Article

Quantum Brain Dynamics: Optical and Acoustic Super-Radiance via a Microtubule

Akihiro Nishiyama 1,*, Shigenori Tanaka 1 and Jack A. Tuszynski 2,3,4

1 Graduate School of System Informatics, Kobe University, 1-1 Rokkodai, Nada-ku, Kobe 657-8501, Japan; tanaka2@kobe-u.ac.jp
2 DIMEAS, Politecnico di Torino, Corso Duca degli Abruzzi 24, I-1029 Turin, Italy; jackt@ualberta.ca
3 Department of Physics, University of Alberta, 11335 Saskatchewan Dr NW, Edmonton, AB T6G 2M9, Canada
4 Department of Data Science and Engineering, The Silesian University of Technology, 44-100 Gliwice, Poland

* Correspondence: anishiyama@people.kobe-u.ac.jp

Abstract: We aim to derive a super-radiance solution of coherent light and sound waves involving water degrees of freedom in the environment of a microtubule. We introduce a Lagrangian density functional of quantum electrodynamics with non-relativistic charged bosons as a model of quantum brain dynamics (QBD) involving water molecular conformational states and photon fields. We also introduce the model of charged boson fields (water degrees of freedom) coupled with phonons. Both optical and acoustic super-radiance solutions are derived in our approach. An acoustic super-radiance mechanism involving information transfer is proposed as an additional candidate to solve the binding problem and to achieve acoustic holography. Our results can be applied to achieve holographic memory storage and information processing in QBD.

Keywords: super-radiance; quantum brain dynamics; microtubule

1. Introduction

What is the physical mechanism of memory in the brain? How do we integrate information processing diffused in a brain? These are still open questions. Memory has several properties distinguished from computer memory [1], such as sequential patterns, auto-associative recalling, storage in a hierarchy, and memorized patterns in an invariant form. We address non-locality in processing of memory and perception, which suggests that even if any part of neocortex in rats is removed, they can perform tasks [2]. Memory is not localized in particular regions in a brain but rather distributed in the whole regions in a brain. To describe these properties of memory, Pribram proposed a holographic brain approach to describe memory and perception [3,4]. Holography is the technique to record 3-dimensional images on holographic plates using interference patterns of reference wave and object wave [5]. Even if we break the holographic plate into several parts, we can reconstruct the whole image from part of the plate, representing fractality in holography. Holographic aspect can describe non-local property of memory suggested in experiments by Lashley, where even if parts of neocortex are damaged, rats can recall the tasks from undamaged parts of neocortex. A holographic approach will be promising to describe the above-mentioned properties of memory in a brain. We then also require physical mechanisms for information transfer to integrate holograms diffused in the whole brain, which is the main topic of this paper.

The collaborators of Pribram were Jibu and Yasue. They studied quantum field theory of the brain called quantum brain dynamics (QBD) to describe memory and consciousness in the brain [6]. Quantum field theory (QFT) provides a powerful approach to investigating both microscopic degrees of freedom in quantum mechanics and macroscopic systems in classical physics. It is applied in a variety of fields, such as cosmology, elementary particle...
physics, nuclear physics, condensed matter physics, and also biology. Quantum field theory has been applied in the past to describe memory formation with interpretation involving subjective experiences in the brain. This approach originated with the monumental work of Ricciardi and Umezawa in 1967 [7]. In this theory, the brain is a mixed system of classical neurons and quantum degrees of freedom in QFT, that is, bosonic corticons and exchange bosons. What corticons and exchange bosons are was not given in this stage. In QFT of the brain, memory represents the vacua emerging in the breakdown of symmetry in QFT. The QFT of the brain was further developed by Stuart et al., applied, for example, to nonlocal memory storage, a memory recall mechanism due to Nambu–Goldstone (NG) bosons, stability due to long-range correlation of NG bosons, etc. [8,9]. Then the non-local property of memory is described by the long-range correlation of NG bosons, which propagate in the whole region of the brain. Around the same time, in 1968 Fröhlich proposed the existence of Bose–Einstein condensation involving long-range correlations in biological systems, which is referred to as the Fröhlich condensation [10,11]. Collective oscillation of a macromolecule may induce phonon condensation, namely coherent oscillation of the entire molecules of proteins, as shown in [12]. Long-lived coherence and the collective motion of condensate might emerge for Fröhlich condensate, where bovine serum albumin and lysozyme are likely the candidates [13]. Azizi et al. also suggested that terahertz vibrational modes of phonons in biological systems, such as frequency 0.3 THz bovine serum albumin protein, might correspond to Fröhlich condensate [14]. In 1976, Davydov and Kislikha derived a special solution representing a solitary wave propagating along the alpha-helix structures of protein chains and also for DNA, which is called the Davydov soliton [15]. The Davydov model has a quantum Hamiltonian, but in the coherent state, it is approximated as a semi-classical Hamiltonian that leads to a non-linear (soliton) equation. The Fröhlich condensation and the Davydov soliton were found to represent static and dynamical features, respectively, for a non-linear Schrödinger equation with an equivalent quantum Hamiltonian [16].

In the 1980s, Del Giudice et al. suggested a QFT-based approach applicable to biological systems, such as Bose–Einstein condensates of dipolar modes in membranes [17,18], coherence domains in proteins [19], water molecule ensembles acting as a laser [20], and Josephson systems [21]. Especially, the size of a coherent domain has been estimated as 14.6 nm [19], which corresponds to the size of the inner diameter of a microtubule in eukaryotic cells, 15 nm. In the 1990s, Jibu and Yasue proposed concrete degrees of freedom in QFT of the brain or quantum brain dynamics (QBD), which are water rotational dipole fields and photon fields [6,22–24]. Memory in this approach is represented as the vacua emerging in the breakdown of rotational symmetry of water dipoles where water dipoles are aligned in the same direction and memory recall processes are represented as excitations of a finite number of incoherent photons [25,26]. Macroscopic order of aligned dipoles is maintained by long-range correlation of NG bosons, namely dipole-wave quanta. These authors introduced optical super-radiance solutions discovered by Dicke in 1954 [27–29] to solve the binding problem in neuroscience [30], namely the problem of how we integrate information processing diffused in the whole brain [31]. Water absorbs light [32] and also sound [33] with various ranges of wavelength. Inversely, water is expected as coherent light and sound sources of various wavelengths, representing various water molecular conformational states. Furthermore, water is also the medium of holographic memory storage via interference patterns of coherent waves of light [34] in which the integrated version of QBD and holography is proposed. A microtubule is expected to be a device for super-radiant emission to achieve holographic memory [30,34,35]. Super-radiance of light (and also sound) might be adopted to integrate information processing among holographic memories diffused in the whole brain, which might solve the binding problem. When these optical and acoustic super-radiant waves are irradiated on holographic memories, simultaneous reconstruction of holograms diffuses in the whole brain. In 1995, Vitiello showed that our brain has a huge memory capacity by considering our brain as an open system and using two-mode squeezed coherent states for Nambu–Goldstone bosons [36].
He also suggested that squeezed coherent states correspond to fractals in nature \[37,38\]. Fractality of holography might be related to squeezed coherent states in dissipative systems.

Electromagnetic fields are well-known to be produced in the human brain, some of which are a consequence of action potential propagation along axons of neurons. The questions of whether these fields carry information or signals and whether they represent internal communication channels in the brain are still open, and further research needs to be done to address these issues. A recent paper by Cavaglia et al. \[39\] has provided an attractive hypothesis that electromagnetic fields within the brain can lead to the formation of holographic images as representations of external sensory inputs. They further postulated that stimulation of membrane dipoles by action potentials carrying electric field gradients causes coherent dipole oscillations in the neuronal membranes. As a result of these oscillations, under suitable conditions of signal strength and frequency, dipolar oscillations may satisfy the weak Bose-Einstein condensate criteria of biological coherence as formulated by Fröhlich. Consequently, such coherent electromagnetic field oscillations can lead to constructive and destructive interference patterns consistent with the holographic image generation hypothesis put forward by Pribram decades ago. Moreover, due to the symmetric distribution of membrane dipoles, a mirror image of such electromagnetic patterns could be formed in the neuronal cytoskeleton, specifically in microtubule bundles, and this could lead to the formation of memory as explored from a purely physical point of view many years ago \[40\].

A microtubule is a cellular structure shaped as a hollow cylinder and is present in all eukaryotic cells, including neurons. Together with actin and intermediate filaments, microtubules represent the protein polymers of the cytoskeleton, whose monomer is a protein called tubulin. The tubulin monomer is about 4 nm high, 5 nm large, and 4 nm wide, while the length of a microtubule is very variable, and it changes continuously due to polymerization and depolymerization processes with a 1 micrometer length that can be taken as an order of magnitude estimate. In neurons, microtubules form parallel bundles, which are stabilized laterally by microtubule-associated proteins (MAPs). Consequently, neuronal microtubules do not change their lengths under normal conditions. Microtubules are an excellent example of a perfectly engineered cellular structure that can accomplish multiple tasks such as providing mechanical strength, pathways for intracellular transport, cell motility, morphogenesis, cell division, and signaling. Lee et al. suggested that cytoskeletons have a dynamic network of proteins that enables significant cellular processes for motility, division, and growth \[41\]. Tryptophans in microtubules might be adopted as information processors. Babcock et al. suggested mega-networks of tryptophans in microtubules involving super-radiant states of tryptophan, where super-radiance can survive even at thermal equilibrium \[42\]. Super-radiant emission can realize the information transfer in a similar way to that for two separated quantum dots establishing photonic information processing in an experimental study \[43\]. Super-radiance reveals long-range atom-atom interactions if atoms are confined in one dimension \[44\]. Microtubules are surrounded by mitochondria. The production of reactive oxygen species (ROS) by mitochondria might induce ultraweak photon emission within cells \[45\]. The absorber of these photons is microtubules. Microtubule networks might transfer ROS-generated photons. Ultra-weak photon emission due to oxidative stress is also investigated in \[46\]. Lin et al. investigated circadian clock activity due to photoreduction mediated by tryptophan \[47\].

In the present paper, we follow up on these concepts and examine their concrete mathematical formulation in terms of electromagnetic modes of excitation in neuronal microtubules. Specifically, we aim to derive super-radiance solutions for both coherent light and sound waves via a microtubule surrounded by the water molecules of the cytoplasm, as shown in Figure 1. Beginning with the Lagrangian density of QFT of charged bosons representing water degrees of freedom coupled with photons and phonons, we derive both optical and acoustic super-radiance solutions by assuming coherence inside a microtubule since the size of a coherent domain corresponds to that of the inner diameter of a microtubule (approximately 15 nm) \[19\]. Bosonic fields are appropriate in describing cooperative
behaviors of multiple molecules representing macroscopic coherent properties that might emerge in life processes such as Bose–Einstein condensates, the breakdown of symmetry involving condensation of NG bosons, super-radiance, and so on. The advantage of QFT is that we can describe cooperative behaviors of molecules or dipoles as the dynamics of coherent fields, such as super-radiance with fast cooperative decay. Instead, we encounter the disadvantage that sub-radiant states with slow decay might not emerge as solutions of coherent fields and that we cannot trace the dynamics of individual qubits, representing molecules or dipoles. These qubits will appear with statistical properties for quantum many-body problems in QFT. Super-radiance, cooperative spontaneous emission of light and sound, propagates along the parallel direction to the cylindrical axis of a microtubule. In our derivation, we refer to [29,48]. We find an analogy in deriving optical and acoustic super-radiance since both photon and phonon fields are coupled with derivatives of charged Bose fields. The time scales of super-radiance are less than a picosecond, which is much smaller than the time scales of thermal noise and dissipative processes ~10 ps, so that super-radiant emission is achieved without thermal loss. Furthermore, since the speed of sound is extremely small compared with the speed of light, the energy level needed to achieve acoustic super-radiance is on the order of 10 cm\(^{-1}\) or ~1 meV. This energy scale corresponds to the energy difference of the rotational motion of water molecules [49]. Phonon density for acoustic super-radiance corresponds to the order of the density of water molecules in a liquid state, namely 10\(^{28}\) m\(^{-3}\). Minimum signal-to-noise ratio required for information transfer for 10\(^4\) bits \(\cdot s\) is on the order of 10\(^{-5}\) for wavelength \(\lambda = 3\) micrometers for sound. Acoustic super-radiance might be an additional candidate for information transfer to solve the binding problem in the brain, namely how cognitive processes are mechanistically carried out in the material substrate of the human brain. We also propose an acoustic holography mechanism based on the modulation of the density of water molecules by interference patterns of acoustic super-radiance for information processing. Since the modulation of the water density affects the transmittance of light, acoustic holography might be tightly correlated with optical holography, providing an attractive explanation of the integration of diverse sensory inputs into a consistent and simultaneous representation of the outside world.

![Image](image_url)

**Figure 1.** Super-radiance via a microtubule due to cooperative spontaneous emission of light and sound. Water molecules (green circles) in the excited state decay cooperatively.

This paper is organized as follows: In Section 2, we introduce a Lagrangian density of QFT of charged bosons coupled with photons or phonons and derive both optical and acoustic super-radiance solutions. In Section 3, we discuss our results by estimating the time scales of super-radiance and the corresponding energy levels. In Section 4, concluding remarks and perspectives are provided. In our calculations, we adopt the natural unit where the light speed, the Planck constant, \(\hbar\) and the Boltzmann constant are all set to unity. We adopt the metric tensor \(\eta_{\mu\nu} = \text{diag}(1, -1, -1, -1)\) with space-time subscript \(\mu, \nu = 0, 1, 2, 3\) and spatial subscript \(i, j, k = 1, 2, 3\) in \(3 + 1\) dimensions.

### 2. Super-Radiance in the Model of Quantum Brain Dynamics

In this section, we begin by proposing a Lagrangian density of quantum electrodynamics (QED) of non-relativistic charged bosons as a model for the interactions involving the water degrees of freedom coupled with photons or phonons in order to derive
time-evolution equations for coherent fields and to then obtain solutions for optical and acoustic super-radiance.

2.1. Optical Super-Radiance

The Lagrangian density of \( \text{(QED)} \) involving photons and charged bosons is given by

\[
\mathcal{L}(x) = -\frac{1}{4} F_{\mu\nu}[A] F^{\mu\nu}[A] - \frac{i}{2} \partial^\mu A_\mu - (\phi^* A_\mu \partial_\mu \phi - \phi A_\mu \partial_\mu \phi^*),
\]

where we have introduced the electromagnetic tensor \( F_{\mu\nu}[A] = \partial_\mu A_\nu - \partial_\nu A_\mu \) represented by scalar potential \( A^0 \) and vector potential \( A^i \), gauge fixing parameter \( \xi \), charged Bose fields \( \phi^*(r), \phi(r) \), potential \( U(x) \), charge \( e \), and mass \( m \). We adopt dressed (evanescent) electromagnetic fields and charged Bose fields to describe the entity of consciousness in the framework of quantum field theory, in which quantum fields are distinguished from classical objects. We adopt the temporal axial gauge \( A^0 = 0 \) with \( \xi = 1 \). The 2-Particle-Irreducible effective action [50,51] for the expectation values of coherent fields \( \bar{A} = \langle A \rangle \) and \( \phi^*(r) = \langle \phi^*(r) \rangle \) is given as,

\[
\Gamma_{2\text{PI}}[\bar{A}, \phi, \phi^*, \text{Green’s functions}] = \int d^3x \left[ -\frac{1}{4} F_{\mu\nu}[\bar{A}] F^{\mu\nu}[\bar{A}] - \frac{i}{2} (\partial^\mu \phi^* A_\mu - \phi A_\mu \partial_\mu \phi^*) \right] \]

\[ + \frac{\phi^* (i \frac{\partial}{\partial \phi} - U + \frac{\nabla_i - ie A_i}{2m}) \phi}{(2\pi)^3} \]

+ Quantum Fluctuations.

In the following, we omit the bar symbol for the expectation values of coherent fields and neglect quantum fluctuations.

Differentiating the effective action by \( A^i, \phi, \) and \( \phi^* \), we derive the Klein–Gordon equation as follows

\[
\left( \frac{\partial_0^2 - \partial_i^2}{m^2} + \frac{\epsilon^2 \phi^* \phi}{m} \right) A_i = -\frac{ie}{2m} (\phi^* \nabla_i \phi - (\nabla_i \phi^*) \phi),
\]

and the Gross–Pitaevskii (GP) equation

\[
\left( i \frac{\partial}{\partial x^0} + \frac{\nabla_i^2}{2m} - U - \frac{\epsilon^2 A_i^2}{2m} - \frac{ie \nabla_i A_i}{2m} - \frac{ie A_i \nabla_i}{m} \right) \phi = 0.
\]

We adopt the cylindrical polar coordinate \( (r, \theta, x^3) \) for the cylindrical microtubule and photon fields \( A_r, A_\theta, \) and \( A_3 \). Due to the relation \( A_r = A_1 \cos \theta + A_2 \sin \theta \) and Equation (3); we find

\[
\frac{\partial_0^2 A_r}{m^2} = \cos \theta \times \partial_0^2 A_1 + \sin \theta \times \partial_0^2 A_2 \]

\[ = \cos \theta \left( \partial_0^2 A_1 - \frac{\epsilon^2 \phi^* \phi}{m} A_1 - \frac{ie}{m} (\phi^* \nabla_1 \phi - \phi \nabla_1 \phi^*) \right) \]

\[ + \sin \theta \left( \partial_0^2 A_2 - \frac{\epsilon^2 \phi^* \phi}{m} A_2 - \frac{ie}{m} (\phi^* \nabla_1 \phi - \phi \nabla_1 \phi^*) \right),
\]

where we have used \( A_1 = A_r \cos \theta - A_\theta \sin \theta, A_2 = A_r \sin \theta + A_\theta \cos \theta \), the relation

\[
\cos \theta (\partial_0^2 + \partial_2^2) A_1 + \sin \theta (\partial_1^2 + \partial_2^2) A_2 = \cos \theta \left( \frac{1}{r} \partial_r (r \partial_r) A_r + \frac{1}{r \sin \theta} \partial_\theta (r \partial_\theta) A_\theta \right) \]

\[ + \sin \theta \left( \frac{1}{r} \partial_r (r \partial_r) A_\theta + \frac{1}{r \sin \theta} \partial_\theta (r \partial_\theta) A_r \right),
\]

\[
= \frac{1}{r} \partial_r (r \partial_r) A_r + \frac{1}{r^2} \partial_\theta (r \partial_\theta) A_\theta - \frac{A_r}{r} - \frac{A_\theta}{r \sin \theta},
\]

and the relation
\[
\cos \theta \partial_1 + \sin \theta \partial_2 = \frac{\partial}{\partial r},
\]

and we have considered the solution \( A_r \) independent of the angle \( \theta \) with \( A_\theta = 0 \) and \( A_\phi = 0 \). Similarly, using the relations \( A_\phi A_r = A_r A_\phi + \frac{1}{r} A_r \), Equation (4) is rewritten by

\[
\left( i \frac{\partial}{\partial x^0} + \frac{\partial^2}{2m} - U - \frac{e^2 A_r^2}{2m} - \frac{ie}{2m} \partial_r A_r - \frac{ie}{2m} \frac{A_r}{r} \right) \varphi = 0. \tag{8}
\]

Next, we expand \( \varphi \) by normalized eigenfunctions \( \phi_n \) as

\[
\varphi(x^0, r, x^3) = \sum_n b_n(x^0) \phi_n(r, x^3), \tag{9}
\]

with

\[
\left( -\frac{\nabla^2}{2m} + U(x) + \frac{e^2 A_r^2}{2m} \right) \phi_n = E_n \phi_n, \tag{10}
\]

where \( E_n \)’s \( (n = 0, 1, 2 \ldots) \) represent energy eigenvalues. We introduce \( \chi_n(x^0) \) as

\[
b_n(x^0) = e^{iE_n x^0} \chi_n(x^0). \tag{11}
\]

We adopt a two-energy level approximation with \( E_0 \) and \( E_1 \) representing the energy eigenvalues for the ground state and first excited state, respectively. We find the solution of the relation:

\[
\left( \partial_r^2 + \frac{1}{r} \partial_r - \frac{1}{r^2} - M^2 + \partial^2_3 \right) A_r = -\omega^2 A_r, \tag{12}
\]

as

\[
A_r = \int_0^\infty \frac{dk}{2\pi} J_1(kr) \left( \alpha^+ e^{ikx^3} + \alpha^- e^{-ikx^3} \right) e^{-i\omega x^0} \\
+ \int_0^\infty \frac{dk}{2\pi} J_1(kr) \left( \beta^+ e^{-ikx^3} + \beta^- e^{ikx^3} \right) e^{i\omega x^0}, \tag{13}
\]

with the Bessel function of the first kind \( J_1 \), \( \omega = \sqrt{k^2 + \alpha^2 + \beta^2} \) (wavenumber \( k \) and parameter \( kr \)), spatial average of the mass for photons \( M^2 = \frac{e^2(q^* q)_{\text{ave}}}{m^2} \) (we only consider the average mass \( M^2 \)) and constant \( \alpha^+ \) and \( \beta^\pm \). Hence, we obtain the solution of Equation (5) as

\[
A_r(x^0, r, x^3) = \int_0^\infty \frac{dk}{2\pi} J_1(kr) \frac{1}{2} \left( \alpha^+_k (x^0) e^{-i(\omega x^0 - kx^3)} + \alpha^-_k (x^0) e^{i(\omega x^0 - kx^3)} \right) \tag{14}
\]

Multiplying \( \phi_0^\dagger(r, x^3) \) in Equation (8) and integrating it over space \( \int dx \), we find the relation

\[
\frac{\partial \chi_0}{\partial x^0} = \frac{e}{m} \int_x \int_k \left[ \frac{1}{2} \left( \alpha_k e^{-i(\omega x^0 - kx^3)} + \alpha_k^* e^{i(\omega x^0 - kx^3)} \right) \right] \\
\times \left[ e^{-if \chi_0} (\phi_0^{\dagger} \partial_r \phi_1) \chi_1 + \phi_0^\dagger (\partial_r \phi_0) \chi_0 \right] \\
+ \frac{e}{2m} \int_x \int_k \left( \partial_r J_1 + \frac{\hbar}{r} \right) \frac{1}{2} \left( \alpha_k e^{-i(\omega x^0 - kx^3)} + \alpha_k^* e^{i(\omega x^0 - kx^3)} \right) \\
\times \left[ e^{-if \chi_0} (\phi_0^{\dagger} \partial_f \phi_1) \chi_1 + (\phi_0^\dagger \partial_f \phi_0) \chi_0 \right], \tag{15}
\]

with \( \int \int \frac{dk}{2\pi} \) and energy difference \( \Omega = E_1 - E_0 \). We only consider the resonant mode \( \Omega = \omega \) by using the rotating-wave approximation. We then find:
\[ i \frac{\partial \chi_1}{\partial x^0} = \frac{e}{4m} \int r dr d\theta dx^3 \int r f_i e^{-ikx^3} \left[ \frac{\partial}{\partial x^0} \left( \frac{1}{2} \epsilon_{ij} \phi_0^i \partial_j \phi_1 + \frac{1}{2} \phi_0^i \partial_j \phi_1 + \frac{\lambda}{2} (\partial_j f_1 + \frac{1}{3} \phi_0^j) \phi_1 \right) \right] \chi_1 \]

\[ = \frac{e}{4m} \int \int r dr d\theta dx^3 e^{ikx^3} \left[ \phi_0^i \partial_j \phi_1 f_1 - r (\partial_j f_1) \phi_0^j \phi_1 - r f_1 (\partial_j \phi_0^j) \phi_1 \right] \chi_1 \]

\[ = \frac{e}{4m} \int \int r dr d\theta dx^3 \left[ r f_1 (\phi_0^i \partial_j \phi_1 - (\partial_j \phi_0^j) \phi_1) \right] \chi_1, \quad (16) \]

where we have used partial integration for \( r \). Similarly, multiplying \( \phi_1^* \) in Equation (8), integrating it over space \( \int dx \), and over the rotating-wave approximation, we derive the following relationship:

\[ i \frac{\partial \chi_1}{\partial x^0} = \frac{e}{4m} \int a_k \int dr d\theta dx^3 e^{ikx^3} \int r f_1 (\phi_0^i \partial_j \phi_0 - (\partial_j \phi_0^j) \phi_0) \chi_0. \quad (17) \]

Substituting Equation (14) in Equation (5), assuming the relation,

\[ \omega |a_k| \gg |\frac{\partial \phi_1}{\partial x^0}|, \quad (18) \]

applying the Fourier transformation \( \int dx^3 e^{-i\omega x^0} \), and considering only the resonant mode, we find

\[ -i \omega \frac{\partial \phi_1}{\partial x^0} e^{-i\omega x^0} f_1(k, r) = \frac{ie}{2m} \int dx^3 e^{-i\omega x^0} \lambda_0^* \chi_1 \left( \phi_0^i \partial_j \phi_1 - \phi_1 \partial_j \phi_0^i \right). \quad (19) \]

We take only the solution satisfying the relation \( f_1(k, r) = 0 \) with an inner radius of a microtubule \( r \) and \( f_1 \) with zero points \( k \cdot \overline{r} = \nu_i \) with \( v_1 = 3.8, 7.0, 10.1, \ldots \) for \( j = 1, 2, 3, \ldots \), multiply \( f_1 \) in the above equation, integrate by space \( \int rdrd\theta \), and use the relation

\[ \int_0^1 d(\frac{r}{R}) \frac{f_1^2}{f_0^2} = \frac{1}{2} f_0^2 (v_j). \quad (20) \]

We then derive the relationship

\[ -i \omega a_k \frac{\partial a_k}{\partial x^0} f_2 \left( \frac{v_j \cdot v_j}{R^2} \right) = \frac{ie}{2m} \int dx^3 e^{-i\omega x^0} \lambda_0^* \chi_1 \left( \phi_0^i \partial_j \phi_1 - \phi_1 \partial_j \phi_0^i \right) \chi_0 \]

\[ = \frac{ie}{2m} \int \int \int dx^3 e^{-i\omega x^0} \lambda_0^* \chi_1 \left( \phi_0^i \partial_j \phi_1 - \phi_1 \partial_j \phi_0^i \right) \chi_0 \]

\[ \chi_0 \chi_1, \quad (21) \]

where \( y \) represents \( y = \frac{v_j \cdot v_j}{R^2} \), \( L_3 \) represents the length of a microtubule, and we have used the dipole approximation \( e^{i k x^3} \approx 1 \) since the wavelength is assumed to be larger than the system size of a microtubule. As a result, we find

\[ \frac{\partial a_k}{\partial x^0} = \frac{-ie \pi L_3}{\lambda_0^* \chi_1} \int_0^1 \bar{y} dy f_1 (v_j, v_j) \left( \phi_0^i \partial_j \phi_1 - \phi_1 \partial_j \phi_0^i \right). \quad (22) \]

Similarly, Equations (16) and (17) are rewritten by

\[ \frac{i \partial \chi_0}{\partial x^0} = \frac{e}{4} \int_{k \omega = \Omega} a_k^* \frac{2\pi \Omega L_3}{m} f_1 \int_0^1 \bar{y} dy f_1 (v_j, v_j) \left( \phi_0^i \partial_j \phi_1 - i (\partial_j \phi_0^j) \phi_1 \right) \chi_1, \quad (23) \]

\[ \frac{i \partial \chi_1}{\partial x^0} = \frac{e}{4} \int_{k \omega = \Omega} a_k^* \frac{2\pi \Omega L_3}{m} f_1 \int_0^1 \bar{y} dy f_1 (v_j, v_j) \left( \phi_1^* \partial_j \phi_0 - i (\partial_j \phi_0^j) \phi_0 \right). \quad (24) \]

We shall define

\[ A = \int_{\omega = \Omega} \frac{dk}{2\pi} a_k. \quad (25) \]

Integrating \( a_k \) by \( k \) with constraint \( \omega = \Omega \) and using \( \int_{k \omega = \Omega} = 2/L_3 \), Equation (22) is rewritten by

\[ \frac{\partial A}{\partial x^0} = -i2C \lambda_0^* \chi_1 (f_0 + f_1), \quad (26) \]
with \( C = \frac{e}{2\pi l_0^2 \Omega J^2} \) and,

\[
J_{01} = \frac{2\pi l_3}{m} \int_0^1 ydy J_1 \phi_0 i\partial_y \Phi_1, \tag{27}
\]

\[
J_{10} = \frac{2\pi l_3}{m} \int_0^1 ydy J_1 \phi_1 i\partial_y \Phi_0. \tag{28}
\]

We also find that Equations (23) and (24) are rewritten by

\[
\partial \chi_0 \partial x_0 = -\frac{i}{4} e (\tilde{J}_{01} + \tilde{J}_{10}^*) \chi_1 A^*, \tag{29}
\]

\[
\partial \chi_1 \partial x_0 = -\frac{i}{4} e (\tilde{J}_{10} + \tilde{J}_{01}^*) \chi_0 A. \tag{30}
\]

Replacing \( A \rightarrow -iA \) and assuming real \( A \), we arrive at

\[
\partial A \partial x_0 = 2C \chi_0 \chi_1 (\tilde{J}_{01} + \tilde{J}_{10}^*), \tag{31}
\]

\[
\partial \chi_0 \partial x_0 = e (\tilde{J}_{01} + \tilde{J}_{10}^*) A \chi_1, \tag{32}
\]

\[
\partial \chi_1 \partial x_0 = -e (\tilde{J}_{10} + \tilde{J}_{01}^*) A \chi_0. \tag{33}
\]

We shall assume real \( \tilde{J}_{01} + \tilde{J}_{10}^* \) and introduce \( g \) by

\[
g = \frac{e}{4} (\tilde{J}_{01} + \tilde{J}_{10}^*). \tag{34}
\]

Then we can rewrite

\[
\partial A \partial x_0 = 2g C \chi_0 \chi_1, \tag{35}
\]

\[
\partial \chi_0 \partial x_0 = g A \chi_1, \tag{36}
\]

\[
\partial \chi_1 \partial x_0 = -g A \chi_0. \tag{37}
\]

with \( C_1 = \frac{2}{\pi l_3 \Omega \tilde{J}_{01}^2} \). We find the number conservation law \( \partial \partial_0 (|\chi_0|^2 + |\chi_1|^2) = 0 \) in the above equations. When we define \( Z = |\chi_1|^2 - |\chi_0|^2 \) and \( R = 2\tilde{J}_{01}^* \chi_1 \) (assumed to be real), we can derive

\[
\partial_0 Z = -2g A R, \tag{38}
\]

\[
\partial_0 R = 2g A Z, \tag{39}
\]

\[
\partial_0 A = g C_1 R. \tag{40}
\]

We then find the energy conservation \( \partial_0 \left( \frac{1}{2} A^2 + \frac{C_1}{2} Z \right) = 0 \) and the number conservation as shown by

\[
\partial_0 \left( Z^2 + R^2 \right) = \partial_0 N^2 = 0, \tag{41}
\]

where \( N \) represents the sum of the number of water molecules in 1st excited state and the ground state.

Due to Equations (38) and (39), we can rewrite \( Z \) and \( R \) by

\[
Z = N \cos \theta, \tag{42}
\]

\[
R = N \sin \theta, \tag{43}
\]

and we find the relation,
In Equation (40), we introduce an additional term \( A/L_3 \) due to the release of radiation from a microtubule with the length of a microtubule \( L_3 \) as

\[
\frac{\partial A}{\partial x^0} + \frac{1}{L_3} A = gC_1 N \sin \bar{\theta}.
\]

(46)

In case \(|A/L_3| \gg |\partial_0 A|\), we arrive at

\[
\partial_0 \bar{\theta} = 2g^2C_1L_3N \sin \bar{\theta}.
\]

(47)

The solution of the above equation is

\[
\bar{\theta} = 2 \tan^{-1} \left( \exp \left( 2g^2C_1L_3N x^0 \right) \tan \frac{\bar{\theta}_0}{2} \right).
\]

(48)

We then arrive at

\[
A = \frac{1}{2g} \frac{\partial \bar{\theta}}{\partial x^0} = gC_1L_3N \left[ \cosh \left( \frac{x^0 - \tau_0}{\tau_R} \right) \right]^{-1},
\]

(49)

with the electric field in the radial direction \( E_r \sim \Omega A \) and

\[
\tau_R = \frac{1}{2g^2C_1L_3N}, \quad \tau_0 = -\tau_R \ln \tan \frac{\bar{\theta}_0}{2}.
\]

(50)

The time scale of radiation is \( \tau_R \propto 1/N \). Since \( N \) water molecules decay in \( 1/N \) time scale, the energy intensity of radiation is \( N^2 \) instantly. We have derived an optical super-radiance solution representing cooperative spontaneous emission of light in the framework of our model.

### 2.2. Acoustic Super-Radiance

In this section, we derive an acoustic super-radiance solution in our model.

We begin with the Lagrangian density given by

\[
\mathcal{L} = \psi^* \left( i \frac{\partial}{\partial x^0} + \frac{\nabla^2}{2m} - U - g_a \left( \partial_i Q_i \right) \right) \psi + \frac{1}{2} \left[ \left( \frac{\partial Q_i}{\partial x^0} \right)^2 - v^2 \left( \frac{\partial Q_i}{\partial x^i} \right)^2 \right],
\]

(51)

where \( \psi^{(*)} \) represents water fields, \( Q_i = -Q_i \) represents acoustic phonon fields, \( U \) represents potential energy, \( m \) represents the mass of a water molecule, \( v \) represents the sound velocity, and \( g_a \) is the coupling constant for acoustic phonons and water degrees of freedom. Phonon fields \( Q_i \) are coupled with density gradients of water \( \nabla_i (\psi^* \psi) \). We do not distinguish sound velocity of transverse and longitudinal modes of phonons.

Using 2-Particle-Irreducible Effective Action technique \([50,51]\) and differentiating it by expectation values \( \bar{\psi} = \langle \psi \rangle \) and \( \bar{Q}_i = \langle Q_i \rangle \), we can derive the Gross–Pitaevskii equation:

\[
\left[ i \frac{\partial}{\partial x^0} + \frac{\nabla^2}{2m} - U + g_a (\partial_i Q_i) \right] \bar{\psi} = 0,
\]

(52)

and the time-evolution equation for coherent phonon fields:

\[
\left( \frac{\partial^2}{\partial t^2} - v^2 \frac{\partial^2}{\partial x^2} \right) Q_i + g_a \nabla_i (\psi^* \psi) = 0,
\]

(53)

where we omit the bar for the expectation values of fields.

We adopt the cylindrical polar coordinates \( r, \theta, x^3 \) for cylindrical microtubule geometry and phonon fields \( Q_r, Q_\theta, \) and \( Q_3 \). Since we can use the relation \( Q_r = Q_1 \cos \theta + Q_2 \sin \theta \) and Equation (53), we find
\[ \partial_0^2 Q_r - \frac{\nu^2}{r} \left( \frac{1}{r} \partial_r (r \partial_r) - \frac{1}{r^2} + \partial_r^2 \right) Q_r = -g_a \partial_r (\psi^* \psi), \tag{54} \]

where we have used similar procedures to Equation (5) in the previous section. We shall write a solution \( Q_r \) for Equation (54) by

\[ Q_r = \int_{-\infty}^{\infty} \frac{dk}{2\pi} J_1(k r) \frac{1}{2} \left( q_k(x^0) e^{-i(\omega x^0 - k x^3)} + q_k^*(x^0) e^{i(\omega x^0 - k x^3)} \right), \tag{55} \]

with \( \omega = \sqrt{\nu^2 + k^2} \). We set \( k_r = \nu_j \) for \( j = 1, 2, 3, \ldots \) are zero-point for \( J_1 \), and \( \bar{r} \) is the inner radius of a microtubule. We investigate cases of \( Q_\theta = 0 \) and solutions for \( Q_r \) independent of \( \theta \).

Next, we expand \( \psi(x^0, r, x^3) \) by normalized eigenfunctions \( \phi_n(r, x^3) \) as

\[ \psi(x^0, r, x^3) = \sum_n b_n(x^0) \phi_n(r, x^3), \tag{56} \]

with

\[ \left( -\frac{\nu^2}{2m} + U \right) \phi_n = E_n \phi_n, \tag{57} \]

where \( E_n \)'s \((n = 0, 1, 2, \ldots)\) are energy eigenvalues. We also introduce \( \chi_n \) as

\[ b_n(x^0) = e^{-i E_n x^0} \chi_n(x^0). \tag{58} \]

We shall adopt two-energy level approximation in which \( E_0 \) and \( E_1 \) are energy eigenvalues for the ground state and 1st excited state, respectively. Multiplying \( \phi_0^* \) for Equation (52), integrating by space \( \int dx \), and using \( \partial_r Q_i = \partial_r Q_r + \frac{1}{r} Q_r \), we can derive

\[ i \frac{\partial \chi_0}{\partial x^0} = g_a \int dxdx' \int dx \frac{r}{2} \left( q_k e^{-i(\omega x^0 - k x^3)} + q_k^* e^{i(\omega x^0 - k x^3)} \right) \]
\[ \times \left[ e^{-i\Omega^2} \partial_r (\phi_0^* \phi_1) \chi_1 + \partial_r (\phi_0^* \phi_0) \chi_0 \right], \tag{59} \]

with \( \int_k = \int \frac{dk}{2\pi} \) and \( \Omega = E_1 - E_0 \). Similarly, multiplying \( \phi_1^* \) for Equation (52) and integrating by space, we find

\[ i \frac{\partial \chi_1}{\partial x^0} = g_a \int dxdx' \int dx \frac{r}{2} \left( q_k e^{-i(\omega x^0 - k x^3)} + q_k^* e^{i(\omega x^0 - k x^3)} \right) \]
\[ \times \left[ e^{i\Omega^2} \partial_r (\phi_1^* \phi_0) \chi_0 + \partial_r (\phi_1^* \phi_1) \chi_1 \right]. \tag{60} \]

We shall adopt rotating-wave approximation, in which we neglect non-resonant terms and quantum fluctuations. Then Equations (59) and (60) are rewritten by

\[ i \frac{\partial \chi_0}{\partial x^0} = \frac{g_a}{2} \int_{k 0 = \Omega} q_k \int_{x 0 = \Omega} \frac{dx}{x 3} \frac{1}{2} J_1(k r) \partial_r (\phi_0^* \phi_1) \chi_1, \tag{61} \]

and

\[ i \frac{\partial \chi_1}{\partial x^0} = \frac{g_a}{2} \int_{k 0 = \Omega} q_k \int_{x 0 = \Omega} \frac{dx}{x 3} \frac{1}{2} J_1(k r) \partial_r (\phi_1^* \phi_0) \chi_0. \tag{62} \]

Furthermore, using the relation

\[ \frac{\partial^2}{\partial (x^3)^2} Q_r = \int_{k} e^{-i(\omega x^0 - k x^3)} J_1 \left( -\omega^2 - 2i \omega \frac{\partial}{\partial x^0} + \cdots \right) \frac{q_k}{2} + (c.c.), \tag{63} \]

in Equation (54) assuming \( |\omega q_k| \gg |\partial_0 q_k| \), Fourier transforming by \( \int dx^3 e^{-il x^3} \) in Equation (54), we derive,

\[ -i \omega \frac{\partial q_0}{\partial x^0} e^{-i\omega x^0} J_1(k r) + \text{(other term } \propto e^{i\omega x^0}) = -g_a \int_{k} e^{-i\Omega^2} \partial_r (\phi_0^* \phi_1) \chi_0 \chi_1 \]
\[ + g_a \int_{k} e^{i\Omega^2} \partial_r (\phi_1^* \phi_0) \chi_1 \chi_0 + \partial_r (\phi_1^* \phi_1) \chi_1 \chi_1 + \partial_r (\phi_1^* \phi_0) \chi_0 \chi_0 \chi_1 \tag{64} \]
Taking the resonant part only in the rotating-wave approximation, we derive
\[ -i\omega \frac{\partial q_k}{\partial \nu} e^{-i\omega \nu} f_1 (k, \nu, r) = -g_a \int_{x^3} e^{-i k x^3} e^{-i \chi_0 \nu} (\chi_0^* \chi_1) \partial_y (\phi_0^* \phi_1). \]  
(65)

Integrating the above equations by \( \int r dr \theta \) and using the relation
\[ \int_0^1 dy f_1^2 (v, y) = \frac{1}{2} f_0^2 (v), \]  
we can derive
\[ -i \omega \frac{\partial q_k}{\partial \nu} \frac{1}{2} f_0^2 (v) 2 \pi r^2 = -g_a L_3 2 \pi r^2 \frac{1}{8} \int_0^1 y dy f_1 (v, y) \partial_y (\phi_0^* \phi_1) \chi_0^* \chi_1, \]  
(67)

where we have used the dipole approximation \( e^{-i k x^3} \approx 1 \) with the length of microtubule \( L_3 \), and we can derive
\[ -i \frac{\partial q_k}{\partial \nu} = \frac{2 L_3}{\omega f_0^2 (v)} \int_0^1 y dy f_1 (v, y) \partial_y (\phi_0^* \phi_1) \chi_0^* \chi_1. \]  
(68)

Using the dipole approximation \( e^{-i k x^3} \approx 1 \) in Equations (61) and (62), we derive
\[ \frac{i \partial \chi_0}{\partial \nu} = \frac{g_a}{2} \int_{k, \nu = \Omega} q_k^* 2 \pi L_3 \int_0^1 y dy f_1 (v, y) \partial_y (\phi_0^* \phi_1) \chi_0^* \chi_1, \]  
(69)

\[ \frac{i \partial \chi_1}{\partial \nu} = \frac{g_a}{2} \int_{k, \nu = \Omega} q_k 2 \pi L_3 \int_0^1 y dy f_1 (v, y) \partial_y (\phi_1^* \phi_0) \chi_0. \]  
(70)

We shall define
\[ Q \equiv \int_{k, \nu = \Omega} q_k, \]  
(71)

and use \( \int_{k, \nu = \Omega} = \frac{2}{L_3} \). Integrating Equation (68) by \( \int_{k, \nu = \Omega} \) we find
\[ -i \partial_0 Q = -g_a \frac{4}{2 \pi L_3 \chi_0^* \chi_1} \int_0^1 y dy f_1 (v, y) \partial_y (\phi_0^* \phi_1). \]  
(72)

The Equations (69) and (70) are written by
\[ i \partial_0 \chi_0 = g_a \pi r L_3 Q^* \int_0^1 y dy f_1 \partial_y (\phi_0^* \phi_1) \chi_1, \]  
(73)

\[ i \partial_0 \chi_1 = g_a \pi r L_3 Q \int_0^1 y dy f_1 \partial_y (\phi_1^* \phi_0) \chi_0. \]  
(74)

We shall define
\[ K_{01} (= K_{10}) \equiv \int_0^1 y dy f_1 \partial_y (\phi_0^* \phi_1) \times \pi r L_3, \]  
(75)

and \( g' \equiv g_a K_{01} \) and assume real \( K_{01} = K_{10} \). We also replace \( Q \to -i Q \) and assume real \( Q \). We then arrive at
\[ \partial_0 \chi_0 = g' Q \chi_1, \]  
(76)

\[ \partial_0 \chi_1 = -g' Q \chi_0, \]  
(77)

\[ \partial_0 Q = 2g' C_1 \chi_0^* \chi_1, \]  
(78)

with \( C_1 = \frac{2}{\pi r L_3 \omega f_0^2 (v)} \). The above relations suggest the number conservation
\[ \partial_0 (|\chi_1|^2 + |\chi_0|^2) = 0 \]  
with \( |\chi_1|^2 + |\chi_0|^2 = N \), where \( N \) represents water molecules inside a microtubule.

We define \( Z \equiv |\chi_1|^2 - |\chi_0|^2 \) and \( R \equiv 2\chi_0^* \chi_1 \) with assuming real \( R \). We then derive
\[
\begin{align*}
\partial_0 Z &= -2g' Q R, \\
\partial_0 R &= 2g' Q Z, \\
\partial_0 Q &= g' C_1 R,
\end{align*}
\]

suggesting the energy conservation \(\partial_0 \left( \frac{1}{2} Q^2 + \frac{1}{2} Z^2 \right) = 0\) and the number conservation \(\partial_0 (Z^2 + R^2) = \partial_0 N^2 = 0\). Due to the number conservation, we write \(Z = N \cos \bar{\theta}(x^0)\) and \(R = N \sin \bar{\theta}(x^0)\). Using relation (79) or (80), we find

\[
\partial_0 \bar{\theta} = 2g' Q, \quad \text{or} \quad \bar{\theta} = \bar{\theta}_0 + 2g' \int_{t_0}^{x^0} Q. \tag{82}
\]

We shall add \(\frac{Q}{L_3}\) in Equation (81) due to the release of radiation for a microtubule as

\[
\partial_0 Q + \frac{Q}{L_3} = g' C_1 N \sin \bar{\theta}, \tag{83}
\]

and assume \(\frac{Q}{L_3} \gg \partial_0 Q\). We then arrive at

\[
\partial_0 \bar{\theta}(x^0) = 2g'^2 C_1 L_3 N \sin \bar{\theta}(x^0). \tag{84}
\]

The solution of the above equation is

\[
\bar{\theta}(x^0) = 2 \tan^{-1} \left( \exp \left( 2g'^2 L_3 C_1 N x^0 \right) \tan \frac{\bar{\theta}_0}{2} \right). \tag{85}
\]

Finally, we arrive at the solution:

\[
Q = \frac{1}{2g'} \partial_0 \bar{\theta} = g' C_1 L_3 N \left[ \cosh \left( \frac{x^0 - \tau_0}{\tau_R} \right) \right]^{-1}, \tag{86}
\]

with

\[
\tau_R = \frac{1}{2g'^2 C_1 L_3 N}, \quad \tau_0 = -\tau_R \ln \tan \frac{\bar{\theta}_0}{2}. \tag{87}
\]

Since \(N\) water molecules decay cooperatively in time scales \(\tau_R \sim 1/N\), the intensity of sound is \(N^2\) instantly. We have derived the acoustic super-radiance solution via a microtubule representing cooperative spontaneous decay of water molecules.

3. Discussion

In this paper, we introduced the Lagrangian density of charged bosons representing water molecular conformational states coupled with photons or phonons, and have derived both optical and acoustic super-radiance solutions of the field dynamics in a microtubule. The oscillations generated in the radial direction in the cylindrical structure of a microtubule might induce coherent super-radiant emission of light and sound in the parallel direction to the cylindrical axis of a microtubule.

The Lagrangian density functional introduced in this paper involves photons or phonons represented by vector fields. These vector fields are coupled with derivatives of charged boson fields representing water degrees of freedom. Photon fields couple with the flow of charge, while phonon fields couple with the gradient of density of charged bosons. Since photon fields and phonon fields are coupled with derivatives of charged boson fields, the analogy in deriving super-radiance solutions for light and sound is found in Sections 2.1 and 2.2. In both cases of light and sound, we found the same forms of time scales \(\tau_R \sim 1/N\) where differences are coupling constants \(g\) and \(g'\).

Energy levels (energy differences between the ground state and the first excited state) necessary for the emission of coherent sound can be estimated as follows: The size of the coherent domain is estimated as 14.6 nm in [19]. It corresponds to the inner diameter of a microtubule 15 of nm. Since the mass squared of photons in Equation (3) is written by
\(e^2|\varphi|^2/m\), we find that the size of the coherent domain by the shielding effect as \(\sqrt{\left(e^2|\varphi|^2/m\right)^{-1}}\) is the order of 10 nm when we set the elementary charge \(e = 0.3\) and the density of the water molecule \(|\varphi|^2 = 3.3 \times 10^{28}\) m^{-3}, and the mass of the electron \(m = 0.5\) MeV even in our model. It is reasonable to consider solutions of coherent sound fields and charged Bose fields, which have zero point in the inner surface of a microtubule due to the size of the coherent domain. We consider the Bessel function of the first kind \(J_1(kr)\), which has zero points on the inner surface \(r = \bar{r} = 7.5\) nm of a microtubule. The zero points \(\nu_j's\) of \(J_1(y)\) are

\[
\nu_j = 3.832, 7.016, 10.17, 13.32, 16.47, 19.62, 22.76, 25.90, 29.05, 32.19, \ldots
\]  

(88)

Using the speed of sound in water \(v = 1510\) m \(\cdot\) s^{-1} at 303 K [52] and the inner radius of a microtubule \(r = 7.5\) nm, we found energy levels necessary for the emission of coherent sound as

\[
\omega > v \times \nu_j/\bar{r} = 0.506, 0.927, 1.34, 1.76, 2.18, 2.59, 3.01, 3.42, 3.84, 4.26 \ldots \text{meV}.
\]  

(89)

These energy levels are lower than but comparable to the energy difference between the ground state and the first excited state of rotational degrees of freedom of water molecules (4 meV), as shown in [34,35]. These energy levels correspond to frequencies that obey the Debye rotational-diffusion theory [49,53], for example 10 cm^{-1} corresponds to 1.24 meV. Rotational degrees of freedom of water molecules might induce acoustic super-radiance via a microtubule.

Time scales of optical and acoustic super-radiance \(\tau_R\) are estimated as follows: Although there is some ambiguity in the coefficient, we can use the relation

\[
\bar{j}_{01} = \frac{1}{\hbar} \int \int d\theta d\phi d^3x J_1(\varphi_0) \partial_x \varphi_1
\]

\[
\sim \frac{1}{\hbar} \int \int d\theta d\phi d^3x J_1(\varphi_0)(-i\varphi_1)
\]

\[
\sim \frac{\hbar}{\mu e}, \quad (\Omega = E_1 - E_0),
\]  

(90)

where \(\mu_e = 2ed_e\) represents the dipole moment of a water molecule with elementary charge \(e = 0.3\) and \(d_e = 0.2\) Å. We then set the energy difference \(\Omega = 0.506\) MeV in the above estimation. Subsequently, we find \(g = \frac{1}{2}(\bar{j}_{01} + \bar{j}_{10}) = \frac{\Omega \mu_e}{2} = 1.54 \times 10^{-8}\). As a result, we derive the following relationship:

\[
\tau_R = \frac{1}{2g^2 - 2\Omega_0(3.8)^2} \times L_3 \frac{N}{\pi r^2 L_3} = 4 \times 10^{-14} \text{ s},
\]  

(91)

where we have used \(\bar{j}_{01}(3.8) = -0.4\), the length of a microtubule \(L_3 = 1\) micrometer, and the density of water molecules \(N/(\pi r^2 L_3) = 3.3 \times 10^{28}\) m^{-3}. Even if we multiply the density of water molecules by 1/10, which might correspond to the partial ratio of the density of water molecules for emission in a particular molecular conformational state representing water molecules with particular structures and functions in the state, we find \(\tau_R = 0.4\) ps, which is much smaller than the time scales characteristic of thermal noise [30]; that is, super-radiant emission is achieved without thermal loss. If the coupling between phonons and charged bosons \(g' = g_0 K_{01}\) in Section 2.2 is in the same order as \(g = \frac{1}{2}(\bar{j}_{01} + \bar{j}_{10})\) in Section 2.1 and the same order energy difference \(\Omega\) is assumed, we find the same order \(\tau_R\) in both optical and acoustic super-radiance. We plot the time-evolution of the amplitudes \(Q(x_0)\) with \(\tau_0 = 0\) in Figure 2. Since we can use \(\bar{j}_{01}(\nu_j) = 0.402, 0.300, 0.250, 0.218, \ldots\) for \(j = 1, 2, 3, 4, \ldots\), the peak of \(Q\) is proportional to \(1/\bar{j}_{01}(\nu_j)^2\), while \(\tau_R\) is proportional to \(\bar{j}_{01}^2/\omega\) with \(\omega = 0.506, 0.927, 1.34, 1.76, \ldots\) meV for \(j = 1, 2, 3, 4, \ldots\). Time scales relevant for \(\tau_R\) are on the order of 0.001 to 0.1 ps. The time scale 0.1 ps is comparable with 0.2 ps, the time scale of optical super-radiance estimated in our preceding work in quantum brain dynamics (QBD) involving water dipoles and photons [34,35]. These time scales are substantially
shorter than those of thermal loss $\sim 10$ ps. Hence super-radiant propagation ends without loss by thermal noise and might be applicable to achieve information transfer in a brain. As the $j$ increases, the $Q$ tends to take sharp peaks on the order $10^6$ eV or $10^{-13}$ J. We here estimate the energy density of acoustic super-radiance. The energy density of acoustic super-radiance is $\omega^2 Q^2 / (\hbar c)^3$ with $\hbar c = 200 \text{ MeV} \cdot \text{fm}$ for the Planck constant $\hbar$ and the speed of light $c$. If we substitute $Q = 10^6$ eV and $\omega = 1$ meV for $\omega^2 Q^2$, we find the energy density $10^{26}$ eV $\cdot$ m$^{-3}$. Since the energy difference between the ground state and the 1st excited state for the rotational motion of a water molecule is 4 meV and the water number density is $3.3 \times 10^{28}$ m$^{-3}$ [35], the energy difference per volume is $10^{26}$ eV $\cdot$ m$^{-3}$ in QBD. The value $10^{26}$ eV $\cdot$ m$^{-3}$ for energy density of acoustic super-radiance is thus comparable with the energy difference per volume for water dipoles ($10^{26}$ eV $\cdot$ m$^{-3}$). Hence, the energy of rotational motion might be used for the emission of acoustic super-radiance. If the energy density of super-radiance is comparable with energy difference per volume for water dipoles and the energy of the phonon for super-radiance is comparable with 4 meV, the density of phonons for super-radiance is the order of $10^{28}$ m$^{-3}$ comparable with the number density of water molecules. Cifra et al. estimated that energy supply from mitochondria to microtubules is $10^{-14}$ W [54]. Dividing this value by the inner volume of a microtubule $\pi r^2 L_3$ with $r = 7.5$ nm and $L_3 = 1$ micrometer, we find the energy supply per second and volume is $10^{26}$ eV $\cdot$ m$^{-3}$ $\cdot$ s$^{-1}$. Since the energy density of acoustic super-radiance is $10^{26}$ eV $\cdot$ m$^{-3}$, an acoustic super-radiance emission occurs per 1 s per cell. (In case $Q = 10^5$ eV, an acoustic super-radiance emission occurs per 0.01 s per cell).

We address the binding problem for the synthesis of cognitive activities in the brain by proposing how information processing diffused in the whole brain can be integrated [31]. Optical super-radiance and the photon tunneling effect are the candidate mechanisms proposed earlier to participate in the integration of processing in quantum brain dynamics [25,30]. We propose acoustic super-radiance as an additional candidate to solve the binding problem. Since the sound velocity $v = 0.5 \times 10^{-5}$ ($= 1510$ m $\cdot$ s$^{-1}$) is extremely small compared with the speed of light unity, the energy $\omega = vk$ ($k$ representing wavenumber) needed for the emission of sound is extremely small compared with photon energy. Hence, we can adopt acoustic super-radiance for low-energy information transfer in the whole brain. Furthermore, electromagnetic signaling is attractive for information trans-
fer among cells [55]. Non-contact and non-chemical cell-to-cell communication might occur by sound, electromagnetic radiation, and electric current, although electromagnetic radiation might be more favorable than sound and electric current [56]. Both optical and acoustic super-radiance shown in this paper might be candidates for information transfer in a brain. We can estimate the signal-to-noise ratio necessary for information transfer by Shannon’s theorem [57]. This theorem suggests that the maximum theoretical capacity $C$ (in bits·s$^{-1}$) is proportional to the bandwidth $B$ (in Hertz) with signal-to-noise rate as $C = B \log_2 \left( 1 + \frac{S}{N} \right)$. Since $B = \frac{\tau}{\lambda}$ with wavelength $\lambda$ and sound velocity $v = 0.5 \times 10^{-5} = 1510 \text{ m} \cdot \text{s}^{-1}$, we derive the minimum signal-to-noise ratio as $\frac{S}{N} \sim 10^{-11}$ for $\lambda = 3$ micrometers (or $B = 0.5 \times 10^9$ Hz) for $C = 10^{-2}$ bits·s$^{-1}$ or $\frac{S}{N} \sim 10^{-5}$ for $\lambda = 3$ micrometers for $C = 10^4$ bits·s$^{-1}$, for example. In case the signal-to-noise ratio is beyond the above value, information transfer necessary for capacity $C$ will be achieved.

Water can be viewed as a source of both coherent light and sound, while it is also the potential medium for memory storage. Super-radiance, therefore, can be regarded as the mechanism for the formation of coherent light and sound in holographic information processing involving both light and sound. Interference patterns of light can serve as storage and processing of information [30, 34, 58]. Water media are adopted for holographic information processing and memory. Acoustic super-radiance introduced in this paper is proposed as the mechanism for information processing and memory formation by coherent sound waves. Since sound waves induce density gradients of water molecules, the transmittance of sound will change due to the density gradients in holograms of water. Density gradients will also change the transmittance of light. We might then encounter the interplay of ‘optical’ holography and ‘acoustic’ holography, and correlation between sound and light could be provided via water degrees of freedom.

We also provide the direction of our future research. While we have investigated super-radiance as coherent light and sound sources irradiated on holograms to achieve reconstruction of optical and acoustic images, we next plan to provide a control theory of optical and acoustic holograms by external light and sound. External light and sound on the scalp might propagate through intermediate layers, such as cerebrospinal fluid (CSF), dura, and skull, and arrive at neocortex. To develop the control theory, we adopt reservoir computing or morphological computation [59–61] applied to the QFT approach in a hierarchy [62]. We then need to investigate the correlation between bosonic quantum fields and neural network, which can be described by fractional-order differential and optimal control, nonlocal evolution equations [63], and the fractional-order neural network [64–66]. If our brain adopts the language of holography, we can manipulate hologram memory involving our subjective experiences by external light and sound fields. Then our approach will be applied to writing memories directly into the brain or deleting traumatic memories for the future experimental study.

4. Concluding Remarks and Perspectives

We have derived both optical and acoustic super-radiance solutions emitted in the parallel direction of a microtubule. Since the time scales of super-radiant emission of light and sound less than picosecond are much smaller than those of thermal effects ($\sim 10$ ps), super-radiance is achieved without thermal loss via a microtubule. The energy scales required for acoustic super-radiance correspond to the energy difference of water rotational degrees of freedom ($\sim 1$ meV). Phonon density for acoustic super-radiance is the order of the density of water molecules in liquid state. Minimum signal-to-noise ratio required to information transfer for $10^4$ bits·s$^{-1}$ is $10^{-5}$ for wavelength and $\lambda = 3$ micrometers for sound. Acoustic super-radiance can be adopted as an additional candidate to solve the binding problem. When coherent super-radiant light and sound waves are irradiated on holograms, images of light and sound will be reconstructed. Water degrees of freedom coupled with sound and light might play a significant role in holographic information processing and memory in the framework of quantum brain dynamics. If our brain adopts the language of holography, we might be able to manipulate hologram memory.
involving our subjective experiences by external light or sound fields. We will adopt the QFT framework adopted in this paper to manipulate hologram memory in the future study.

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