fractal and fractional

Editorial

The Craft of Fractional Modeling in Science and Engineering 2017

Jordan Hristov

Department of Chemical Engineering, University of Chemical Technology and Metallurgy, 8 Kliment Ohridsky Blvd, Sofia 1756, Bulgaria; jordan.hristov@mail.bg

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Fractional calculus has performed an important role in the fields of mathematics, physics, electronics, mechanics, and engineering in recent years. The modeling methods involving fractional operators have been continuously generalized and enhanced, especially during the last few decades. Many operations in physics and engineering can be defined accurately by using systems of differential equations containing different types of fractional derivatives.

This book is a result of the contributions of scientists involved in the special collection of articles organized by the journal *Fractal and Fractional* (MDPI), most of which have been published at the end of 2017 and the beginning 2018. In accordance with the initial idea of a Special Issue, the best published have now been consolidated into this book.

The articles included span a broad area of applications of fractional calculus and demonstrate the feasibility of the non-integer differentiation and integration approach in modeling directly related to pertinent problems in science and engineering. It is worth mentioning some principle results from the collected articles, now presented as book chapters, which make this book a contemporary and interesting read for a wide audience:

The fractional velocity concept developed by Prodanov [1] is demonstrated as tool to characterize Hölder and in particular, singular functions. Fractional velocities are defined as limits of the difference quotients of a fractional power and they generalize the local notion of a derivative. On the other hand, their properties contrast some of the usual properties of derivatives. One of the most peculiar properties of these operators is that the set of their nontrivial values is disconnected. This can be used, for example, to model instantaneous interactions, such as Langevin dynamics. In this context, the local fractional derivatives and the equivalent fractional velocities have several distinct properties compared to integer-order derivatives.

The classical pantograph equation and its generalizations, including fractional order and higher order cases, is developed by Bhalekar and Patade [2]. The special functions are obtained from the series solution of these equations. Different properties of these special functions are established, andtheir relations with other functions are developed.

The new direction in fractional calculus involving nonsingular memory kernels, developed in the last three years following the seminar articles of Caputo and Fabrizio in 2015 [3], is hot research topic. Two studies in the collection clearly demonstrate two principle directions: operators with nonsingular exponential kernels, i.e., the so-called Caputo-Fabrizio derivatives [4,5] (Hristov, 2016, Chapter 10), and operators with nonsingular memory kernels based on the Mittag-Leffler function [6,7] (Atangana, Baleanu, 2016; Baleanu, Fennandez, 2018).

Yavuz and Ozdemir [8] demonstrate a novel approximate-analytical solution method, called the Laplace homotopy analysis method (LHAM), using the Caputo-Fabrizio (CF) fractional derivative operator based on the exponential kernel. The recommended method is obtained by combining Laplace transform (LT) and the homotopy analysis method (HAM). This study considers the application of LHAM to obtain solutions of the fractional Black-Scholes equations (FBSEs) with the Caputo-Fabrizio

(CF) fractional derivative and appropriate initial conditions. The authors demonstrate the efficiencies and accuracies of the suggested method by applying it to the FBS option pricing models with their initial conditions satisfied by the classical European vanilla option. Using real market values from finance literature, it is demonstrated how the option is priced for fractional cases of European call option pricing models. Moreover, the proposed fractional model allows modeling of the price of different financial derivatives such as swaps, warrant, etc., in complete agreement with the corresponding exact solutions.

In light of new fractional operators, Gomez-Aguilar and Atangana [9] present alternative representations of the Freedman model considering Liouville-Caputo and Atangana-Baleanu-Caputo fractional derivatives. The solutions of these alternative models are obtained using an iterative scheme based on the Laplace transform and the Sumudu transform. Moreover, special solutions via the Adams-Moulton rule are obtained for both fractional derivatives.

In light of certain applied problems, the thermal control of complex thermal interfaces and heat conduction are principle issues to which classical fractional calculus is widely applicable. The control of thermal interfaces has gained importance in recent years because of the high cost of heating and cooling materials in many applications. The main focus in the work of Moreau et al. [10] is to compare the second and third generations of the CRONE controller (French acronym of Commande Robusted'Ordre Non Entier) in the control of a non-integer plant by means a fractional order controller. The idea is that, as a consequence of the fractional approach, all of the systems of integer order are replaced by the implementation of a CRONE controller. The results reveal that the second generation CRONE controller is robust when the variations in the plant are modeled with gain changes, whereas the phase remains the same for all of the plants (even if not constant). However, the third generation CRONE controller demonstrates a good, feasible robustness when the parameters of the plant are changed as well as when both gain and phase variations are encountered.

Thermistors are part of a larger group of passive components. They are temperature-dependent resistors and come in two varieties, negative temperature coefficients (NTCs) and positive temperature coefficients (PTCs), although NTCs are most commonly used. NTC thermistors are nonlinear, and their resistance decreases as the temperature increases. The self-heating may affect the resistance of an NTC thermistor, and the work of Vivek et al. [11] focuses on the existence and uniqueness and Ulam-Hyers stability types of solutions for Hilfer-type thermistor problems.

A heat conduction inverse problem of the fractional (Caputo fractional derivative) heat conduction problem is developed by Brociek et al. [12] for porous aluminum. In this case, the Caputo fractional derivative is employed. The direct problem is solved using a finite difference method and approximations of Caputo derivatives, while the inverse problem, the heat transfer coefficient, thermal conductivity coefficient, initial condition, and order of derivative are sought and the minimization of the functional describing the error of approximate solution is carried out by the Real Ant Colony Optimization algorithm.

Process monitoring represents an important and fundamental tool aimed at process safety and economics while meeting environmental regulations. Lenzi et al. [13] present an interesting approach to the quality control of different olive and soybean oil mixtures characterized by image analysis with the aid of an RGB color system by the algebraic fractional model. The model based on the fractional calculus-based approach could better describe the experimental dataset, presenting better results of parameter estimation quantities, such as objective function values and parameter variance. This model could successfully describe an independent validation sample, while the integer order model failed to predict the value of the validation sample.

The classical Stokes' first problem for a class of viscoelastic fluids with the generalized fractional Maxwell constitutive model was developed by Bazhlekova and Bazhlekov [14]. The constitutive equation is obtained from the classical Maxwell stress-strain relation by substituting the first-order derivatives of stress and strain by derivatives of non-integer orders in the interval $(0, 1)$. Explicit integral representation of the solution is derived and some of its characteristics are discussed: non-negativity and monotonicity, asymptotic behavior, analyticity, finite/infinite propagation speed, and absence of wave front.

Summing up, this special collection presents a detailed picture of the current activity in the field of fractional calculus with various ideas, effective solutions, new derivatives, and solutions to applied problems. As editor, I believe that this will be continued as series of Special Issues and books released to further explore this subject.

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