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Relativistic Time-of-Arrival Measurements: Predictions, Post-Selection and Causality Problems

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Abstract: We analyze time-of-arrival probability distributions for relativistic particles in the context of quantum field theory (QFT). We show that QFT leads to a unique prediction, modulo post-selection that incorporates properties of the apparatus into the initial state. We also show that an experimental distinction of different probability assignments is possible especially in near-field measurements. We also analyze causality in relativistic measurements. We consider a quantum state obtained by a spacetime-localized operation on the vacuum, and we show that detection probabilities are typically characterized by small transient non-causal terms. We explain that these terms originate from Feynman propagation of the initial operation, because the Feynman propagator does not vanish outside the light cone. We discuss possible ways to restore causality, and we argue that this may not be possible in measurement models that involve switching the field–apparatus coupling on and off.

Keywords: quantum measurement theory; relativistic quantum theory; causality; time in quantum mechanics

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1. Introduction

Finding a consistent quantum description for time-of-arrival measurements is a classic problem in the foundations of quantum theory. In the simplest time-of-arrival measurement, a particle is prepared on an initial state $|\psi_0\rangle$ with positive momentum, and localized around $x = 0$. A particle detector is placed at $x = L$. The issue is to determine probability $P(t, L)dt$ that the detector clicks at some moment between t and $t + \delta t$. Despite the apparent simplicity of the problem, no unique time-of-arrival probability exists [1–3]. Fundamentally, this is due to the fact that time is not a quantum observable. There is no self-adjoint operator for time [4]; hence, we cannot rely on Born's rule for a unique answer.

The time-of-arrival problem is exacerbated for relativistic particles, because it becomes entangled with another foundational problem, the issue of localization. Particle localization is essential to any description of measurements, because all particle-detection records are localized in space and in time. However, the existence of observables associated to spatial localization is in conflict with the requirement of causality, as evidenced by several theorems [5–7].

The most well-known set-up where this conflict appears is Fermi's two-atom problem. Fermi studied information transmission through a quantum field in a system of two localized atoms at distance r [8]. He assumed that at time $t = 0$, atom A is in an excited state and atom B is in the ground state. He posed a question of when the influence of A will cause B to leave its ground state. In accordance with the locality principle that space-like separated events cannot influence each other, he found that this happens only at time greater than r. However, Fermi's result was shown to be an artefact of approximation [9]. Later studies of the problem came to different results that were heavily dependent on approximations. Eventually, Hegerfeldt showed that the conflict of localization and causality is generic in relativistic systems [10,11]; it only requires energy positivity and the treatment of atoms as being localized in disjoint spatial regions—see also the clarification of this result in [12].

In this paper, we analyze a broad class of probability distributions for the time of arrival of relativistic particles. This class was identified in ref. [13] through an analysis of measurements in QFT. Part of our motivation is the possibility of experimentally distinguishing between different proposals for time of arrival. We also analyze the structure of small apparently super-luminal transient terms—analogous to the ones in the Fermi-atom problem—that appear in the time-of-arrival probability distribution. We identify their origin and examine their implications for relativistic quantum theory of measurement.

An important reason for studying the time of arrival is that it forces us to re-conceptualize the description of quantum measurements. Ever since von Neumann [14], measurements have been described as almost instantaneous processes that occur at a single moment of time t. In von Neumann-type measurements, the interaction of the apparatus with the measured quantum system is switched on for a pre-determined time interval—the exact timing of the measurement is determined by the shape of the switching function. Hence, the timing of the measurement is an *external parameter* of the measurement scheme, and not a *random variable* of the experiment. This logic has recently been extended to QFT measurements [15,16]: the measured field and the apparatus are initially uncorrelated, and they interact only in a finite, predetermined spacetime region.

Time-of-arrival measurements challenge this measurement paradigm. Actual particle detectors (e.g., photographic plates, silicon strips) have a fixed location in space and they are made sensitive for a long time interval during which particles may be detected. Therefore, the location of a detection event is a fixed parameter of the experiment; the actual random variable is the detection time. Von Neumann-type measurements are fundamentally incapable of describing time as a random variable, but it turns out that they can mimic some aspects of time-of-arrival measurements. However, imitations have limitations: they work only to the lowest order in perturbation theory and eventually lead to causality problems.

We note that we use the word "causality" exclusively in its temporal sense, namely to denote temporal relations between two or more events. In relativistic physics, this means that there should exist no faster-than-light signals. In this paper, the word "causality" does not have the connotation of one event being a cause of another, a concept that is ill defined in standard quantum theory.

Our results are the following. First, we re-derive the relativistic time-of-arrival probabilities of ref. [13] in a von Neumann measurement scheme for quantum fields. The original derivation involved the Quantum Temporal Probabilities (QTP) method [13,17–20] that explicitly constructed a probability density with respect to time. In the von Neumann measurement scheme, we use a switching function that is localized in a compact spacetime region, and we reinterpret the probabilities in order to define a probability density. This works only to leading the order in perturbation theory. We use this alternative derivation in order to identify the problems that persist in this treatment of measurements.

Time-of-arrival probability distributions are post selected, i.e., they refer only to the fraction of particles that has been detected. They depend on the apparatus through specific operator $\hat{S}$ that describes the localization of the detection records. We show that this operator can be absorbed into a post selection of the initial state. *A unique time-of-arrival probability measure* ensues, modulo post selection. This measure was first derived by Leon [21] and then rederived from a QFT analysis in [17]. In the non-relativistic limit, it coincides with Kijowski's time-of-arrival distribution [22].

The measured time-of-arrival probability distribution does depend on the properties of the apparatus. We analyze the probability measure for an operationally meaningful class of initial states that was recently proposed, in relation to experiments for distinguishing between different time-of-arrival proposals. We find that the different distributions are in principle distinguishable in near-field experiments.

Then, we analyze locality in the derived time-of-arrival probabilities. We consider an initial state that is generated by a localized external source acting on the quantum field vacuum. By "localized" we mean that the source has support in a compact spacetime region.

We find that the time-of-arrival probability has a small but non-zero contribution outside the light cone of the source's support. We analyze the origin of this term, and we find that it originates from the well-known fact that in QFT, sources evolve with the Feynman propagator. The problem is that Feynman propagator is non-zero outside the light cone. We argue that von Neumann-type models that rely on switching on the interaction may not be able to resolve this problem.

Finally, we revisit the QTP description of relativistic measurements and present possible strategies through which the super-luminal transient terms can be consistently removed.

2. Relativistic Time-of-Arrival Probabilities

In this section, we first re-derive the time-of-arrival probabilities of ref. [13] using a von Neumann-type account of measurements. We show that these probabilities are unique modulo post selection, and that the effect of the detector—as determined by its localization properties—can be distinguished in near-field measurements.

The probabilities derived here follow from a QFT analysis that constraints the couplings of the field to the apparatus. This leads to a specific family of POVMs for the time-of-arrival probability distribution. At the level of free fields, the QFT analysis provides no further calculational benefit; in particular, it does not affect post selection. However, it is essential in order to guarantee that the analysis of causality occurs at the most fundamental level.

2.1. Detection Probability from a Von Neumann-Type Measurement

We consider the measurement of a free scalar field $\hat{\phi}(x)$ on Hilbert space $\mathcal{F}$, interacting with an apparatus described by Hilbert space $\mathcal{H}$. As our focus is time-of-arrival measurements, we work only in one spatial dimension. The reason is that only particles that propagate along the axis that connects the source with the detector contribute to the total probability.

The Hamiltonian of the total system is

$$\hat{H} = \hat{H}_\phi \otimes \hat{I} + \hat{I} \otimes \hat{H}_A + \hat{H}_I, \tag{1}$$

where $\hat{H}_\phi$ is the quantum field Hamiltonian and $\hat{H}_A$ the Hamiltonian of the apparatus. We consider a von Neumann type of measurement, in which the interaction is switched on for a finite time. Then, we choose an interaction Hamiltonian

$$\hat{H}_I = \int dx F_{\bar{t},\bar{x}}(t,x)\hat{\phi}(x) \otimes \hat{J}(x), \tag{2}$$

where $F_{\bar{t},\bar{x}}(t,x)$ is a switching function centered around the spacetime point $(\bar{t},\bar{x})$, and $\hat{J}(x)$ is a current operator on $\mathcal{H}$. Certainly, the idea of a switching interaction is not realistic in QFT. Fields interact with the apparatuses through terms defined by the Standard Model of particle physics, and in a Poincaré covariant theory, the interaction terms are always present.

We assume initial state $|\psi\rangle$ for the field and initial state $|\Omega\rangle$ for the detector. It is convenient to identify $|\Omega\rangle$ with the ground state of Hamiltonian $\hat{H}_A$, i.e., to assume $\hat{H}_A|\Omega\rangle = 0$, and also to be annihilated by the generator of space translations $\hat{P}_A$ of the apparatus. Then, the probability that the detector is found in an excited state, when measured after the interaction is switched off, is given by

$$\text{Prob}(\bar{t},\bar{x}) = \langle\psi_0,\Omega|\hat{S}^\dagger_{\bar{t},\bar{x}}\hat{I} \otimes (\hat{I} \otimes |\Omega\rangle\langle\Omega|)\hat{S}_{\bar{t},\bar{x}}|\psi_0,\Omega\rangle, \tag{3}$$

expressed in terms of the S-matrix $\hat{S}_{\bar{t},\bar{x}} = \mathcal{T}\exp\left[-i\int dtdx F_{\bar{t},\bar{x}}(t,x)\hat{\phi}(t,x) \otimes \hat{J}(t,x)\right]$; $\hat{\phi}(t,x)$ and $\hat{J}(t,x)$ are Heisenberg-picture operators and $\mathcal{T}$ stands for time-ordering.

To leading order in perturbation theory,

$$\text{Prob}(\bar{t},\bar{x}) = \int dt_1dx_1dt_2dx_2F_{\bar{t},\bar{x}}(t_1,x_1)F_{\bar{t},\bar{x}}(t_2,x_2)G(t_x,t_1;t_2,x_2)\langle\Omega|\hat{J}(t_1,x_1)\hat{J}(t_2,x_2)|\Omega\rangle, \tag{4}$$

where

$$G(t_1, x_1; t_2, x_2) = \langle \psi_0 | \hat{\phi}(t_1, x_1) \hat{\phi}(t_2, x_2) | \psi_0 \rangle \quad (5)$$

is a two-point correlation function for the field.

We take reference point $x_0 = 0$ on the apparatus and we define $|\omega\rangle := \hat{J}(0)|\Omega\rangle$. Then, we can write $\Omega|\hat{J}(t_1, x_1)\hat{J}(t_2, x_2)|\Omega\rangle = R(t_2 - t_2, x_2 - x_1)$, where

$$R(t, x) = \langle \omega | e^{i\hat{H}_A t - i\hat{P}_A x} | \omega \rangle$$

is the detector kernel. The key property of $R(t, x)$ is that its Fourier transform

$$\tilde{R}(E, K) := \int dt dx R(t, x) e^{-iEt + iKx} = 2\pi \langle \omega | \delta(E - \hat{H}_A) \delta(K - \hat{P}_A) | \omega \rangle \quad (6)$$

vanishes for $E < 0$, by energy positivity for apparatus Hamiltonian $\hat{H}_A$.

In Equation (4), $\bar{x}$ and $\bar{t}$ appear as parameters, not as random variables. However, Equation (4) has a natural interpretation in terms of a probability density. We consider a homogeneous switching function $F_{\bar{t},\bar{x}}(t, x) = f(t - \bar{t}, x - \bar{x})$, where $f(t, x) = \exp\left[-\frac{t^2}{2\delta_t^2} - \frac{x^2}{2\delta_x^2}\right]$, for space and time spreads δ_x and δ_t, respectively. Gaussians satisfy identity

$$f(t, x) f(t', x') = f^2\left(\frac{t + t'}{2}, \frac{x + x'}{2}\right) \sqrt{f}(t - t', x - x'). \quad (7)$$

Then, Equation (4) takes the form

$$\text{Prob}(\bar{t}, \bar{x}) = \int dt dx f^2(\bar{t} - t, \bar{x} - x) P(t, x), \quad (8)$$

i.e., it is a convolution of probability density

$$P(t, x) = \int ds dy R(s, y) \sqrt{f}(s, y) G(t - \frac{s}{2}, x - \frac{1}{2}y; t + \frac{1}{2}s, x + \frac{1}{2}y). \quad (9)$$

Typically, the detector kernel decays to zero for $|t|$ larger than temporal scale $\hat{\sigma}_T$ or $|x|$ larger than length scale σ_x. Both scales depend on properties of the apparatus. Taking $\delta_t >> \sigma_t$ and $\delta_x >> \sigma_x$, the contribution of $\sqrt{f}$ can be dropped from Equation (9), so that

$$P(t, x) = \int ds dy R(s, y) G(t - \frac{s}{2}, x - \frac{1}{2}y; t + \frac{1}{2}s, x + \frac{1}{2}y). \quad (10)$$

It is important to emphasize that probability density $P(t, x)$ of Equation (10) is meaningful only to the leading order in perturbation theory. We cannot construct a spacetime density out of higher-order terms in this procedure. This is to be constructed with the QTP method, briefly described in Section 3, that leads to probability densities at all orders of perturbation theory.

Equation (10) holds for any scalar field theory. For a free field of mass m,

$$\hat{\phi}(t, x) = \int \frac{dp}{2\pi\sqrt{2\epsilon_p}} [\hat{a}(k) e^{ipx - i\epsilon_p t} + \hat{a}^\dagger(k) e^{-ipx + i\epsilon_p t}], \quad (11)$$

where $\epsilon_p = \sqrt{p^2 + m^2}$.

Equation (10) becomes

$$P(t, x) = P_0 + \int \frac{dp dp'}{2\pi} \frac{\rho(p, p')}{2\sqrt{\epsilon_p \epsilon_{p'}}} \tilde{R}\left(\frac{p + p'}{2}, \frac{\epsilon_p + \epsilon_{p'}}{2}\right) e^{i(p - p')x - i(\epsilon_p - \epsilon_{p'})t}, \quad (12)$$

where $\hat{\rho}(p,p') = \langle\psi|\hat{a}^\dagger(p')\hat{a}(p)|\psi\rangle$ is the single-particle reduced density matrix. Term

$$P_0 = \int \frac{dp}{4\pi\epsilon_p}\tilde{R}(p,\epsilon_p) \tag{13}$$

is constant and state independent. It corresponds to vacuum noise, i.e., a background rate of false alarms. Hence, a detection signal exists only as long as $P(t,x)$ is larger than P_0. In what follows, we ignore P_0, except for checking whether transient terms that appear in $P(t,x)$ are actual detection signals.

2.2. Post Selection with Respect to Recorded Events

Equation (12) is an unnormalized probability density. To normalize, we consider a set-up where the particle source is in the vicinity of $x=0$ and the detector is at $x=L$, where $L>0$ is a macroscopic distance. In this set-up, the contribution of negative momenta to $P(t,L)$ is negligible. It is therefore convenient to normalize over initial states with support only over positive momenta. Then, integrating over $t\in\mathbb{R}$, we obtain the total detection probability for constant L,

$$P_{tot} = \int dp\rho(p,p)\,\frac{\tilde{R}(p,\epsilon_p)}{2p}. \tag{14}$$

This means that quantity

$$\alpha(p) := \frac{\tilde{R}(p,\epsilon_p)}{2p} \tag{15}$$

is the *absorption coefficient* of the detector, i.e., it expresses the fraction of particles of momentum p that are absorbed, and hence detected. We note that we integrated t over the full real axis, because the contribution of $t<0$ is very small if the initial state has no negative momenta. This is an approximation, as only positive values of t are operationally meaningful.

Conditional probability distribution $P_c(t,L) := P(t,L)/P_{tot}$ is normalized to unity, as long as the initial state contains positive momenta. Then, we define the post-selected density matrix

$$\rho_{ps}(p,p') := \frac{\sqrt{\alpha(p)}\sqrt{\alpha(p')}}{P_{tot}}\rho(p,p'). \tag{16}$$

We note that for an initial pure state, the post-selected state remains pure. Then,

$$P_c(t,L) = \int \frac{dpdp'}{2\pi}\rho_{ps}(p,p')\sqrt{v_pv_{p'}}S(p,p')e^{i(p-p')L-i(\epsilon_p-\epsilon_{p'})t}, \tag{17}$$

where $v_p = p/\epsilon_p$ is particle velocity. $S(p,p')$ are matrix elements $\langle p|\hat{S}|p'\rangle$ of *localization operator* $\hat{S}$ defined by

$$\langle p|\hat{S}|p'\rangle := \frac{\tilde{R}\left(\frac{p+p'}{2},\frac{\epsilon_p+\epsilon_{p'}}{2}\right)}{\sqrt{\tilde{R}(p,\epsilon_p)\tilde{R}(p',\epsilon_{p'})}}. \tag{18}$$

By definition, $\langle p|\hat{S}|\hat{p}'\rangle \geq 0$ and $S(p,p)=1$. Its name originates from the fact that $\hat{S}$ describes the localization of an elementary measurement event. $\hat{S}$ is a positive operator if $\ln\tilde{R}(p,\epsilon_p)$ is a convex function of p. Then, the Cauchy–Schwarz inequality applies:

$$\langle p|\hat{S}|p'\rangle \leq \sqrt{\langle p|\hat{S}|p\rangle\langle p'|\hat{S}|p'\rangle} = 1. \tag{19}$$

Maximal localization is achieved when Equation (19) is saturated, i.e., for $\langle p|\hat{S}|p'\rangle = 1$. For a pure initial state,

$$P_c(t,L) = \left| \int_0^\infty dp \psi_{ps}(p) \sqrt{v_p} e^{ipL - i\epsilon_p t} \right|^2. \quad (20)$$

Equation (20) was first derived in [21]. In the non-relativistic limit, it coincides with Kijowski's formula [22]. These expressions differ from those obtained by the current operator $\hat{J} = \frac{1}{2}[\hat{v}\delta(\hat{x} - L) + \delta(\hat{x} - L)\hat{v}]$,

$$P_J(t,L) = \int \frac{dpdp'}{2\pi} \rho_{ps}(p,p') \frac{1}{2}(v_p + v_{p'}) e^{i(p-p')L - i(\epsilon_p - \epsilon_{p'})t}. \quad (21)$$

Unlike Equation (17), Equation (21) does not guarantee the positivity of probabilities.

To understand the meaning of the localization operator, we evaluate its Wigner–Weyl transform,

$$\tilde{S}(x,p) := \int \frac{d\xi}{2\pi} S\left(p - \frac{\xi}{2}, p + \frac{\xi}{2}\right) e^{-i\xi x}. \quad (22)$$

It is straightforward to show that $\int dx \tilde{S}(x,p) = S(p,p) = 1$. Furthermore, by Bochner's theorem, $\tilde{S}(x,p) \geq 0$. Hence, $\tilde{S}(x,p)$ is a p-dependent probability distribution with respect to x, which we denote by $u_p(x)$. This probability distribution defines the irreducible spread of the measurement record. Maximum localization corresponds to $u_p(x) = \delta(x)$ [13]. We note that the general form (17) is fixed by the requirement of Poincaré covariance [23]; however, this requirement does not fix the properties of the localization operator or its physical interpretation in terms of properties of the apparatus.

We can absorb the localization operator $\hat{S}$ through further post selection of the initial state,

$$\tilde{\rho}_{ps}(p,p') = S(p,p')\rho_{ps}(p,p'). \quad (23)$$

It is straightforward to show that $\tilde{\rho}_{ps}$ satisfies all properties of a density matrix. Hence, all time-of-arrival probabilities can be brought in the form of (19) modulo post selection of the initial state.

This uniqueness result has to be interpreted carefully. Uniqueness is not reflected in the measured probability distributions, which depend strongly on the properties of the detector. The latter are incorporated in two quantities, absorption coefficient $\alpha(p)$ and localization operator $\hat{S}$. In principle, we can determine both quantities for any detector: $\alpha(p)$ is determined by the attenuation coefficient of the detecting medium, while $\hat{S}$ can be determined through time-of-arrival experiments. Hence, both redefinitions, (23) and (16), make operational sense.

2.3. Wigner Representation

We gain some insight into the structure of the time-of-arrival probabilities by expressing them in terms of the Wigner function associated to density matrix $\rho_{ps}(p,p')$,

$$W(x,p) = \int \frac{d\xi}{2\pi} \rho_{ps}\left(p - \frac{\xi}{2}, p + \frac{\xi}{2}\right) e^{-i\xi x}. \quad (24)$$

With these definitions, Equation (17) becomes

$$P_c(t,L) = \int dp dx_0 dx_f W(x_0,p) u_p(x_f) F_t(L - x_f - x_0, t), \quad (25)$$

where

$$F_t(x,p) = \int_{-2p}^{2p} d\xi \sqrt{|v_{p+\xi/2} v_{p-\xi/2}|} e^{ix\xi - i(\epsilon_{p+\xi/2} - \epsilon_{p-\xi/2})t} \quad (26)$$

is a function on the classical phase space that represents the quantum time-of-arrival observable.

Changing variables to $x := x_f - x_0$, we can write Equation (25) as

$$P_c(t, L) = \int dp dx \tilde{W}(x, p) F_t(x, p), \tag{27}$$

where $\tilde{W}(x, p) = \int dx_f u_p(x_f) W(x - x_f, p)$ is the Wigner function associated to the post-selected quantum state $\tilde{\rho}_{ps}$. Hence, the localization operator implements a position smearing of the particle's Wigner function.

In the non-relativistic limit,

$$F_t(x, p) = m^{-1} \int_{-2p}^{2p} d\xi \sqrt{p^2 - \frac{\xi^2}{4}} e^{ix\xi - ip\xi t/m} = v_p \frac{\pi J_1(2p|x - v_p t|)}{|x - v_p t|}, \tag{28}$$

where J_1 is the Bessel function. We note that F_t takes negative values. This fact does not contradict the positivity of time-of-arrival probabilities. No probability distribution that is concentrated in the regions where F_t is negative is a Wigner function for a quantum state.

In the ultra-relativistic limit ($m = 0$),

$$F_t(x, p) = \int_{-2p}^{2p} d\xi e^{ix\xi - i\xi t} = \frac{\sin[2p(t - x)]}{p(t - x)}. \tag{29}$$

These expressions are to be compared with the corresponding classical expression for the time-of-arrival observable,

$$F_t^{cl}(x, p) = \delta(t - x/v_p), \tag{30}$$

and the expression obtained from the non-relativistic current operator

$$F_t^J(x, p) = \int_{-2p}^{2p} d\xi \frac{1}{2}(v_{p+\xi/2} + v_{p-\xi/2}) e^{ix\xi - i(\epsilon_{p+\xi/2} - \epsilon_{p-\xi/2})t} = v_p \frac{2\sin[2p(x - v_p t)]}{(x - v_p t)}. \tag{31}$$

The different versions of the time-of-arrival functions in the non-relativistic regime are plotted in Figure 1 as a function of $s := p(x - v_p t)$. The classical time-of-arrival observable is a delta function at $s = 0$. Equations (28) and (31) have finite spread around $s = 0$, and they differ strongly near $s = 0$.

2.4. Probability Dependence on the Detector Kernel

One motivation of this paper is the possibility of experimental comparison between different proposals for time-of-arrival probabilities. To achieve this, we need to specify an operationally meaningful initial condition. Such a condition was proposed in ref. [24]. In this work, a strong divergence was suggested of the prediction by Bohmian mechanics from the predictions of the semi-classical theory for the time of arrival.

The idea is that we prepare a number of particles in a box of width a, and then we remove one wall of the box (or both) at time $t = 0$. Then, the initial state is an eigenfunction of the Hamiltonian for a particle in a box, $\psi_n(x) = \sqrt{\frac{2}{a}} \sin(\pi x/a_n)$ for $x \in [0, a]$, and zero otherwise; here, $a_n = a/n$ and $n = 1, 2, \ldots$. Equivalently, in the momentum representation,

$$\psi_n(p) = \pi \sqrt{\frac{2}{a}} a_n \frac{1 + e^{-ia_n p}}{\pi^2 - a_n^2 p^2}. \tag{32}$$

To understand the dependence of probability density $P_c(t)$ on the detection kernel, we consider a simple model, where $S(p, p') = \exp[-\sigma^2(p - p')^2]$, i.e., a detection kernel that is determined by localization length σ. In Figure 2, we plot $P_c(t)$ for different values of σ; we also plot the pseudo-probability density (21). We see that the probability densities differ significantly at early times, and they have the same asymptotic fall-off behavior.

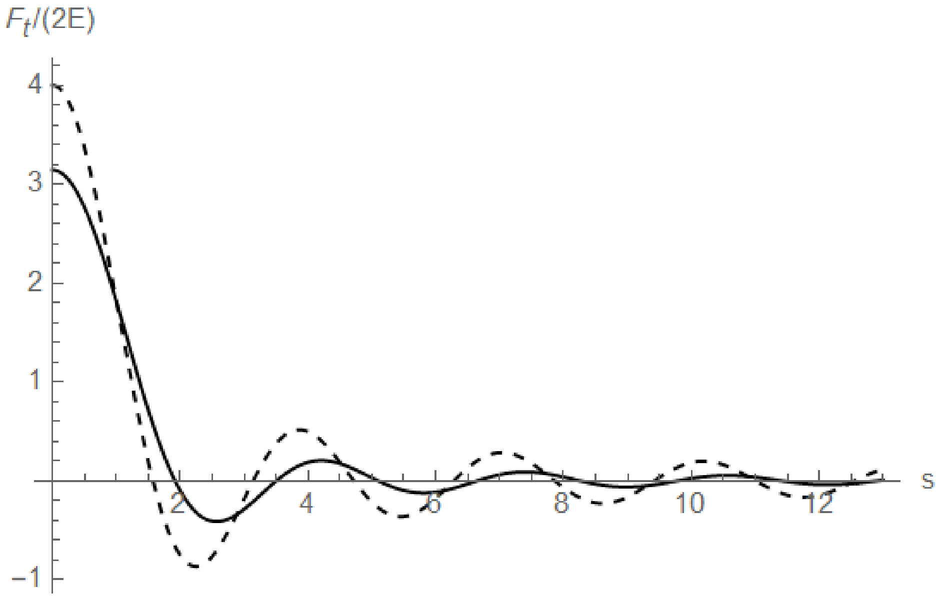

Figure 1. The time-of-arrival functions (28) (solid) and (31) (dashed) divided by $2E = p^2/m$ as a function of $s := p(x - v_p t)$.

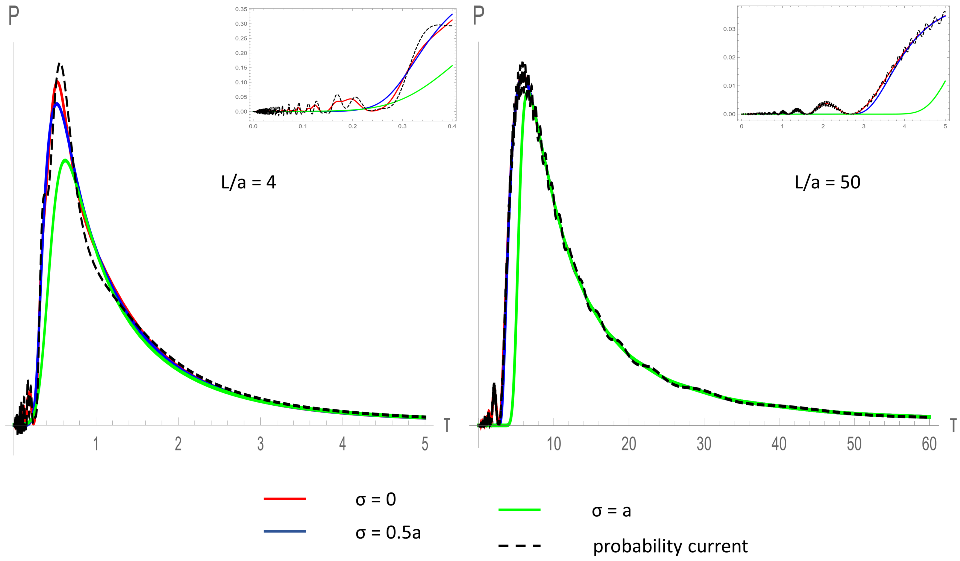

Figure 2. Probability density $P_c(\tau)$ as a function of $\tau := t/(2ma^2)$ for different values of localization length σ and for different distances, as well as for pseudo-probability density (21). The plots correspond to the non-relativistic regime. The insets show the early-time behavior of $P_c(t)$.

We also find that if $\sigma/\alpha << 1$, probability distribution $P_c(t)$ essentially coincides with the maximal localization probability distribution given by Equation (20). We also note that Distribution (21), defined by the current operator, becomes negative at early times; hence, it has no operational significance. The differences between the probability distributions are more pronounced for short distances L, i.e., in the near-field regime. At large distances L, only distributions with σ/α of order one or larger are distinguishable. The analysis suggests that it is, in principle, possible to distinguish experimentally between different probability distributions for the time of arrival in near-field measurements, as it was also shown in ref. [13].

3. Causality Issues

In this section, we show that the time-of-arrival probabilities suffer from small transient super-luminal terms. These terms originate from the fact that unitary evolution implies the Feynman propagation of sources, and the Feynman propagator does not vanish outside the light cone. We discuss possible strategies for removing such terms from the theoretical description, and we argue that this may not be possible in von Neumann-type models of measurement.

3.1. Apparent Causality Violation

Any theory of relativistic measurements for quantum systems must respect causality, in the sense that if the system is initially prepared within a localized spacetime region A, then there should be no detection signal in any region B space-like to A.

To implement this condition, we must first identify initial states that are localized in a spacetime region. This is a difficult task. First, we know that spatial localization at a moment of time does not work. Any notion of spatial localization in the single-time quantum state (even approximate ones) leads to faster-than-light signals [6,7]. This conclusion is not an artefact of specific models, but a consequence of fundamental properties of relativistic quantum systems, namely Poincaré covariance and energy positivity.

Rather than spatially localized states, we consider spacetime localized operations. We assume that the field is initially in the vacuum state $|0\rangle$, and that we prepare an initial state for the time-of-arrival measurement by an external intervention. This intervention has support in a compact spacetime region that lies wholly before the Cauchy surface $t = 0$. Then, we take the resulting state as $|\psi_0\rangle$ in Equation (5).

The simplest type of external intervention on a field involves the switching on of a source. This intervention is implemented through the inclusion of a time-dependent term $\int dx\hat{C}(x)J(x,t)$ in the field Hamiltonian. Here, $\hat{C}(x)$ is a local composite operator and $J(t,x)$ is a classical source with support in a compact spacetime region. Then, $|\psi_0\rangle = \mathcal{T}\exp[-i\int dtdx\hat{C}(x,t)J(x,t)]|0\rangle$.

For $\hat{C}(t,x) = \hat{\phi}(t,x)$, $|\psi_0\rangle$ coincides with the field coherent state $e^{-i\int dtdxJ(x,t)\hat{\phi}(t,x)}|0\rangle$, modulo a phase. Hence, the single-particle wave function is identified with

$$\psi_0(p) = \frac{1}{\sqrt{2\epsilon_p}}\int dtdxJ(t,x)e^{-ipx+i\epsilon_p t}. \quad (33)$$

We note that states of this form cannot have support only on positive momenta; hence, the probability density $P_c(t)$ is not normalized to unity. Any state with only positive momenta has a tail in a position that extends up to the location of the detector, and is therefore not appropriate for testing causality).

We consider a source that vanishes outside the spatial interval $[-a,0]$ and $[-T,0]$, where $a > 0$ and $T > 0$. We work in regime $a << T$, i.e., a spatially localized interaction of long duration. For concreteness, we take

$$J(t,x) = \begin{cases} \delta(x)\sin(\pi t/T), & t \in [-T,0] \\ 0, & \text{otherwise} \end{cases}. \quad (34)$$

Then,

$$\psi_0(p) = -\frac{\pi T(1+e^{-i\epsilon_p T})}{\pi^2 - \epsilon_p^2 T^2}. \quad (35)$$

Causality requires that the detection probability at $x = L$ vanishes for all $t < L - T$. This condition guarantees that the detector is only influenced by events that happen after the onset of the external intervention. This turns out not to be the case. Figure 3 shows that $P_c(t)$ starts becoming appreciably larger than zero slightly before $t = L - T$ and, in fact, it is non-zero for all times $t > 0$. The non-causal component is more pronounced in the near-field regime, i.e., small L.

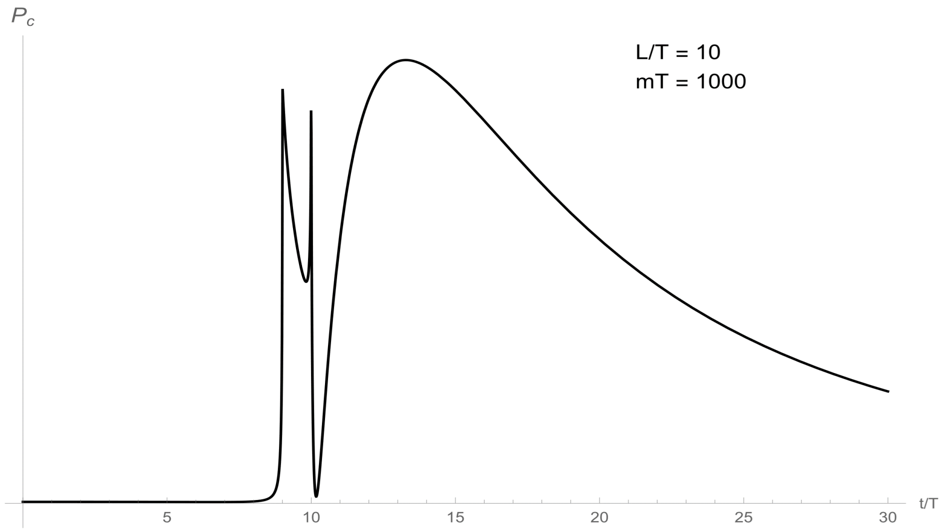

Figure 3. The probability density $P_c(t)$ for maximal localization as a function of $\tau := t/T$ for $L/T = 10$ and $mT = 10^3$. The non-causal behavior is manifested in the jump of the probability density prior to $t = L - T$.

We emphasize that this type of apparent causality violation is different from ones that have so far been considered in the literature. Past results focus on the evolution of spatially localized quantum states at a single moment of time. In contrast, here, we considered a *time-extended* external localized intervention. Hence, the causality problem is not an artefact of using idealizations from non-relativistic theory in describing a fundamentally relativistic system. It is also not an artefact of perturbation theory. The conditional probability density $P_c(t)$ that manifests the causality violating terms is of order λ^0 with respect to the field-apparatus coupling λ; higher perturbative corrections are negligible for sufficiently weak coupling.

This result does not indicate a fundamental failure of causality; rather, it points at the limitations of von Neumann's description of measurements, i.e., the description of measurements in terms of an interaction that is switched on. If we take the quantum formalism literally, Probability density (3) does not describe a measurement that takes place at time $\bar{t}$; $\bar{t}$ specifies the moment when the field-apparatus is on. Rather, Equation (3) refers to a measurement that takes place *after the field-apparatus interaction is switched-off*. Certainly, this distinction may be unimportant when the interaction time is very small. Indeed, in the models presented here, the width of the switching function can be made arbitrarily small.

In contrast, if we take the duration of the interaction larger than the distance between source and detector, no causality problem arises. However, the switching function is supposed to model a defining feature of a measuring apparatus; it cannot change arbitrarily if the distance between the source and the apparatus is changed. Hence, no matter how large the duration of the interaction is, we can always move the detector at a sufficiently large distance from the source so that the causality problem reappears.

In S-matrix theory, the coupling is assumed to be switched on and off adiabatically. This means that the effective duration of the interaction is arbitrarily large, and measurement takes place effectively at $t \to \infty$. In this description, transient terms disappear. However, this is not a solution of the problem, merely a sidestep. In any particle detector, particles are recorded at specific moments of time, i.e., *particle detection is a temporally localized process*.

An adiabatic switching of the interaction fails to localize the measurement event in time.

Hence, we conclude that von Neumann-type measurement models cannot account for a fundamental observed feature of the particle detection process, namely that particle detection is localized in time.

One possible way of restoring causality is by showing that the causality violating terms are impossible to measure, as a matter of principle. This resolution is conceivable, because QFT measurements are invariably characterized by false alarms, i.e., spurious detection events. Such events essentially define an irreducible background noise that persists even if the field is in the vacuum state. A detection signal exists only if it leads to a significant spike of recorded events over the noise background. This means that the super-luminal transient terms may be ignored if they contribute to probability less than the noise term due to vacuum.

In the present model, this noise is expressed by the constant background P_0 of Equation (13). This term is quite strong; in fact, it drowns a large part of the signal in the ultra-relativistic regime, and not only the transient terms. The strength of this term is an artefact of coupling switching. A realistic model for the apparatus must take into account that the initial state of the apparatus is correlated with the field vacuum, i.e., it is a dressed state. Term P_0 of Equation (13) incorporates also the equilibration of the apparatus with the field vacuum when the coupling is switched on. This equilibration is a spurious process. In the full description, the initial state of the apparatus is already equilibrated with the field. This implies that the restoration of causality through a signal/noise analysis requires a more nuanced approach to quantum measurements, beyond the use of switching functions. In Section 3.3, we argue that such an analysis is feasible in the QTP approach.

3.2. Retarded Propagator versus Feynman Propagator

For the initial state (33), the detection probability becomes

$$P(t,x) = P_0 + \int dsdyR(s,y)\Big[2\mathrm{Re}\Big[(\Delta_F J)(t-\frac{s}{2},x-\frac{1}{2}y)(\Delta_F J)(t+\frac{1}{2}s,x+\frac{1}{2}y)\Big] + (\Delta_F J)(t-\frac{s}{2},x-\frac{1}{2}y)(\Delta_F J)^*(t-\frac{s}{2},x-\frac{1}{2}y)\Big]. \quad (36)$$

Here, $\Delta_F J(t,x)$ stands for $\int dt'dx'\Delta_F(t-t',x-x')J(t',x')$, where

$$\Delta_F(t,x) = \int_{-\infty}^{\infty} \frac{dp}{4\pi\epsilon_p} e^{ipx-i\epsilon_p|t|} \quad (37)$$

is the Feynman propagator. The reason for the non-causal terms in the probability assignment is that the Feynman propagator does not vanish outside the light cone.

The causal solutions to the Klein–Gordon equation with a source, $\Box\phi - m^2\phi = J$, are retarded solutions, i.e., they are generated from J through the retarded propagator, $\phi(t,x) = \int dx'\Delta_{ret}(t',x')J(t,x)$. The retarded propagator Δ_{ret} is simply the imaginary part of Feynman propagator

$$\Delta_{ret}(t,x) = \int_{-\infty}^{\infty} \frac{dp}{4\pi\epsilon_p} e^{ipx} \sin(\epsilon_p|t|), \quad (38)$$

and it vanishes outside the light cone, i.e., for $|x| > |t|$.

In classical field theory, the restriction to retarded solutions is implemented as a boundary condition. This condition breaks the time-reversal invariance of the evolution equation; it defines an arrow of time. There are two main explanations for the emergence of this arrow: either it is a consequence of thermodynamic time asymmetry, or the boundary conditions must be specified at the cosmological level—see the analysis in Chapter 2 of [25].

Certainly, if we substitute Δ_F with Δ_{ret} in Equation (36), the problem of causality is resolved. Analogous substitutions have been proposed in the context of photo-detection theory, in order to cure Glauber's model from the transient super-luminal terms [26–29]). This amounts to a substitution of $e^{-i(\epsilon_p-\epsilon_{p'})t}$ with $\sin[(\epsilon_p-\epsilon_{p'})|t|]$ in Equation (12).

If we make this substitution, we cannot normalize probability density $P_c(t)$ by integrating t along the whole real axis. The error due to the inclusion of negative times exactly equals the error in the substitution of Δ_F with Δ_{ret} in $P_c(t)$. The two error terms do not cancel, but they add up. Still, for a large class of initial states, the correction in the total probability is small, so it is meaningful to define the post-selected density matrices (14) and (16), and write the causally corrected version of Equation (18),

$$P_c(t,L) = \left| \int_0^\infty dp \psi_{ps}(p) \sqrt{v_p} e^{ipL - i\epsilon_p t} \right|^2 - \left| \int_0^\infty dp \psi_{ps}(p) \sqrt{v_p} e^{ipL + i\epsilon_p t} \right|^2, \tag{39}$$

modulo the fact that Probability density (39) is not normalized to unity.

The problem is that the substitution of Δ_F with Δ_{ret} is equivalent to the substitution of evolution operator $e^{-i\hat{H}_\phi t}$ for the field with operator $\hat{K}_t = e^{-i\hat{H}_\phi t} - e^{i\hat{H}_\phi t}$ which is non-unitary. Not only is such a substitution completely ad hoc, it also contradicts the basic rules of quantum theory. Furthermore, $\hat{K}_t$ makes no sense as the evolution of an initial state, because $\hat{K}_0 = 0$. As long as we use a measurement model in which the instant of detection is determined by the Hamiltonian, the sources are Feynman propagated, and probabilities cannot be cured from super-luminal transients. A resolution requires, at the very least, an incorporation of time-irreversibility in the description of measurement in order to obtain some version of causal propagation.

3.3. Causal Propagation versus Restricted Propagation

Next, we describe the QTP approach to quantum field measurements [13,17,20] because it has the potential to resolve the spurious superluminality problem from first principles. In QTP, detection probabilities are genuine densities with respect to the space-time coordinates, and the interaction between system and apparatus is always present. The key point here is that the propagator for an initial source is not the Feynman propagator. A resolution of the superluminality problem within QTP requires an explicit modeling of the macroscopic apparatus in a way consistent with thermodynamic irreversibility. This is taken up in a different publication.

In QTP, we describe measurement events as a transition between two complementary subspaces $\mathcal{H}_+$ and $\mathcal{H}_-$ of Hilbert space $\mathcal{H} = \mathcal{H}_+ \oplus \mathcal{H}_-$. Subspace $\mathcal{H}_+$ describes the states of the system that are compatible with the realization of the event under consideration. If the event is a detection of a microscopic particle by a measuring apparatus, then subspace $\mathcal{H}_+$ corresponds to all states of the apparatus compatible with a macroscopic detection record. We denote the projection operator onto $\mathcal{H}_+$ as $\hat{P}$ and the projector onto $\mathcal{H}_-$ as $\hat{Q} := 1 - \hat{P}$.

Transitions that are correlated with the emergence of a macroscopic record of observation are *logically irreversible*. Once they occur, and a measurement outcome is recorded, further time evolution of the system does not affect our knowledge.

After transition occurs, a pointer variable λ of the measurement apparatus takes a definite value. We let $\hat{\Pi}(\lambda)$ be positive operators that correspond to the different values of λ. For example, when considering transitions associated with particle detection, the projectors $\hat{\Pi}(\lambda)$ may be correlated to the position or to the momentum of the microscopic particle. Since λ has a value only under the assumption that a detection event occurred, alternatives $\hat{\Pi}(\lambda)$ span subspace $\mathcal{H}_+$ and not the full Hilbert space $\mathcal{H}$. Hence, $\sum_\lambda \hat{\Pi}(\lambda) = \hat{P}$.

Assuming that the systems is prepared in state $|\Psi_0\rangle \in \mathcal{H}_-$, and that the Hamiltonian is $\hat{H}$, probability distribution $P(\lambda, t)$ that the transition took place at time t and an outcome λ was recorded is

$$P(\lambda, t) = \int d\tau \langle \Psi_0 | \hat{C}^\dagger(\lambda, t - \frac{\tau}{2}) \hat{C}(\lambda, t + \frac{\tau}{2}) | \Psi_0 \rangle, \tag{40}$$

where the *class operator*

$$\hat{C}(\lambda, t) := e^{i\hat{H}t} \sqrt{\hat{\Pi}}(\lambda) \hat{H} \hat{S}_t \tag{41}$$

is defined in terms of restricted evolution operator $\hat{S}_t$ which is the continuous limit of product $\hat{Q}e^{-i\hat{H}t/N}\hat{Q}e^{-i\hat{H}t/N}\hat{Q}\ldots\hat{Q}e^{-i\hat{H}t/N}\hat{Q}$ as number of steps N continues to infinity. By the Mishra–Sudarshan theorem [30], $\hat{S}_t$ is a unitary operator on $\mathcal{H}_-$ in this limit. However, one may modify the definition, for example, by regularizing the product so that there is an effective minimal time τ—see, for example, the analysis of Ref. [31]. Then, $\hat{S}_t$ may be non-unitary.

It is important to emphasize that Equation (40) is a genuine probability density with respect to both λ and t [17]. The derivation of Equation (40) requires the assumption of decoherence in the measuring apparatus.

We note that $P(\lambda, t)$ vanishes if for $[\hat{P}, \hat{H}] = 0$. We consider Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_I$, where $[\hat{H}_0, \hat{P}] = 0$, and H_I a perturbing interaction. Since all transitions are due to $\hat{H}_I$, to the leading order in the perturbation,

$$\hat{C}(\lambda, t) = e^{i\hat{H}_0 t}\sqrt{\hat{\Pi}}(\lambda)\hat{H}_I e^{-i\hat{H}_0 t}. \tag{42}$$

When we use Equation (42) for the measurement model of Section 2.1, and we take λ to coincide with position x of a record, we obtain Equation (12). The derivation is conceptually more rigorous here, because Equation (40) is a genuine probability density to all orders of perturbation theory. Furthermore, the interaction is always present; no switching is necessary in order to specify the instant of detection.

There are two distinct ways that the QTP approach to measurements could resolve the superluminality problem. First, in QTP, we can employ a dressed state for the apparatus, i.e., we may employ a non-factorized initial state $|\Psi_0\rangle$ for the field–apparatus system. In perturbation theory, $|\Psi_0\rangle = |\Omega, \psi_0\rangle + |\chi\rangle$, where $|\chi\rangle$ is a correction to the leading order in the coupling. A first-principle identification of $|\chi\rangle$ allow us identification of the vacuum noise for the measurement. Thus, we are able to check whether the super-luminal transients are strongly dominated by noise and are hence unobservable.

Second, in QTP, the initial state is propagated by restricted propagator $\hat{S}_t$. This implies that for states derived by localized source $J(x, t)$, the source is Feynman-propagated only at the tree level; higher-order terms incorporate contributions from the apparatus through projector $\hat{Q}$. We conjecture that these contributions will introduce desired time asymmetry in the evolution of $J(x, t)$ that may lead to a suppression of transient terms.

4. Conclusions

We presented our motivation and our results in the introduction. Here, we want to iterate the importance of a consistent formulation of a QFT measurement theory in order to provide a first-principle construction of relativistic quantum information [19,20]. Such formulations are also important for the consistent description of proposed quantum optics experiments in space [32,33]. These are characterized by very long baselines, and they can explore fundamental issues of relativistic locality and causality at a fundamental level.

The issues analyzed in this paper are challenges to any QFT measurement theory. Any such theory must provide specific predictions for time-of-arrival experiments, because in most of our measurements, the location of the detector is fixed and detection time is a random variable. The theory must also be consistent with locality and causality. This means that it must provide an explicit recipe for writing the initial state that corresponds to a given measurement set-up and then specify the associated probabilities. Superluminal transients must either be removed from the predicted probabilities or drowned by vacuum noise.

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