A Transition-Matrix Growth Model and Equilibrium Curve for Natural Forests in Jiangxi Province

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Abstract: Researchers build growth models to predict the growth of forest stands and propose management measures to improve the overall quality of these stands. In this study, data collected from 91 sample plots from the eighth (2010) and ninth (2015) Chinese National Forest Inventories in Jiangxi Province were used to establish a transition-matrix growth model. Then, 12 potential equilibrium curves were set to guide forest management, and a transition-matrix growth model was used to predict stand growth in Jiangxi Province. In each 10-year management period, trees with diameters that exceeded the equilibrium curve were cut down. The results show that species diversity ($H_1$), size diversity ($H_2$), and basal area (B) have statistically significant influences on growth, mortality, and recruitment. Moreover, the high accuracy of the transition-matrix growth model is demonstrated. According to the simulation results, $B = 35 \text{ m}^2/\text{ha}$, the maximum diameter of retained trees $D_{\text{max}} = 45 \text{ cm}$ and the adjacent diameter ratio $q = 1.7$ constitute the optimal equilibrium curve to guide forest management. The diameter distribution guided by the equilibrium curve is reverse J-shaped and is associated with significant increases in the hardwood stock volume and current annual growth. Under the guidance of the equilibrium curve, the forests in Jiangxi Province can be reasonably managed, produce more high-economic-value timber, and achieve a more stable species composition. This study will help maximize the ecological and economic benefits of forests and provide a reference for the realization of the sustainable development of forestry. Furthermore, the results can be used to improve the facility and accuracy of natural forest harvesting.

Keywords: transition-matrix growth model; equilibrium curve; forest management; stock growth; natural forest

1. Introduction

Forests constitute 60% of the area of Jiangxi Province, China, which has a total area of 10.8 million hm$^2$ and a volume of 506,658,300 m$^3$ [1–3]. The natural forest area in Jiangxi Province is 6.52 million hectares, accounting for 60.49% of the total forest area with a total volume of 367,721,800 m$^3$. Due to historical anthropogenic disturbance, however, the natural forests in the province are of poor quality. For example, the average stocking per hectare is 70.66 m$^3$ compared to healthier forests in Germany (320 m$^3$) [4]. Furthermore, the natural forests are dominated by coppicing trees, which are short with small diameters, curved trunks, and low utilization. Ganatsas found that high forests exhibit significantly higher productivity and carbon storage capacity than coppice forests [5].

In this context, since 2017, China has suspended the commercial logging of natural forests. In the past, suspended logging meant that forests were completely protected, and no cuttings were allowed, which resulted in excessive stand density, as well as the increased risk of pests, diseases, and forest fires [6–8]. Peng and Qiao argued that natural forest protection requires active management to promote restoration [9]. The “Suggestions...
on Problems and Countermeasures in the Protection and Sustainable Management of Natural Forests” issued by the Chinese National Forestry and Grassland Administration (CNFGA) stressed that natural forests must be scientifically managed, rather than simply “fenced in”. Thus, the determination of how to sustainably manage natural forests and formulate optimal forest management strategies is an urgent issue [10]. In contrast to even-aged plantations, the optimal forest management measures for natural forests are not easily determined due to their complex structure and species composition [11].

System structure determines system function [12–14]; thus, forest structure determines forest function [15,16]. The quantification of the optimal forest structure is, therefore, of great importance for informing forest management. Due to the simple forest structure of even-aged plantations, forest density is often quantified as a unique index to guide plantation management. Stand density management diagrams have been developed for different tree species worldwide [17–19], e.g., the China fir [20], Scots pine, Austrian black pine [21], Pinus massoniana [22], Radiata pine [23], and Norway spruce [24].

Compared to even-aged plantations, natural forests are characterized by complex species compositions and forest structures; thus, the stand density cannot be used as the sole index by which to prescribe forest management regimes. Equilibrium curves are diameter distributions that consider the growth, mortality, harvesting, and recruitment of a stand under a specific harvesting strategy. They ensure a stable diameter distribution and allow for sustainable forest management [25,26]. For example, Brzeziecki used equilibrium curves to guide stocking and structure regulation in mixed stands in northeast Poland [27]. Furthermore, López determined the equilibrium diameter distribution of the European beech and identified the appropriate harvesting intensity [28].

Forest growth models are widely used to predict the dynamics of forest growth under different management programs and serve as a basis for forest management measures [29–35]. The determination of the equilibrium curve requires the construction of a forest growth model. For example, Sterba used an individual-tree model to determine the equilibrium curve [26], whereas López produced a transition-matrix model to derive the equilibrium curve [28].

The forest growth and yield models can be divided into the whole-stand, individual-tree, and size-class models [36–39]. Whole-stand models developed for even-aged monocultures cannot predict the dynamics of uneven-aged, mixed-species forests [40,41]. Although individual-tree models have robust predictive performance for uneven-aged, mixed-species forests [42–46], their high costs hinder development and application. Size-class growth models represent a trade-off between the whole-stand and individual-tree model and are widely used to inform uneven-aged, mixed-species forest management [47]. Size-class growth models can be classified into projection methods and matrix models, among which projection methods are only used for short-term forecasting. Due to the need to make long-term forecasts in the present study, the matrix model was adopted to determine the optimal equilibrium curve. Furthermore, many scholars have documented that transition-matrix growth models achieve similar predictive performance as individual-tree growth models when modeling uneven-aged mixed forests [48–52].

Because of the current Chinese policy on natural forest management, there is an urgent need for the rational management of natural forests to improve forest quality and increase carbon storage capacity. There is a lack of relative growth modeling of natural forests in Jiangxi Province and the absence of an instrument with which to guide the management of natural forests in this area. In particular, with carbon stock as a current research hotspot, the findings of this study can enhance the carbon sequestration capacity of forests by addressing the low quality of forest stands, providing assistance in mitigating greenhouse gas emissions. The findings of this research can be used to resolve the low quality of natural forests in Jiangxi Province, compensate for the absence of natural forest management in this area, effectively predict the change in forest growth, and make timely adjustments to the management of harvesting. The following three steps were taken to achieve these objectives: (1) modeling the transition-matrix growth, (2) determining the
optimal equilibrium curves for natural forests in Jiangxi Province, and (3) simulating future forest growth.

The main novelty of this study is the determination of the optimal equilibrium curve to quantitatively guide the management of the natural forests in Jiangxi Province. Although the general methodology of previous studies is adopted to construct the transfer-matrix growth model, this study presents the first model built for the natural forests in Jiangxi Province, which is of great importance for local decision-making. A transition-matrix growth model is first constructed, and 12 potential equilibrium curves are produced using different values of the basal area (B), the maximum diameter of retained trees (D_{max}), and the adjacent diameter ratio (q). Next, the optimal equilibrium curves are determined using the transition-matrix model. Finally, under the guidance of the optimal equilibrium curve, the change in forest growth in the next 50 years is simulated, the changes in the composition of tree species, the stock volume, the diameter distribution, etc., are observed, and the change in the forest is analyzed. Increasing the forest stock is found to ensure the stability of the forest structure, improve the timber productivity, and increase the valuation of the forest trees.

2. Methods

2.1. Study Area

The study area is located in Jiangxi Province (113°34′18″–118°28′56″ E, 24°29′14″–30°04′43″ N) in southern China (Figure 1). Jiangxi Province is dominated by hills and mountains, with an average altitude of 350 m. The region has a subtropical warm and humid monsoon climate, with mild winters and cool summers and an annual average precipitation of 1300–1950 mm. The main soil types are red and yellow soils [53].

![Figure 1. Jiangxi Province, China.](image)

2.2. Data and Analysis

Data from the eighth and ninth Chinese National Forest Inventories (CNFIs) in Jiangxi Province were utilized. The plots were distributed within 4 km × 6 km grids, and each plot was 0.067 hm². Each tree in the plot was numbered, and its identity, diameter at breast height (DBH), slope, aspect, and origin were recorded over a 5-year survey cycle. The data collected from 91 sample plots from the eighth (2010) and ninth (2015) CNFIs in Jiangxi Province over 5-year measurement cycles were used; of these 91 sample plots, 64 were used for model construction and 27 were used for model validation. All the selected plots were natural forests without any management measures, such as logging and artificial cultivation. The trees were categorized by family into six tree species groups: oak, fir, softwood, *Schima superba*, hardwood, and others (the frequency of trees in this class was less than 5%). The tree species diversity (H_{1}), size diversity (H_{2}), DBH (D), basal area
(B), slope (SLcos), and trees per hectare (N) were modeled as independent variables, as summarized in Table 1.

Table 1. The statistics of the sample plots.

<table>
<thead>
<tr>
<th></th>
<th>(H_1)</th>
<th>(H_2)</th>
<th>B ((\text{m}^2/\text{ha}))</th>
<th>SLcos</th>
<th>N (Trees/ha)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>1.78</td>
<td>2.13</td>
<td>57.61</td>
<td>40.00</td>
<td>4575</td>
</tr>
<tr>
<td>Min</td>
<td>0.01</td>
<td>0.23</td>
<td>5.27</td>
<td>−48.00</td>
<td>750</td>
</tr>
<tr>
<td>Mean</td>
<td>1.12</td>
<td>1.47</td>
<td>26.87</td>
<td>−0.99</td>
<td>2483</td>
</tr>
<tr>
<td>SD</td>
<td>0.37</td>
<td>0.36</td>
<td>10.63</td>
<td>21.44</td>
<td>816</td>
</tr>
</tbody>
</table>

2.3. Model Structure

The general expression of the transition-matrix model is given by:

\[
y_{t+1} = G_t (y_t - h_t) + R_t + \epsilon_t,
\]

where \(y_t = [y_{ijt}]\) represents the number of surviving trees of species \(i\) (\(i = 1, 2, 3, \ldots, \text{sp}\)) in diameter class \(j\) (\(j = 1, 2, 3, \ldots, \text{dc}\)) at time \(t\), \(t\) represents a certain moment in time, and \(G_t\) represents the growth and mortality of different tree species between \(t\) and \(t + 1\). Moreover, \(h_t = [h_{ijt}]\) represents the number of harvested trees of species \(i\) in diameter class \(j\) at time \(t\) (if there is no harvest at time \(t\), the value is recorded as 0), and \(R_t\) represents the number of trees that grow into the minimum diameter class during the interval (the interval of this survey is 5 years). Finally, \(\epsilon_t\) represents the independent and normally distributed random error, the mean value of which is 0.

\(G\) and \(R\) are defined as follows:

\[
G = \begin{bmatrix}
G_1 \\
G_2 \\
\vdots \\
G_m
\end{bmatrix},
\]

\[
G_i = \begin{bmatrix}
\alpha_{i1} \\
\beta_{i1} \\
\alpha_{i2} \\
\beta_{i2} \\
\vdots \\
\alpha_{i,n-1} \\
\beta_{i,n-1} \\
\alpha_{in}
\end{bmatrix},
\]

\[
R = \begin{bmatrix}
R_1 \\
R_2 \\
\vdots \\
R_m
\end{bmatrix},
\]

\[
R_i = \begin{bmatrix}
R_i \\
0 \\
\vdots \\
0
\end{bmatrix},
\]

where \(\alpha_{ij}\) represents the probability that a tree of species \(i\) in diameter class \(j\) maintains its diameter class during the interval period, whereas \(b_{ij}\) represents the probability that a tree in species \(i\) and in diameter class \(j\) grows into diameter class \(j + 1\) between \(t\) and \(t + 1\). \(b_{ij}\) can be calculated by the annual growth (\(g\)) divided by the width of the diameter class. The relationship between \(\alpha_{ij}\) and \(b_{ij}\) can be expressed as follows:

\[
\alpha_{ij} = 1 - b_{ij} - m_{ij},
\]

where \(\alpha_{ij}, b_{ij},\) and \(m_{ij}\) represent the survival rate, advancement rate, and mortality rate, respectively. These parameters were once considered to be fixed in these models, but this is associated with problems with long-term prediction [41]. These problems can be overcome by the inclusion of more variables in the model, such as biodiversity and stand density [54].

The species diversity index (\(H_1\)), diameter diversity index (\(H_2\)), DBH (D), basal area (B), and slope (SLcos) were selected as independent variables. The parameter \(b_{ij}\) is calculated as the growth increment \(g_{ij}\) divided by the width of the diameter class, and \(g_{ij}\) is calculated as follows:

\[
\log(g_{ij} + 1) = \gamma_{i1} + \gamma_{i2}D_j + \gamma_{i3}D_j^2 + \gamma_{i4}H_1 + \gamma_{i5}H_2 + \gamma_{i6}B + \gamma_{i7}\text{SLcos} + \mu_{ij},
\]
where \( D_j \) represents the diameter of trees, \( H_1 \) represents the species diversity index, \( H_2 \) represents the diameter diversity index, \( \text{SLcos} \) represents \( \text{Slope} \times \cos(\text{Aspect}) \) [55], \( \gamma \) represents a parameter, and \( \mu_{ij} \) is the error term.

\( H_1 \) and \( H_2 \) can be calculated using the following formulas:

\[
H_1 = - \sum_{i=1}^{m} \frac{B_i}{B} \ln \left( \frac{B_i}{B} \right), \tag{5}
\]

\[
H_2 = - \sum_{j=1}^{n} \frac{B_j}{B} \ln \left( \frac{B_j}{B} \right), \tag{6}
\]

where \( B_i \) and \( B_j \) represent the basal area of tree species \( i \) in diameter class \( j \), \( m \) represents the number of tree species, \( n \) represents the number of diameter classes, and \( B \) represents the basal area of all trees in the plot. The tree species were classified into six groups, and trees with a proportion of less than 5% were combined and labeled as “other species”.

As the recruitment of \( R \) values is greater than or equal to zero, the Tobit model was used in the calculation \([56,57]\).

\[
R = \Phi \left( \frac{\beta_1 X_1}{\sigma_1} \right) \beta_2 X_1 + \sigma_1 \varphi \left( \frac{\beta_1 X_1}{\sigma_1} \right), \tag{7}
\]

\[
\beta_1 X_1 = \beta_{11} + \beta_{12} N_i + \beta_{13} B + \beta_{14} H_1 + \beta_{15} H_2 + \beta_{16} \text{SLcos} + \nu_i, \tag{8}
\]

where \( N_i \) represents the number of trees per hectare for species \( i \), \( \Phi \) and \( \varphi \) represent the standard normal cumulative distribution function and density function, respectively, and \( \sigma_1 \) is the standard deviation of the residuals \( \nu_i \) obtained in the estimation of the parameter \( \beta \).

The Probit model was used to estimate the mortality rate \( m_{ij} \):

\[
m_{ij} = \frac{M_{ij}}{T} = \frac{1}{T} \Omega \left( \delta_{11} + \delta_{12} D_j + \delta_{13} D^2_j + \delta_{14} B + \delta_{15} H_1 + \delta_{16} H_2 + + \delta_{17} \text{SLcos} + \xi_{ij} \right), \tag{9}
\]

where \( M_{ij} \) represents the probability of tree mortality of species \( i \) and diameter class \( j \) during time \( T \), \( \delta \) is a parameter, and \( \xi_{ij} \) is an error term. The parameter for diameter growth was calculated by the least-squares method based on data from individual trees and plot-level data. The Tobit model was employed using maximum likelihood estimation to calculate the recruitment equation for trees, while the Probit model was used to estimate the mortality equation. For statistical calculations, dead trees were assigned a value of 1, and others were assigned a value of 0. The definitions of each variable are presented in Table 2.

**Table 2.** The variables in the diameter growth model, recruitment model, and mortality model.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g )</td>
<td>The annual increment of trees over 5 years (cm)</td>
</tr>
<tr>
<td>( M )</td>
<td>The 5-year mortality rate of trees</td>
</tr>
<tr>
<td>( R )</td>
<td>The number of trees that reached the minimum measurable diameter class for growth within 5 years (trees/ha·year(^{-1}))</td>
</tr>
<tr>
<td>( D )</td>
<td>The median diameter class (cm)</td>
</tr>
<tr>
<td>( D^2 )</td>
<td>The square of the median diameter class</td>
</tr>
<tr>
<td>( B )</td>
<td>The average stand basal area per hectare (m(^2)/ha)</td>
</tr>
<tr>
<td>( H_1 )</td>
<td>The tree species diversity</td>
</tr>
<tr>
<td>( H_2 )</td>
<td>The tree size class diversity</td>
</tr>
<tr>
<td>Slope</td>
<td>The plot slope (°)</td>
</tr>
<tr>
<td>Aspect</td>
<td>Aspect is recorded as 0 for north, 90 for west, 180 for south, and 270 for east</td>
</tr>
</tbody>
</table>

The variance inflation factor (VIF) was used to test for multicollinearity in the growth, recruitment, and mortality models of the forest stand. VIF < 10 indicated no multicollinearity, and all VIF values in this study were <10.
2.4. Model Validation

The prediction results were compared to the measured data from the same natural forest sample plots in the ninth CNFI in Jiangxi Province. The predicted and measured basal areas of each sample plot were compared in terms of various performance indicators to evaluate the accuracy of the model. The root-mean-square error (RMSE) is a statistical measure of the average magnitude of the differences or errors between predicted values and actual observed values. The mean absolute error (MAE) is a statistical metric used to quantify the average magnitude of differences between predicted values and actual observed values. Finally, the total relative error (TRE) is a measure used to assess the overall relative accuracy of predictions or estimates in relation to actual observed values.

\[
RMSE = \sqrt{\frac{1}{m} \sum_{n=1}^{m} (\hat{y}_n - y_n)^2}
\]  
(10)

\[
MAE = \frac{1}{m} \sum_{n=1}^{m} |\hat{y}_n - y_n|
\]  
(11)

\[
TRE = \frac{\sum_{n=1}^{m} (\hat{y}_n - y_n)^2}{\sum_{n=1}^{m} y_n} \times 100
\]  
(12)

In these equations, \( y_n \) represents the predicted value, \( \hat{y}_n \) represents the measured value, and \( m \) represents the sample size.

2.5. Constructing Equilibrium Curves

The diameter distribution of natural forest trees tends to follow a reverse J-shaped pattern, which can be expressed by the following formula [58].

\[
N_j = k_0 e^{-k_1 d_j}
\]  
(13)

The ratio of the number of trees between two consecutive diameter classes \( q \) is expressed as:

\[
q = N_{j-1} / N_j
\]  
(14)

where \( d_j \) is the median diameter of trees in diameter class \( j \), \( N_j \) is the number of trees in diameter class \( j \), \( N_{j-1} \) is the number of trees in the previous diameter class \( (j - 1) \), \( k_0 \) is the intercept, and \( k_1 \) represents the probability of a decrease in the number of trees in diameter class \( j + 1 \) compared to the number of trees in diameter class \( j \).

The integration of Equations (13) and (14) yields the following equation:

\[
q = \frac{k_0 e^{-k_1 (d_j - h)}}{k_0 e^{-k_1 d_j}} = e^{k_1 h},
\]  
(15)

where \( h \) represents the diameter class width. If the number of trees in the smallest diameter class (the first diameter class) is assumed to be \( N_1 \), then the number of trees in each diameter class can be expressed as follows.

\[
N_j = q^{j-1} N_1
\]  
(16)

If the desired residual basal area \( B \) of a natural secondary forest after selective cutting is known, then:

\[
B = k_2 \sum_{i=1}^{m} N_i d_i^2,
\]  
(17)

where \( m \) represents the number of diameter classes and \( k_2 = \pi / 40,000 \). The substitution of Equation (16) into Equation (17) yields the following expression:

\[
B = k_2 \sum_{j=1}^{m} N_1 q^{j-1} d_j^2 = k_2 N_1 \sum_{j=1}^{m} q^{j-1} d_j^2 = k_3 N_1
\]  
(18)
where \( k_3 = k_2 \sum_{j=1}^{m} q^{-1}d_j^2 \),
\[
N_1 = \frac{B}{k_3}.
\] (19)

Based on Equations (16) and (19), an equilibrium curve can be obtained with the post-harvest stand basal area \( B \), the adjacent diameter ratio \( q \), and the maximum diameter of retained trees \( D_{max} \). Considering that multiple combinations of \( B, D_{max} \), and \( q \) can be used to construct equilibrium curves, the range of \( B \) values was set as 35–40 m\(^2\)/ha, the \( q \) values were set as 1.3, 1.5, and 1.7, and the \( D_{max} \) values were set as 45 and 50 cm based on investigations of the growth status of forest stands in Jiangxi Province and previous research results.

### 2.6. Determining the Optimal Equilibrium Curve

Based on the established transition-matrix growth model, the management effects under different potential equilibrium curves were predicted. The rotation period was set to 10 years based on domestic and international research [59]. The diameter distribution of the forest stand was compared with the equilibrium curve, and the trees that exceeded the equilibrium curve were harvested. It was assumed that the selected natural forests in Jiangxi Province had not experienced human disturbance or serious natural disasters. The prediction period was set to 50 years and the harvesting sequence for the trees was as follows: softwood > fir > others > Schima superba > oak > hardwood.

By comparing the current annual growth \( iv \), the stock volume \( \Delta V \), the quadratic mean diameter of the harvested trees \( D_g \), var, and the species composition between the forest diameter distribution and the equilibrium curves, the optimal curve was sought from twelve potential curves. Among these, var is calculated as:
\[
\text{var} = \frac{\sum (\ln N_{actual} - \ln N_{target})^2}{k},
\] (20)

where var is the variance of the deviation between the observed and target numbers of trees under the equilibrium condition, \( N_{actual} \) is the actual or simulated number of trees in the corresponding diameter class, \( N_{target} \) is the number of trees in the corresponding diameter class under the equilibrium condition, and \( k \) is the number of diameter classes. A larger var value indicates a greater deviation of the stand from the equilibrium structure, whereas a smaller value indicates closer proximity to the equilibrium state. In this study, var < 0.5 was considered to indicate equilibrium, 0.5 < var < 1.0 reflected a critical state, and var > 1.0 was considered non-equilibrium. The optimal equilibrium curve is that which first reaches a stable state with the maximum volume, maximum annual growth, and maximum average diameter of the harvested trees.

### 3. Results

#### 3.1. Model Parameters and Validation

The diameter growth of most tree species in the forest stands was found to be significantly negatively correlated with \( B \) and \( D_j^2 \) and positively correlated with \( D_j \) (Table 3). For all species groups, except for oak and hardwood, the growth of the tree diameter had a significantly negative relationship with \( H_1 \). Furthermore, the diameter growth of oak and fir species was significantly positively correlated with \( H_2 \).

The results of the recruitment model parameters are reported in Table 4. Except for hardwood species, the recruitment of the forest stand was found to be significantly negatively correlated with the basal area \( B \) of all tree species groups \( (p < 0.01) \). \( N \) and \( H_1 \) were positively correlated with softwood, hardwood, and Schima superba and negatively correlated with other tree species groups. \( H_2 \) exhibited a significant positive relationship with softwood and other species, and SLcos had a significant correlation with all tree species groups except Schima superba.
Table 3. The statistical analysis of the diameter growth model parameters.

<table>
<thead>
<tr>
<th></th>
<th>Oak</th>
<th>Fir</th>
<th>Softwood</th>
<th>Schima superba</th>
<th>Hardwood</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>6.21 × 10⁻³</td>
<td>1.45 × 10⁰</td>
<td>5.72 × 10⁻¹</td>
<td>1.27 × 10⁰</td>
<td>4.96 × 10⁻¹</td>
<td>6.88 × 10⁻³</td>
</tr>
<tr>
<td>D₁</td>
<td>4.19 × 10⁻²</td>
<td>−3.50 × 10⁻³</td>
<td>6.32 × 10⁻²</td>
<td>5.28 × 10⁻²</td>
<td>4.69 × 10⁻²</td>
<td>7.46 × 10⁻²</td>
</tr>
<tr>
<td>D₁²</td>
<td>−5.42 × 10⁻⁴</td>
<td>1.14 × 10⁻³</td>
<td>−1.14 × 10⁻³</td>
<td>−7.05 × 10⁻⁴</td>
<td>−6.14 × 10⁻⁴</td>
<td>−1.51 × 10⁻³</td>
</tr>
<tr>
<td>H₁</td>
<td>6.65 × 10⁻²</td>
<td>−4.07 × 10⁻¹</td>
<td>−1.49 × 10⁻¹</td>
<td>−3.02 × 10⁻¹</td>
<td>6.90 × 10⁻²</td>
<td>−1.94 × 10⁻¹</td>
</tr>
<tr>
<td>H₂</td>
<td>1.35 × 10⁻¹</td>
<td>2.02 × 10⁻¹</td>
<td>1.95 × 10⁻¹</td>
<td>−2.33 × 10⁻¹</td>
<td>6.78 × 10⁻³</td>
<td>1.08 × 10⁻²</td>
</tr>
<tr>
<td>B</td>
<td>−1.69 × 10⁻²</td>
<td>−2.07 × 10⁻²</td>
<td>−1.20 × 10⁻²</td>
<td>9.36 × 10⁻⁴</td>
<td>−9.02 × 10⁻³</td>
<td>−1.23 × 10⁻²</td>
</tr>
<tr>
<td>SLcos</td>
<td>−4.03 × 10⁻³</td>
<td>1.48 × 10⁻³</td>
<td>3.20 × 10⁻⁶</td>
<td>5.14 × 10⁻³</td>
<td>3.32 × 10⁻⁴</td>
<td>1.58 × 10⁻³</td>
</tr>
<tr>
<td>AIC</td>
<td>371.91</td>
<td>1203.15</td>
<td>585.03</td>
<td>394.73</td>
<td>1403.75</td>
<td>1347.98</td>
</tr>
<tr>
<td>BIC</td>
<td>411.20</td>
<td>1242.32</td>
<td>616.36</td>
<td>427.17</td>
<td>1443.46</td>
<td>1387.22</td>
</tr>
<tr>
<td>R²</td>
<td>0.27</td>
<td>0.19</td>
<td>0.13</td>
<td>0.15</td>
<td>0.08</td>
<td>0.11</td>
</tr>
<tr>
<td>logLik</td>
<td>−177.96</td>
<td>−593.58</td>
<td>−284.52</td>
<td>−189.37</td>
<td>−693.88</td>
<td>−665.99</td>
</tr>
</tbody>
</table>

Level of significance: *** p < 0.01; ** p < 0.05; * p < 0.1; SLcos = Slope × cos (Aspect); R²: Nagelkerke’s pseudo r-squared; logLik: log-likelihood value.

Table 4. The statistics of the recruitment model parameters.

<table>
<thead>
<tr>
<th></th>
<th>Oak</th>
<th>Fir</th>
<th>Softwood</th>
<th>Schima superba</th>
<th>Hardwood</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>5.01 × 10⁰</td>
<td>7.75 × 10⁻²</td>
<td>−1.25 × 10⁻²</td>
<td>2.28 × 10⁰</td>
<td>−4.31 × 10⁻²</td>
<td>4.09 × 10⁻³</td>
</tr>
<tr>
<td>N</td>
<td>1.78 × 10⁻¹</td>
<td>−1.48 × 10⁻²</td>
<td>4.18 × 10⁻²</td>
<td>1.12 × 10⁻¹</td>
<td>1.74 × 10⁻¹</td>
<td>−5.34 × 10⁻²</td>
</tr>
<tr>
<td>H₁</td>
<td>−1.48 × 10⁰</td>
<td>−3.41 × 10⁻²</td>
<td>7.07 × 10⁻⁶</td>
<td>1.05 × 10⁻⁴</td>
<td>4.25 × 10⁻⁶</td>
<td>−1.35 × 10⁻⁵</td>
</tr>
<tr>
<td>H₂</td>
<td>−4.58 × 10⁰</td>
<td>1.77 × 10⁻⁶</td>
<td>8.87 × 10⁻⁶</td>
<td>−1.01 × 10⁻¹</td>
<td>1.11 × 10⁻¹</td>
<td>8.88 × 10⁻⁶</td>
</tr>
<tr>
<td>B</td>
<td>−1.88 × 10⁻⁴</td>
<td>−1.17 × 10⁻⁶</td>
<td>−1.38 × 10⁻⁶</td>
<td>−5.03 × 10⁻⁶</td>
<td>−2.12 × 10⁻⁶</td>
<td>−7.88 × 10⁻⁶</td>
</tr>
<tr>
<td>SLcos</td>
<td>−3.13 × 10⁻⁶</td>
<td>1.88 × 10⁻⁶</td>
<td>−1.14 × 10⁻⁶</td>
<td>7.30 × 10⁻⁶</td>
<td>6.19 × 10⁻⁶</td>
<td>1.86 × 10⁻⁶</td>
</tr>
<tr>
<td>logSigma</td>
<td>5.54 × 10⁰</td>
<td>5.10 × 10⁰</td>
<td>4.01 × 10⁰</td>
<td>4.49 × 10⁰</td>
<td>5.47 × 10⁰</td>
<td>5.17 × 10⁰</td>
</tr>
<tr>
<td>AIC</td>
<td>25.683</td>
<td>14.309</td>
<td>30.50</td>
<td>58.265</td>
<td>16.970</td>
<td>15.382</td>
</tr>
<tr>
<td>BIC</td>
<td>25.723</td>
<td>14.345</td>
<td>30.79</td>
<td>58.568</td>
<td>17.007</td>
<td>15.419</td>
</tr>
<tr>
<td>R²</td>
<td>0.43</td>
<td>0.45</td>
<td>0.20</td>
<td>0.26</td>
<td>0.36</td>
<td>0.16</td>
</tr>
<tr>
<td>logLik</td>
<td>−12.834</td>
<td>−714.67</td>
<td>−1518.15</td>
<td>−2906.18</td>
<td>−8478.26</td>
<td>−7684.43</td>
</tr>
</tbody>
</table>

Level of significance: ** p < 0.05; *** p < 0.01; logSigma: log of the standard deviation of residuals; N: the total number of trees in each plot with recruitment.

The parameters of the mortality model are exhibited in Table 5. The mortality of the forest stand was negatively correlated with the D₁ value of all tree species, whereas H₁ and H₂ had positive relationships with stand mortality for all species except oak.

Table 5. The statistics of the mortality model parameters.

<table>
<thead>
<tr>
<th></th>
<th>Oak</th>
<th>Fir</th>
<th>Softwood</th>
<th>Schima superba</th>
<th>Hardwood</th>
<th>Others</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>−1.45 × 10⁻²</td>
<td>−2.75 × 10⁻⁷</td>
<td>−1.94 × 10⁻⁷</td>
<td>−1.65 × 10⁻⁷</td>
<td>−1.87 × 10⁻⁷</td>
<td>−1.78 × 10⁻⁷</td>
</tr>
<tr>
<td>D₁</td>
<td>−4.76 × 10⁻²</td>
<td>−4.36 × 10⁻³</td>
<td>−5.33 × 10⁻²</td>
<td>−4.73 × 10⁻²</td>
<td>−3.57 × 10⁻²</td>
<td>−3.30 × 10⁻²</td>
</tr>
<tr>
<td>H₁</td>
<td>2.99 × 10⁻¹</td>
<td>1.14 × 10⁻⁸</td>
<td>3.35 × 10⁻¹</td>
<td>1.30 × 10⁻⁸</td>
<td>2.72 × 10⁻¹</td>
<td>3.32 × 10⁻¹</td>
</tr>
<tr>
<td>H₂</td>
<td>−1.09 × 10⁻²</td>
<td>3.46 × 10⁻¹</td>
<td>1.01 × 10⁻²</td>
<td>7.05 × 10⁻¹</td>
<td>3.26 × 10⁻¹</td>
<td>5.67 × 10⁻¹</td>
</tr>
<tr>
<td>B</td>
<td>1.72 × 10⁻²</td>
<td>−7.10 × 10⁻⁶</td>
<td>−3.13 × 10⁻²</td>
<td>−8.93 × 10⁻⁶</td>
<td>6.55 × 10⁻³</td>
<td>−1.40 × 10⁻²</td>
</tr>
<tr>
<td>SLcos</td>
<td>−3.39 × 10⁻³</td>
<td>−4.51 × 10⁻⁴</td>
<td>7.30 × 10⁻⁴</td>
<td>−1.12 × 10⁻⁴</td>
<td>−4.09 × 10⁻³</td>
<td>−6.85 × 10⁻³</td>
</tr>
<tr>
<td>AIC</td>
<td>1494.71</td>
<td>557.18</td>
<td>328.35</td>
<td>412.43</td>
<td>992.05</td>
<td>917.10</td>
</tr>
<tr>
<td>BIC</td>
<td>1526.21</td>
<td>588.14</td>
<td>353.17</td>
<td>438.43</td>
<td>1023.32</td>
<td>948.09</td>
</tr>
<tr>
<td>R²</td>
<td>0.05</td>
<td>0.15</td>
<td>0.07</td>
<td>0.17</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>logLik</td>
<td>−741.36</td>
<td>−272.59</td>
<td>−158.18</td>
<td>−200.22</td>
<td>−490.02</td>
<td>−452.55</td>
</tr>
</tbody>
</table>

Level of significance: * p < 0.10; ** p < 0.05; *** p < 0.01.

Based on the calculation of the RMSE, MAE, and TRE, the transition-matrix growth model for natural forests in Jiangxi Province achieved good accuracy and a high R² value. Therefore, this growth model is suitable for predicting the growth of natural forests. The specific calculation values of each index are shown in Table 6.

Table 6. The accuracy test results of the model.

<table>
<thead>
<tr>
<th></th>
<th>RMSE</th>
<th>MAE</th>
<th>TRE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.52 m²/ha</td>
<td>0.50 m²/ha</td>
<td>1.38%</td>
</tr>
</tbody>
</table>
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3.2. Optimal Equilibrium Curve Selection

The simulation results of each potential equilibrium curve are summarized in Table 7. The value of var for most potential equilibrium curves was less than 0.5, suggesting that most of these curves could reach an equilibrium state after 50 years of growth simulation. Although equilibrium curve 8 had the maximum current annual growth of 29.14 m$^3$/ha and the maximum stand volume of 553.93 m$^3$/ha after 50 years, the value of var was greater than 0.5. Equilibrium curve 7 had the maximum quadratic mean diameter of the harvested trees (37.06 cm), but the species composition was unstable over 30 years (2038–2068), and the var value was close to the unstable state. Equilibrium curve 3 ($B = 35$ m$^2$/ha, $D_{max} = 45$ cm, $q = 1.7$) was determined as the optimal equilibrium curve because its var value was always below 0.4. The curve reached equilibrium, and the species composition, stand volume (AV), quadratic mean diameter of the harvested trees ($D_g$), and current annual growth (iv) remained stable over 30 years (2038–2068).

Table 7. The performance of the equilibrium curves after 40 years.

<table>
<thead>
<tr>
<th>No.</th>
<th>$B$ (m$^2$/ha)</th>
<th>$D_{max}$ (cm)</th>
<th>q</th>
<th>var (m$^{3} \cdot a^{-1} \cdot$hm$^{-2}$)</th>
<th>QU%</th>
<th>CF%</th>
<th>SW%</th>
<th>SS%</th>
<th>OT%</th>
<th>HW%</th>
<th>AV</th>
<th>$D_g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>35</td>
<td>45</td>
<td>1.3</td>
<td>0.35</td>
<td>10.36 ↓</td>
<td>7.32 ↓</td>
<td>4.26 ↑</td>
<td>4.70 ↑</td>
<td>21.53 ↓</td>
<td>4.20 ↑</td>
<td>57.99 -</td>
<td>376.43 -</td>
</tr>
<tr>
<td>2</td>
<td>35</td>
<td>45</td>
<td>1.5</td>
<td>0.33</td>
<td>9.82 ↓</td>
<td>5.23 ↓</td>
<td>5.22 ↑</td>
<td>2.43 ↑</td>
<td>25.32 ↓</td>
<td>3.54 ↑</td>
<td>58.26 -</td>
<td>398.26 -</td>
</tr>
<tr>
<td>3</td>
<td>35</td>
<td>45</td>
<td>1.7</td>
<td>0.32</td>
<td>13.44 ↓</td>
<td>5.48 ↓</td>
<td>5.53 ↑</td>
<td>3.12 ↓</td>
<td>27.06 -</td>
<td>6.00 -</td>
<td>52.81 -</td>
<td>392.87 ↑</td>
</tr>
<tr>
<td>4</td>
<td>35</td>
<td>50</td>
<td>1.3</td>
<td>0.49</td>
<td>3.69 ↓</td>
<td>8.38 ↓</td>
<td>6.41 ↑</td>
<td>3.35 ↓</td>
<td>14.96 ↓</td>
<td>2.05 ↑</td>
<td>64.85 ↑</td>
<td>388.64 ↓</td>
</tr>
<tr>
<td>5</td>
<td>35</td>
<td>50</td>
<td>1.5</td>
<td>0.66</td>
<td>26.35 ↑</td>
<td>5.99 ↓</td>
<td>3.32 ↑</td>
<td>5.01 ↑</td>
<td>17.33 ↑</td>
<td>4.96 ↓</td>
<td>63.39 ↑</td>
<td>486.67 ↓</td>
</tr>
<tr>
<td>6</td>
<td>35</td>
<td>50</td>
<td>1.7</td>
<td>0.37</td>
<td>20.31 ↑</td>
<td>8.30 ↓</td>
<td>6.70 ↑</td>
<td>4.02 ↓</td>
<td>18.32 ↑</td>
<td>6.33 ↓</td>
<td>56.33 ↑</td>
<td>419.64 ↓</td>
</tr>
<tr>
<td>7</td>
<td>40</td>
<td>50</td>
<td>1.3</td>
<td>0.48</td>
<td>26.30 ↑</td>
<td>7.23 ↓</td>
<td>4.98 ↓</td>
<td>2.33 ↓</td>
<td>16.17 ↑</td>
<td>3.55 ↓</td>
<td>65.74 ↑</td>
<td>482.33 ↓</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
<td>50</td>
<td>1.5</td>
<td>0.76</td>
<td>29.14 ↑</td>
<td>6.54 ↓</td>
<td>3.74 ↑</td>
<td>4.10 ↓</td>
<td>15.84 ↑</td>
<td>4.22 ↓</td>
<td>65.56 ↑</td>
<td>553.93 ↓</td>
</tr>
<tr>
<td>9</td>
<td>40</td>
<td>50</td>
<td>1.7</td>
<td>0.36</td>
<td>20.33 ↓</td>
<td>7.25 ↓</td>
<td>7.75 ↑</td>
<td>5.62 ↓</td>
<td>16.03 ↑</td>
<td>4.01 -</td>
<td>59.34 -</td>
<td>469.38 ↓</td>
</tr>
<tr>
<td>10</td>
<td>40</td>
<td>45</td>
<td>1.3</td>
<td>0.47</td>
<td>10.58 ↓</td>
<td>10.03 ↓</td>
<td>3.99 ↓</td>
<td>4.36 ↓</td>
<td>17.22 ↓</td>
<td>3.96 ↑</td>
<td>60.44 ↑</td>
<td>503.46 ↓</td>
</tr>
<tr>
<td>11</td>
<td>40</td>
<td>45</td>
<td>1.5</td>
<td>0.38</td>
<td>18.54 ↓</td>
<td>6.92 ↓</td>
<td>8.01 ↑</td>
<td>5.30 ↓</td>
<td>16.82 ↓</td>
<td>4.33 ↓</td>
<td>58.62 ↓</td>
<td>539.88 ↓</td>
</tr>
<tr>
<td>12</td>
<td>40</td>
<td>45</td>
<td>1.7</td>
<td>0.33</td>
<td>16.35 -</td>
<td>5.53 ↓</td>
<td>7.85 ↑</td>
<td>5.03 ↓</td>
<td>17.40 ↓</td>
<td>4.88 -</td>
<td>59.31 -</td>
<td>472.11 ↓</td>
</tr>
</tbody>
</table>

var represents the variance of the logarithmic stem number around the target; iv represents the current annual growth; QU represents the proportion of oak species among the total standing volume; CF represents the proportion of fir species among the total standing volume; SW represents the proportion of hardwood species among the total standing volume; OT represents the proportion of other species in the total standing volume; SS represents the proportion of Schina superba among the total standing volume; AV represents the forest stock volume; $D_g$ represents the quadratic mean diameter of the harvested trees. “−”, “↑” and “↓” represent values that remain essentially stable, increasing and decreasing in the last 30 years of the simulation period, respectively.

3.3. The Prediction of Forest Dynamics under the Optimal Equilibrium Curve

The species composition, diameter distribution, stock volume, and current annual growth of the forests managed under the optimal equilibrium curve ($B = 35$ m$^2$/ha, $D_{max} = 45$ cm, and $q = 1.7$) were predicted. Before prediction, hardwood species accounted for 50.2% and softwood species accounted for 34.3% (Figure 2). By comparison, the tree species composition was found to have changed significantly by 2068, e.g., the hardwood tree species increased from 50.2% to 52.8%, while the softwood species decreased from 34.3% to 3.1%.

![Figure 2](https://example.com/figure2.png)
In the current diameter distribution, the trees are constrained in the small-diameter class (5–20 cm). Moreover, each diameter class is dominated by softwood tree species, e.g., in the 5 cm class, softwood species account for 54.5%. After 50 years of growth simulation, the forest diameter distribution showed a typical reverse J-shaped distribution, and large-diameter trees appeared, e.g., the large diameter class (35–45 cm) was found to account for 7.03% in 2068 (Figure 3). Furthermore, hardwood tree species accounted for large proportions of the large-diameter class, e.g., in the 5 cm diameter class, they accounted for 88.89% (2068). By contrast, the softwood tree species were found to be significantly decreased in each diameter class, e.g., in the 5 cm diameter class, the softwood species decreased from 54.49% to 18.75%, and there were no large-diameter softwood tree species.

Figure 3. The initial diameter distribution (A) and the simulated diameter distribution after 50 years (B) based on the optimized equilibrium curve.

For hardwood tree species, the stand volume increased, whereas decreasing trends were observed for other tree species. For instance, in the 50th year of the simulation (2068), hardwood species exceeded 50%, whereas softwood species only accounted for 3.12% (Figure 4). The trend of the current annual growth was similar to the stand volume, i.e., the current annual growth gradually increased for hardwood tree species but decreased for other tree species.

Figure 4. The variation of the stock volume (A) and current annual growth (B) over 50 years for different tree species.

Additionally, at the beginning of the simulation (2018), the stand volume was observed to be dominated by small-diameter trees (5–15 cm), but with continuous management guided by the optimal equilibrium curves, middle-diameter trees (20–30 cm) and large-diameter trees (35–45 cm) appeared and eventually dominated the forest stand (Figure 5). The current annual growth of the large-diameter trees was also found to increase and account for an overwhelming proportion of the total current annual growth, e.g., in 2068, the current annual growth of large-diameter trees accounted for 57.56% of the total.
which states that plant species or functional groups occupy functionally distinct niches. Thus, detailed geographic coordinates and other data could not be accessed. Moreover, if the location of each stand was considered, the location of the emergence of regenerated trees in the later stages of the growth simulation process could not be determined. Moreover, this indicator would not be able to be calculated in the follow-up investigations. Thus, the climate and other related variables were unfortunately not considered in this study. In follow-up research, climate factors will be taken into consideration if more detailed data can be accessed.

With the inclusion of $H_1$, $H_2$, and $B$ in the transition growth model, statistically significant influences were found on growth, mortality, and recruitment. Many studies have reported that species diversity is positively correlated with tree growth [60–63] but negatively correlated with tree mortality [64]. For example, Ruan reported that the growth rate increased by 17% after the conversion of single-tree species plantations to mixed-species forests [65]. They attributed the relationship to niche complementarity theory, which states that plant species or functional groups occupy functionally distinct niches in an ecosystem and use resources in a complementary way [66–68]. In the present study, species diversity ($H_1$) was found to be negatively related to diameter growth (Table 3) and positively related with tree mortality (Table 5). These findings are consistent with those of other studies. For example, Edgar and Burke also found that species diversity is negatively correlated with tree growth [69]. Using a plotted network across Canada and the United States, Searle also documented that higher tree diversity is linked to higher tree mortality [70]. They attributed the higher mortality to the increases in stand density and tree size variation caused by high tree diversity.

In the present work, size diversity ($H_2$) was found to have a positive correlation with tree mortality for all tree species (except oak). This finding is consistent with those of other studies [71,72] and suggests that tree size diversity can increase the mortality of trees. This could be explained by asymmetric competition, in which trees of different sizes have variable competitive abilities for acquiring nutrients [70]. Size diversity ($H_2$)
was also found to be positively related to tree recruitment (for softwood and others) and growth (for oak and fir). Similarly, Wang also reported that size diversity can promote tree recruitment and growth [73]. Lei and Tan also found that the size diversity of forest trees has significant positive effects on net growth and recruitment [74,75]. The positive relationships between tree size diversity and tree growth and recruitment can be attributed to niche complementarity theory [76,77].

The total stand basal area (B) of trees was found to have significant negative relationships with tree growth and recruitment, indicating a significant relationship with mortality. Similar findings have been documented in many studies. For instance, Rozendaal found that greater reductions in growth occur in forests with high basal areas [78]. Kaber reported that the basal area negatively impacts tree recruitment [79]. Consistent with the present work, Chen found that the basal area has a positive relationship with mortality [80], which can be explained by the fact that the strength of competition increases with the basal area, and various species reflect competition in different ways.

4.2. Equilibrium Curve

In 2017, China imposed a total ban on commercial logging in natural forests, which strictly forbids commercial logging but encourages positive management to restore degraded areas [81]. Additionally, China has implemented the “precision forestry project”, which requires quantitative management with the objective of fully improving the ecological and economic functions of the forest [82,83]. Due to the simple structure of single-species, even-aged plantations, stand density management diagrams have been developed to guide forest management [20–22]. However, for natural, uneven-aged, mixed-species forests, it is more difficult to prescribe a quantitative management strategy because of the complex forest structure and species composition.

Structure-based forest management (SBFM) describes the management of uneven-aged, mixed-species forests to achieve continuous cover [84]. SBFM assumes that the system structure determines function and aims to cultivate a healthy, stable, high-quality, and high-efficiency forest using structural parameters to quantitatively guide adjustments and optimize the forest structure [85]. The most frequently employed structural parameters include the uniform angle index (W), mingling degree (M), and neighborhood comparison (U) [86–88]. For instance, based on the complex forest structure and species intermingling, Li determined the harvesting priority of trees according to stand structural indices for a mixed Korean pine-broadleaved forest in northeast China and a mixed pine-oak forest in northwest China [89]. Li employed the same approach to determine the harvest of trees, providing a reference for the management of uneven-aged, mixed-species forests in Jilin Province and Gansu Province [90]. Although SBFM provides a promising option for the precise management of natural, uneven-aged, mixed-species forests, it requires detailed individual-level information, e.g., the spatial coordinates and competitive status of individual trees. Thus, it is expensive to implement [89,91].

In addition to SBFM, equilibrium curves are widely used to quantitatively inform uneven-aged, mixed-species forest management [92]. For example, Drozdowski developed equilibrium curves for Norway spruce (Picea abies) mixed-species forests in Poland [93]. Sterba first constructed six potential equilibrium curves based on the equilibrium model of Schütz [94] and Cancino and von Gadow [92] and then determined the optimal equilibrium curves using the PROGNAUS simulator, an individual-tree growth model [26]. Following the same methodology, in the present work, 12 potential equilibrium curves were first constructed for natural, uneven-aged, mixed-species forests in Jiangxi Province. Then, using the produced transition-matrix model, the forest dynamics under all the potential equilibrium curves were predicted. The var values of all potential equilibrium curves were found to be less than 0.5 (except for those of equilibrium curves 5 and 8), achieving an equilibrium state at the 20th year of forest growth projection (2038). In the subsequent 30 years (2038–2068), changes in the forest stock volume (ΔV), current annual growth (iv), quadratic mean diameter of the harvested trees (D₉), and species composition were...
observed under equilibrium curve 3 (B = 35 m²/ha, Dₘₐₓ = 45 cm, and q = 1.7), which showed a stable state, while the other equilibrium curves exhibited pronounced increasing or decreasing trends. Therefore, equilibrium curve 3 (B = 35 m²/ha, Dₘₐₓ = 45 cm, q = 1.7) was ultimately identified as the optimal equilibrium curve to guide natural forest management in Jiangxi Province.

The current and predicted species composition in 2068 are shown in Figure 2. In 2018, the hardwood species accounted for 50.2%; however, after 50 years of management under the optimal equilibrium curve, the hardwood species increased from 50.2% to 52.8%, the *Schima superba* species increased from 7.4% to 27.1%, and the softwood species decreased from 34.3% to 3.1% (Figure 2). This suggests that sustainable forest management could accelerate natural forest succession, i.e., fast-growing, shade-intolerant tree species (softwood tree species) first dominated and then were gradually replaced by late-seral, shade-tolerant species (hardwood tree species) [95,96]. A similar conclusion was reached by Dodson, who found that thinning treatments can accelerate late-successional structures in young coniferous stands in western Oregon, USA [97]. Furthermore, Iverson documented that low-intensity logging can increase the recruitment of late-seral, shade-tolerant species, and the sprouting ability of hardwood is more powerful in brighter environments [98].

Numerous studies have argued that a reverse J-shaped distribution is a more desirable forest structure [92,99,100]. In 2018, although the diameter distribution exhibited a reverse J shape, the small-diameter trees in the 5 cm diameter class dominated the forest, representing 76% of the total individual trees (Figure 3). Moreover, there were only four diameter classes. By contrast, after 50 years of prediction, under the guidance of equilibrium curve management, the diameter distribution exhibited a typical reverse J-shaped distribution, with the diameter class ranging from 5 to 45 cm. Many scholars [101–104] have found similar results. For instance, Meng reported that after four years of transformation of a degraded *Pinus massoniana* stand, the diameter distribution exhibited a reverse J shape, ranging from 6 to 12 cm [101]. Wang and Liu argued that sustainable thinning not only could increase species diversity and promote stand growth but could also convert the diameter distribution into a sustainable reverse J shape [102]. Additionally, the species composition and species richness showed significant differences in the diameter distribution. For instance, in 2018, the softwood tree species dominated in each diameter class, whereas in 2068, the hardwood and *Schima superba* tree species dominated, especially in the large-diameter class (35–40 cm).

Under the guidance of the equilibrium curve, the forest stand volume and current annual growth increased. The simulation indicated that in 2053, after 35 years of management, the current annual stock growth would reach 20.73 m³/ha and the stock volume would reach 311.00 m³/ha; these are similar to the values for close-to-nature forests in Germany reported by Wang and Albert, namely 336 and 15 m³/ha, respectively [105]. With regular harvesting guided by the optimal equilibrium curve, the proportion of hardwood and the *Schima superba* stand volume were found to gradually increase to 52.81% and 27.06%, respectively, whereas the proportions of the other tree species (mainly softwood tree species) gradually decreased toward the end of the simulation (2068) (Figure 4). This trend indicates that harvesting guided by the optimal equilibrium curve could promote forest succession by facilitating the recruitment and growth of hardwood tree species. Other studies have also found that a suitable intensity of harvesting can promote forest succession [106,107].

A decrease in softwood tree species was also reported by Verburg and van Eijk-Bos, who found that after 20 years of selective thinning, the percentage of softwood in the large-diameter class decreased, and no recruitment occurred [108]. The decrease or disappearance of softwood tree species (pioneer species) could be attributed to their high light requirements, which could not be met after the growth of the highly competitive hardwood tree species [109]. Compared to softwood tree species, hardwood tree species are shade-tolerant, which means that their seedlings and saplings can survive—but with slow growth—in the understory, waiting for release through thinning. Once forest gaps are created through thinning, the seedlings and saplings of hardwood tree species rapidly
develop [110–112]. Additionally, in the present work, a significant increase in forest stock volume after harvesting was observed, as harvesting reduces competition within the stand, opens space, and provides nutrients for forest growth [113–117].

Large-diameter trees were found to begin to appear after 2033 and increase in stock volume and current annual growth, suggesting that harvesting promotes the growth of large-diameter trees (Figure 5). Pothier analyzed the growth of balsam fir forests and found that harvesting could significantly promote the growth of large-diameter trees [118]. Caihong reported that thinning can positively affect soil nutrients and fungal community structures and promote the growth of large-diameter trees [119]. Furthermore, the increase in large-diameter trees suggests the increase in the economic value of the forests under management guided by the equilibrium curves.

The findings of this study revealed the effective improvement of forest timber yield and the increase in the timber harvest based on the optimized equilibrium curve. With the trend of global warming, carbon sequestration is a key priority. Under the guidance of the equilibrium curve, the forest stock and the proportion of large-diameter trees can be increased, which further enhances the carbon storage capacity of forests and helps to mitigate global warming. Although it is convenient to carry out forest management under the guidance of the equilibrium curve, the implementation of precision forestry would be further facilitated if distance-related competition indicators between trees could be taken into account. However, these indicators were not considered in this study because it is not possible to predict the location of the regenerated trees. Thus, if subsequent conditions allow, climate and distance-related indicators will be taken into account, and regeneration simulations will be conducted to further determine the relative locations of the trees.

5. Conclusions

Management and harvesting operations were conducted on natural forests in Jiangxi Province, China, by establishing a transfer-matrix growth model to predict forest growth conditions, based on which an equilibrium curve was constructed. The transition-matrix growth model showed good accuracy, with tree species diversity ($H_1$), size diversity ($H_2$), and basal area ($B$) having extremely significant effects on growth, recruitment, and mortality. The results are similar to those of previous studies and are consistent with forest growth patterns.

Unfortunately, this study did not take climate into account due to data confidentiality; however, due to the drastic change in the climate, related indicators should be considered. Furthermore, an evaluation index of the competitive ability of individual trees was also unavailable. Thus, the accuracy of calculating individual trees in the simulation process was reduced, and the transfer matrix was used to simulate forest growth by calculating the number of trees in each diameter class. The goals of future research are to obtain climatic data and find a suitable method by which to calculate the competition index of individual trees and introduce these factors into long-term prediction.

Although climate was not taken into account, some studies have shown that in the short term, there is no significant difference between the predictions of climate and non-climate models [72,73]. The optimal equilibrium curve was determined to comprise $B = 35 \text{ m}^2/\text{ha}$, $D_{\text{max}} = 45 \text{ cm}$, and $q = 1.7$. Using a 10-year rotation cycle, the continuous growth volume and total growth increment of forest stands were found to be superior to those of natural forests. This equilibrium curve can serve as a reference for subsequent management, promote positive forest succession, increase forest stock volume and growth, and allow for the full utilization of ecological and economic benefits. It can also be used to regulate the stand density and ensure that the forest maintains an optimal diameter distribution. Finally, it can provide a model to increase resilience to natural disasters, such as fires, pests, and diseases, and facilitate the improved facility and accuracy of natural forest harvesting in Jiangxi Province.
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