaningful results of finite-time H_∞ observer-based controller designing schemes have been successfully attained; see [37–41] and references therein.

At present, the study of continuous-time MJSs has obtained rich results. With the popularization of digital controllers and the development of computer science and technology, the research on discrete-time systems has attracted much attention. Discrete-time MJSs provide a framework for modeling and analyzing a variety of complex systems in the real world [42]. Through the discrete description of the system, it is easier to analyze the dynamic behavior, stability, and convergence of the system [43]. This kind of modeling and analysis is essential for understanding and predicting the behavior of systems [44], and is widely used in control systems.

Inspired by the preceding analysis, this article presents the designing schemes of a stochastic finite-time H_∞ state feedback controller and a stochastic finite-time H_∞ observer-based state feedback controller for a discrete-time MJS. Different from [17,38], the MJS considered in this paper is influenced by a time delay and stochastic white noise, which is more in line with the actual demand, but also increases the difficulty of the article derivation. Compared with the existing literature, the primary contributions of this study include the following:

(I) In this paper, the state feedback control strategy and the observer-based state feedback control strategy are adopted. The concepts of SFT H_∞ state feedback stabilization and SFT H_∞ observer-based state feedback stabilization for time-delay MJSs are defined simultaneously for the first time. The results of [17,38,44] are extended to time-delay MJSs with partially unknown TPs.

(II) By constructing a delay-dependent LKF, several sufficient conditions are given to ensure that the CLS is SFT H_∞-bounded under two control strategies.

The article is structured as follows: Section 2 presents an introduction to the system along with some preliminary knowledge. In Section 3, a state feedback controller is de-

signed, and some sufficient conditions for the MJS to be SFT H_∞ state feedback stabilization are obtained through the LKF and LMI methods. Similar to Section 3, in Section 4 we design an observer-based state feedback controller and verify that the MJS is SFT H_∞ observer-based state feedback stabilization. In Section 5, the feasibility and effectiveness of this work are validated through two simulation examples. Section 6 summarizes the entire article and provides an outlook on future research directions.

Notation: A^{-1} and A^T represent the matrix inverse and transpose of matrix A, respectively. The expression for a real positive definite matrix A is denoted as $A > 0$. $diag\{P_1, P_2, \cdots, P_n\}$ is the block diagonal matrix with $P_1, P_2, \cdots, P_n$ on the diagonal. We denote $I_{n\times n}$ as the identity matrix with $n \times n$ dimensions. $\mathbb{N}^+$ is the set of positive integers and $\mathbb{R}$ is the real number set. $\mathbb{R}^m$ and $\mathbb{R}^{m\times n}$ are the m-dimensional Euclidean space with 2-norm $\|\cdot\|$ and the vector space of all $m \times n$ matrices with entries in $\mathbb{R}$, respectively. $\mathbb{E}\{\sigma\}$ represents the mathematical expectation of σ. The symbol $*$ implies the symmetric hidden matrix entries. This paper presupposes that every specified matrix possesses the necessary dimensions. For ease of understanding, the acronyms in this paper and their corresponding meanings are shown in Table 1.

Table 1. The acronyms used in this article and their meanings.

Acronyms	Meaning of Acronyms
MJS	Markovian jump system
SFT	Stochastic finite-time
LKF	Lyapunov–Krasovskii functional
CLS	Closed-loop system
LMI	Linear matrix inequality
TP	Transition probability

2. System Description and Preliminary Knowledge

Consider an MJS with a time delay, as outlined below:

$$\begin{cases} x(k+1) = A_1(m_k)x(k) + A_{d1}(m_k)x(k-\tau) + B_1(m_k)u(k) + C_1(m_k)v(k) \\ \qquad\quad + [A_2(m_k)x(k) + A_{d2}(m_k)x(k-\tau) + B_2(m_k)u(k) + C_2(m_k)v(k)]\omega(k), \\ y(k) = D(m_k)x(k) + G(m_k)u(k), \\ z(k) = D_1(m_k)x(k) + D_{d1}(m_k)x(k-\tau) + G_1(m_k)u(k) + G_2(m_k)v(k), \ k \in \{0,1,2,\cdots,\widehat{T}\}, \\ x(n) = \psi(n), \ n \in \{-\tau, -\tau+1, \cdots, 0\}, \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the system state, $y(k) \in \mathbb{R}^p$ is the measured output, $z(k) \in \mathbb{R}^r$ is the control output, and $u(k) \in \mathbb{R}^q$ is the control input. $\psi(n)$, $n \in \{-\tau, -\tau+1, \cdots, 0\}$ are the initial conditions. τ is a positive integer that signifies the fixed time delay. The sequence $\omega(k)$ denotes one-dimensional white noises on the complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$, and satisfies $\mathbb{E}\{\omega(k)\} = 0$ and $\mathbb{E}\{\omega(k)\omega(s)\} = \delta_{ks}$, where δ_{ks} is the Kronecker delta. $v(k) \in \mathbb{R}^l$ stands for the external disturbance, which satisfies the following:

$$\sum_{k=0}^{\widehat{T}} v^T(k)v(k) \le h, \ h \ge 0. \tag{2}$$

$A_1(m_k)$, $A_2(m_k)$, $A_{d1}(m_k)$, $A_{d2}(m_k)$, $B_1(m_k)$, $B_2(m_k)$, $C_1(m_k)$, $C_2(m_k)$, $D(m_k)$, $D_1(m_k)$, $D_{d1}(m_k)$, $G(m_k)$, $G_1(m_k)$, and $G_2(m_k)$ are coefficient matrices with appropriate dimensions. These matrices depend upon the Markovian jump process $\{m_k, k \ge 0\}$, which is a discrete-time, discrete-state Markovian chain taking values in a finite state space $S = \{1, 2, \cdots, N\}$ with transition probabilities π_{ij}, where $\pi_{ij} = Pr\{m_{k+1} = j | m_k = i\}$, $i, j \in S$, denotes

the transition probability from mode i at time k to mode j at time $k+1$, and satisfies $\sum_{j=1}^{N}\pi_{ij}=1, \pi_{ij}\geq 0\ (i\in S)$. When $m_k=i,\ i\in S$, the system parameter matrices are expressed by A_{1i}, A_{2i}, A_{d1i}, A_{d2i}, B_{1i}, B_{2i}, and so on. In addition, $\omega(k)$ and m_k are independent of each other.

In this paper, it is presumed that the information in the TP matrix is partially available. In this situation, the TP matrix Π for an MJS with N modes may be represented by:

$$\Pi=\begin{bmatrix}\hat{\pi}_{11} & \pi_{12} & \cdots & \pi_{1N}\\ \hat{\pi}_{21} & \pi_{22} & \cdots & \hat{\pi}_{2N}\\ \vdots & \cdots & \ddots & \vdots\\ \hat{\pi}_{N1} & \pi_{N2} & \cdots & \pi_{NN}\end{bmatrix}, \tag{3}$$

where $\hat{\pi}_{ij}$ is the unknown TP, for all $i\in S$, and the set S is defined as $S=S_k^i\cup S_{uk}^i$, where:

$$S_k^i=\{j:\pi_{ij}\ is\ known\},\ S_{uk}^i=\{j:\pi_{ij}\ is\ unknown\}. \tag{4}$$

Moreover, when $S_k^i\neq\emptyset$, then it can be described as:

$$S_k^i=\{\zeta_1,\zeta_2,\cdots,\zeta_{p_i}\},\ p_i\in\{1,2,\cdots,N-2\}, \tag{5}$$

where $\zeta_g\in\mathbb{N}^+,\ g\in\{1,2,\cdots,p_i\}$ denotes the g-th known element in the i-th row of the TP matrix Π. Similarly, when $S_{uk}^i\neq\emptyset$, it can be expressed as follows:

$$S_{uk}^i=\{\zeta_{u1},\zeta_{u2},\cdots,\zeta_{uq_i}\},\ q_i\in\{2,\cdots,N\}, \tag{6}$$

where $\zeta_{ug}\in\mathbb{N}^+,\ g\in\{1,2,\cdots,q_i\}$ is the g-th unknown element in the i-th row of the TP matrix Π.

Remark 1. *Since $\sum_{j=1}^{N}\pi_{ij}=1$, there are at least two unknown elements in (3), and if there are unknown elements in a certain row, their quantity is at least two.*

Lemma 1. *(Schur's complement [38]) The LMI*

$$S=\begin{bmatrix}S_{11} & S_{12}^T\\ S_{12} & S_{22}\end{bmatrix}<0$$

is equivalent to $S_{11}-S_{12}^TS_{22}^{-1}S_{12}<0$, where $S_{22}<0$.

Definition 1. *(SFT stability)*

The MJS (1) with $v(k)=0$ is said to be SFT-stable with respect to $(\rho_1,\rho_2,R_i,\widehat{T})$, if:

$$\sup_{k_0\in\{-\tau,\cdots,0\}}\mathbb{E}\{x^T(k_0)R_ix(k_0)\}\leq\rho_1\Rightarrow\mathbb{E}\{x^T(k)R_ix(k)\}<\rho_2,\ \ \forall k\in\{0,1,2,\cdots,\widehat{T}\} \tag{7}$$

holds for matrix $R_i>0,\ i\in S$, and given scalars $0<\rho_1<\rho_2$.

Remark 2. *Definition 1 means that if the initial state is bounded, then the state trajectory of the system does not exceed a predetermined boundary in a finite time interval under certain conditions, which is different from asymptotic stability. An asymptotically stable system may not be finite-time stable, if its state trajectory exceeds the given upper bound in a finite-time interval, and vice versa.*

Definition 2. *(SFT boundedness)*

The MJS (1) is said to be SFT-bounded with respect to $(\rho_1,\rho_2,R_i,\widehat{T},h)$ if the system state $x(k)$ and the external disturbance $v(k)$ satisfy (7) and (2), respectively.

3. Finite-Time H_∞ State Feedback Control

3.1. State Feedback Controller

We design the following state feedback controller for MJS (1):

$$u(k) = K(m_k)x(k), \tag{8}$$

where $K(m_k)$ is the state feedback controller gain to be designed and $K(m_k)$ is denoted by K_i when $m_k = i, i \in S$. Then, the resulting CLS can be described as follows:

$$\begin{cases} x(k+1) &= \bar{A}_{1i}x(k) + A_{d1i}x(k-\tau) + C_{1i}v(k) \\ & \quad + [\bar{A}_{2i}x(k) + A_{d2i}x(k-\tau) + C_{2i}v(k)]\omega(k), \\ z(k) &= \bar{D}_{1i}x(k) + D_{d1i}x(k-\tau) + G_{2i}v(k), \end{cases} \tag{9}$$

where $\bar{A}_{1i} = A_{1i} + B_{1i}K_i$, $\bar{A}_{2i} = A_{2i} + B_{2i}K_i$, $\bar{D}_{1i} = D_{1i} + G_{1i}K_i$.

Definition 3. *(SFT H_∞ boundedness)*
The CLS (9) is said to be SFT H_∞-bounded with respect to $(\rho_1, \rho_2, R_i, \widehat{T}, h, \gamma)$ if the subsequent two conditions hold:

(a) The CLS (9) satisfies SFT boundedness with respect to $(\rho_1, \rho_2, R_i, \widehat{T}, h)$;
(b) Under the zero initial condition, for any external disturbance $v(k)$ satisfying (2), the control output $z(k)$ satisfies

$$\mathbb{E}\{\sum_{k=0}^{\widehat{T}} z^T(k)z(k)\} < \gamma^2\mathbb{E}\{\sum_{k=0}^{\widehat{T}} v^T(k)v(k)\}, \tag{10}$$

where $R_i > 0$, $i \in S$, $\gamma > 0$, $0 < \rho_1 < \rho_2$.

Definition 4. *(SFT H_∞ state feedback stabilization)*
The MJS (1) is said to be SFT H_∞ state feedback stabilization with respect to $(\rho_1, \rho_2, R_i, \widehat{T}, h, \gamma)$ if there exists a state feedback controller (8) such that the CLS (9) satisfies SFT H_∞ boundedness. Moreover, the controller (8) is called the SFT H_∞ state feedback controller.

3.2. Main Results

This section will present some sufficient conditions for the existence of a state feedback controller (8) for system (1).

Theorem 1. *The CLS (9) with partly unknown TPs is SFT H_∞-bounded with respect to $(\rho_1, \rho_2, R_i, \widehat{T}, h, \gamma)$ if there exist the scalars $\alpha > 1$ and $\gamma > 0$ and matrices $M > 0$ and $P_i > 0$ for all $i \in S$, satisfying the following:*

$$\begin{bmatrix} \Theta_{1i}^T - \alpha P_i + M + \bar{D}_{1i}^T\bar{D}_{1i} & \Theta_{2i}^T + \bar{D}_{1i}^T D_{d1i} & \Theta_{3i}^T + \bar{D}_{1i}^T G_{2i} \\ * & \Theta_{4i}^T - M + D_{d1i}^T D_{d1i} & \Theta_{5i}^T + D_{d1i}^T G_{2i} \\ * & * & \Theta_{6i}^T - \gamma^2 I + G_{2i}^T G_{2i} \end{bmatrix} < 0, \tag{11}$$

$$\alpha^{\widehat{T}}\rho_1\left[\sup_{i\in S}\{\lambda_{max}(\bar{P}_i)\} + \sup_{i\in S}\{\lambda_{max}(\bar{M}_i)\}\tau\right] + \gamma^2\alpha^{\widehat{T}}h < \inf_{i\in S}\{\lambda_{min}(\bar{P}_i)\}\rho_2, \tag{12}$$

where $\Theta_{1i}^T = \bar{A}_{1i}^T\Psi_i\bar{A}_{1i} + \bar{A}_{2i}^T\Psi_i\bar{A}_{2i}$, $\Theta_{2i}^T = \bar{A}_{1i}^T\Psi_i A_{d1i} + \bar{A}_{2i}^T\Psi_i A_{d2i}$,
$\Theta_{3i}^T = \bar{A}_{1i}^T\Psi_i C_{1i} + \bar{A}_{2i}^T\Psi_i C_{2i}$, $\Theta_{4i}^T = A_{d1i}^T\Psi_i A_{d1i} + A_{d2i}^T\Psi_i A_{d2i}$,
$\Theta_{5i}^T = A_{d1i}^T\Psi_i C_{1i} + A_{d2i}^T\Psi_i C_{2i}$, $\Theta_{6i}^T = C_{1i}^T\Psi_i C_{1i} + C_{2i}^T\Psi_i C_{2i}$,
$\Psi_i = \sum_{j\in S_k^i}\pi_{ij}P_j + (1-\pi_k^i)(\sum_{j\in S_{uk}^i}P_j)$, $\bar{P}_i = R_i^{-\frac{1}{2}}P_iR_i^{-\frac{1}{2}}$, $\bar{M}_i = R_i^{-\frac{1}{2}}MR_i^{-\frac{1}{2}}$.

Proof. For the CLS (9), we consider the following LKF:

$$V(x(k), m_k = i) = x^T(k)P_i x(k) + \sum_{l=k-\tau}^{k-1} x^T(l)Mx(l). \tag{13}$$

Then, we compute the following:

$$\begin{aligned}\mathbb{E}\{\Delta V(x(k), m_k = i)\} =& \mathbb{E}\{V(x(k+1), m_{k+1} = j)\} - \mathbb{E}\{V(x(k), m_k = i)\} \\ =& \sum_{j\in S} \pi_{ij} x^T(k+1)P_j x(k+1) + \sum_{l=k+1-\tau}^{k} x^T(l)Mx(l) \\ & - x^T(k)P_i x(k) - \sum_{l=k-\tau}^{k-1} x^T(l)Mx(l) \\ =& x^T(k+1)\sum_{j\in S} \pi_{ij}P_j x(k+1) + x^T(k)[M - P_i]x(k) - x^T(k-\tau)Mx(k-\tau).\end{aligned} \tag{14}$$

Since the TP matrix Π contains partly accessible information, not all the probabilities π_{ij} $(j \in S)$ are known. Thus, we denote $\pi_k^i = \sum_{j\in S_k^i} \pi_{ij}$. $\hat{\pi}_{ij}$ are the unknown TPs of Π. Moreover, from $\sum_{j=1}^{N} \pi_{ij} = 1$, it is obvious that $\sum_{j\in S_{uk}^i} \hat{\pi}_{ij} = 1 - \pi_k^i \geq 0$. Supposing that $\pi_k^i < 1$, we can obtain the following:

$$\begin{aligned}\sum_{j\in S} \pi_{ij}P_j &= \sum_{j\in S_k^i} \pi_{ij}P_j + \sum_{j\in S_{uk}^i} \hat{\pi}_{ij}P_j \\ &= \sum_{j\in S_k^i} \pi_{ij}P_j + (1 - \pi_k^i) \sum_{j\in S_{uk}^i} \frac{\hat{\pi}_{ij}}{(1 - \pi_k^i)} P_j \\ &\leq \sum_{j\in S_k^i} \pi_{ij}P_j + (1 - \pi_k^i)(\sum_{j\in S_{uk}^i} P_j) = \Psi_i.\end{aligned} \tag{15}$$

By (15), we can rewrite (14) as follows:

$$\begin{aligned}\mathbb{E}\{\Delta V(x(k), m_k = i)\} &\leq x^T(k+1)\Psi_i x(k+1) + x^T(k)[M - P_i]x(k) - x^T(k-\tau)Mx(k-\tau) \\ &= \begin{bmatrix} x(k) \\ x(k-\tau) \\ v(k) \end{bmatrix}^T \begin{bmatrix} \Theta_{1i}^T - P_i + M & \Theta_{2i}^T & \Theta_{3i}^T \\ * & \Theta_{4i}^T - M & \Theta_{5i}^T \\ * & * & \Theta_{6i}^T \end{bmatrix} \begin{bmatrix} x(k) \\ x(k-\tau) \\ v(k) \end{bmatrix}.\end{aligned} \tag{16}$$

From (16) and (11), we have the following:

$$\begin{aligned}\mathbb{E}\{\Delta V(x(k), m_k = i)\} <& (\alpha - 1)\mathbb{E}\{x^T(k)P_i x(k)\} + \gamma^2 \mathbb{E}\{v^T(k)v(k)\} - \mathbb{E}\{z^T(k)z(k)\} \\ <& (\alpha - 1)\mathbb{E}\{x^T(k)P_i x(k)\} + \gamma^2 \mathbb{E}\{v^T(k)v(k)\} \\ \leq& (\alpha - 1)\mathbb{E}\{V(x(k), m_k = i)\} + \gamma^2 \mathbb{E}\{v^T(k)v(k)\}.\end{aligned}$$

Thus, we can obtain the following:

$$\mathbb{E}\{V(x(k+1), m_{k+1} = j)\} < \alpha \mathbb{E}\{V(x(k), m_k = i)\} + \gamma^2 \mathbb{E}\{v^T(k)v(k)\}. \tag{17}$$

Observing that $\alpha > 1$, from (17) we obtain the following:

$$\begin{aligned}\mathbb{E}\{V(x(k), m_k = i)\} <& \alpha^k \mathbb{E}\{V(x(0), m_0)\} + \gamma^2 \sum_{l=0}^{k-1} \alpha^{k-1-l} \mathbb{E}\{v^T(l)v(l)\} \\ <& \alpha^{\widehat{T}} \mathbb{E}\{V(x(0), m_0)\} + \gamma^2 \alpha^{\widehat{T}} h.\end{aligned} \tag{18}$$

Letting $\bar{P}_i = R_i^{-\frac{1}{2}} P_i R_i^{-\frac{1}{2}}$, $\bar{M}_i = R_i^{-\frac{1}{2}} M R_i^{-\frac{1}{2}}$. According to (7), we have the following:

$$\begin{aligned}\mathbb{E}\{V(x(0), m_0)\} &\leq \sup_{i\in S}\{\lambda_{max}(\bar{P}_i)\}\mathbb{E}\{x^T(0)R_i x(0)\} + \sup_{i\in S}\{\lambda_{max}(\bar{M}_i)\}\mathbb{E}\{\sum_{l=-\tau}^{-1} x^T(l)R_i x(l)\} \\ &\leq \left[\sup_{i\in S}\{\lambda_{max}(\bar{P}_i)\} + \sup_{i\in S}\{\lambda_{max}(\bar{M}_i)\}\tau\right]\rho_1,\end{aligned} \tag{19}$$

and

$$\begin{aligned}\mathbb{E}\{V(x(k), m_k = i)\} &\geq \mathbb{E}\{x^T(k)P_i x(k)\} = \mathbb{E}\{x^T(k)R_i^{\frac{1}{2}}\bar{P}_i R_i^{\frac{1}{2}} x(k)\} \\ &\geq \inf_{i\in S}\{\lambda_{min}(\bar{P}_i)\}\mathbb{E}\{x^T(k)R_i x(k)\}.\end{aligned} \tag{20}$$

Combining with (18)–(20), it can be inferred that:

$$\mathbb{E}\{x^T(k)R_i x(k)\} < \frac{\alpha^{\widehat{T}}\rho_1\left[\sup_{i\in S}\{\lambda_{max}(\bar{P}_i)\} + \sup_{i\in S}\{\lambda_{max}(\bar{M}_i)\}\tau\right] + \gamma^2\alpha^{\widehat{T}}h}{\inf_{i\in S}\{\lambda_{min}(\bar{P}_i)\}}. \tag{21}$$

Together with (12) and (21), it is clear that $\mathbb{E}\{x^T(k)R_i x(k)\} < \rho_2$, $k \in \{0, 1, 2, \cdots, \widehat{T}\}$. This implies that the CLS (9) satisfies SFT boundedness. Next, we demonstrate that the H_∞ condition (10) holds under the zero initial condition. From (13), we can obtain the following:

$$\mathbb{E}\{V(x(k+1), m_{k+1} = j)\} < \alpha\mathbb{E}\{V(x(k), m_k = i)\} - \mathbb{E}\{z^T(k)z(k)\} + \gamma^2\mathbb{E}\{v^T(k)v(k)\}. \tag{22}$$

Then, we have the following:

$$\begin{aligned}\mathbb{E}\{V(x(k), m_k = i)\} <& \alpha^k\mathbb{E}\{V(x(0), m_0)\} - \sum_{l=0}^{k-1}\alpha^{k-1-l}\mathbb{E}\{z^T(l)z(l)\} \\ &+\gamma^2\mathbb{E}\{\sum_{l=0}^{k-1}\alpha^{k-1-l}v^T(l)v(l)\}.\end{aligned} \tag{23}$$

Assuming a zero initial condition and recognizing that $V(x(k), m_k = i) \geq 0$ for all $k \in \{0, 1, 2, \cdots, \widehat{T}\}$, we have the following:

$$\sum_{l=0}^{k-1}\alpha^{k-1-l}\mathbb{E}\{z^T(l)z(l)\} < \gamma^2\mathbb{E}\{\sum_{l=0}^{k-1}\alpha^{k-1-l}v^T(l)v(l)\}. \tag{24}$$

Noting that $\alpha > 1$, from (24) we obtain the following:

$$\mathbb{E}\{\sum_{k=0}^{\widehat{T}} z^T(k)z(k)\} < \gamma^2\mathbb{E}\{\sum_{k=0}^{\widehat{T}} v^T(k)v(k)\}. \tag{25}$$

Therefore the closed-loop MJS (9) is SFT H_∞-bounded. □

Remark 3. *It is important to note that Theorem 1 is preliminary, and since it does not provide a way to choose K_i, one can check (11), (12) on the closed-loop matrices, but this requires that K_i has already been chosen. The problem is solved in Theorem 2.*

Theorem 2. *Consider the state feedback controller (8); if there exist scalars $\alpha > 1$, $\gamma > 0$, $\rho_2 > 0$, $\sigma_1 > 0$, $\xi_1 > 0$, $\xi_2 > 0$ and matrices $J > 0$, $X_i > 0$, and Y_i, for all $i \in S$, satisfying the following conditions:*

$$\begin{bmatrix} -\alpha X_i & 0 & \Omega^T_{\bar{A}_{1i}} & \Omega^T_{\bar{A}_{2i}} & \bar{D}^T_{1i} & X_i \\ * & -\gamma^2 I & \Omega^T_{C_{1i}} & \Omega^T_{C_{2i}} & G^T_{2i} & 0 \\ * & * & -X+\Omega_{A_{d1i}} J \Omega^T_{A_{d1i}} & 0 & 0 & 0 \\ * & * & * & -X+\Omega_{A_{d2i}} J \Omega^T_{A_{d2i}} & 0 & 0 \\ * & * & * & * & -I+D_{d1i} J D^T_{d1i} & 0 \\ * & * & * & * & * & -J \end{bmatrix} < 0, \tag{26}$$

$$\sigma_1 R_i^{-1} < X_i < R_i^{-1}, \tag{27}$$

$$\xi_1 R_i^{-1} < J < \xi_2 R_i^{-1}, \tag{28}$$

$$\begin{bmatrix} -\alpha^{-\widehat{T}}\rho_2 + \gamma^2 h & \sqrt{\rho_1} & \sqrt{\tau\rho_1} \\ * & -\sigma_1 & 0 \\ * & * & -\xi_1 \end{bmatrix} < 0, \tag{29}$$

where $\Omega^T_{\bar{A}_{1i}} = [\sqrt{\pi_{i\zeta_1}}(A_{1i}X_i + B_{1i}Y_i)^T \ \sqrt{\pi_{i\zeta_2}}(A_{1i}X_i + B_{1i}Y_i)^T \ \cdots \ \sqrt{\pi_{i\zeta_{p_i}}}(A_{1i}X_i + B_{1i}Y_i)^T$
$\sqrt{1-\pi^i_k}(A_{1i}X_i + B_{1i}Y_i)^T \ \sqrt{1-\pi^i_k}(A_{1i}X_i + B_{1i}Y_i)^T \ \cdots \ \sqrt{1-\pi^i_k}(A_{1i}X_i + B_{1i}Y_i)^T]$,
$\Omega^T_{\bar{A}_{2i}} = [\sqrt{\pi_{i\zeta_1}}(A_{2i}X_i + B_{2i}Y_i)^T \ \sqrt{\pi_{i\zeta_2}}(A_{2i}X_i + B_{2i}Y_i)^T \ \cdots \ \sqrt{\pi_{i\zeta_{p_i}}}(A_{2i}X_i + B_{2i}Y_i)^T$
$\sqrt{1-\pi^i_k}(A_{2i}X_i + B_{2i}Y_i)^T \ \sqrt{1-\pi^i_k}(A_{2i}X_i + B_{2i}Y_i)^T \ \cdots \ \sqrt{1-\pi^i_k}(A_{2i}X_i + B_{2i}Y_i)^T]$,
$\Omega^T_{A_{d1i}} = [\sqrt{\pi_{i\zeta_1}}A^T_{d1i} \ \sqrt{\pi_{i\zeta_2}}A^T_{d1i} \ \cdots \ \sqrt{\pi_{i\zeta_{p_i}}}A^T_{d1i} \ \sqrt{1-\pi^i_k}A^T_{d1i} \ \sqrt{1-\pi^i_k}A^T_{d1i} \ \cdots \ \sqrt{1-\pi^i_k}A^T_{d1i}]$,
$\Omega^T_{A_{d2i}} = [\sqrt{\pi_{i\zeta_1}}A^T_{d2i} \ \sqrt{\pi_{i\zeta_2}}A^T_{d2i} \ \cdots \ \sqrt{\pi_{i\zeta_{p_i}}}A^T_{d2i} \ \sqrt{1-\pi^i_k}A^T_{d2i} \ \sqrt{1-\pi^i_k}A^T_{d2i} \ \cdots \ \sqrt{1-\pi^i_k}A^T_{d2i}]$,
$\Omega^T_{C_{1i}} = [\sqrt{\pi_{i\zeta_1}}C^T_{1i} \ \sqrt{\pi_{i\zeta_2}}C^T_{1i} \ \cdots \ \sqrt{\pi_{i\zeta_{p_i}}}C^T_{1i} \sqrt{1-\pi^i_k}C^T_{1i} \ \sqrt{1-\pi^i_k}C^T_{1i} \ \cdots \ \sqrt{1-\pi^i_k}C^T_{1i}]$,
$\Omega^T_{C_{2i}} = [\sqrt{\pi_{i\zeta_1}}C^T_{2i} \ \sqrt{\pi_{i\zeta_2}}C^T_{2i} \ \cdots \ \sqrt{\pi_{i\zeta_{p_i}}}C^T_{2i} \sqrt{1-\pi^i_k}C^T_{2i} \ \sqrt{1-\pi^i_k}C^T_{2i} \ \cdots \ \sqrt{1-\pi^i_k}C^T_{2i}]$,
$X = diag\{X_{\zeta_1}, X_{\zeta_2}, \cdots, X_{\zeta_{p_i}}, X_{\zeta_{u1}}, X_{\zeta_{u2}}, \cdots, X_{\zeta_{uq_i}}\}$, $\bar{D}_{1i} = D_{1i}X_i + G_{1i}Y_i$,
then the CLS (9) with the partly unknown TPs is SFT H_∞*-bounded with respect to* $(\rho_1, \rho_2, R_i, \widehat{T}, \gamma, h)$, *i.e., MJS (1) is SFT* H_∞ *state feedback stabilization, and the controller gain* $K_i = Y_i X_i^{-1}$.

Proof. First, we demonstrate the equivalence between condition (26) and condition (11). According to Lemma 1, (26) is equivalent to the following:

$$\begin{bmatrix} -\alpha X_i + X_i J^{-1} X_i & 0 & 0 & \Omega^T_{\bar{A}_{1i}} & \Omega^T_{\bar{A}_{2i}} & \bar{D}^T_{1i} \\ * & -J^{-1} & 0 & \Omega^T_{A_{d1i}} & \Omega^T_{A_{d2i}} & D^T_{d1i} \\ * & * & -\gamma^2 I & \Omega^T_{C_{1i}} & \Omega^T_{C_{2i}} & G^T_{2i} \\ * & * & * & -X & 0 & 0 \\ * & * & * & * & -X & 0 \\ * & * & * & * & * & -I \end{bmatrix} < 0. \tag{30}$$

Letting $X_i = P_i^{-1}$, $J = M^{-1}$, $X = P^{-1}$, $K_i = Y_i X_i^{-1}$. Pre- and post-multiplying (30) by $diag\{X_i^{-1}, I, I, I, I, I\}$, we can observe that (30) is equivalent to (11) by using Lemma 1.

On the other hand, from Lemma 1, (29) can be expressed as the following inequality:

$$\alpha^{\widehat{T}}\rho_1(\sigma_1^{-1} + \xi_1^{-1}\tau) + \gamma^2\alpha^{\widehat{T}}h < \rho_2. \tag{31}$$

We note that $\bar{P}_i = R_i^{-\frac{1}{2}} P_i R_i^{-\frac{1}{2}}$ and $\bar{M}_i = R_i^{-\frac{1}{2}} M R_i^{-\frac{1}{2}}$; combined with conditions (27) and (28), we can obtain that:

$$\sup_{i\in S}\{\lambda_{max}(\bar{P}_i)\} < \sigma_1^{-1}, \ \inf_{i\in S}\{\lambda_{min}(\bar{P}_i)\} > 1, \ \sup_{i\in S}\{\lambda_{max}(\bar{M}_i)\} < \xi_1^{-1}.$$

Therefore, it is easy to observe that (12) holds. This completes the proof. □

Remark 4. *Theorem 2 generalizes the results of [17] to a time-delay MJS, and gives sufficient conditions for MJS (1) to be SFT H_∞ state feedback stabilization.*

Remark 5. *It can be seen from Theorem 2 that the controller gain K_i depends on X_i and Y_i, that in turn depend on the system matrices of mode i. It is necessary to consider the dynamic characteristics of the system in different modes and the transition probabilities between modes to ensure the stability and the controller in each mode.*

4. Finite-Time H_∞ Observer-Based State Feedback Control

4.1. Observer-Based State Feedback Controller

In the presence of a system state that is not fully measurable, the following observer-based state feedback controller is designed:

$$\begin{cases} \widehat{x}(k+1) &= A_1(m_k)\widehat{x}(k) + A_{d1}(m_k)\widehat{x}(k-\tau) + B_1(m_k)u(k) + H(m_k)[y(k) - \widehat{y}(k)], \\ \widehat{y}(k) &= D(m_k)\widehat{x}(k) + G(m_k)u(k), \\ u(k) &= \widehat{K}(m_k)\widehat{x}(k), \\ \widehat{x}(n) &= \psi(n), \quad n \in \{-\tau, -\tau+1, \cdots, 0\}, \end{cases} \tag{32}$$

where $\widehat{x}(k)$ is the estimated state and $\widehat{y}(k)$ is the estimated output, and $\widehat{K}(m_k)$ and $H(m_k)$ denote the state feedback gain and observer gain to be determined, respectively. The estimated state error is defined as $e(k) = x(k) - \widehat{x}(k)$, and $\eta^T(k) = [\ x^T(k) \ \ e^T(k)\ ]$. For $m_k = i\ (i \in S)$, the CLS is represented by the following:

$$\begin{cases} \eta(k+1) &= \widehat{A}_{1i}\eta(k) + \widehat{A}_{d1i}\eta(k-\tau) + \widehat{C}_{1i}v(k) \\ &\quad + [\widehat{A}_{2i}\eta(k) + \widehat{A}_{d2i}\eta(k-\tau) + \widehat{C}_{2i}v(k)]\omega(k), \\ z(k) &= \widehat{D}_{1i}\eta(k) + \widehat{D}_{d1i}\eta(k-\tau) + G_{2i}v(k), \end{cases} \tag{33}$$

where $\widehat{A}_{1i} = \begin{bmatrix} A_{1i} + B_{1i}\widehat{K}_i & -B_{1i}\widehat{K}_i \\ 0 & A_{1i} - H_iD_i \end{bmatrix}$, $\widehat{A}_{d1i} = \begin{bmatrix} A_{d1i} & 0 \\ 0 & A_{d1i} \end{bmatrix}$, $\widehat{C}_{1i} = \begin{bmatrix} C_{1i} \\ C_{1i} \end{bmatrix}$,
$\widehat{A}_{2i} = \begin{bmatrix} A_{2i} + B_{2i}\widehat{K}_i & -B_{2i}\widehat{K}_i \\ A_{2i} + B_{2i}\widehat{K}_i & -B_{2i}\widehat{K}_i \end{bmatrix}$, $\widehat{A}_{d2i} = \begin{bmatrix} A_{d2i} & 0 \\ A_{d2i} & 0 \end{bmatrix}$, $\widehat{C}_{2i} = \begin{bmatrix} C_{2i} \\ C_{2i} \end{bmatrix}$,
$\widehat{D}_{1i} = [\ D_{1i} + G_{1i}\widehat{K}_i \quad -G_{1i}\widehat{K}_i\ ]$, $\widehat{D}_{d1i} = [\ D_{d1i} \quad 0\ ]$.

Definition 5. *(SFT H_∞ boundedness)*

The CLS (33) is said to be SFT H_∞-bounded with respect to $(\rho_1, \rho_2, \widehat{R}_i, \widehat{T}, h, \gamma)$ if the following two conditions hold:

(a) The MJS (33) satisfies SFT boundedness with respect to $(\rho_1, \rho_2, \widehat{R}_i, \widehat{T}, h)$;

(b) Under the zero initial condition, for the external disturbance $v(k)$ satisfying condition (2), the control output $z(k)$ satisfies the following:

$$\mathbb{E}\{\sum_{k=0}^{\widehat{T}} z^T(k)z(k)\} < \gamma^2\mathbb{E}\{\sum_{k=0}^{\widehat{T}} v^T(k)v(k)\}, \tag{34}$$

where $\widehat{R}_i > 0,\ i \in S, 0 < \rho_1 < \rho_2, \gamma > 0$.

Definition 6. *(SFT H_∞ observer-based state feedback stabilization)*

The MJS (1) is said to be SFT H_∞ observer-based state feedback stabilization with respect to $(\rho_1, \rho_2, \widehat{R}_i, \widehat{T}, h, \gamma)$ if there exists an observer-based state feedback controller (32) such that the CLS (33) satisfies SFT H_∞ boundedness, and the controller (32) is called the SFT H_∞ observer-based state feedback controller.

4.2. Main Results

Theorem 3. *The CLS (33) with the partly unknown TPs is SFT H_∞-bounded with respect to $(\rho_1, \rho_2, \widehat{R}_i, \widehat{T}, \gamma, h)$ if there exist the scalars $\beta > 1$ and $\gamma > 0$, matrix $\widehat{M} > 0$, and positive-definite matrices $\widehat{P}_i$ for all $i \in S$, satisfying the following conditions:*

$$\begin{bmatrix} -\beta\widehat{P}_i + \widehat{M} + \Gamma_{1i}^T + \widehat{D}_{1i}^T\widehat{D}_{1i} & \Gamma_{2i}^T + \widehat{D}_{1i}^T\widehat{D}_{d1i} & \Gamma_{3i}^T + \widehat{D}_{1i}^T G_{2i} \\ * & \Gamma_{4i}^T - \widehat{M} + \widehat{D}_{d1i}^T\widehat{D}_{d1i} & \Gamma_{5i}^T + \widehat{D}_{d1i}^T G_{2i} \\ * & * & \Gamma_{6i}^T - \gamma^2 + G_{2i}^T G_{2i} \end{bmatrix} < 0, \tag{35}$$

$$\beta^{\widehat{T}}\rho_1\left[\sup_{i\in S}\{\lambda_{max}(\widetilde{P}_i)\} + \sup_{i\in S}\{\lambda_{max}(\widetilde{M}_i)\}\tau\right] + \gamma^2\beta^{\widehat{T}}h < \inf_{i\in S}\{\lambda_{min}(\widetilde{P}_i)\}\rho_2, \tag{36}$$

where $\widehat{P}_i = diag\{P_i, P_i\}$, $\widehat{M} = diag\{M, M\}$, $\widetilde{P}_i = \widehat{R}_i^{-\frac{1}{2}}\widehat{P}_i\widehat{R}_i^{-\frac{1}{2}}$, $\widetilde{M}_i = \widehat{R}_i^{-\frac{1}{2}}\widehat{M}\widehat{R}_i^{-\frac{1}{2}}$,
$\Gamma_{1i}^T = \widehat{A}_{1i}^T\widehat{\Psi}_i\widehat{A}_{1i} + \widehat{A}_{2i}^T\widehat{\Psi}_i\widehat{A}_{2i}$, $\Gamma_{2i}^T = \widehat{A}_{1i}^T\widehat{\Psi}_i\widehat{A}_{d1i} + \widehat{A}_{2i}^T\widehat{\Psi}_i\widehat{A}_{d2i}$, $\Gamma_{3i}^T = \widehat{A}_{1i}^T\widehat{\Psi}_i\widehat{C}_{1i} + \widehat{A}_{2i}^T\widehat{\Psi}_i\widehat{C}_{2i}$,
$\Gamma_{4i}^T = \widehat{A}_{d1i}^T\widehat{\Psi}_i\widehat{A}_{d1i} + \widehat{A}_{d2i}^T\widehat{\Psi}_i\widehat{A}_{d2i}$, $\Gamma_{5i}^T = \widehat{A}_{d1i}^T\widehat{\Psi}_i\widehat{C}_{1i} + \widehat{A}_{d2i}^T\widehat{\Psi}_i\widehat{C}_{2i}$, $\Gamma_{6i}^T = \widehat{C}_{1i}^T\widehat{\Psi}_i\widehat{C}_{1i} + \widehat{C}_{2i}^T\widehat{\Psi}_i\widehat{C}_{2i}$,
$\widehat{\Psi}_i = \sum_{j\in S_k^i}\pi_{ij}\widehat{P}_j + (1-\pi_k^i)(\sum_{j\in S_{uk}^i}\widehat{P}_j)$.

Proof. The proof procedure is similar to Theorem 1, and thus will not be reiterated. □

From the above discussion, system (33) is SFT H_∞-bounded. Then, the following theorem will develop the observer-based state feedback controller for system (33).

Theorem 4. *The CLS (33) with the partly unknown TPs is SFT H_∞-bounded with respect to $(\rho_1, \rho_2, \widehat{R}_i, \widehat{T}, \gamma, h)$ if there exist the scalars $\beta > 1$, $\gamma > 0$, $\rho_2 > 0$, $\sigma_2 > 0$, $\varrho_1 > 0$, $\varrho_2 > 0$, matrix $J > 0$, positive-definite matrices X_i, nonsingular matrices Z_i, and matrices Y_i and F_i for all $i \in S$, satisfying the following conditions:*

$$\begin{bmatrix} -\beta\widehat{X}_i & 0 & \Xi_{\widetilde{A}_{1i}}^T & \Xi_{\widetilde{A}_{2i}}^T & \widetilde{D}_{1i}^T & \widehat{X}_i \\ * & -\gamma^2 I & \Xi_{\widehat{C}_{1i}}^T & \Xi_{\widehat{C}_{2i}}^T & G_{2i}^T & 0 \\ * & * & -\widehat{X} + \Xi_{\widehat{A}_{d1i}}\widehat{J}\Xi_{\widehat{A}_{d1i}}^T & 0 & 0 & 0 \\ * & * & * & -\widehat{X} + \Xi_{\widehat{A}_{d2i}}\widehat{J}\Xi_{\widehat{A}_{d2i}}^T & 0 & 0 \\ * & * & * & * & -I + \widehat{D}_{d1i}\widehat{J}\widehat{D}_{d1i}^T & 0 \\ * & * & * & * & * & -\widehat{J} \end{bmatrix} < 0, \tag{37}$$

$$D_iX_i = Z_iD_i, \tag{38}$$

$$\sigma_2\widehat{R}_i^{-1} < \widehat{X}_i < \widehat{R}_i^{-1}, \tag{39}$$

$$\varrho_1\widehat{R}_i^{-1} < \widehat{J} < \varrho_2\widehat{R}_i^{-1}, \tag{40}$$

$$\begin{bmatrix} -\beta^{-\widehat{T}}\rho_2 + \gamma^2 h & \sqrt{\rho_1} & \sqrt{\tau\rho_1} \\ * & -\sigma_2 & 0 \\ * & * & -\varrho_1 \end{bmatrix} < 0, \tag{41}$$

where $\Xi_{\widetilde{A}_{1i}}^T = [\sqrt{\pi_{i\zeta_1}}\widetilde{A}_{1i}^T\ \sqrt{\pi_{i\zeta_2}}\widetilde{A}_{1i}^T\ \cdots\ \sqrt{\pi_{i\zeta_{p_i}}}\widetilde{A}_{1i}^T\ \sqrt{1-\pi_k^i}\widetilde{A}_{1i}^T\ \sqrt{1-\pi_k^i}\widetilde{A}_{1i}^T\ \cdots\ \sqrt{1-\pi_k^i}\widetilde{A}_{1i}^T]$,
$\Xi_{\widetilde{A}_{2i}}^T = [\sqrt{\pi_{i\zeta_1}}\widetilde{A}_{2i}^T\ \sqrt{\pi_{i\zeta_2}}\widetilde{A}_{2i}^T\ \cdots\ \sqrt{\pi_{i\zeta_{p_i}}}\widetilde{A}_{2i}^T\ \sqrt{1-\pi_k^i}\widetilde{A}_{2i}^T\ \sqrt{1-\pi_k^i}\widetilde{A}_{2i}^T\ \cdots\ \sqrt{1-\pi_k^i}\widetilde{A}_{2i}^T]$,
$\widetilde{A}_{1i} = \begin{bmatrix} A_{1i}X_i + B_{1i}Y_i & -B_{1i}Y_i \\ 0 & A_{1i}X_i - F_iD_i \end{bmatrix}$, $\widetilde{A}_{2i} = \begin{bmatrix} A_{2i}X_i + B_{2i}Y_i & -B_{2i}Y_i \\ A_{2i}X_i + B_{2i}Y_i & -B_{2i}Y_i \end{bmatrix}$,
$\Xi_{\widehat{A}_{d1i}}^T = [\sqrt{\pi_{i\zeta_1}}\widehat{A}_{d1i}^T\ \sqrt{\pi_{i\zeta_2}}\widehat{A}_{d1i}^T\ \cdots\ \sqrt{\pi_{i\zeta_{p_i}}}\widehat{A}_{d1i}^T\ \sqrt{1-\pi_k^i}\widehat{A}_{d1i}^T\ \sqrt{1-\pi_k^i}\widehat{A}_{d1i}^T\ \cdots\ \sqrt{1-\pi_k^i}\widehat{A}_{d1i}^T]$,
$\Xi_{\widehat{A}_{d2i}}^T = [\sqrt{\pi_{i\zeta_1}}\widehat{A}_{d2i}^T\ \sqrt{\pi_{i\zeta_2}}\widehat{A}_{d2i}^T\ \cdots\ \sqrt{\pi_{i\zeta_{p_i}}}\widehat{A}_{d2i}^T\ \sqrt{1-\pi_k^i}\widehat{A}_{d2i}^T\ \sqrt{1-\pi_k^i}\widehat{A}_{d2i}^T\ \cdots\ \sqrt{1-\pi_k^i}\widehat{A}_{d2i}^T]$,
$\Xi_{\widehat{C}_{1i}}^T = [\sqrt{\pi_{i\zeta_1}}\widehat{C}_{1i}^T\ \sqrt{\pi_{i\zeta_2}}\widehat{C}_{1i}^T\ \cdots\ \sqrt{\pi_{i\zeta_{p_i}}}\widehat{C}_{1i}^T\ \sqrt{1-\pi_k^i}\widehat{C}_{1i}^T\ \sqrt{1-\pi_k^i}\widehat{C}_{1i}^T\ \cdots\ \sqrt{1-\pi_k^i}\widehat{C}_{1i}^T]$,
$\Xi_{\widehat{C}_{2i}}^T = [\sqrt{\pi_{i\zeta_1}}\widehat{C}_{2i}^T\ \sqrt{\pi_{i\zeta_2}}\widehat{C}_{2i}^T\ \cdots\ \sqrt{\pi_{i\zeta_{p_i}}}\widehat{C}_{2i}^T\ \sqrt{1-\pi_k^i}\widehat{C}_{2i}^T\ \sqrt{1-\pi_k^i}\widehat{C}_{2i}^T\ \cdots\ \sqrt{1-\pi_k^i}\widehat{C}_{2i}^T]$,

$\widetilde{D}_{1i} = [\, D_{1i}X_i + G_{1i}Y_i \;\; -G_{1i}Y_i \,]$, $\widehat{X}_i = diag\{X_i, X_i\}$, $\widehat{J} = diag\{J, J\}$, $\widehat{R}_i = diag\{R_i, R_i\}$, $\widehat{X} = diag\{\widehat{X}_{\zeta_1}, \widehat{X}_{\zeta_2}, \cdots, \widehat{X}_{\zeta_{p_i}}, \widehat{X}_{\zeta_{u1}}, \widehat{X}_{\zeta_{u2}}, \cdots, \widehat{X}_{\zeta_{uq_i}}\}$.

Then, MJS (1) is called SFT H_∞ observer-based state feedback stabilization, and the controller gain $\widehat{K}_i$ as well as the observer gain H_i are represented as follows:

$$\widehat{K}_i = Y_i X_i^{-1},\; H_i = F_i Z_i^{-1}. \tag{42}$$

Proof. Defining $\widehat{R}_i = diag\{R_i, R_i\}$, $\widehat{P}_i = diag\{P_i, P_i\}$, $X_i = P_i^{-1}$, $\widehat{X} = \widehat{P}^{-1}, \widehat{M} = diag\{M, M\}$, $M^{-1} = J$, $\widehat{K}_i = Y_i X_i^{-1}$, $H_i = F_i Z_i^{-1}$, and taking into account condition (38), (35) will be obtained from (37) via Lemma 1. In addition, we denote $\widetilde{P}_i = \widehat{R}_i^{-\frac{1}{2}} \widehat{P}_i \widehat{R}_i^{-\frac{1}{2}}$, $\widetilde{M}_i = \widehat{R}_i^{-\frac{1}{2}} \widehat{M} \widehat{R}_i^{-\frac{1}{2}}$. According to the proof of Theorem 2, it is obvious that condition (36) will be guaranteed by (39) to (41). □

Remark 6. *Theorems 3 and 4 extend the results of [38,44] to MJSs with partly unknown TPs.*

Remark 7. *Addressing condition (38) through the application of the LMI toolbox is a challenging task. As a solution, constraint (38) can be approximated by the following inequality:*

$$[D_i X_i - Z_i D_i]^T [D_i X_i - Z_i D_i] < \varpi I, \tag{43}$$

where ϖ represents an exceedingly small positive scalar. According to Lemma 1, the above inequality can be formulated as follows:

$$\begin{bmatrix} -\varpi I & [D_i X_i - Z_i D_i]^T \\ * & -I \end{bmatrix} < 0. \tag{44}$$

Remark 8. *We can note that conditions (26), (29), (37), and (41) are not strict LMIs; however, once we fix the parameters α and β, the conditions can be turned into LMI-based feasibility problems. Therefore, the feasibility of the conditions stated in Theorems 2 and 4 can be turned into the following feasibility problems with the fixed parameters α and β, respectively:*

$$\begin{aligned} &min\ (\rho_2 + \gamma^2) \\ &s.t.\ LMIs\ (26),\ (27),\ (28)\ and\ (29), \end{aligned} \tag{45}$$

$$\begin{aligned} &min\ (\rho_2 + \gamma^2) \\ &s.t.\ LMIs\ (37),\ (39),\ (40),\ (41)\ and\ (44). \end{aligned} \tag{46}$$

5. Numerical Examples

In this section, we present two examples to validate the effectiveness and practicality of the proposed method. The first example is used to show the effectiveness of the state feedback controller (8) design approach developed in Theorem 2 for MJSs (1) with partly unknown transition probabilities.

Example 1. *Consider MJS (1) with three modes, and the coefficient matrices are given as follows:*

Mode 1 ($i = 1$):
$A_{11} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $A_{d11} = \begin{bmatrix} 0.8 & 0 \\ -0.2 & 0.1 \end{bmatrix}$, $B_{11} = \begin{bmatrix} 1 \\ -0.1 \end{bmatrix}$, $C_{11} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$, $A_{21} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.1 \end{bmatrix}$, $A_{d21} = \begin{bmatrix} -0.1 & 1 \\ 0 & 0.1 \end{bmatrix}$, $B_{21} = \begin{bmatrix} -0.1 \\ 0.3 \end{bmatrix}$, $C_{21} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$, $D_{11} = [\; -0.2 \quad 1 \;]$, $D_{d11} = [\; -0.1 \quad 0 \;]$, $D_1 = [\; 1 \quad 1 \;]$, $G_1 = 1$, $G_{11} = -1$, $G_{21} = 0.1$.

Mode 2 ($i = 2$):
$A_{12} = \begin{bmatrix} 1 & 0 \\ 0.3 & 1 \end{bmatrix}$, $A_{d12} = \begin{bmatrix} 0 & 0.1 \\ 0.1 & -0.1 \end{bmatrix}$, $B_{12} = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}$, $C_{12} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$, $A_{22} =$

$\begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $A_{d22} = \begin{bmatrix} -0.1 & 1 \\ 0 & 0.1 \end{bmatrix}$, $B_{22} = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix}$, $C_{22} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$, $D_{12} = \begin{bmatrix} 1 & 0.1 \end{bmatrix}$, $D_{d12} = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}$, $D_2 = \begin{bmatrix} 2 & 1 \end{bmatrix}$, $G_2 = 1$, $G_{12} = -1$, $G_{22} = 0.2$.

Mode 3 ($i = 3$):
$A_{13} = \begin{bmatrix} -1 & 0.1 \\ 0 & 0.1 \end{bmatrix}$, $A_{d13} = \begin{bmatrix} -0.1 & 0.1 \\ 0 & 0.1 \end{bmatrix}$, $B_{13} = \begin{bmatrix} 1.1 \\ -0.1 \end{bmatrix}$, $C_{13} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$, $A_{23} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.1 \end{bmatrix}$, $A_{d23} = \begin{bmatrix} 0.2 & 1 \\ 0 & 0.1 \end{bmatrix}$, $B_{23} = \begin{bmatrix} -0.1 \\ 0.4 \end{bmatrix}$, $C_{23} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$, $D_{13} = \begin{bmatrix} 0.1 & 1 \end{bmatrix}$, $D_{d13} = \begin{bmatrix} -0.1 & 0.1 \end{bmatrix}$, $D_3 = \begin{bmatrix} 1 & 3 \end{bmatrix}$, $G_3 = 1$, $G_{13} = -1$, $G_{23} = 0.1$.

The partly unknown TP matrix Π with three modes is given as follows:

$$\Pi = \begin{bmatrix} \hat{\pi}_{11} & 0.2 & \hat{\pi}_{13} \\ \hat{\pi}_{21} & \hat{\pi}_{22} & 0.8 \\ 0.1 & \hat{\pi}_{32} & \hat{\pi}_{33} \end{bmatrix},$$

where $\hat{\pi}_{ij}(i, j = 1, 2, 3)$ is the unknown element. One possible mode evolution is given in Figure 1.

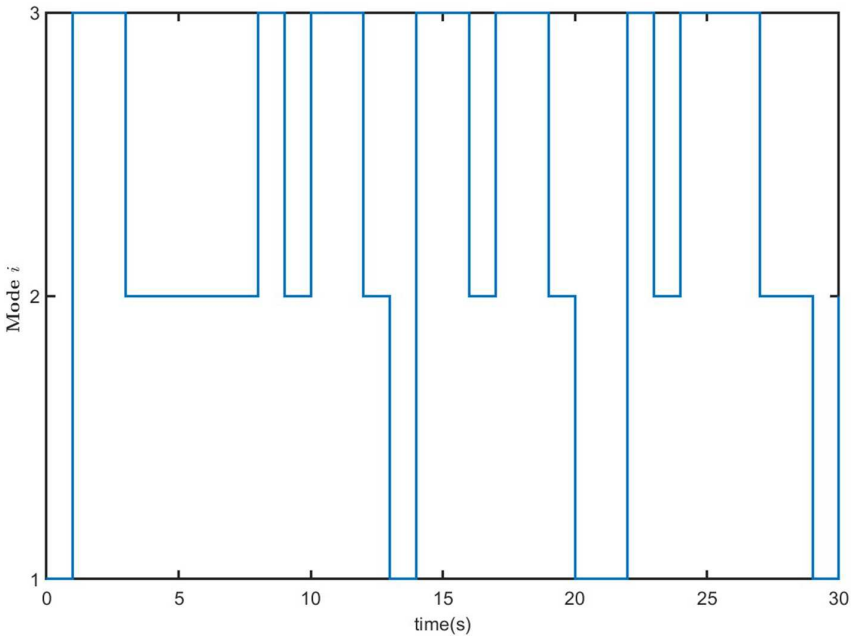


Figure 1. Markovian switching process of MJS (1) and CLS (9).

According to Remark 8, the minimum value of $\rho_2 + \gamma^2$ relies on the parameter α. We can obtain the feasible solution of (45) when $1.01 \leq \alpha \leq 2.01$. Figures 2 and 3 show the optimal values of ρ_2, γ^2 and $\rho_2 + \gamma^2$ with different α values. We can see that the optimal values $\rho_2 = 2.6437$, $\gamma^2 = 7.5107$, and $\gamma^2 + \rho_2 = 10.1544$ when $\alpha = 1.02$.

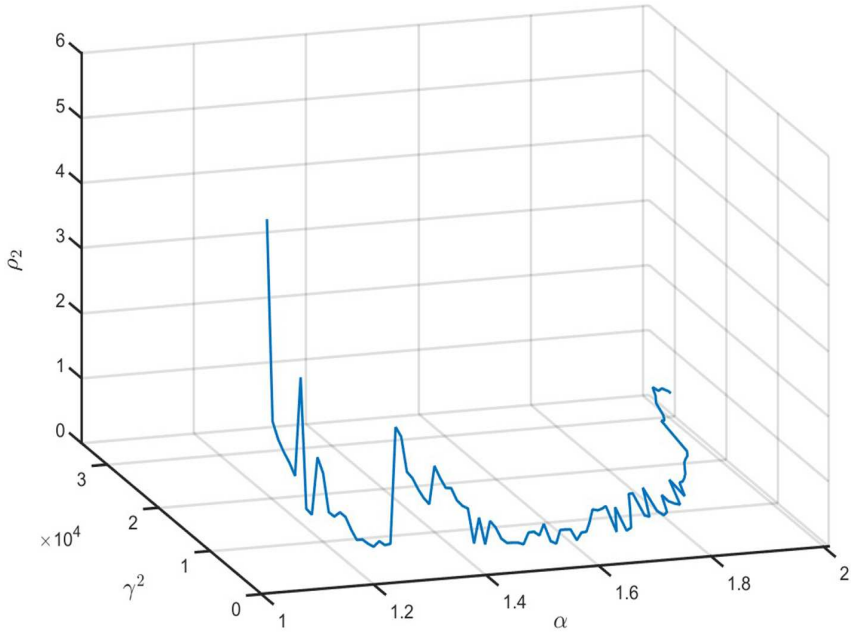


Figure 2. The values of ρ_2 and γ^2 with different α values.

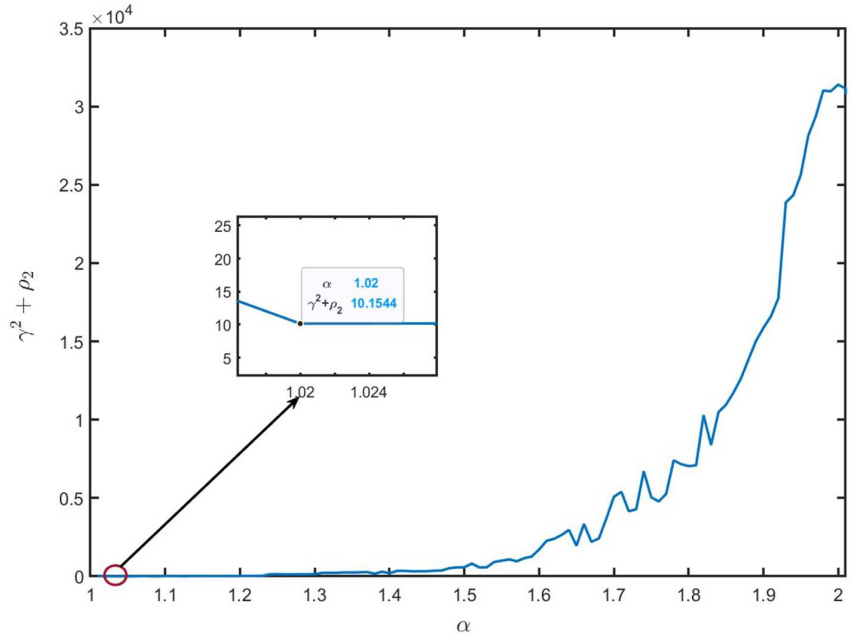


Figure 3. The values of $\gamma^2 + \rho_2$ with different α values.

Next, letting $\tau = 3$, $h = 3$, $\widehat{T} = 30$, $\rho_1 = 0.1$, and $R_i = I_{2\times 2}$ $(i = 1, 2, 3)$, and solving LMIs (26) to (29), we obtain $\sigma_1 = 0.1888$, $\xi_1 = 0.3535$, and $\xi_2 = 21.5646$, and the gains of the state feedback controller (8) are as follows:

$$K_1 = \begin{bmatrix} -0.1176 & 0.2082 \end{bmatrix}, K_2 = \begin{bmatrix} 0.9916 & 0.1625 \end{bmatrix}, K_3 = \begin{bmatrix} 0.4783 & 0.3299 \end{bmatrix}.$$

Then, we set the initial value $x(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ for MJS (1) and CLS (9), and the external disturbance signal $v(k) = 0.4\sin k$, which satisfies $\sum_{k=0}^{\widehat{T}} v^T(k)v(k) \leq h = 3$.

Figure 4 shows the trajectories of $x^T(k)R_ix(k)$ (50 curves) and $\mathbb{E}\{x^T(k)R_ix(k)\}$ of the open-loop system (1) ($u(k) = 0$). It can be seen that the trajectory of $\mathbb{E}\{x^T(k)R_ix(k)\}$ exceeds the upper bound ρ_2, despite $\mathbb{E}\{x^T(0)R_ix(0)\} = 0 < \rho_1 = 0.1$. This implies that the open-loop system (1) is not finite-time bound.

Figure 5 shows the trajectories of the system state $x(k)$ for closed-loop system (9) and the control input $u(k)$ of MJS (1). The trajectories of $x^T(k)R_ix(k)$ (50 curves) and $\mathbb{E}\{x^T(k)R_ix(k)\}$ of closed-loop system (9) are illustrated in Figure 6. From Figure 6, it is seen that when $\mathbb{E}\{x^T(0)R_ix(0)\} = 0 < \rho_1 = 0.1$, $\mathbb{E}\{x^T(k)R_ix(k)\} < \rho_2 = 2.6437$, which means that the CLS (9) is SFT H_∞-bounded, that is to say, MJS (1) is SFT H_∞ state feedback stabilization. Therefore, it is proven that the state feedback controller (8) designed in this paper is effective.

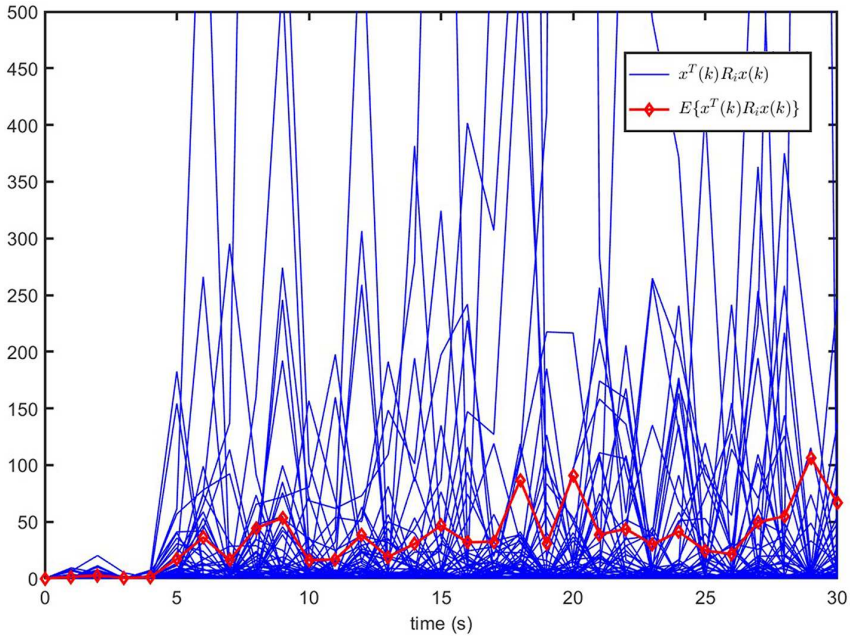


Figure 4. The trajectories of $x^T(k)R_ix(k)$ and $\mathbb{E}\{x^T(k)R_ix(k)\}$ for open-loop system (1) ($u(k) = 0$).

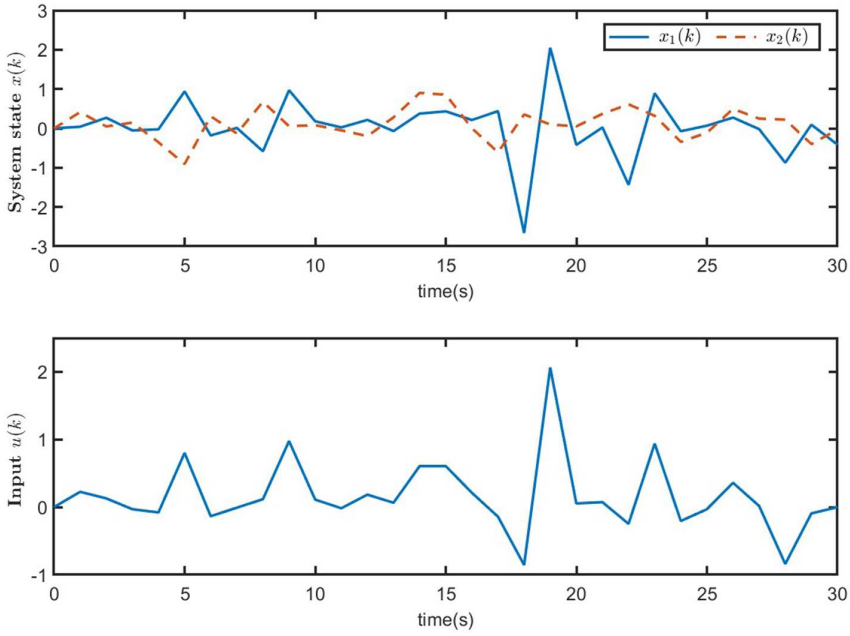


Figure 5. The trajectories of system state $x(k)$ for CLS (9) and control input $u(k)$ (8).

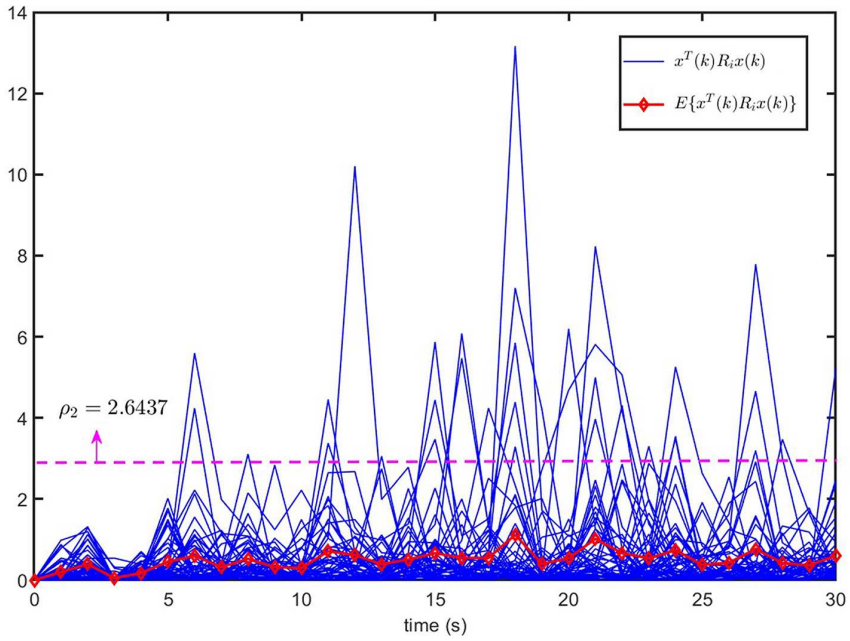


Figure 6. The trajectories of $x^T(k)R_ix(k)$ and $\mathbb{E}\{x^T(k)R_ix(k)\}$ for MJS (1) and CLS (9).

Next, the second example focuses on the effectiveness of the observer-based state feedback controller (32) designed in Theorem 4 for MJS (1) with partly unknown transition probabilities.

Example 2. *The parameters of MJS (1) with three modes and partly unknown TPs are given as follows:*

Mode 1 ($i = 1$):
$A_{11} = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.1 \end{bmatrix}$, $A_{d11} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $B_{11} = \begin{bmatrix} 1 \\ -0.1 \end{bmatrix}$, $C_{11} = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}$, $A_{21} = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.2 \end{bmatrix}$, $A_{d21} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $B_{21} = \begin{bmatrix} -0.1 \\ 0.1 \end{bmatrix}$, $C_{21} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$, $D_{11} = \begin{bmatrix} 0.5 & 0.1 \end{bmatrix}$, $D_{d11} = \begin{bmatrix} 0 & 0.5 \end{bmatrix}$, $D_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $G_1 = 1$, $G_{11} = 0.9$, $G_{21} = -0.1$.

Mode 2 ($i = 2$):
$A_{12} = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}$, $A_{d12} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}$, $B_{12} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$, $C_{12} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$, $A_{22} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}$, $A_{d22} = \begin{bmatrix} 0.1 & 0 \\ 0.2 & -0.1 \end{bmatrix}$, $B_{22} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$, $C_{22} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$, $D_{12} = \begin{bmatrix} 1 & 0.1 \end{bmatrix}$, $D_{d12} = \begin{bmatrix} 0.3 & 0.1 \end{bmatrix}$, $D_2 = \begin{bmatrix} 2 & 1 \end{bmatrix}$, $G_2 = 1$, $G_{12} = 1$, $G_{22} = 0.1$.

Mode 3 ($i = 3$):
$A_{13} = \begin{bmatrix} 0.1 & 0 \\ -0.2 & 0.1 \end{bmatrix}$, $A_{d13} = \begin{bmatrix} -0.2 & 0.2 \\ 0.1 & 0.2 \end{bmatrix}$, $B_{13} = \begin{bmatrix} 0.1 \\ -0.1 \end{bmatrix}$, $C_{13} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$, $A_{23} = \begin{bmatrix} -0.2 & 0 \\ 0 & -0.2 \end{bmatrix}$, $A_{d23} = \begin{bmatrix} 0 & 0.2 \\ 0.1 & 0.1 \end{bmatrix}$, $B_{23} = \begin{bmatrix} 1 \\ 0.1 \end{bmatrix}$, $C_{23} = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}$, $D_{13} = \begin{bmatrix} 0.1 & -0.2 \end{bmatrix}$, $D_{d13} = \begin{bmatrix} 0.2 & 0 \end{bmatrix}$, $D_3 = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $G_3 = 2$, $G_{13} = 2$, $G_{23} = 0.1$.

Then, letting $\varpi = 10^{-10}$, $\widehat{R}_i = I_{4\times4}$ $(i = 1, 2, 3)$, the partly unknown TP matrix Π and the remaining parameters have identical values to those in Example 1. Similar to Example 1, we can obtain the feasible solution of (46) when $1.01 \leq \beta \leq 2.04$. The relationships between β and γ^2 and ρ_2, and between β and $\gamma^2 + \rho_2$ are shown in Figures 7 and 8, respectively. From Figures 7 and 8, we can see that the optimal values are $\rho_2 = 28.5424$ and $\gamma^2 = 92.5307$ with $\beta = 1.03$.

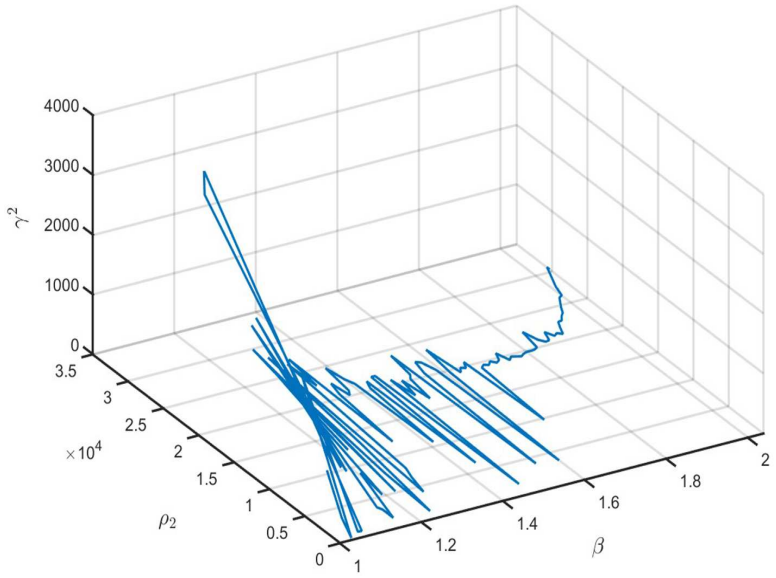


Figure 7. The optimal values of ρ_2 and γ^2 with different β values.

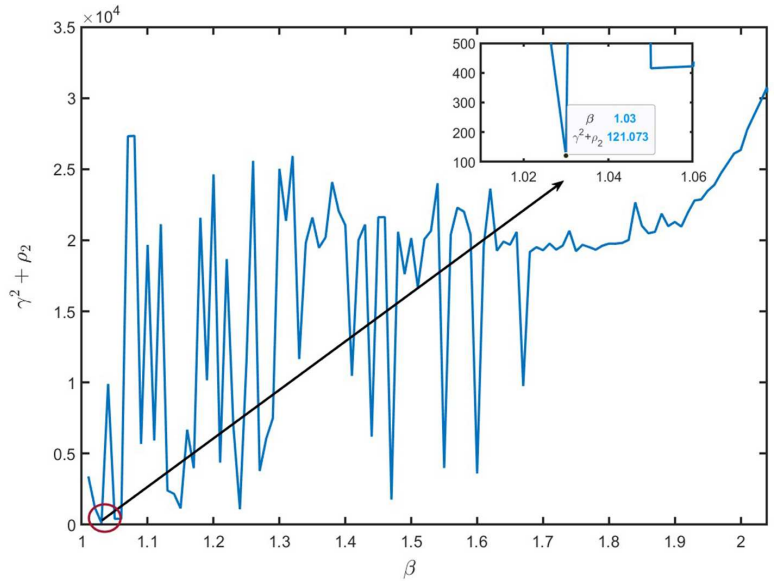


Figure 8. The values of $\gamma^2 + \rho_2$ with different β values.

Then, we compare the results of Theorem 4 with Theorem 2 in [44]. The optimal values of ρ_2 (i.e., τ in [44]) and γ obtained from the two works are shown in Table 2.

Table 2. The optimal values of γ, ρ_2, and $\gamma^2 + \rho_2$.

Method	Theorem 4 in This Paper	Theorem 2 in Reference [44]
γ	9.6193	105.9526
ρ_2	28.5424	150.4146
$\gamma^2 + \rho_2$	121.0731	11,376.368

From Table 2, it appears that the optimal values of γ, ρ_2, and $\gamma^2 + \rho_2$ obtained in this paper are smaller than those of [44], which indicates that the results of this paper are better. In addition, ref. [44] assumed that the transition probabilities were completely known, which means that the results of [44] are special cases of this paper.

In addition, by solving LMIs (37), (39), (40), (41) ,and (44), we have $\sigma_2 = 0.1779$, $\varrho_1 = 0.9901$, and $\varrho_2 = 5.2268$, and the gains of observer-based state feedback controller (32) are as follows:

$$\widehat{K}_1 = \begin{bmatrix} -0.2337 & -0.0433 \end{bmatrix}, \widehat{K}_2 = \begin{bmatrix} -0.4995 & -0.0534 \end{bmatrix}, \widehat{K}_3 = \begin{bmatrix} 0.0599 & 0.0414 \end{bmatrix},$$

$$H_1 = \begin{bmatrix} -0.0501 \\ 0.0500 \end{bmatrix}, H_2 = \begin{bmatrix} -0.0541 \\ -0.0623 \end{bmatrix}, H_3 = \begin{bmatrix} 0.0429 \\ 0.0527 \end{bmatrix}.$$

Next, we set the initial value $x(0) = \widehat{x}(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ for systems (1) and (33), respectively. The external disturbance signal $v(k)$ is the same as in Example 1. The Markovian switching process of MJS (1) and CLS (33) is shown in Figure 9. Figure 10 shows the trajectories of $x^T(k)R_ix(k)$ (50 curves) and $\mathbb{E}\{x^T(k)R_ix(k)\}$ of open-loop system (1) $(u(k) = 0)$, which implies that open-loop system (1) is not finite time-bound.

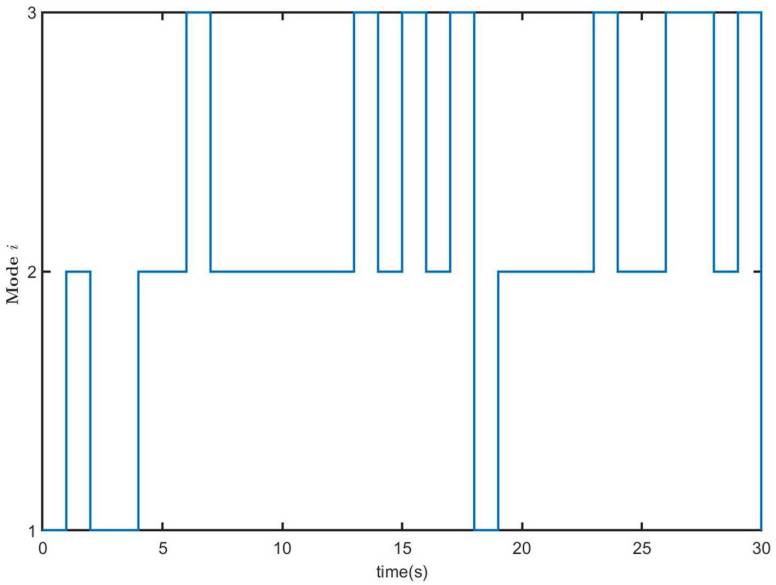


Figure 9. Markovian switching process of MJS (1) and CLS (33).

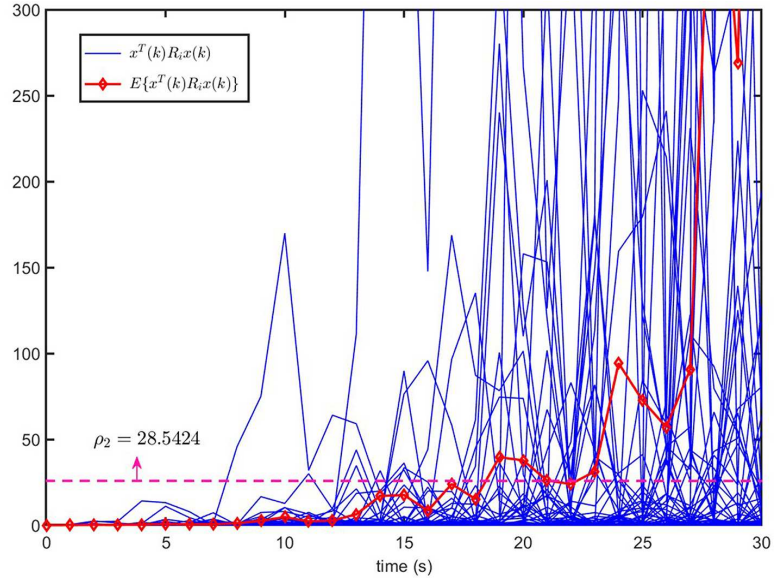


Figure 10. The trajectories of $x^T(k)R_ix(k)$ and $\mathbb{E}\{x^T(k)R_ix(k)\}$ for open-loop system (1) $(u(k) = 0)$.

The trajectories of system state $\eta(k)$ for CLS (33) and the curve of the control input $u(k)$ of (32) are illustrated in Figure 11. Moreover, Figure 12 shows the trajectories of $\eta^T(k)\widehat{R}_i\eta(k)$ (50 curves) and $\mathbb{E}\{\eta^T(k)\widehat{R}_i\eta(k)\}$ of closed-loop system (33). From Figure 12, it can be observed that CLS (33) is SFT H_∞-bounded, i.e., MJS (1) is SFT H_∞ observer-based state feedback stabilization. Furthermore, by comparing Figures 10 and 12, it can be proven that observer-based state feedback controller (32) is effective.

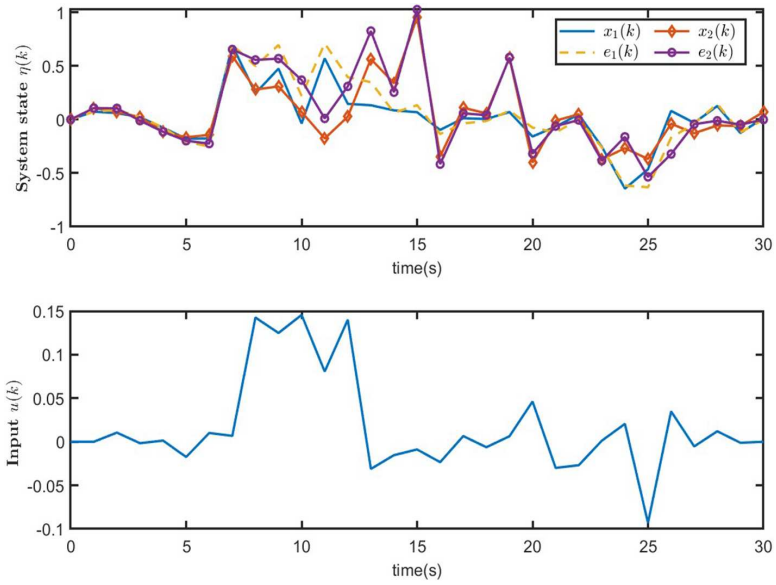


Figure 11. The trajectories of system state $\eta(k)$ for CLS (33) and control input $u(k)$ (32).

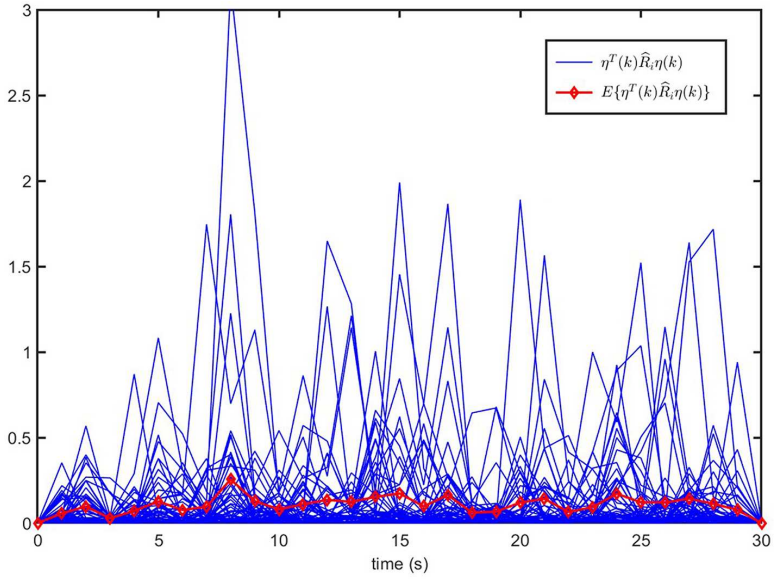


Figure 12. The trajectories of $\eta^T(k)\widehat{R}_i\eta(k)$ and $\mathbb{E}\{\eta^T(k)\widehat{R}_i\eta(k)\}$ for CLS (33).

6. Conclusions

Based on existing results, the design schemes of a stochastic finite-time H_∞ state feedback controller and a stochastic finite-time H_∞ observer-based state feedback controller for MJSs with a time delay and partly unknown TPs were studied in this paper. A state feedback controller and an observer-based state feedback controller were designed and some sufficient conditions for the CLSs to satisfy SFT H_∞ boundedness were presented via LKF technology. Then, the controller gains were obtained by using the LMI method. Lastly, two examples were provided to verify the validity of the proposed design schemes.

In the following work, the finite-time guaranteed cost control and event-triggered control of discrete-time MJSs will be studied on the basis of this paper.

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