fractal and fractional

Editorial

Editorial for the Special Issue "Operators of Fractional Calculus and Their Multidisciplinary Applications"

Hari Mohan Srivastava [1,2,3,4,5]

[1] Department of Mathematics and Statistics, University of Victoria, Victoria, BC V8W 3R4, Canada; harimsri@math.uvic.ca
[2] Department of Medical Research, China Medical University Hospital, China Medical University, Taichung 40402, Taiwan
[3] Center for Converging Humanities, Kyung Hee University, 26 Kyungheedae-ro, Dongdaemun-gu, Seoul 02447, Republic of Korea
[4] Department of Mathematics and Informatics, Azerbaijan University, 71 Jeyhun Hajibeyli Street, AZ1007 Baku, Azerbaijan
[5] Section of Mathematics, International Telematic University Uninettuno, I-00186 Rome, Italy

Citation: Srivastava, H.M. Editorial for the Special Issue "Operators of Fractional Calculus and Their Multidisciplinary Applications". *Fractal Fract.* **2023**, *7*, 415. https://doi.org/10.3390/fractalfract7050415

Received: 10 May 2023
Accepted: 17 May 2023
Published: 22 May 2023

1. Introduction

This Special Issue of the MDPI journal, ***Fractal and Fractional***, on the subject area of "Operators of Fractional Calculus and Their Multidisciplinary Applications" consists of 19 peer-reviewed papers, including some invited feature articles, originating from all over the world.

Current widespread interest in various families of fractional-order integral operators and fractional-order derivative operators, such as those that are named after Riemann–Liouville, Weyl, Hadamard, Grunwald–Letnikov, Riesz, Erdélyi–Kober, Liouville–Caputo, and so on, has stemmed essentially from their demonstrated applications in numerous diverse areas of the mathematical, physical, biological, chemical, engineering, and statistical sciences. Each of these fractional-order operators has been fruitfully applied to provide interesting and potentially useful tools for solving ordinary and partial differential equations, as well as integral, differintegral, and integro-differential equations, the fractional-calculus analogues and extensions of each of these equations, and various other problems involving special functions of mathematical physics, applicable analysis, and applied mathematics, as well as their extensions and generalizations in one, two, or more variables.

This Special Issue includes invited review, expository, and other research articles dealing with the recent advances in the theory of integrals and derivatives of fractional order and their multidisciplinary applications, and also on various potentially useful families of higher transcendental functions (or special functions or mathematical functions) of mathematical analysis and on their applications in the above-mentioned fields. Indeed, in both the recent literature and the current literature, several higher transcendental functions have also been involved in the theory and applications of various families of fractional-order integral operators and the corresponding fractional-order derivative operators, such as those that aid in finding solutions for a large variety of problems in the applied sciences, which are modeled mathematically by ordinary and partial differential equations, as well as integral, differintegral, and integro-differential equations.

2. An Overview of the Special Issue

Investigations based upon the theory and applications of fractional calculus (that is, integrals and derivatives of non-integer real or complex order) are remarkably widespread in many diverse areas of the mathematical, physical, biological, chemical, engineering and statistical sciences. The suggested topics of interest for the call for papers for this Special Issue included, but were not limited to, the following keywords:

- Operators of fractional integrals and fractional derivatives and their applications;
- Chaos and dynamical systems based upon fractional calculus;
- Fractional-order ODEs and PDEs;
- Fractional-order differintegral and integro-differential equations;
- Integrals and derivatives of fractional order associated with special functions of mathematical physics and applied mathematics;
- Special functions of mathematical physics and applied mathematics;
- Identities and inequalities involving fractional-order integrals and fractional-order derivatives.

A considerably large number of well-established international scientific research journals, which are published by several widely-recognized publishing houses, have published and continue to publish a number of Special Issues of many of their journals on recent and state-of-the-art advances in various multidisciplinary aspects, especially related to the above-mentioned keywords.

3. Contributors and Contributions to the Special Issue

The geographical distribution of the contributors to this Special Issue is remarkably widely-scattered. Their contributions (see [1–19]) originated in many different countries on every continent of the world. The subject matter of these nineteen publications (see [1–19]) deals extensively with such topics as fractional-order complex Ginzburg-Landau equations, fractional modeling for the treatment of cancer by using radiotherapy, fractional-order fuzzy complex-valued neural networks, the fractal–fractional Michaelis-Menten enzymatic reaction model, fractional-calculus operators involving the (p,q)-extended Bessel and Bessel-Wright functions, fractional-order diffusion-wave equations, Abel integral equations and their fractional-order analogues, nonlinear integro-differential equations, fractional-order investigations of a number of celebrated integral inequalities, such as those that are popularly called the Pólya–Szegö inequality, the Grüss inequality, the Hermite-Hadamard inequality, and so on.

We will now briefly describe the developments which are reported in this Special Issue. The authors of [1] have studied a fractional-order complex Ginzburg-Landau equation by using the parabolic law and the law of weak non-local non-linearity. The study presented in [2] is based upon a modification of a well-known predator-prey equation or the Lotka–Volterra competition model. Herein, by investigating a system of differential equations, the authors have discussed the population of healthy and cancerous cells within the tumor tissue of a patient struggling with cancer. In [3], the authors have addressed the problem of uniform stability for a family of fractional-order fuzzy impulsive complex-valued neural networks with mixed delays in infinite dimensions. The article in [4] has derived some properties involving the logarithmic growth of entire functions which are represented by the Laplace-Stieltjes transform of order 0. The authors of [5] have compared the design of a fractional-order proportional integral (FOPI) controller and an integer-order proportional integral (IOPI) controller for the permanent-magnet synchronous motor (PMSM) speed regulation system. Three new models of the fractal-fractional Michaelis-Menten enzymatic reaction (FFMMER) are investigated in [6] by basing these models upon different kernels that involve the power law, exponential decay, and the Mittag-Leffler-type functions.

Some general forms of fractional integral operators are used in [7] for finding several fractional-order integral inequalities in the Hermite–Hadamard and the Minkowski settings. In [8], the authors have studied a fractional-order logistic differential equation by making use of several appropriate limit relations. The authors in [9] derive some asymptotic properties of non-oscillatory solutions of the even-order delay differential equation, thereby deducing criteria for oscillation. The article in [10] considers and analyzes a large number of fractional-calculus operators that are accessible in the current literature. The authors in [11] establish a number of presumably new results, which are related to the Marichev-Saigo-Maeda fractional integral and fractional derivative operators, which involve the two-variable Appell F_3-function in the kernel, and apply their derived results to the (p,q)-

extended Bessel function and the Bessel-Wright function. In [12], a modified form of the Adomian decomposition method is developed for solving some initial- and boundary-value problems involving fractional-order diffusion-wave equations.

The so-called (k, Ψ)-proportional fractional-order integral inequalities of the Pólya-Szegö and Grüss types are investigated in [13]. Several fractional-order integral inequalities based upon some general families of fractional integrals are investigated by the authors in [14,15]. The uniqueness problem for solutions of some general Abel-type integral equations in a related coupled system in Banach spaces has been addressed in [16]. The uniqueness problem for solutions of several nonlinear Liouville-Caputo integro-differential equations, which are equipped with variable coefficients and initial conditions as well as an associated coupled system in Banach spaces, can be found in [17]. Some iterative methods are presented in [18] for solving nonlinear equations in the quantum calculus (or the q-calculus).

The article in [19] is essentially the third part of a series of essays in which the author advocates the use of (non-integer) fractional calculus in order to capture the dynamics of complex networks in the twilight of the Newtonian era. In addition to the widely-cited monograph [20] on fractional differential, integral, differintegral and integro-differential equations and their widespread applications, the interested reader can potentially benefit by the recently-published survey-cum-expository review articles [21–23] on the developments of the theory and applications of fractional calculus. In particular, in [23], one can find a systematic overview of some developments involving a hybrid version of several known extensions and generalizations of the Mittag-Leffler-type functions as well as the Hurwitz-Lerch-type zeta functions, together with their associated fractional integrals and fractional derivatives (see also [24,25]).

Funding: This research received no external funding.

Acknowledgments: I take this opportunity to sincerely thank all of the participating authors, and the referees and the peer-reviewers, for their invaluable contributions toward the remarkable success of this Special Issue. I hope that the Second Edition of this Special Issue in ***Fractal and Fractional*** will prove to be successful as well. I do also express my appreciation for the editorial and managerial help and assistance which was provided efficiently and generously by the colleagues and associates in the Editorial Office of ***Fractal and Fractional***. The dedicated and wholehearted support and help of one and all are indeed greatly appreciated.

Conflicts of Interest: The author declares that there is no conflict of interest.

References

1. Alabedalhadi, M.; Al-Smadi, M.; Al-Omari, S.; Karaca, Y.; Momani, S. New Bright and Kink Soliton Solutions for Fractional Complex Ginzburg-Landau Equation with Non-Local Nonlinearity Term. *Fractal Fract.* **2022**, *6*, 724. [CrossRef]
2. Momenzadeh, M.; Obi, O.A.; Hincal, E. A Bi-Geometric Fractional Model for the Treatment of Cancer Using Radiotherapy. *Fractal Fract.* **2022**, *6*, 287. [CrossRef]
3. Liu, X.; Chen, L.; Zhao, Y. Uniform Stability of a Class of Fractional-Order Fuzzy Complex-Valued Neural Networks in Infinite Dimensions. *Fractal Fract.* **2022**, *6*, 281. [CrossRef]
4. Xu, H.; Li, H.; Xuan, Z. Some New Inequalities on Laplace-Stieltjes Transforms Involving Logarithmic Growth. *Fractal Fract.* **2022**, *6*, 233. [CrossRef]
5. Wang, B.; Wang, S.; Peng, Y.; Pi, Y.; Luo, Y. Design and High-Order Precision Numerical Implementation of Fractional-Order PI Controller for PMSM Speed System Based on FPGA. *Fractal Fract.* **2022**, *6*, 218. [CrossRef]
6. Alqhtani, M.; Saad, K.M. Fractal–Fractional Michaelis–Menten Enzymatic Reaction Model via Different Kernels. *Fractal Fract.* **2022**, *6*, 13. [CrossRef]
7. Hyder, A.-A.; Barakat, M.A.; Fathallah, A.; Cesarano, C. Further Integral Inequalities through Some Generalized Fractional Integral Operators. *Fractal Fract.* **2021**, *5*, 282. [CrossRef]
8. Area, I.; Nieto, J.J. Fractional-Order Logistic Differential Equation with Mittag-Leffler-Type Kernel. *Fractal Fract.* **2021**, *5*, 273. [CrossRef]
9. Moaaz, O.; Cesarano, C.; Askar, S. Asymptotic and Oscillatory Properties of Noncanonical Delay Differential Equations. *Fractal Fract.* **2021**, *5*, 259. [CrossRef]
10. Torres-Hernandez, A.; Brambila-Paz, F. Sets of Fractional Operators and Numerical Estimation of the Order of Convergence of a Family of Fractional Fixed-Point Methods. *Fractal Fract.* **2021**, *5*, 240. [CrossRef]

11. Srivastava, H.M.; AbuJarad, E.S.A.; Jarad, F.; Srivastava, G.; AbuJarad, M.H.A. The Marichev-Saigo-Maeda Fractional-Calculus Operators Involving the (p, q)-Extended Bessel and Bessel-Wright Functions. *Fractal Fract.* **2021**, *5*, 210. [CrossRef]
12. Mustafa, S.; Hajira; Khan, H.; Shah, R.; Masood, S. A Novel Analytical Approach for the Solution of Fractional-Order Diffusion-Wave Equations. *Fractal Fract.* **2021**, *5*, 206. [CrossRef]
13. Aljaaidi, T.A.; Pachpatte, D.B.; Abdo, M.S.; Botmart, T.; Ahmad, H.; Almalahi, M.A.; Redhwan, S.S. (k, Ψ)-Proportional Fractional Integral Pólya–Szegö- and Grüss-Type Inequalities. *Fractal Fract.* **2021**, *5*, 172. [CrossRef]
14. Srivastava, H.M.; Kashuri, A.; Mohammed, P.O.; Nonlaopon, K. Certain Inequalities Pertaining to Some New Generalized Fractional Integral Operators. *Fractal Fract.* **2021**, *5*, 160. [CrossRef]
15. Ahmad, H.; Tariq, M.; Sahoo, S.K.; Baili, J.; Cesarano, C. New Estimations of Hermite-Hadamard Type Integral Inequalities for Special Functions. *Fractal Fract.* **2021**, *5*, 144. [CrossRef]
16. Li, C.; Srivastava, H.M. Uniqueness of Solutions of the Generalized Abel Integral Equations in Banach Spaces. *Fractal Fract.* **2021**, *5*, 105. [CrossRef]
17. Li, C.; Beaudin, J. On the Nonlinear Integro-Differential Equations. *Fractal Fract.* **2021**, *5*, 82. [CrossRef]
18. Sana, G.; Mohammed, P.O.; Shin, D.Y.; Noor, M.A.; Oudat, M.S. On Iterative Methods for Solving Nonlinear Equations in Quantum Calculus. *Fractal Fract.* **2021**, *5*, 60. [CrossRef]
19. West, B.J. The Fractal Tapestry of Life: III Multifractals Entail the Fractional Calculus. *Fractal Fract.* **2022**, *6*, 225. [CrossRef]
20. Kilbas, A.A.; Srivastava, H.M.; Trujillo, J.J. *Theory and Applications of Fractional Differential Equations*; North-Holland Mathematical Studies; Elsevier: Amsterdam, The Netherlands; London, UK; New York, NY, USA, 2006; Volume 204.
21. Srivastava, H.M. Fractional-Order Derivatives and Integrals: Introductory Overview and Recent Developments. *Kyungpook Math. J.* **2020**, *60*, 73–116.
22. Srivastava, H.M. Some Parametric and Argument Variations of the Operators of Fractional Calculus and Related Special Functions and Integral Transformations. *J. Nonlinear Convex Anal.* **2021**, *22*, 1501–1520.
23. Srivastava, H.M. An Introductory Overview of Fractional-Calculus Operators Based Upon the Fox-Wright and Related Higher Transcendental Functions. *J. Adv. Engrg. Comput.* **2021**, *5*, 135–166. [CrossRef]
24. Srivastava, H.M. A Survey of Some Recent Developments on Higher Transcendental Functions of Analytic Number Theory and Applied Mathematics. *Symmetry* **2021**, *13*, 2294. [CrossRef]
25. Wright, E.M. The Asymptotic Expansion of Integral Functions Defined by Taylor Series. *Philos. Trans. Roy. Soc. Lond. Ser. A Math. Phys. Sci.* **1940**, *238*, 423–451.