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On the Physical Fractional Modulations on Langmuir Plasma Structures

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Abstract: Langmuir waves propagate in fractal complex plasma with fractal characteristics, which may cause some plasma particles to be trapped or causes wave turbulences. This phenomenon appears in the form of fractional order equations. Using an effective unified solver, some new solitary profiles such as rational, trigonometrically and hyperbolic functions forms are discussed, using fractional derivatives in conformable sense. The fractional order modulates the solitary properties, such as amplitudes and widths. The proposition technique can be executed to study many applied science models.

Keywords: FISALWs equation; conformable derivative; ion sound wave; Langmuir waves; unified solver

MSC: 26A33; 35C07; 34K37; 35Q35; 35Q40



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1. Introduction

Fractional calculus—which means differentiation and integration in any order—has many interesting applications in applied science and new physics. It characterizes real-life problems better than classical integer-order ones. Many complex phenomena can be modeled by nonlinear fractional partial differential equations (NFPDEs). Recently, NFPDEs have played a key role in many vital fields of applied analysis, such as plasma physics, control theory, optical physics, quantum mechanics, fractional dynamics, solid state physics, super conductivity and many others [1–8]. Hence, searching for traveling wave solutions for these types of equations has attracted the attention of many researchers. Thus, various methods are proposed to find these types of solutions; see, for example [9–16].

There are many definitions for fractional differential equations, such as Grunwald–Letnikov, Caputo's fractional derivatives and Riemann–Liouville [17]. Khalil et al. [18] proposed a very interesting conformable fractional derivative. This definition is easier to consider, and also satisfies several conventional properties which cannot be satisfied by the known fractional derivatives; for example, the product of two functions, Rolle's theorem, mean value theorem and chain rule [19]. The conformable fractional derivative has attracted important attention due to its simplicity. So, many powerful works have been carried out using the conformable fractional definition [20–23]. There are also some other interesting definitions of fractional orders, such as fractal derivative order, which is directly connected to the Hausdorff fractal dimension of the diffusion trajectory [24], He's fractional derivative [25] and local fractional derivative [26].

It is convenient to give a definition of the conformable fractional derivative and some notions of this definition [18]:

Definition 1 ([18]). Let a function $\mathfrak{F} : (0, \infty) \rightarrow \mathbb{R}$; thus, the conformable fraction derivative of $\mathfrak{F}$ of order ς is

$$D_t^\varsigma(\mathfrak{F}(t)) = \lim_{\varepsilon \rightarrow 0} \frac{\mathfrak{F}(t + \varepsilon t^{1-\varsigma}) - \mathfrak{F}(t)}{\varepsilon}, \quad t > 0, \quad 0 < \varsigma \leq 1.$$

This conformable derivative obeys:

- (i) $D_t^\varsigma(\alpha \mathfrak{E} + \beta \mathfrak{F}) = \alpha D_t^\varsigma(\mathfrak{E}) + \beta D_t^\varsigma(\mathfrak{F}), \quad \alpha, \beta \in \mathbb{R};$
- (ii) $D_t^\varsigma(t^n) = n t^{n-\varsigma}, \quad n \in \mathbb{R};$
- (iii) $D_t^\varsigma(\mathfrak{E} \mathfrak{F}) = \mathfrak{F} D_t^\varsigma(\mathfrak{E}) + \mathfrak{E} D_t^\varsigma(\mathfrak{F});$
- (vi) $D_t^\varsigma\left(\frac{\mathfrak{E}}{\mathfrak{F}}\right) = \frac{\mathfrak{F} D_t^\varsigma(\mathfrak{E}) - \mathfrak{E} D_t^\varsigma(\mathfrak{F})}{\mathfrak{F}^2};$
- (v) If $\mathfrak{F}$ is differentiable, thus $D_t^\varsigma(\mathfrak{F})(t) = t^{1-\varsigma} \frac{d\mathfrak{F}}{dt}.$

Theorem 1 ([18]). Let $\mathfrak{E}, \mathfrak{F} : (0, \infty) \rightarrow \mathbb{R}$ be differentiable and also ς -differentiable; hence:

$$D_t^\varsigma(\mathfrak{E} \circ \mathfrak{F})(t) = t^{1-\varsigma} \mathfrak{F}'(t) \mathfrak{E}'(\mathfrak{F}(t)).$$

Nonlinear quantum mechanical models are deduced for Langmuir waves in isotropic electron collisionless plasma. In this work, we take into account the system of the ion sound wave via the action of the ponderomotive force due to the high-frequency field, as well as for the Langmuir wave. Zakharov et al. introduced the modeling of Langmuir turbulence [27]. Dyachenko et al. presented some computer simulations for Langmuir collapse [28]. Rubenchik et al. investigated the strong Langmuir turbulence in plasma acceleration [29]. Ratcliffe et al. presented the Langmuir wave-spectral-energy densities from the electric field [30]. Our study aims to introduce new closed form of solutions to the fractional ion sound and Langmuir waves (FISALWs) equation, given as follows [31–33]:

$$i D_t^\varsigma E + \frac{1}{2} E_{xx} - \eta E = 0, \tag{1}$$

$$D_t^{2\varsigma} \eta + \eta_{xx} - 2(|E|^2)_{xx} = 0, \quad 0 < \varsigma \leq 1,$$

$E e^{-i\omega_p t}$ is the normalized electric field of the Langmuir oscillation, whereas η is the normalized density perturbation. Younas et al. [32] produced the some wave structures for model (1) using the generalized exponential rational function method. Durur et al. [33] implemented the ShGFM and $1/G'$ -expansion methods for model (1) and produced trigonometric and hyperbolic-type traveling wave solutions. Rezazadeh et al. implemented the new auxiliary equation scheme via the Atangana–Baleanu (AB) fractional and extracted hyperbolic and trigonometric function solutions for model (1).

The subsistence of fractal modes in the study of nonlinear dynamical fluid systems leads to an abundance of fractal structure properties in nonlinear propagating waves [34,35]. Additionally, in space environmental plasmas, the fractal solitary structure appears in many events concerning the dynamical conservative dissipative and dispersive systems [35–38]. On the other hand, the ion waves propagate in fractal complex medium with fractal properties, which may cause some ions to be trapped or cause wave turbulence [36,37,39]. This appears in the form of fractional order equations. This mathematical order modulates wave properties such as amplitude and width [36,37]. It was reported that the fractal parameter plays an operative role in solving the problems caused by the absence of higher-order perturbed effects [36]. The magnetic Lagrangian fractal structures in ergodic limiter tokamak have been studied near its wall. Additionally, the chaotic line regions near the walls have been discussed [35].

Abdelrahman et al. studied the mathematical model of ionic sound Langmuir waves in fractional form [39]. They obtained different fractal solutions via a mathematical method depending on He's semi-inverse analysis without using any specified real parameters. In [40], we presented a unified solver technique to solve NFPDE, based on the Riccati–Bernoulli sub-ODE technique [41]. In the present work, we implement this solver in order to obtain some new solutions for the Langmuir plasma waves model. These solutions have a significant contribution in quantum plasma, solar–wind and many other fields. Namely, new field structures were obtained and specified parameters related to solar–wind were used [42,43]. In contrast with other methods, the proposed solver admits various advantages, such as giving the complete construction of waves for the proposed model. It avoids tedious computations and provides crucial solutions in an explicit form. Indeed, this solver is forthright, robust, functional and convenient. Our study shows that the proposed solver will be important for engineers, physicists and mathematicians, since it can be used as a box solver. Our analysis depicts the efficiency of the proposed technique in order to deal with many real life problems in applied science. To the best of our knowledge, no previous study has been performed using the presented solver to solve the FISALWs equation.

The structure of this study is as follows: Section 2 presents some new solutions for the model of FISALWs. Section 3 introduces the physical interpretation for the presented results. Some 3D graphs for some selected solutions are illustrated. Finally, Section 4 reports detailed conclusions.

2. Mathematical Analysis

In this section, we introduce some new solutions for the FISALWs equation. Utilizing the wave transformation [31]:

$$E(x, t) = e^{i\omega} q(\xi), \quad \eta(x, t) = v(\xi), \quad \xi = rx + a \frac{t^\zeta}{\zeta}, \quad \omega = kx + b \frac{t^\zeta}{\zeta}. \quad (2)$$

Superseding Equation (2) into Equation (1) gives

$$i(a + kr)q' = 0, \quad (3)$$

$$r^2 q'' - (2b + k^2)q - 2qv = 0, \quad (4)$$

$$(a^2 - r^2)v'' - 2r^2(q^2)'' = 0. \quad (5)$$

Integrating Equation (5) twice with respect to ξ , with an integration constant of zero and using Equation (3), we get

$$v = \frac{2r^2}{a^2 - r^2} q = \frac{2}{k^2 - 1} q. \quad (6)$$

Setting Equation (6) into Equation (4) gives

$$\Gamma_1 q'' + \Gamma_2 q^3 + \Gamma_3 q = 0, \quad (7)$$

where $\Gamma_1 = (k^2 - 1)r^2$, $\Gamma_2 = -4$ and $\Gamma_3 = -(k^2 - 1)(2b + k^2)$.

Solution of Equation (1) via a Unified Solver Approach

In the light of the solver technique [40], the solutions of Equation (7) are:

Rational solutions:

$$q_{1,2}(x, t) = \left(\mp \sqrt{\frac{2}{(k^2 - 1)r^2}} \left(rx + a \frac{t^\zeta}{\zeta} + \varrho \right) \right)^{-1}. \quad (8)$$

Thus, the solutions of Equation (1) are

$$E_{1,2}(x, t) = e^{i(kx + b \frac{t^\zeta}{\zeta})} \left(\mp \sqrt{\frac{2}{(k^2 - 1)r^2}} \left(rx + a \frac{t^\zeta}{\zeta} + \varrho \right) \right)^{-1}, \quad (9)$$

$$\eta_{1,2}(x, t) = \frac{2}{k^2 - 1} \left(\mp \sqrt{\frac{2}{(k^2 - 1)r^2}} \left(rx + a \frac{t^\zeta}{\zeta} + \varrho \right) \right)^{-1}. \quad (10)$$

Trigonometric solutions:

$$q_{3,4}(x, t) = \pm \frac{\sqrt{(k^2 - 1)(2b + k^2)}}{2} \tan \left(\frac{\sqrt{2b + k^2}}{\sqrt{2}r} \left(rx + a \frac{t^\zeta}{\zeta} + \varrho \right) \right) \quad (11)$$

and

$$q_{5,6}(x, t) = \pm \frac{\sqrt{(k^2 - 1)(2b + k^2)}}{2} \cot \left(\frac{\sqrt{2b + k^2}}{\sqrt{2}r} \left(rx + a \frac{t^\zeta}{\zeta} + \varrho \right) \right). \quad (12)$$

Thus, the solutions of Equation (1) are

$$E_{3,4}(x, t) = \pm \frac{\sqrt{(k^2 - 1)(2b + k^2)}}{2} e^{i(kx + b \frac{t^\zeta}{\zeta})} \tan \left(\frac{\sqrt{2b + k^2}}{\sqrt{2}r} \left(rx + a \frac{t^\zeta}{\zeta} + \varrho \right) \right), \quad (13)$$

$$\eta_{3,4}(x, t) = \pm \frac{1}{k^2 - 1} \sqrt{(k^2 - 1)(2b + k^2)} \tan \left(\frac{\sqrt{2b + k^2}}{\sqrt{2}r} \left(rx + a \frac{t^\zeta}{\zeta} + \varrho \right) \right) \quad (14)$$

and

$$E_{5,6}(x, t) = \pm \frac{\sqrt{(k^2 - 1)(2b + k^2)}}{2} e^{i(kx + b \frac{t^\zeta}{\zeta})} \cot \left(\frac{\sqrt{2b + k^2}}{\sqrt{2}r} \left(rx + a \frac{t^\zeta}{\zeta} + \varrho \right) \right), \quad (15)$$

$$\eta_{5,6}(x, t) = \pm \frac{1}{k^2 - 1} \sqrt{(k^2 - 1)(2b + k^2)} \cot \left(\frac{\sqrt{2b + k^2}}{\sqrt{2}r} \left(rx + a \frac{t^\zeta}{\zeta} + \varrho \right) \right). \quad (16)$$

Hyperbolic solutions:

$$q_{7,8}(x, t) = \pm \frac{\sqrt{(1 - k^2)(2b + k^2)}}{2} \tanh \left(\frac{\sqrt{-(2b + k^2)}}{\sqrt{2}r} \left(rx + a \frac{t^\zeta}{\zeta} + \varrho \right) \right) \quad (17)$$

and

$$q_{9,10}(x, t) = \pm \frac{\sqrt{(1 - k^2)(2b + k^2)}}{2} \coth \left(\frac{\sqrt{-(2b + k^2)}}{\sqrt{2}r} \left(rx + a \frac{t^\zeta}{\zeta} + \varrho \right) \right). \quad (18)$$

Thus, the solutions of Equation (1) are

$$E_{7,8}(x, t) = \pm \frac{\sqrt{(1 - k^2)(2b + k^2)}}{2} e^{i(kx + b \frac{t^\zeta}{\zeta})} \tanh \left(\frac{\sqrt{-(2b + k^2)}}{\sqrt{2}r} \left(rx + a \frac{t^\zeta}{\zeta} + \varrho \right) \right), \quad (19)$$

$$\eta_{7,8}(x, t) = \pm \frac{1}{k^2 - 1} \sqrt{(1 - k^2)(2b + k^2)} \tanh \left(\frac{\sqrt{-(2b + k^2)}}{\sqrt{2}r} \left(rx + a \frac{t^\zeta}{\zeta} + \varrho \right) \right) \quad (20)$$

and

$$E_{9,10}(x, t) = \pm \frac{\sqrt{(1 - k^2)(2b + k^2)}}{2} e^{i(kx + b \frac{t^\zeta}{\zeta})} \coth \left(\frac{\sqrt{-(2b + k^2)}}{\sqrt{2}c} \left(cx + a \frac{t^\zeta}{\zeta} + \varrho \right) \right), \quad (21)$$

$$\eta_{9,10}(x,t) = \pm \frac{1}{k^2 - 1} \sqrt{(1 - k^2)(2b + k^2)} \coth \left(\frac{\sqrt{-(2b + k^2)}}{\sqrt{2}c} \left(cx + \alpha \frac{t^\zeta}{\zeta} + \varrho \right) \right). \quad (22)$$

Here, ϱ is an arbitrary constant.

Remark 1.

1. The presented solver in this study can be applied for the huge classes of NFPDEs.
2. This solver can be easily extended to solve stochastic NFPDEs.

Remark 2. The proposed solver solves all classes of NFPDEs reduced to Equation (7), but it fails to solve other classes of NFPDEs, which is considered a disadvantage of the proposed method.

3. Physical Interpretation

In this work, new fractional solutions for the FLWs equations were derived and mathematically solved using a unified solver. The achieved fractional profiles appear in rational forms, trigonometric profiles and hyperbolic structures. Here, it is more appropriate for us to study the effects of the fractal parameters on the shapes, structures and properties of electrostatic waves, and the consequent plasma density. Given the importance of electrostatic rational solutions (9), we study the effect of the fractional order parameter on the solution (9), as shown in Figures 1 and 2. It was found that the electric field is an explosive soliton, with ζ and x , as shown in Figure 1. It was noted that increasing ζ produces phase change. The variation in Equation (9) with ζ and t is depicted in Figure 2. It was considered that for $\zeta = 0.3$ a supershock-like wave is produced, but for $\zeta = 0.4$ – 0.5 , the wave converts to a super soliton profile, as seen in Figure 2 (on the left). Another increasing ζ from 0.8 – 1 produces stationary soliton, which converts to explosive soliton for $\zeta = 1$.

The solution (15) produces a blow-up explosive profile with changing path difference with ζ , as given in Figure 3. On the other hand, an envelope periodic solitary wave (19) is obtained, as seen in Figure 4. Finally, the sensitive changes in Equation (21) with ζ , t and x are given in Figure 5. It is reported that a periodic wave is produced for t in Figure 5, and an envelope periodic wave is produced for x in Figure 6. The varying ζ does not affect the wave amplitude while increasing the frequency in Figure 5 and changing the phase in Figure 6.

To check our results against real space plasma data, they were compared to observed STEREO instruments and spacecraft data in solar wind. Figure 7 indicated that the mode results agree with observed structures $k = 1.3$, $r = 0.39$, $a = 1.7$, $b = -1.6$, $\zeta = 0.75$.

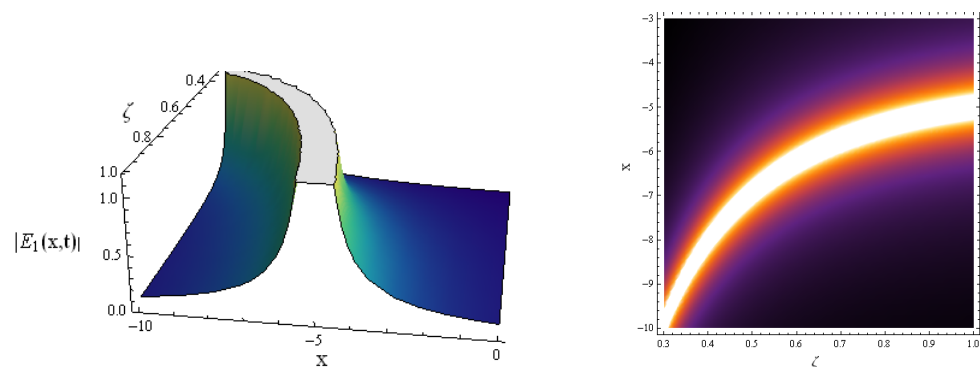


Figure 1. Variation of solution (9) versus x and ζ .

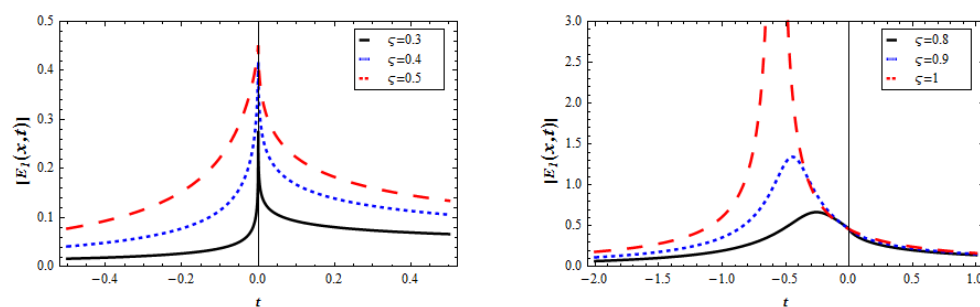


Figure 2. Variation of solution (9) versus t and ζ .

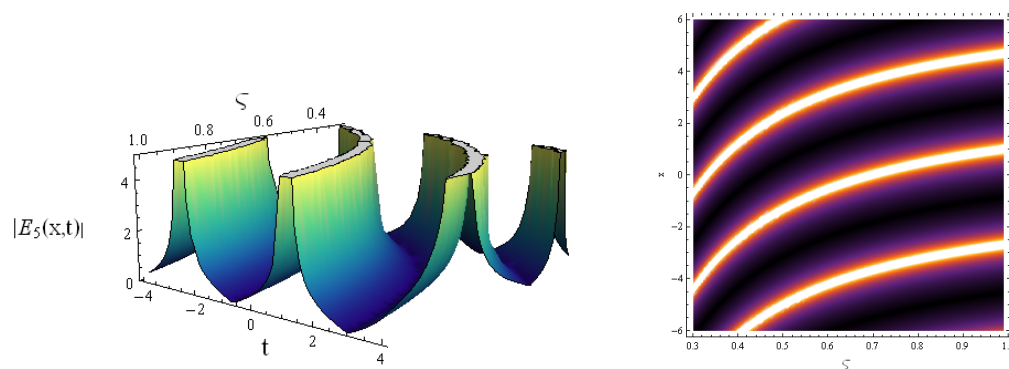


Figure 3. Variation of solution (15) versus x and ζ .

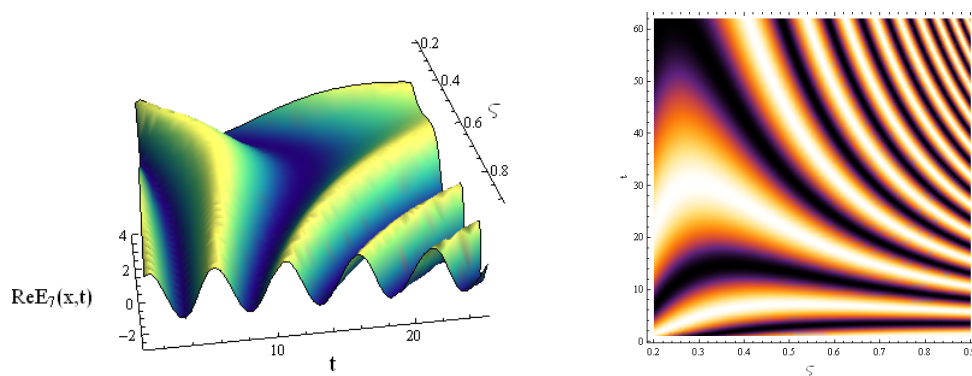


Figure 4. Variation of solution (19) versus t and ζ .

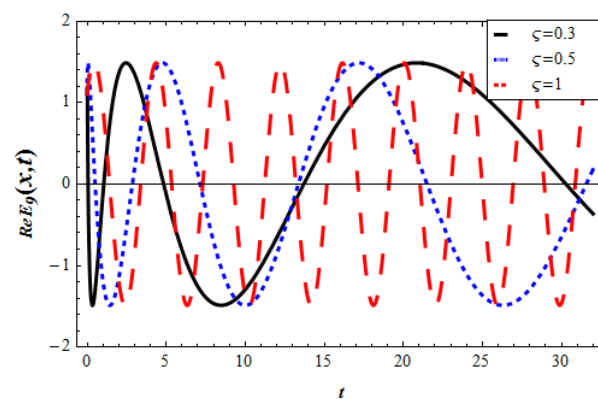


Figure 5. Variation of solution (21) versus t and ζ .

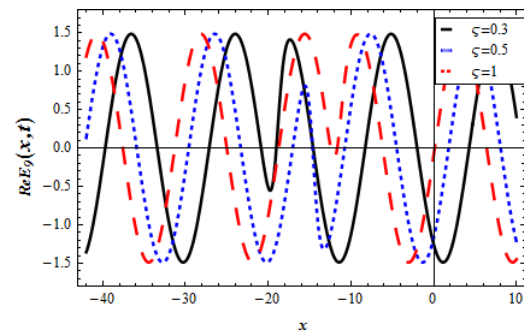


Figure 6. Variation of solution (21) versus x and ζ .

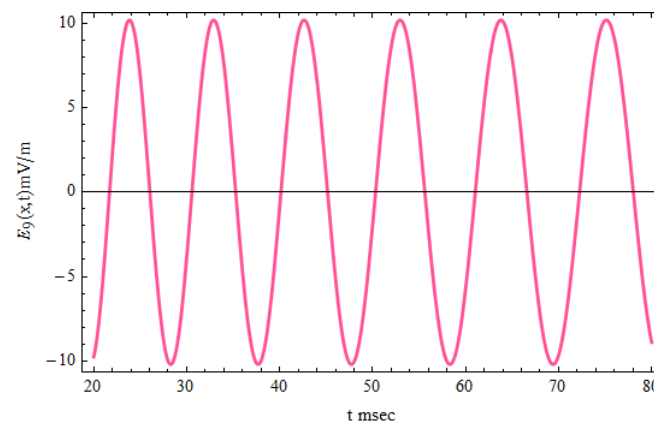


Figure 7. The variation of periodic Langmuir electrostatic E_9 wave with t .

4. Conclusions

We have successfully extracted some new solutions to the FISALWs equation, utilizing the proposed solver. Our results show that this solver is a convenient, efficacious and robust technique method for NFPDEs. Indeed, this solver avoids complicated and tedious algebraic calculations. Thus, one can say that the proposed solver has a key role in obtaining vital solutions for NFPDEs. Finally, some graphical simulations are given to clarify the behavior of the solutions. Finally, the acquired Langmuir electrostatic solutions with fractional parameters are in agreement with the solar wind solitary forms observed by spacecrafts and STEREO instruments.

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