An Improved Marine Predators Algorithm-Tuned Fractional-Order PID Controller for Automatic Voltage Regulator System

Mohd Zaidi Mohd Tumari 1, Mohd Ashraf Ahmad 2,*, Mohd Helmi Suid 2 and Mok Ren Hao 2

1 Faculty of Electrical and Electronics Engineering Technology, Universiti Teknikal Malaysia Melaka, Durian Tunggal 76100, Melaka, Malaysia; mohdzaidi.tumari@utem.edu.my
2 Faculty of Electrical and Electronics Engineering Technology, Universiti Malaysia Pahang, Pekan 26600, Pahang, Malaysia; mhelmi@ump.edu.my (M.H.S); pek19003@stdmail.ump.edu.my (M.R.H)
* Correspondence: mashraf@ump.edu.my

Abstract: One of the most popular controllers for the automatic voltage regulator (AVR) in maintaining the voltage level of a synchronous generator is the fractional-order proportional–integral–derivative (FOPID) controller. Unfortunately, tuning the FOPID controller is challenging since there are five gains compared to the three gains of a conventional proportional–integral–derivative (PID) controller. Therefore, this research work presents a variant of the marine predators algorithm (MPA) for tuning the FOPID controller of the AVR system. Here, two modifications are applied to the existing MPA: the hybridization between MPA and the safe experimentation dynamics algorithm (SEDA) in the updating mechanism to solve the local optima issue, and the introduction of a tunable step size adaptive coefficient ($\delta$) to improve the searching capability. The effectiveness of the proposed method in tuning the FOPID controller of the AVR system was assessed in terms of the convergence curve of the objective function, the statistical analysis of the objective function, Wilcoxon’s rank test, the step response analysis, stability analyses, and robustness analyses where the AVR system was subjected to noise, disturbance, and parameter uncertainties. We have shown that our proposed controller has improved the AVR system’s transient response and also produced about two times better results for objective function compared with other recent metaheuristic optimization-tuned FOPID controllers.

Keywords: marine predators algorithm; automatic voltage regulator; fractional-order PID controller; metaheuristic algorithms; optimization

1. Introduction

The main concern among the power system distributors is to stabilize and maintain the nominal voltage level produced by the synchronous generator at all times in order to increase the electrical energy supply, therefore increasing their profits. However, failing to stabilize the nominal voltage level can lead to the performance degradation of all connected equipment and devices, thus reducing the quality of the electrical energy. Furthermore, an unstable voltage level can disturb the real and reactive power flows, increasing the real line losses during the distribution. Thus, the automatic voltage regulator (AVR) is designed to maintain and stabilize the voltage produced by a synchronous generator at a certain level. It also helps in controlling the reactive power flow by ensuring the proper reactive power distribution amongst all connected generators. In other words, AVR is able to maintain the consistency of the terminal voltage, although the exciter voltage of the alternator is varied [1]. Unfortunately, controlling the AVR system with a fast and stable response is hard to achieve because of the load variation and the high inductance of the alternator field windings. Additionally, the disturbance that may occur during the power system distribution can cause insulation breakdowns in different parts and damage the connected...
equipment. Hence, it is essential to improve the control quality of the AVR system to ensure the stability and security of the power system distribution.

Over the past few decades, the most popular controller among researchers has been the proportional–integral–derivative (PID) controller. A PID controller has several advantages, including robust performance, a simple structure, and low execution effort [2]. Therefore, it has been extensively utilized in many engineering areas, such as a piezoelectric ultrasonic motor [3], doubly fed induction motor [4], and 2nd-order non-linear time-invariant plants [5]. Nevertheless, to achieve excellent PID controller performance, the PID gains, which are proportional gain, $K_p$, integral gain, $K_i$, and derivative gain, $K_d$, should be optimally tuned first. Unfortunately, tuning PID gains is not an easy task for operators and researchers, especially when tuning using classical techniques such as Ziegler–Nichols (ZN) [6], Cohen–Coon [7], and gain–phase margin [8]. These classical techniques are based on the ‘trial and error’ method, which requires extra effort to attain optimal PID gains, especially for complex plants with complicated mathematical computing and time-varying dynamics. Additionally, in terms of the AVR system that faces inconstant operating points and non-linear load features, the implementation of classical techniques can hinder the tuning process in obtaining the optimal PID gains.

Alternatively, current researchers have given more attention to modern heuristic optimization techniques in finding the optimal PID gains for AVR systems. This technique tunes the PID gains automatically to minimize the objective function. One of the pioneers in implementing modern heuristic optimization for PID tuning in AVR systems was Gaing [9] in 2004. He proposed particle swarm optimization (PSO) and compared the results with a genetic algorithm (GA). However, the most recent studies in PID tuning for AVR systems focus on the state-of-the-art metaheuristic optimization algorithm. These studies include teaching–learning-based optimization (TLBO) [10], symbiotic organisms search (SOS) algorithm [11], ant colony optimization (ACO) [12], cuckoo search (CS) algorithm [13,14], sine–cosine algorithm (SCA) [15], kidney-inspired algorithm (KIA) [16], whale optimization algorithm (WOA) [17], tree seed algorithm (TSA) [18], crow search algorithm (CSA) [19], equilibrium optimizer (EO) algorithm [20], and hybrid simulated annealing–manta ray foraging optimization (SA-MRFO) [21].

In the literature mentioned above, the PID controller has excellent performance in the AVR system towards maintaining and stabilizing the nominal voltage level produced by the synchronous generator. However, using the PID controller alone without modification and improving the basic PID structure may degrade the power system’s steady-state and transient performance since the current technology requires more precise and robust control. That is the reason that motivates researchers and practitioners to create variants of the PID controller in order to improve the performance of the AVR system. These PID variants comprised proportional–integral–derivative–acceleration (PIDA), which includes additional filter elements [17], PID with additional second-order derivative terms (PIDD$^2$) [22], sigmoid PID, in which the controller parameters are varied according to the changes in error [23], 2DOF PI controller that consists of two PI controllers connected at set point and feedback [24], and a real PID with derivative filter (PIDN) [21].

Meanwhile, the fractional-order method can be utilized on the conventional PID since the fractional-order method has been proven to produce more precise representations for complex systems [25,26]. Therefore, fractional-order PID (FOPID) controllers have become a prevalent choice among researchers since they introduce two more controller gains, which are fractional exponential terms of integral $\lambda$ and derivative $\mu$. Thus, by having five controller gains, the FOPID controller can provide new control possibilities for researchers in designing the control system for various engineering fields [27]. In many cases, the FOPID controller outperformed the conventional PID in terms of time domain specifications, robustness, and stability, as presented in [27]. Based on the aforementioned advantages of the FOPID controller, this paper focuses on this type of controller. Unfortunately, finding the optimal gains for a FOPID controller is more challenging with the increment of controller
gains compared to a conventional PID. Therefore, there is a high demand for tuning the FOPID controller gains using suitable optimization methods.

Various attempts have been made to control the AVR systems using the FOPID controller. This was initially studied by Karimi-Ghartemani et al. [28] in 2007. They proposed the PSO to tune the FOPID controller gains for the AVR system. The objective function is based on a combination of overshoot, rise time, settling time, steady-state error, integral absolute error (IAE), integral square error (ISE) of control input, phase margin, and gain margin with corresponding weighting coefficients. They illustrated that the FOPID controller could provide better results in terms of transient response, stability assessment (Bode diagram), and robustness analysis (parameter variations) compared to the conventional PID controller. Later on, a further method was proposed by Pan and Das [29] in 2012. Their study considers various multi-objective functions, which consist of the integral time square error (ITSE) of the set point, the ISE of the control input, and the ITSE of the load disturbance. They have implemented a non-dominated sorting genetic algorithm II (NSGA II) for tuning the FOPID controller gains. They have shown that the NSGA II has produced superior results for transient response and robustness analysis (parameter variations, load disturbance rejection) compared to other algorithms. In the same year, Tang et al. [30] suggested a chaotic ant swarm (CAS) optimization method to find the optimum FOPID controller gains that produce a high-performance result for AVR systems. The objective function is based on function of demerit (FOD), and the analysis involved the convergence curve, transient response, and robustness analysis (parameter variations). In the subsequent year, Ramezanian et al. [31] proposed PSO to tune the FOPID controller gains for AVR systems. This work used the same algorithm as [28], which was PSO, but with a different objective function, which is a combination of FOD and ITSE. The analysis involves the convergence curve and transient response. Then, in 2017, Lahcene et al. [32] applied the simulated annealing (SA) optimization algorithm to optimize the FOPID controller gains by considering the combination of ITAE, overshoot, steady-state error, settling time, rise time, and peak time with corresponding weighting coefficients as an objective function. They only consider transient response analysis in their work.

Meanwhile, Sikander et al. [33] proposed a cuckoo search (CS) algorithm in an effort to enhance the operation of the AVR system in 2018. They have used the FOD as an objective function, with analysis based on transient response and robustness analysis (parameter variations). Apart from that, Khan et al. [34] introduced a salp swarm optimization algorithm (SSA)-based FOPID controller for enhancing the stability and dynamic response of the AVR system in 2019. They only considered integrated time absolute error (ITAE) as an objective function. The simulation works involve the convergence curve, transient response, stability assessment (Bode diagram, pole-zero map), and robustness analysis (parameter variations). Following that, Micev et al. [35] introduced a chaotic yellow saddle goatfish algorithm (C-YSGA) in 2020 by hybridizing the existing YSGA and chaos optimization algorithm (COA). The tuning process of the FOPID controller is based on the objective function, which consists of ITAE, overshoot, steady-state error, and settling time with corresponding weighting coefficients. The improved version of YSGA showed superior results for convergence curve, transient response, and robustness analysis (parameter variations, control signal disturbance and load disturbance rejection, measurement noise) compared to the existing one.

Next, in the same year, Jumani et al. [36] proposed the Jaya optimization algorithm (JOA) to tune FOPID controller gains. However, they only considered ITAE as an objective function in optimizing the performance of the AVR system. The JOA produces better results for convergence curve, transient response, stability assessment (Bode diagram, pole-zero map), and robustness analysis (parameter variations) compared to other algorithms. In 2021 and 2022, researchers were still focused on tuning the FOPID controller gains for AVR systems using metaheuristic optimization. For instance, Munagala and Jatoth [37] introduced a chaotic black widow optimization (ChBWO). Furthermore, they used a new objective function by combining the FOD [9] and ITAE. The results of the analysis are based
on convergence curve, transient response (step response and trajectory tracking), stability
assessment (Bode diagram), and robustness analysis (parameter variations, impulse
disturbance rejection). In comparison, Altbawi et al. [38] suggested a FOPID tuning method
using gradient-based optimization (GBO) and implementing ITAE as an objective function.
Their simulation works involve convergence curve, transient response, stability assessment
(Bode diagram, pole-zero map, and Nyquist diagram), and robustness analysis (parameter
variations, load disturbance rejection, and noise tests). Furthermore, in [21], they utilized
SA-MRFO as a tuning method for the FOPID controller. They introduced a new objective
function, which consists of ITAE, overshoot, steady-state error, and settling time with
respect to the PID controller. The proposed SA-MRFO scores excellent results
for convergence curve, transient response, and robustness analysis (parameter variations,
control signal disturbance, and load disturbance rejection) compared to other variants of the
PID controller. Based on the reported FOPID controller-tuning method above, it is justified
that the metaheuristic optimization algorithm is regarded as an excellent tool for finding
the optimal FOPID controller gains for the AVR system, thus providing a more accurate
nominal voltage level. However, although such metaheuristic optimization techniques
have improved the performance of many AVR systems, the obtained results are still low in
accuracy and need to be improved. Thus, other state-of-the-art metaheuristic optimization
algorithms can be applied to tune the FOPID controller gains to improve the AVR system’s
overall performance.

Meanwhile, the marine predators algorithm (MPA) is one of the state-of-the-art meta-
heuristic optimization algorithms invented by Faramarzi et al. [39] in 2020. MPA is a
nature-inspired metaheuristic algorithm that mimics the foraging strategy of ocean preda-
tors by considering the interaction between predator and prey. A large volume of published
studies describes the role of the MPA in solving optimization problems in various fields.
For example, Sadiq et al. [40] used the MPA to find the optimal fair power allocation
in non-orthogonal multiple access (NOMA) and visible light communications (VLC) for
Beyond 5G (B5G) networks. In [41], the MPA is employed in renewable energy fields
to solve the optimal reactive power dispatch (ORPD) problem under system uncertainties.
Sowmya et al. [42] utilized the MPA to minimize the charging and discharging cycles of
electric vehicles in order to decrease the electricity cost. Furthermore, the MPA is applied
in wind plant systems to predict the total power production using an enhanced variant
of the adaptive neuro-fuzzy inference system (ANFIS) [43]. The MPA has also been a
popular optimization tool in control engineering fields. For example, the MPA is used to
fine-tune the proportional–integral–derivative–acceleration (PIDA) controller for the static
synchronous compensator (STATCOM) to improve the frequency response of the power
systems [44]. In [45], the authors adopted the MPA to fine-tune the PID cascaded controller
for the infinite bus power system and IEEE-39 bus system. Besides that, Sobhy et al. [46]
have implemented the MPA as an optimizer to find the optimum value of PID gains for
load frequency control in modern interconnected power systems. Furthermore, in [47],
they have utilized the MPA as a robust, coordinated tuning method of damping controllers
for improving the small-signal stability of high-wind integrated systems. Additionally,
the MPA is utilized to tune the multiple-node hormone regulation neuroendocrine-PID
(MnHR-NEPID) controller of the gantry crane system [48]. Also, in [49], MPA is responsible
for tuning the controller parameters of a single row of ten turbines to improve the wind
plant’s power production.

From the above-mentioned studies, the MPA shows itself to be effective in solving a
variety of engineering problems as well as control engineering problems and producing
better performances than other state-of-the-art metaheuristic-based methods. Moreover, it
has also been verified in the original study [39] that the MPA produces better convergence
accuracy in most of the benchmark functions than its rival. However, although the MPA has
become a promising optimization tool for solving various problems, like other metaheuristic
algorithms, it still poses some drawbacks. Firstly, in the existing MPA, there is a high
possibility of the solution getting stuck in local optima. This is because, during the transition
from exploration to exploitation in Phase 2, each prey updates its location at each iteration only based on either its previous location or the location of the current best predator. Therefore, if the location of the current best predator is suddenly trapped in a local region, it may cause the other prey to be trapped in the same region. Similarly, if the current location of the prey is suddenly trapped in the local optima region, it is difficult for it to jump out from the region since it mostly depends on the information from its previous location. Secondly, our preliminary investigation has shown that the existing adaptive coefficient to control the step size (defined as $\text{CF}^{[39]}$) is too restrictive. Thus, it is not able to properly control the exploration and exploitation phases more effectively. Therefore, it must be noted that if the existing MPA is solely used, less impressive control performance in the AVR system may be observed.

In order to solve the issues of the existing MPA, a new variant of the hybrid algorithm formed by a combination of multi-agent and single-agent algorithms between MPA and the safe experimentation dynamics algorithm (SEDA), named MP-SEDA, with a tunable adaptive coefficient for controlling the step size ($\text{CF}^{[39]}$), is proposed. Firstly, the MP-SEDA is introduced to solve the local optima problem in Phase 2 of the existing MPA, where the next location of the prey is selected by changing some of the elements of the updated location vector of the prey with the elements of the best predator’s location vector randomly according to the predefined probability. As a result, the current best predator or the existing prey can assist any outlier’s prey and predator in jumping out from the local optima region and continuing a new searching track. Secondly, the tunable $\text{CF}^{[39]}$ is adopted in the existing MPA to improve the improper balance of exploration and exploitation phases. Note that the tunable $\text{CF}^{[39]}$ is expected to provide more flexibility in retaining well-balanced exploration and exploitation phases, thus improving the searching capability.

This paper introduces a new optimization method for the FOPID controller of AVR systems based on MP-SEDA with tunable $\text{CF}^{[39]}$. In particular, the hybridization between MPA and SEDA solves the local optima problem, while the tunable $\text{CF}^{[39]}$ was introduced to obtain a proper balance between the exploration and exploitation phases. The proposed MP-SEDA with tunable $\text{CF}^{[39]}$ was then used to tune the FOPID controller gains of the AVR system. Here, the effectiveness of the proposed MP-SEDA-FOPID controller was compared with the existing MPA-FOPID controller, as well as other FOPID controller metaheuristic-based methods, viz. SA-MRFO [21], GBO [38], ChBWO [37], JOA [36], C-YGSA [35], SSA [34], CS [33], SA [32], PSO [28,31], CAS [30], and NSGA II [29]. In this study, several case studies were conducted to evaluate the effectiveness of the MP-SEDA-FOPID controller.

Firstly, the convergence curve of the objective function is analyzed to evaluate the overall performance of the proposed optimization method. Then, the statistical analysis of the objective function between the existing MPA-FOPID and the MP-SEDA-FOPID controller is presented to highlight the superiority of the improved version of MPA. Next, the non-parametric statistical test is demonstrated using Wilcoxon’s rank test to estimate the statistical difference between the existing MPA-FOPID and the MP-SEDA-FOPID controller. Then, the step response analysis was conducted to examine the control performance of the FOPID controller obtained by the MP-SEDA-based method with tunable $\text{CF}^{[39]}$ in terms of time response specification in comparison with existing MPA as well as the aforementioned compared algorithms. Furthermore, the stability analyses were implemented by evaluating the Bode plot. Finally, the robustness analyses were tested on all compared methods by assessing the trajectory tracking (including the measurement noise), disturbance rejection, time-invariant, and time-varying parameter uncertainties. At this point, the robustness analyses were evaluated in terms of the integral absolute error (IAE), integral square error (ISE), integral time absolute error (ITAE), and integral time square error (ITSE). Overall, the significant contributions of this study can be identified as follows:

(i) Note that the main challenge of the existing MPA in [39] is that the solution tends to get stuck in local optima since the prey updates its location at each iteration only based on either its previous location or the location of the current best predator. To overcome this difficulty, a new method for the updating mechanism based on the
SEDA characteristic was adopted in the existing MPA, where the following location of the prey would be selected by changing some of the elements of the updated location vector of the prey with the elements of the best predator’s location vector randomly according to the predefined probability. The merit of such a method is that the location of the current best predator or the location of the current prey can help any trapped prey or predator to jump out from the local optima region and continue a new search track.

(ii) Another challenge of the existing MPA in [39] is that the CF is too restrictive and has no flexibility in adjusting the exploration and exploitation phases. Thus, to solve this difficulty, the tunable CF is introduced to give more flexibility to the user in finding well-balanced exploration and exploitation phases according to the given optimization problem, thus improving the searching capability.

(iii) The proposed MP-SEDA-based method with tunable CF was a new variant of the hybrid algorithm formed by a combination of multi-agent and single-agent algorithms applied to tune the FOPID controller gains for the AVR system. Moreover, it was shown in this study that the MP-SEDA-FOPID controller could provide better control accuracy with significant results than other recent multi-agent-based methods, such as SA-MRFO-FOPID [21], ChBWO-FOPID [37], GBO-FOPID [38], and JOA-FOPID [36].

(iv) A new objective function was introduced by modifying the existing FOD in [9] with an additional weighting coefficient to give more flexibility for users in adjusting the overshoot.

(v) A new reference signal, which is the combination of the trapezoidal and sinusoidal signals, was introduced in both trajectory tracking and disturbance rejection analyses to show the superiority of the proposed method in comparison with other existing FOPID controllers. This reference signal differs from the previous studies conducted in [32], which utilized different step changes.

(vi) Time-varying parameter uncertainties were introduced in robustness analyses to further test the effectiveness of the proposed FOPID controller and to reflect the real application of the AVR system. Unlike [21,37,38], they are investigated based on time-invariant parameter uncertainties.

The outline of the paper is as follows: the problem formulation of the FOPID controller for AVR systems is presented in Section 2. Section 3 describes the existing MPA-based method and the proposed MP-SEDA-based method with tunable CF. The procedure to apply the MP-SEDA-based method with tunable CF is also discussed in the same section. The effectiveness of the proposed method is validated in Section 4. Finally, some concluding remarks are given in Section 5.

2. Problem Formulation of FOPID Controller for AVR System

In this section, the working principle and mathematical model of the AVR system are initially interpreted and described. Then, the closed-loop block diagram of the AVR system with the FOPID controller is provided. Lastly, the problem formulation of the AVR system with the FOPID controller is explained at the end of this section.

AVR is designed to stabilize the power system’s operation by regulating the output voltage produced by a synchronous generator. AVR is responsible for ensuring the output voltage variations do not exceed 5% of the rated voltage under all operating conditions to meet the IEEE standards. Figure 1 shows the AVR system for a conventional synchronous generator voltage control mechanism, while Figure 2 shows the block diagram representation of the AVR system. As we can see in Figure 2, the AVR system consists of five major components, i.e., controller, amplifier, generator, exciter, and sensor [21]. To further understand the working principle of the AVR system, we start with the measurement of the output voltage of the synchronous generator that the voltage sensor has sensed. Then, the voltage from the sensor is rectified and filtered before being sent to the comparator circuit. From the comparator circuit, the difference between measurement voltage ($V_m$) and reference voltage ($V_{ref}$) known as an error signal ($e_v$) is produced. Next, this error signal
is fed to the controller, which is responsible for providing a stable controlled signal to the amplifier according to the predetermined controller gains. Therefore, accurately tuning the controller gains is vital to avoid any unwanted response of the AVR system, such as higher overshoots and a longer time to achieve the desired voltage. Finally, after the appropriate amplification, the exciter circuit generates the controlled flux for the synchronous generator and produces the output voltage \( V_{\text{out}} \) which follows the desired voltage level [14].

![Figure 1. Conventional AVR system.](image1)

![Figure 2. Block diagram of the AVR system.](image2)

In this study, we use a linearized mathematical model of the AVR system proposed by Gaing et al. [9], derived from the frequency domain using the Laplace transformation. Note that the controller used in this study is the FOPID controller introduced by Pudlubny [50]. The FOPID controller is a variant of the conventional PID, which introduces two new terms, namely, fractional exponential terms of integral \( \lambda \) and derivative \( \mu \). The transfer functions for each component of the AVR system in Figure 2, i.e., FOPID controller, amplifier, exciter, generator, and sensor, are depicted in Equations (1)–(5), respectively.

\[
G_{\text{FOPID}}(s) = K_p + K_i s^{-\lambda} + K_ds^\mu \\
G_{\text{Amp}}(s) = \frac{K_A}{1 + T_A s} \\
G_{\text{Ext}}(s) = \frac{K_E}{1 + T_E s} \\
G_{\text{Gen}}(s) = \frac{K_G}{1 + T_G s} \\
G_{\text{Sen}}(s) = \frac{K_S}{1 + T_S s}
\]
In Equation (1), $K_p$, $K_I$, $K_D$, $\lambda$, and $\mu$ are proportional gain, integral gain, derivative gain, exponent of integral term, and exponent of differential term, respectively. Meanwhile, the gains of the amplifier, exciter, generator, and sensor in Equations (2)–(5) are denoted by $K_A$, $K_E$, $K_G$, and $K_S$, respectively. Moreover, the time constant for all components is denoted by $T_A$, $T_E$, $T_G$, and $T_S$. Therefore, the closed-loop block diagram of the AVR system with the FOPID controller is represented in Figure 3.

![Figure 3. Closed-loop block diagram of the AVR system with FOPID controller.](image)

To guarantee the stability of the AVR system, the range of gains and time constants are shown in Table 1, as described in the literature [37]. Although those values are set to ensure the stability of the system, the response of the AVR system without the controller is exceptionally oscillating (see blue line in Figure 4) and requires considerable time to settle within the desired voltage (see green dotted-line in Figure 4). As seen in Figure 4, the response has a high overshoot of 65.21%, a rise time $T_r = 0.26$ s, a settling time $T_{set} = 7.02$ s, and a steady-state error $E_{ss} = 0.091$. This response indicates that there is an unusual escalation happening in the generator’s reactive power load and dropping the voltage of the exciter. This situation causes high oscillations at the terminal voltage. Furthermore, it is noticeable that during the steady-state condition, the terminal voltage deviates from the nominal value by more than 9%. This deviation is too large to be handled by sophisticated equipment susceptible to voltage variations and can cause malfunctions or even trips when voltage variations happen for more than a second. Also, large overshoots in power system networks may force the system to be unstable. Thus, it is expected to have a response with a settling time of less than a second and no overshoot to reduce the cost of downtime caused by equipment trips [14]. Therefore, it is important to employ a controller in the AVR system to mitigate this problem and optimize the response.

<table>
<thead>
<tr>
<th>AVR Component</th>
<th>Parameter Ranges</th>
<th>Used Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplifier</td>
<td>$10 \leq K_A \leq 400$</td>
<td>$K_A = 10$, $T_A = 0.1$</td>
</tr>
<tr>
<td></td>
<td>$0.02 \leq T_A \leq 1.0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$1.0 \leq K_E \leq 10$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$0.4 \leq T_E \leq 1.0$</td>
<td>$K_E = 1$, $T_E = 0.4$</td>
</tr>
<tr>
<td>Exciter</td>
<td>$0.7 \leq K_G \leq 1.0$</td>
<td>$K_G = 1$, $T_G = 1$</td>
</tr>
<tr>
<td></td>
<td>$1.0 \leq T_G \leq 2.0$</td>
<td></td>
</tr>
<tr>
<td>Generator</td>
<td>$1.0 \leq K_S \leq 2.0$</td>
<td>$K_S = 1$, $T_S = 0.01$</td>
</tr>
<tr>
<td></td>
<td>$0.001 \leq T_S \leq 0.06$</td>
<td></td>
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</tbody>
</table>

Moreover, in order to evaluate the performance of the AVR system controlled by the FOPID controller, the objective function, as presented by Gaing et al. [9], is modified by introducing the weighting coefficient, $w$. Note that the merit of the proposed weighting coefficient is to give more flexibility for the user to solely adjust the overshoot without affecting other coefficients, which is formulated by

$$J = (1 - e^{-\eta}) \times (w \ast M_p + E_{ss}) + e^{-\eta}(T_{set} - T_r).$$

(6)
In Equation (6), $M_p$ represents overshoot, $E_{ss}$ represents steady-state error, $T_{set}$ represents settling time, and $T_r$ represents rise time. The symbol $\eta$ is a weighting factor that can be adjusted to meet the requirements of a system. For example, if $\eta$ is set to higher than 0.7, then the system will have low $M_p$ and less $E_{ss}$. Meanwhile, if $\eta$ is set to less than 0.7, then the values of $T_{set}$ and $T_r$ can be reduced. In this study, the value of $\eta$ was set to 1.0, which follows the value of other studies such as in [23,30,33]. In the meantime, the weighting coefficient, $w$, is set to 0.3 after conducting several preliminary investigations. Specifically, adjusting $w$ is different from adjusting $\eta$ where, by adjusting $\eta$, all time response coefficients ($M_p$, $E_{ss}$, $T_r$, and $T_{set}$) will be affected. Whereas, by adjusting $w$, only the overshoot will be affected. Thus, depending on the application, the users can decide whether to have a high overshoot or a low overshoot without sacrificing the performance of other coefficients. Finally, the problem can be described as:

**Problem 1.** Based on the given closed-loop system block diagram in Figure 3, the values of the FOPID controller gains $K_p$, $K_i$, $K_d$, $\lambda$, and $\mu$ are obtained such that the objective function $I$ is minimized.

### 3. Improved Marine Predators Algorithm

This section describes the improved marine predators algorithm for tuning the FOPID controller of the AVR system. Firstly, a brief explanation of the existing MPA-based method is given. Then, the proposed MP-SEDA-based method with tunable CF is discussed.

#### 3.1. An Existing Marine Predators Algorithm (MPA)-Based Method

The MPA, introduced in [39], is mainly inspired by the *survival of the fittest* of ocean predators in finding the optimal strategy during foraging for food. Generally, the foraging pattern of marine predators follows a random walk strategy, which includes the Levy walk and Brownian walk. Therefore, the main inspiration of MPA is to find the best optimal strategy through a tradeoff between the Levy and Brownian strategies. In the MPA, the predator is foraging for food, and the prey is foraging for its food.

Firstly, the initial solution of predators and prey is randomly distributed over the given search space to solve the given optimization problem

\[
\arg \min_{X_i(1),X_i(2),...} f_i(X_i(k))
\]
for iterations \( k = 1, 2, \ldots, k_{\text{max}} \), where \( f_i \) is the objective function of the agent \( i \), \( X_i \) is the position vector of the agent \( i \), and \( k_{\text{max}} \) is the maximum number of iterations. As both predators and prey are considered search agents, two main matrices should be defined. There is an elite matrix \( E \) that consists of the best predator and the prey matrix \( P \). The elite matrix and prey matrix are defined as follows

\[
E = \begin{bmatrix}
X'_{1,1} & X'_{1,2} & \cdots & X'_{1,d} \\
X'_{2,1} & X'_{2,2} & \cdots & X'_{2,d} \\
\vdots & \vdots & \ddots & \vdots \\
X'_{n,1} & X'_{n,2} & \cdots & X'_{n,d}
\end{bmatrix},
\]  

(8)

\[
P = \begin{bmatrix}
X_{1,1} & X_{1,2} & \cdots & X_{1,d} \\
X_{2,1} & X_{2,2} & \cdots & X_{2,d} \\
\vdots & \vdots & \ddots & \vdots \\
X_{n,1} & X_{n,2} & \cdots & X_{n,d}
\end{bmatrix},
\]  

(9)

where \( n \) is the total number of agents and \( d \) is the number of dimensions. For simplicity of explanation, let \( X_{i,j}' \) in Equation (8) be the \( j \)-th element of the best predator vector \( X_i' \), which are replicated \( n \) times in the matrix \( E \). Here, \( X_i' \) is chosen from the best position vector among the search agent in each iteration. In Equation (9), \( X_{i,j} \) presents the \( j \)-th dimension of the \( i \)-th prey or agent \( X_i \) in Equation (7). Then, the predators and prey are updated based on three main phases as well as the environmental issues and marine memory to find the global optimum solution. The details of the MPA structure are described as follows.

A. Phase 1: Exploration phase

During Phase 1, the prey is moving faster than the predator, which indicates the prey is in a hurry to search for their food while the predator waits or remains immobile. This situation happens in the first third of the maximum iterations \((k_{\text{max}})\), where the full exploration stage occurs. The prey location is updated using the following equation:

\[
P_i = P_i + Q_1 \otimes \left[ R_B \otimes \left( E_i - R_B \otimes P_i \right) \right], \quad \text{if} \quad k < \frac{1}{3} k_{\text{max}},
\]  

(10)

for \( i = 1, 2, \ldots, n \). In Equation (10), \( P_i \) and \( E_i \) represent the \( i \)-th row of matrices \( P \) and \( E \), respectively, and \( R_B \) is a vector of random numbers for Brownian motion. Moreover, the notation \( \otimes \) indicates the element-wise multiplication, \( Q \) is a constant number that is set to 0.5, and \( r_1 \) is a random number withdrawn from a uniform distribution in the range of \([0, 1]\).

B. Phase 2: Transition from exploration to exploitation phase

In Phase 2, both predator and prey are moving at the same pace. This situation indicates that some of the prey perturb their current position towards the exploitation phase, while other prey update their position based on the movement of predators towards the exploration phase. This phase is considered the transition from exploration to exploitation phase. For such a case, half of the agents are moving in a Brownian walk for exploration, while the other half are moving in a Levy walk for exploitation. This stage happens after one-third of \( k_{\text{max}} \) until two-thirds of \( k_{\text{max}} \). Therefore, the updated location of prey for \( i = 1, 2, \ldots, n/2 \) is

\[
P_i = P_i + Q_1 \otimes \left[ R_L \otimes \left( E_i - R_L \otimes P_i \right) \right], \quad \text{if} \quad \frac{1}{3} k_{\text{max}} < k < \frac{2}{3} k_{\text{max}},
\]  

(11)

while for \( i = n/2, n/2 + 1, \ldots, n \) is given by

\[
P_i = E_i + Q.CF \otimes \left[ R_B \otimes \left( R_B \otimes E_i - P_i \right) \right], \quad \text{if} \quad \frac{1}{3} k_{\text{max}} < k < \frac{2}{3} k_{\text{max}}.
\]  

(12)
In Equation (11), \( R_L \) is a random number based on Levy distribution, while \( R_B \) and \( CF \) in Equation (12) are a random number based on Brownian distribution and an adaptive coefficient that controls the step size of the predator motion, respectively. The detailed expression of \( CF \) is given as follows

\[
CF = \left( 1 - \frac{k}{k_{\text{max}}} \right)^{(2 \times \frac{k}{k_{\text{max}}})}. \tag{13}
\]

C. Phase 3: Exploitation phase

In the third phase of the MPA structure, the predators move faster than the prey, indicating the high exploitation phase. This phase is executed on the last third of the maximum iterations, where the predators follow the Levy walk. To respond to the faster motion of predators, each prey \( (i = 1, 2, \ldots, n) \) updates its location using the following equation:

\[
P_i = E_i + Q.CF \otimes (R_L \otimes (R_L \otimes E_i - P_i)), \quad \text{if} \quad k > \frac{2}{3}k_{\text{max}}. \tag{14}
\]

D. Eddy Formation or Fish Aggregating Devices (FADs) Effect

Besides the three main phases stated above, the MPA also considers the behavior of marine predators in different environmental conditions, which is the eddy formation or fish aggregating devices (FADs) effect. The predators not only move in Levy and Brownian walks while searching for prey but they also perform longer vertical jumps into the sea to hopefully find a prey-abundant area. This vertical jump can help the predators avoid stagnation in the local optima region. The FADs effect can be presented in the following equation:

\[
P_i = \begin{cases} 
P_i + CF[L \otimes (UB - LB)] \otimes U, & \text{if} \quad r_2 \leq \text{FADs}, \\
(P_i + \text{FADs}(1 - r_2) + r_2)(P_{r_3} - P_{r_4}), & \text{if} \quad r_2 > \text{FADs},
\end{cases} \tag{15}
\]

where \( \text{FADs} = 0.2, r_1 \) and \( r_2 \) are the random numbers that are generated randomly in the range of \([0, 1]\), and \( UB \) and \( LB \) represent the upper bound and the lower bound vectors, respectively. The symbol \( U \) is a binary vector where its element consists of the value 0 or 1. Specifically, \( U \) produces a zero array if \( r_2 \leq 0.2 \). Otherwise, \( U \) generates its array to 1. Furthermore, \( r_3 \) and \( r_4 \) are random indexes of the column of the prey matrix.

E. Marine Memory

Another feature of MPA is memorizing the location of the high-production foraging sites of predators, thus avoiding local solutions. After updating the prey matrix and implementing the FADs effect, the fitness of each prey is evaluated to update the elite matrix. Specifically, the fitness of each solution in the current iteration is compared to its equivalent in the prior iteration, and the current one replaces the solution if it is more fit. This method simulates the return of predators to the successful high-production foraging area. Thus, the marine memory method is able to improve the quality of the solution for the entire iteration.

A detailed explanation of the MPA can be obtained in [39].

3.2. An Improved MP-SEDA-Based Method with Tunable CF

Although the MPA has its advantages and functions well for various optimization problems, in some cases, its ability to escape from the local optima is weakened, and the balancing between exploration and exploitation phases is poor for some optimization problems. This limitation is supported by the no free lunch (NFL) theorem, which proves that no optimization algorithm can solve all optimization problems with equally good results. In other words, if Algorithm A can perform better than Algorithm B in solving certain optimization problems, Algorithm B can also perform better than Algorithm A in finding a suitable solution for another optimization problem. Therefore, many researchers are currently active in exploring different optimization algorithms to deal with various
optimization problems. Hence, this study proposes the following modifications to solve the MPA limitations. Firstly, to avoid stagnation in local optima, the hybridization of MPA and SEDA will be explained in detail in Section 3.2.1. Secondly, to improve the capability to balance the exploration and exploitation phases, a tunable CF will be introduced in Section 3.2.2.

3.2.1. Hybridization of MPA and SEDA

In the existing MPA, during Phase 2, the location of the prey is updated based on either its previous location or the location of the current best predator. As a result, if the current best predator is suddenly trapped in the local optima region, it will remain in that region, and in the worst-case scenario, it will pull other prey towards the same local optima region. Similarly, if the previous location of the prey is in the local optima region, the subsequent location of the prey will also remain in the same local optima region without any direct interactions with the current best predator. In order to solve this problem, the proposed MP-SEDA-based method is designed with a strategy to hybridize the existing MPA with the safe updating mechanism of the SEDA. With this hybridization, higher-accuracy results can be obtained while maintaining a reasonable complexity level. As far as we can ascertain, this is a pioneering study that makes use of a multi-agent optimization algorithm and SEDA hybridizing concepts in the AVR system.

The SEDA architecture is first explained as a preliminary to our hybrid design. SEDA is a single-agent algorithm first introduced by Marden et al. [51]. The inspiration for SEDA is based on a game-theoretic method that implements the concept of repeated multi-player games where the players choose strategies from a finite set of available strategies in a repeated and simultaneous manner, depending on the strategy adjustment process. Each player moves in a random motion that is decided based on a predetermined probability. Since its first debut, SEDA has been utilized as a tool to cater to various engineering applications such as flexible joint robots [52], pantograph–catenary systems [53], underactuated container cranes [54], and DC/DC buck–boost converter–inverter–DC motors [55]. In the domain of a single agent, systematic studies have shown that SEDA has solid performance and is very simple to implement, with a safe updating mechanism. The safe updating mechanism means the SEDA only updates a part of a design variable element depending on a predetermined probability, compared to most available algorithms that update all design variable elements during the entire iteration. This merit shows that SEDA is better in terms of complexity level and computational time.

SEDA defines each element of the design variable as a player. Here, each player’s random motion is decided based on a predetermined probability such that an optimal goal or design variable is achieved, corresponding to the objective function’s minimum value. Let \( X \in \mathbb{R}^n \) be the design variable. The SEDA algorithm iteratively updates \( X \in \mathbb{R}^n \) using the updated law:

\[
X_j(k+1) = \begin{cases} 
    h \left( X'_j - \xi r_6 \right), & \text{if } r_5 \leq C, \\
    X'_j, & \text{if } r_5 > C, 
\end{cases}
\]

(16)

for \( k = 0, 1, \ldots, k_{\max} \). The symbols \( r_5 \) and \( r_6 \) in Equation (16) represent independent random numbers uniformly distributed in the range of \([0, 1]\), \( C \) is a coefficient that defines the probability of using the newly updated design variable, \( \xi \) represents the perturbation step size gain of the design variable, \( X_j \) is the \( j \)-th element of \( X \in \mathbb{R}^n \), \( X'_j \) is the element of the current best design variable \( X' \in \mathbb{R}^n \) during the course of an iteration. In Equation (16), the function \( h(.) \) is given by:

\[
h(.) = \begin{cases} 
    UB, & \text{if } X'_j - \xi r_6 > UB, \\
    X'_j - \xi r_6, & \text{if } LB \leq X'_j - \xi r_6 \leq UB, \\
    LB, & \text{if } X'_j - \xi r_6 < LB, 
\end{cases}
\]

(17)
where \( LB \) and \( UB \) are predefined lower and upper bound values of the design variable, respectively.

The vital feature of SEDA is that it can provide stable convergence due to its capability to keep the best design variable during the tuning process. Moreover, the SEDA uses a fixed interval step size, independent of the number of iterations. Therefore, it would possibly be a useful tool to solve the existing MPA that suffered from local optima stagnation. Technically, this problem can be said to stem from its working principle, in which MPA rejects all the fitness values that exceed the global best and never preserves the possible set of solutions, which may have the potential to escape from a local optima region. In order to get rid of this problem, the safe updating mechanism in Equation (16) is adopted in Phase 2 of the existing MPA after executing Equations (11) and (12). Specifically, if the random number \( r_5 \) is greater than the predetermined probability \( C \), the element of the updated location vector of the prey \( P_i(k + 1) \) will be substituted by the element of the best predator’s location vector \( E_i \). Otherwise, the element of the prey \( P_i(k + 1) \) will maintain the element of the previous updated location as executed in Equations (11) and (12). As a result, the adopted safe updating mechanism can prevent the current best predator from pulling the current prey into the local optima region. After several iterations, the current best predator or the current prey can indirectly help any previously trapped prey or predator to jump out from the local optima region and continue a new search track.

Figure 5 illustrates the graphical representation of a predefined contour plot with a two-dimensional location \((d = 2)\) for better understanding the purpose of MP-SEDA. Let us assume that during Phase 2, the current best predator \( E_i \) (blue predator) is suddenly trapped in the local optima region. According to Equation (12) of the existing MPA, there is a high possibility of \( E_i \) remaining trapped in that region, and in the worst-case scenario, it may pull the current prey \( P_i \) (red prey) to the same local optima region as illustrated by \( P_{old i} \) (purple prey) and this perturbation is denoted by the red dotted arrow. A similar situation can also happen in Equation (11), which is not illustrated in this paper due to limited space. Nonetheless, this problem can be solved with the presence of a safe updating mechanism where only some elements in \( P_i \) will be changed according to the element in \( E_i \). In the example given, consider that the predetermined probability \( C = 0.5 \) and the generated random number \( r_5 \) is greater than 0.5. Then, the second element of the agent \( P_i \) (i.e., represented by the \( y \)-axis), is changed to the second element in \( E_i \) which allows \( P_i \) to be perturbed to a new location \( P_{new i} \) (green prey) that is denoted by the black dotted arrow. In particular, the resultant \( P_{new i} \) has more possibility to avoid the trapped \( E_i \) or \( P_i \), and then it can continue searching other regions with better solutions. Note that, for easy understanding, this example uses only two elements. The same concept can be extended to a larger number of elements.

### 3.2.2. Tunable Adaptive Coefficient for Controlling the Step Size (CF)

The suggested tunable adaptive coefficient for controlling the step size \((CF)\) is elaborated on in this section. Here, it is shown that the exploration and exploitation phases of the existing MPA can be enhanced further by modifying the existing \( CF \). Hypothetically, exploration and exploitation have opposing relationships to each other, where excessive exploration will result in an inaccurate global optimum value. Conversely, excessive exploitation can lead to local optima stagnation. Previously, the existing version of the MPA-based method placed the restricted \( CF \) in Phase 2, Phase 3, and \( FADs \), which can lead to an improper balance between the exploration and exploitation phases. As we can see, the value of \( CF \) in Equation (13) was non-linearly decreasing from 1 to 0. However, such a setting is too restrictive since there is no flexibility for the user to control or manipulate the exploration and exploitation phases. In addition, the existing \( CF \) might limit the applications of the MPA, so it is essential to have a more generic equation of \( CF \) which can cover a broader class of applications. To solve this issue, the tunable \( CF \) is adopted in the existing MPA, which can provide more flexibility in retaining well-balanced exploration
and exploitation phases, thus improving the searching capability. Therefore, the equation of $CF$ in Equation (13) is modified as

$$CF = \left( 1 - \frac{k}{k_{\text{max}}} \right)^{(\beta \times \frac{k}{k_{\text{max}}})}, \quad (18)$$

where $\beta$ is the newly proposed tunable coefficient employed to alter the ratio of exploration and exploitation phases throughout the tuning process. As a result, it is expected that the introduction of the MP-SEDA-based method with tunable $CF$ might deliver further promising performance for optimizing the FOPID controller of the AVR system. Furthermore, our new tunable $CF$ can provide more choices for exploration and exploitation portions compared to the existing $CF$, which can cover a variety of real application optimization problems. Technically, the proposed MP-SEDA-based method with tunable $CF$ will replace $CF$ in the existing MPA in Equations (12), (14) and (15) with the $CF$ in Equation (18). Figure 6 shows the effect of varying the value of $\beta$ to the value of the proposed $CF$. Precisely, in order to possess high exploitation ability, $\beta$ can be set greater than 2, e.g., $\beta = 10$. In opposition, if $\beta$ is set less than 2, e.g., $\beta = 0.3$, the algorithm is expected to possess high exploration capability. The proposed MP-SEDA-based method with tunable $CF$ pseudocode is shown in Algorithm 1. Based on the pseudocode, the proposed hybrid MP-SEDA is represented in lines 13–19, while the proposed tunable $CF$ is demonstrated in lines 12, 21, and 24.

![Figure 5. Graphical representation of the safe updating mechanism in the SEDA.](image-url)
Here, the prey or predator can assist any outlier predator or prey to jump out from the local optima region and continue in the new search space.

![Figure 6. Value of CF for different β.](image)

**Algorithm 1** Proposed MP-SEDA with tunable CF algorithm

1. Initialize $UB$, $LB$, $k_{\text{max}}$, $n$, $d$, $Q$, $FADs$, $\beta$, and $C$
2. Randomly initialize the search agents $P_i$ populations $i = 1, 2, \ldots, n$
3. $k = 1$
4. While $k < k_{\text{max}}$
   5. Evaluate the fitness of all $P_i$, then construct the $E_i$ matrix and accomplish memory saving
   6. If $k \leq \frac{1}{3} k_{\text{max}}$
      7. Update $P_i$ based on Equation (10)
   8. Else if $\frac{1}{3} k_{\text{max}} < k \leq \frac{2}{3} k_{\text{max}}$
      9. For the first half of the populations ($i = 1, 2, \ldots, n/2$)
         10. Update $P_i$ based on Equation (11)
      11. For the second half of the populations ($i = n/2, n/2 + 1, \ldots, n$)
         12. Update $P_i$ based on Equation (12) where $CF$ is based on Equation (18)
         13. for $j = 1 : d$
            14. for $i = 1 : n$
               15. If $r_5 > C$
               16. $P_{ij}(k+1) = E_{ij}$
               17. end if
            18. end for
         19. end for
   20. Else if $k > \frac{2}{3} k_{\text{max}}$
      21. Update $P_i$ based on Equation (14) where $CF$ is based on Equation (18)
   22. End if
23. Evaluate memory saving and update $E_i$
24. Applying $FADs$ effect and update based on Equation (15) where $CF$ is based on Equation (18)
25. End while

Furthermore, unlike in Phase 1, the prey’s velocity is higher than the predator’s, indicating that the prey is the most active agent. Thus, the prey is responsible for exploring the new search space and moving in a Brownian walk while the predator only waits for the
prey. So, the possibility of the prey being stuck in local optima is very low since they are always searching for a new search area. That is why the SEDA characteristic is not applied during Phase 1.

Similarly, in Phase 3, the SEDA characteristic is not worth applying since the predator is moving in a Levy walk and has a high capability to exploit the prey. Thus, the possibility of the prey being stuck in local optima is minimal since the SEDA characteristic was implemented in the previous phase. Secondly, the tunable CF is only applied in Phase 2, Phase 3, and FADs. In Phase 2, the tunable CF is implemented for the second half of the population since the predator is moving in a Brownian manner. Thus, there is an opportunity to control the step size of the predator in order to balance the exploration and exploitation phases properly. Meanwhile, in the high exploitation stage (Phase 3), although the predator starts to move in the Levy walk, there is still an opportunity to adjust the step size of the predator so that it can improve the global optimum searching capability. Meanwhile, during FADs, the new tunable CF can offer a variation of a longer vertical jump motion for predators to find more promising prey-abundant areas. Finally, the proposed MP-SEDA-based method with tunable CF is expected to improve the existing MPA in terms of local optima avoidance and well-balanced exploration and exploitation phases.

3.3. Optimization of Benchmark Functions Using MP-SEDA-Based Method with Tunable CF

The efficiency verification of the proposed MP-SEDA-based method with tunable CF in optimizing the benchmark functions is discussed in this section. Specifically, nine different benchmark functions were used, and they were categorized into three groups: uni-modal benchmark functions, multi-modal benchmark functions, and fixed-dimension multi-modal benchmark functions. The uni-modal functions ($F_1$–$F_4$) are suitable for benchmarking the exploitation ability or the algorithm’s precision since they only have one global solution. Meanwhile, ($F_5$–$F_8$) are multi-modal functions that are helpful in examining exploration and poor local optima avoidance of an algorithm, as these multi-modal functions have many local optima. A fixed-dimension multi-modal function ($F_9$) has a vast number of local optima, so it is used to test the local optima avoidance capability and the balance between exploration and exploitation of the algorithm. The function name, dimension, search range (i.e., the boundary of the function search space), and theoretical optimal value, $F_{min}$, are shown in Table 2. For verifying the results, the proposed MP-SEDA-based method with tunable CF is compared with the existing MPA, SCA [56], ant lion optimizer (ALO) [57], multi-verse optimization (MVO) [58], spiral dynamic optimization algorithm (SDA) [59], moth–flame optimization (MFO) [60], and grasshopper optimization algorithm (GOA) [61]. The coefficients for all compared algorithms are set as default, as stated in the cited articles. Note that the MP-SEDA-based method with tunable CF coefficients is set as $β = 3.99$ and predetermined probability $C = 0.67$, after performing several initial investigations. Meanwhile, the default MPA-based method coefficients that are also used in the MP-SEDA-based method with tunable CF remain as $Q = 0.5$ and $FADs = 0.2$. All these optimization algorithms were run 30 times on each benchmark function using 30 agents and 1000 iterations. Then, the average minimization results are presented in Table 3.

<table>
<thead>
<tr>
<th>Function</th>
<th>Dim</th>
<th>Range</th>
<th>$F_{min}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_1$ (Sphere)</td>
<td>30</td>
<td>$[-100, 100]$</td>
<td>0</td>
</tr>
<tr>
<td>$F_2$ (Schwefel 2.22)</td>
<td>30</td>
<td>$[-10, 10]$</td>
<td>0</td>
</tr>
<tr>
<td>$F_3$ (Resenbrock)</td>
<td>30</td>
<td>$[-30, 30]$</td>
<td>0</td>
</tr>
<tr>
<td>$F_4$ (Step)</td>
<td>30</td>
<td>$[-100, 100]$</td>
<td>0</td>
</tr>
<tr>
<td>$F_5$ (Penalized)</td>
<td>30</td>
<td>$[-50, 50]$</td>
<td>0</td>
</tr>
<tr>
<td>$F_6$ (Penalized 2)</td>
<td>30</td>
<td>$[-50, 50]$</td>
<td>0</td>
</tr>
<tr>
<td>$F_7$ (Kowalik)</td>
<td>4</td>
<td>$[-5, 5]$</td>
<td>0.00030</td>
</tr>
<tr>
<td>$F_8$ (Six-hump Camel Back)</td>
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<td>$[-5, 5]$</td>
<td>-1.0316</td>
</tr>
<tr>
<td>$F_9$ (Shekel 10)</td>
<td>4</td>
<td>$[0, 10]$</td>
<td>-10.5363</td>
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Table 3. Minimization results of benchmark functions.

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<thead>
<tr>
<th></th>
<th></th>
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<tbody>
<tr>
<td>$F_1$</td>
<td>1.7713 $\times 10^{-37}$</td>
<td>1.0011 $\times 10^{-47}$</td>
<td>3.5000 $\times 10^{-2}$</td>
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<td>5.0800 $\times 10^{3}$</td>
<td>4.2200 $\times 10^{-1}$</td>
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<td>$-1.0316$</td>
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<td>$-1.0316$</td>
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</table>

As shown in Table 3, the MP-SEDA-based method with tunable $CF$ dominates the best results for most benchmark functions, as highlighted in bold. In the uni-modal test function, the MP-SEDA-based method with tunable $CF$ manages to provide the best results for all uni-modal functions ($F_1$–$F_4$). These results show that the proposed MP-SEDA-based method with tunable $CF$ has immense strength in searching for precision uni-modal problems. In the meantime, from the experimental results of the multi-modal functions, it is clearly seen that the MP-SEDA-based method with tunable $CF$ highly outperforms other algorithms on $F_5$ and $F_6$. For function $F_7$, the MP-SEDA-based method with tunable $CF$ gives a superior result compared to other algorithms except for existing MPA, which produces a similar result. Meanwhile, for function $F_8$, the MP-SEDA-based method with tunable $CF$ manages to produce similar results to other algorithms. Therefore, it can be said here that the proposed algorithm also has a high exploration ability and is able to escape from local optima. The rest of the results, which belong to the fixed-dimension multi-modal benchmark function, can be observed in function $F_9$. The resultant readings are consistent with those of other test functions, in which the proposed MP-SEDA-based method with tunable $CF$ shows a similar result to the existing MPA and dominates other optimization algorithms. Thus, it is proven that the MP-SEDA-based method with tunable $CF$ can avoid the local optima and balance between exploration and exploitation.

Additionally, the statistical analysis of the benchmark functions is conducted to emphasize the improvement of the MP-SEDA-based method with tunable $CF$ compared to the existing MPA-based method. This is to counter the randomization effect due to generated random values at the initial stage of the algorithm; both algorithms are simulated for 30 trials to achieve the best statistical results. The statistical analysis of the benchmark functions is based on the mean, best, worst, and standard deviation (std). Meanwhile, to evaluate the statistical difference of the objective function between both methods, a non-parametric statistical test using Wilcoxon’s rank test with a significance level of 5% is implemented. Table 4 shows the statistical results over 30 trials and $p$-value for the MP-SEDA-based method with tunable $CF$ and the existing MPA. As we can see, the MP-SEDA-based method with tunable $CF$ has superior results for functions $F_1$–$F_7$ compared to the existing MPA by dominating all statistical results and producing a $p$-value less than 0.05, as highlighted in bold, which indicates there is a significant difference between the algorithms. Meanwhile, for functions $F_8$ and $F_9$, both algorithms produce similar statistical results except for standard deviation, where the MP-SEDA-based method with tunable $CF$ produces more consistent results, as shown by a smaller value of std. Also, the $p$-values for functions $F_8$ and $F_9$ are less than 0.05. Thus, it is proven that the algorithms are significantly different. Overall, the modifications of the existing MPA by introducing hybridization between MPA and SEDA, as well as tunable $CF$, have been proven significantly valuable for improving the results of the aforementioned benchmark functions. Therefore, it is worth applying the proposed MP-SEDA-based method with tunable $CF$ to tune the FOPID controller to solve the AVR system’s problem.
Table 4. Statistical results and p-values between MP-SEDA-based method with tunable CF and existing MPA.

<table>
<thead>
<tr>
<th>Function</th>
<th>MP-SEDA</th>
<th>MPA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F1</td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>$1.7733 \times 10^{-37}$</td>
<td>$1.0011 \times 10^{-49}$</td>
</tr>
<tr>
<td>Best</td>
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<tr>
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<td>p-value</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>F2</td>
<td></td>
</tr>
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<td>Best</td>
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<td>F3</td>
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<td></td>
<td>F5</td>
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<td>Best</td>
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<td>$3.0749 \times 10^{-4}$</td>
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<tr>
<td>Worst</td>
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<td>p-value</td>
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<td>$-1.0316$</td>
</tr>
<tr>
<td>Worst</td>
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<td>$-1.0316$</td>
</tr>
<tr>
<td>Std</td>
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<td>p-value</td>
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<td></td>
<td>F9</td>
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</tr>
<tr>
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<td>$-10.5364$</td>
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<tr>
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<td>$-10.5364$</td>
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<td>p-value</td>
<td>$0.021614$</td>
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3.4. Application of the Proposed MP-SEDA-Based Method with Tunable CF for Tuning the FOPID Controller of the AVR System

In this section, the procedure of optimizing the FOPID controller using the MP-SEDA-based method with tunable CF is explained, which strives to decrease the objective function as presented in Equation (6). Firstly, each prey or agent position vector \( X_i \in \mathbb{R}^d \) in Equation (7) is mapped to the design variable of the FOPID controller

\[
\psi = [K_P, K_I, K_D, \lambda, \mu] \in \mathbb{R}^d.
\]  

In the meantime, the objective function \( f_i \) in Equation (7) is mapped with the objective function in Equation (6) as \( f_i(X_{i,1} \ X_{i,2} \ \ldots \ X_{i,d}) = J(\psi) \). Technically, the proposed method updates the agent position \( X_i(k) \) which corresponds to the prey location \( P_i \) such that the value of the objective function \( f_i \) is minimized. Usually, the updated \( X_i(k) \) (from \( P_i \)) produces a different \( f_i \) value, which corresponds to the overshoot \( M_p \), steady-state error \( E_{ss} \), settling time \( T_{set} \), and rise time \( T_r \) values. Then, each \( f_i \) is ranked, the best \( f_i \) value is recorded, and its corresponding best position vector is kept as \( X_i'(k) \) in the matrix \( E \). The detailed procedure of updating the agent position vector \( X_i(k) \) is given in Algorithm 1, which also corresponds to the process of tuning the value for the FOPID controller gains. This duplex coordination between the MP-SEDA-based method with tunable CF and the FOPID controller of the AVR system is recursively executed until the maximum iteration.

In a nutshell, the summary of the step-by-step procedure for tuning the FOPID controller is given by:

Step 1: The mappings of \( f_i = J(\psi) \) and \( X_{i,j} = \psi_j (j = 1, 2, \ldots, d) \) are established. Then, \( UB, LB, k_{max}, n, d, Q, FADs, \beta, \) and \( C \) are determined.

Step 2: The MP-SEDA-based method with tunable CF in Algorithm 1 is executed.

Step 3: After reaching the maximum iteration \( k_{max} \), the optimal design parameter \( X_i' \) is obtained from the elite matrix \( E \). Then, these optimal design parameters are applied to the FOPID controller as the solution for the AVR control system.

Remark 1. In this paper, it is highlighted that the tuning method of the FOPID controller is based on a data-driven or model-free approach. This method treats the AVR system as a ‘black box’ model. It means that all FOPID controller gains used in AVR control systems are tuned up solely based on the input and output information without knowing the plant model. This data-driven method improves the feasibility of the proposed controller and makes it possible to utilize it in a real AVR system. As for the computational execution of the proposed algorithm, the process of tuning can be expedited by the existence of supercomputers.

Figure 7 shows a detailed flow diagram of the FOPID controller tuning approach using the MP-SEDA-based method with tunable CF. There are two main blocks: the implementation of the FOPID controller for the AVR system block and the MP-SEDA-based method with a tunable CF block. In the first block, the aim is to obtain the minimal objective function \( J(\psi) \) using the given overshoot \( M_p \), steady-state error \( E_{ss} \), settling time \( T_{set} \), and rise time \( T_r \) values. By mapping \( f_i \) to \( J(\psi) \), Algorithm 1 can be executed in the second block to obtain the updated design variable of each prey \( X_i \), which is obtained from \( P_i \). This updated design variable is then applied to the first block by defining \( \psi_j = X_{i,j} (j = 1, 2, \ldots, d) \). This bidirectional flow between these two main blocks is repeated until reaching the maximum number of iterations to obtain the optimal design parameter \( X_i' \).
Figure 7. Block diagram of MP-SEDA-based method with tunable CF implementation for FOPID controller of the AVR system.

4. Results and Discussion

The performance investigation of the AVR system using a FOPID controller based on the MP-SEDA-based method with tunable CF is presented in this section. There are five case studies conducted to investigate the performance of the proposed method, i.e., (i) step response analysis, (ii) Bode plot analyses, (iii) trajectory tracking, (iv) disturbance rejection, and (v) parameter variations of the AVR system. Here, the effectiveness of the MP-SEDA-based method with tunable CF is compared with the existing MPA [39], as well as other metaheuristics-based methods, viz. SA-MRFO [21], GBO [38], ChBWO [37], JOA [36], C-YGSA [35], SSA [34], CS [33], SA [32], PSO [28,31], CAS [30], and NSGA II [29], in tuning the
FOPID controller for the AVR system. The following performance criteria are considered in this study:

(i) The performance comparison of the best objective function convergence curve (out of 25 trials) between the MP-SEDA-FOPID and the existing MPA-FOPID controller. Specifically, the ability of the algorithms to minimize the objective function is observed.

(ii) The statistical analysis of the objective function $J$ in Equation (6) from 25 independent trials based on the mean, best, worst, and standard deviation (std). Those statistical performances are compared between the MP-SEDA-FOPID and the existing MPA-FOPID controller.

(iii) The non-parametric statistical test uses Wilcoxon’s rank test to estimate the statistical difference between the MP-SEDA-FOPID and the existing MPA-FOPID controller with a significance level of 5%. The two different controllers must undergo the statistical test and the mean values are compared to find the $p$-value, which indicates the significance level. When $p$-value $< 0.05$, the experimental results and efficiency of both controllers are considered significantly different. On the other hand, when $p$-value $> 0.05$, the experimental results of both controllers are considered not significantly different, or in other words, the experimental results of the controllers are the same.

(iv) The performance comparison of step response analysis of the AVR system in terms of overshoot $M_p$, steady-state error $E_{ss}$, settling time $T_{set}$, and rise time $T_r$ between the MP-SEDA-FOPID controller and the aforementioned compared FOPID-based controllers.

(v) The stability analyses between the MP-SEDA-FOPID controller and the aforementioned compared FOPID-based controllers need to be performed by evaluating the Bode plot.

(vi) The robustness analysis is applied to all compared controllers by assessing the trajectory tracking (including measurement noise), disturbance rejection, time-invariant, and time-varying parameter uncertainties. At this point, the robustness analyses are evaluated in terms of the integral absolute error (IAE), integral square error (ISE), integral time absolute error (ITAE), and integral time square error (ITSE), which are mathematically formulated as

$$\text{ISE} = \int_0^{t_s} \left( V_{\text{ref}} - V_m \right)^2 dt,$$

$$\text{IAE} = \int_0^{t_s} \left| V_{\text{ref}} - V_m \right| dt,$$

$$\text{ITSE} = \int_0^{t_s} t \left( \left( V_{\text{ref}} - V_m \right)^2 \right) dt,$$

$$\text{ITAE} = \int_0^{t_s} t \left| V_{\text{ref}} - V_m \right| dt,$$

where $t_s$ is the final simulation time.

In this study, the simulation works were performed based on MATLAB/Simulink R2020a on a personal computer with the specifications of Microsoft Windows 10, 8 GB RAM, and Intel Core i7-6700 Processor (3.41 GHz). In finding the optimal values of the FOPID of the AVR system, the range of FOPID gains is set as $K_p = (0.1, 3)$, $K_i = (0.1, 1)$, $K_d = (0.1, 1.5)$, $\lambda = (0.5, 1.5)$, and $\mu = (0.5, 1.5)$ by referring to the study in [37]. Then, the maximum number of iterations and the number of agents are set as $k_{\text{max}} = 100$ and $n = 40$, respectively, which contribute to a total of 4000 function evaluations (NFEs), which is similar to the work in [23]. Next, the coefficients for the MP-SEDA-based method with tunable CF are set as $\beta = 1.89$ and a predetermined probability $C = 0.67$ after several initial investigations are performed. Finally, the default MPA-based method coefficients that are also used in the MP-SEDA-based method with tunable CF remain as $Q = 0.5$ and $FADs = 0.2$, which is similar to the original work in [39]. Next, the fractional order transfer functions are designed based on the 5th-order Oustaloup with a frequency range of $\omega \in \left[ 10^{-5}, 10^5 \right]$.
The obtained results are categorized into the following case studies.

4.1. Step Response Analysis

In this subsection, the first case study based on the applied unit step input for the FOPID controller of the AVR system is presented and discussed. Firstly, the effectiveness of the proposed MP-SEDA-FOPID controller is observed by comparing the best objective function convergence curve (out of 25 trials) with the existing MPA-FOPID controller. Furthermore, the statistical analysis of the objective function \( J \) from 25 independent trials is evaluated in terms of mean, best, worst, and standard deviation (std) to highlight the superiority of the improved version of MPA. Moreover, a non-parametric statistical test using Wilcoxon’s rank test is also conducted to estimate the statistical difference between the MP-SEDA-FOPID and the existing MPA-FOPID controller. Lastly, the step response analysis is evaluated in terms of time response specifications, such as overshoot \( M_p \), steady-state error \( E_{ss} \), settling time \( T_{set} \), and rise time \( T_r \), in comparison with other FOPID-based controllers.

Table 5 shows the statistical outcomes over 25 trials for the MP-SEDA-FOPID and the existing MPA-FOPID controller. As seen from Table 5, the proposed controller provides superior statistical results by producing the lowest values for mean, best, and worst, as highlighted in bold, compared to the existing MPA-FOPID controller. Meanwhile, the existing MPA-FOPID scores a slightly lower value for standard deviation (std), which indicates it has slightly better quality in producing a consistent result for each trial compared to the proposed MP-SEDA-FOPID controller. In the meantime, Figure 8 shows the best convergence curves of the objective function for MP-SEDA-FOPID and the existing MPA-FOPID controller out of 25 trials. As we can see, for MP-SEDA (blue line), the objective function \( J \) has successfully converged from \( J = 0.1019 \) and achieved the minimum \( J = 0.01107 \) during 97 iterations. Meanwhile, for MPA (red line), it takes the entire iteration for the objective function to converge from \( J = 0.0472 \) to the minimum \( J = 0.01108 \). If we investigate the MP-SEDA convergence curve further, we can see that for the first tier of the maximum iterations, the algorithm is in the exploration phase (Phase 1), where the convergence curve behavior is approximately similar to MPA. This is because there was no modification to the MPA algorithm during Phase 1. After implementing the MP-SEDA algorithm during Phase 2 \( \left( \frac{1}{3}k_{max} < k < \frac{2}{3}k_{max} \right) \), a steeper response of the convergence curve is produced compared to MPA. It justifies that the hybridization of MPA and SEDA can improve the exploration and exploitation phases of the algorithm, especially during Phase 2. Then, during Phase 3 \( k > \frac{2}{3}k_{max} \), it is clearly shown that the tunable CF in Equation (18) can further help the algorithm exploit the minimum objective function. The results prove that the proposed modification of MPA can reach a good solution with an excellent convergence rate and solution quality.

### Table 5. Statistical analysis over 25 trials for MP-SEDA and existing MPA.

<table>
<thead>
<tr>
<th>Objective Function</th>
<th>MP-SEDA</th>
<th>MPA</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J ) Mean</td>
<td>0.01269131</td>
<td>0.01402836</td>
</tr>
<tr>
<td>Best</td>
<td>0.01107177</td>
<td>0.01108495</td>
</tr>
<tr>
<td>Worst</td>
<td>0.01512334</td>
<td>0.01524987</td>
</tr>
<tr>
<td>Std</td>
<td>0.00156338</td>
<td>0.00129324</td>
</tr>
</tbody>
</table>

Meanwhile, a non-parametric statistical test using Wilcoxon’s rank test with a significance level of 5% is implemented to evaluate the statistical difference in the objective function between each method. The pairwise results of Wilcoxon’s rank test between the MP-SEDA-FOPID and the existing MPA-FOPID controller are shown in Table 6. In this table, \( S^+ \) is defined as the sum of ranks in which the proposed algorithm performed better than its competitor, and \( S^- \) is vice versa [23]. Based on the results, \( S^+ \) is dominated by the MP-SEDA-FOPID with a score of 179, as highlighted in bold, compared to the
MPA-FOPID with a score of 139. This indicates a big enough difference between them. Moreover, it clearly shows that the $p$-value between the MP-SEDA-FOPID and the existing MPA-FOPID controller is less than 0.05. This result shows that the superior performance of the MP-SEDA-FOPID compared to the existing MPA-FOPID is statistically significant.

Eventually, from the best convergence curve, the optimum FOPID gains obtained by the MP-SEDA-based method with tunable $CF$ at the end of the simulation, i.e., $K_p = 2.948671163710841$, $K_i = 0.453269823092194$, $K_d = 0.439096791042741$, $\lambda = 1.401573912220059$, and $\mu = 1.415394312394095$, are found. Meanwhile, for MPA, the optimum FOPID gains obtained from the best convergence curve, i.e., $K_p = 2.940878878596037$, $K_i = 0.451028297446648$, $K_d = 0.438562480134299$, $\lambda = 1.402697639316959$, and $\mu = 1.414674798488237$, are found. The FOPID gains for both algorithms are simplified to four decimal places, as shown in Table 7. Table 7 also presents the optimal FOPID gains obtained from other compared algorithms taken directly from their respective papers. Subsequently, the terminal voltage step responses attained by the proposed MP-SEDA-FOPID controller and other compared FOPID-based controllers are illustrated in Figure 9. Here, the simulation time is set as $t_s = 5$ s and the desired terminal voltage is set to 1.0 p.u. It is noted that all compared controllers are rerun by considering the experimental settings in this study, such as Oustaloup, settling time tolerance, and simulation time.
Table 7. Optimal setting of FOPID controller gains obtained from different algorithms.

<table>
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<td>$K_i$</td>
<td>$K_d$</td>
<td>$\lambda$</td>
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<td>0.6124</td>
<td>0.4932</td>
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<td>CS [33]</td>
<td>2.5150</td>
<td>0.1629</td>
<td>0.3888</td>
<td>0.9700</td>
</tr>
<tr>
<td>SA [32]</td>
<td>0.7837</td>
<td>0.5027</td>
<td>0.2307</td>
<td>1.0103</td>
</tr>
<tr>
<td>PSO1 [31]</td>
<td>1.2623</td>
<td>0.5331</td>
<td>0.2382</td>
<td>1.1827</td>
</tr>
<tr>
<td>PSO2 [28]</td>
<td>0.3265</td>
<td>0.2506</td>
<td>0.2050</td>
<td>0.9680</td>
</tr>
<tr>
<td>CAS [30]</td>
<td>1.0597</td>
<td>0.4418</td>
<td>0.2510</td>
<td>1.0624</td>
</tr>
<tr>
<td>NSGA II [29]</td>
<td>0.8399</td>
<td>1.3359</td>
<td>0.3512</td>
<td>0.9147</td>
</tr>
</tbody>
</table>

Figure 9. Step responses of the AVR system obtained from the FOPID controller by different algorithms.

Furthermore, the time response specifications and objective function $J$ for all optimization-based FOPID controllers are depicted in Table 8. Here, the rise time is defined as the time taken by the response to increase from 10% to 90% of its final value, while the settling time is defined as the time taken to achieve 5% of the final value of the response. The step response analysis starts with the comparison of $M_p$ where the proposed MP-SEDA produces a fourth smaller $M_p$, pioneered by CS, then followed by SA and MPA. Hence, it shows that the value of $M_p$ for MP-SEDA is slightly comparable with CS, where the difference is only 0.5%. Nevertheless, the 0.56% overshoot produced by MP-SEDA is still acceptable, and it may not affect the output performance of the AVR system. Interestingly, the proposed controller performed brilliantly for other time response specifications, where it produced the fastest $T_r$, the fastest $T_{set}$, and the smaller $E_{ss}$ compared to other algorithms, as highlighted in bold. Lastly, to evaluate the overall performance, the objective function $J$ in Equation (6) is obtained by using the $M_p$, $T_r$, $T_{set}$, and $E_{ss}$ produced by each algorithm. As expected, by winning three out of four time response specifications, the MP-SEDA has clearly generated an excellent result for $J$ with the lowest value of 0.01107177. This result shows that the MP-SEDA yields the best results by 0.12% in comparison to the existing MPA. Meanwhile, for the other controllers, MP-SEDA scores a lower objective function of 1.86 times in comparison to SA-MRFO, 16.53 times in comparison to GBO, 1.82 times
in comparison to ChBWO, 15.09 times in comparison to JOA, 2.07 times in comparison to C-YSGA, 15.34 times in comparison to SSA, 9.62 times in comparison to CS, 3.41 times in comparison to SA, 3.93 times in comparison to PSO [26], 81.6 times in comparison to PSO [23], 3.28 times in comparison to CAS, and 56.5 times in comparison to NSGA II. Overall, the MP-SEDA-based method with tunable $\text{CF}$ is the best tool for FOPID controller tuning in terms of time response specification and objective function.

Table 8. Time response specifications and objective function $J$ obtained by FOPID controller from different algorithms.

<table>
<thead>
<tr>
<th>Type of Algorithm</th>
<th>$M_p$ (%)</th>
<th>$T_r$ (s)</th>
<th>$T_{set}$ (s) (5%)</th>
<th>$E_{ss}$</th>
<th>$J$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP-SEDA</td>
<td>0.56</td>
<td>0.083</td>
<td>0.1103</td>
<td>$1.24 \times 10^{-7}$</td>
<td>0.01107177</td>
</tr>
<tr>
<td>MPA</td>
<td>0.55</td>
<td>0.0833</td>
<td>0.1106</td>
<td>$1.83 \times 10^{-6}$</td>
<td>0.01108495</td>
</tr>
<tr>
<td>SA-MRFO [21]</td>
<td>1.95</td>
<td>0.1311</td>
<td>0.1760</td>
<td>$6.6745 \times 10^{-4}$</td>
<td>0.02060125</td>
</tr>
<tr>
<td>GBO [38]</td>
<td>12.46</td>
<td>0.1081</td>
<td>0.5161</td>
<td>0.0148</td>
<td>0.18305555</td>
</tr>
<tr>
<td>ChBWO [37]</td>
<td>3.89</td>
<td>0.0956</td>
<td>0.1266</td>
<td>0.0022</td>
<td>0.02011246</td>
</tr>
<tr>
<td>JOA [36]</td>
<td>19.49</td>
<td>0.0966</td>
<td>0.4468</td>
<td>0.0020</td>
<td>0.16703187</td>
</tr>
<tr>
<td>C-YSGA [35]</td>
<td>1.93</td>
<td>0.1382</td>
<td>0.1858</td>
<td>0.0028</td>
<td>0.02294091</td>
</tr>
<tr>
<td>SSA [34]</td>
<td>13.91</td>
<td>0.1008</td>
<td>0.4858</td>
<td>0.0028</td>
<td>0.16979448</td>
</tr>
<tr>
<td>CS [33]</td>
<td>0.06</td>
<td>0.1039</td>
<td>0.3479</td>
<td>0.0262</td>
<td>0.10646490</td>
</tr>
<tr>
<td>SA [32]</td>
<td>0.50</td>
<td>0.2656</td>
<td>0.3656</td>
<td>$1.5454 \times 10^{-5}$</td>
<td>0.03776948</td>
</tr>
<tr>
<td>PSO1 [31]</td>
<td>1.37</td>
<td>0.2231</td>
<td>0.3227</td>
<td>0.0067</td>
<td>0.04346518</td>
</tr>
<tr>
<td>PSO2 [28]</td>
<td>8.38</td>
<td>1.6453</td>
<td>4.0385</td>
<td>0.0112</td>
<td>0.90340689</td>
</tr>
<tr>
<td>CAS [30]</td>
<td>3.63</td>
<td>0.2205</td>
<td>0.2989</td>
<td>$8.4339 \times 10^{-4}$</td>
<td>0.03627426</td>
</tr>
<tr>
<td>NSGA II [29]</td>
<td>42.69</td>
<td>0.2025</td>
<td>1.6800</td>
<td>0.0016</td>
<td>0.62550586</td>
</tr>
</tbody>
</table>

4.2. Bode Plot Analyses

In this subsection, the second case study, which is the frequency response analysis of the MP-SEDA-FOPID controller and other FOPID-based controllers, is conducted in terms of a Bode plot. The stability characteristic is analyzed from the Bode plot by analyzing the phase margin $P_m$, delay margin $D_m$, peak gain, and bandwidth frequency. Specifically, the $P_m$ is a characteristic that indicates the amount of phase shift that can be handled by a system without causing any unstable conditions. Meanwhile, the $D_m$ is the maximum amount of time delay that can be tolerated by the system to retain its stability. Peak gain indicates the overshoot performance of a system, while the bandwidth frequency is defined as the frequency at which the closed-loop magnitude drops 3 dB below its magnitude at DC (magnitude as the frequency approaches zero). Generally, the system is said to be stable if the $P_m$ is a positive value, while a system with a larger $P_m$, longer $D_m$, smaller peak gain, and higher bandwidth frequency is considered more stable. In particular, the stability performance of the closed-loop system response using the MP-SEDA-FOPID controller is observed in comparison with other FOPID-based controllers.

Figure 10 shows the Bode plots of magnitude and phase for the proposed MP-SEDA-FOPID controller and other compared FOPID-based controllers, while Table 9 represents the comparative stability characteristic results obtained by all controllers. According to the results, a good frequency response is obtained by CAS with the highest $P_m$ (i.e., 178.4217 degrees) and longer $D_m$ (i.e., 21.6891 s), as highlighted in bold. Meanwhile, a good frequency response has also been obtained by SA, with the second highest $P_m$ (i.e., 178.3847 degrees), second longest $D_m$ (i.e., 20.4419 s), and the smallest peak gain of 0.000109 dB. However, both CAS and SA algorithms produce low bandwidth values, which only score 9.9114 Hz and 8.3791 Hz, respectively. Theoretically, the system’s output response will face a higher distortion when the bandwidth value is small. This situation may lead to the performance degradation of the AVR system. In contrast, the proposed MP-SEDA scores the highest bandwidth value of 26.9955 Hz, 2.72 and 3.22 times better than the CAS and SA bandwidth values, respectively. Overall, we can say that the MP-SEDA-FOPID controller has produced a good frequency response by considering the highest
bandwidth value and other tolerable phase margins, delay margins, and peak gain values (i.e., 168.8267 degrees, 0.9986 s, and 0.0645 dB, respectively).

Figure 10. Bode plots for AVR systems obtained by FOPID controller from different algorithms.

Table 9. Stability characteristic results obtained by FOPID controller from different algorithms.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$P_m$ (deg)</th>
<th>$D_m$ (s)</th>
<th>Peak Gain (dB)</th>
<th>Bandwidth (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP-SEDA</td>
<td>168.8267</td>
<td>0.9986</td>
<td>0.0645</td>
<td>26.9595</td>
</tr>
<tr>
<td>MPA</td>
<td>168.8416</td>
<td>1.0008</td>
<td>0.0651</td>
<td>26.92</td>
</tr>
<tr>
<td>SA-MRFO [21]</td>
<td>161.7465</td>
<td>0.8503</td>
<td>0.157</td>
<td>17.3936</td>
</tr>
<tr>
<td>GBO [38]</td>
<td>88.4794</td>
<td>0.0957</td>
<td>1.2</td>
<td>20.0106</td>
</tr>
<tr>
<td>ChBWO [37]</td>
<td>163.6947</td>
<td>0.7181</td>
<td>0.194</td>
<td>23.4647</td>
</tr>
<tr>
<td>JOA [36]</td>
<td>81.4664</td>
<td>0.0804</td>
<td>1.88</td>
<td>21.3722</td>
</tr>
<tr>
<td>C-YSGA [35]</td>
<td>163.0984</td>
<td>0.9687</td>
<td>0.118</td>
<td>16.5389</td>
</tr>
<tr>
<td>SSA [34]</td>
<td>88.2128</td>
<td>0.0895</td>
<td>1.32</td>
<td>21.2305</td>
</tr>
<tr>
<td>CS [33]</td>
<td>166.7140</td>
<td>0.9244</td>
<td>0.0476</td>
<td>22.5192</td>
</tr>
<tr>
<td>SA [32]</td>
<td>178.3847</td>
<td>20.4419</td>
<td>0.00109</td>
<td>8.3791</td>
</tr>
<tr>
<td>PSO1 [31]</td>
<td>175.3853</td>
<td>5.6949</td>
<td>0.0606</td>
<td>10.0939</td>
</tr>
<tr>
<td>PSO2 [28]</td>
<td>162.4166</td>
<td>4.2886</td>
<td>0.00515</td>
<td>1.1579</td>
</tr>
<tr>
<td>NSGA II [29]</td>
<td>40.7036</td>
<td>0.0857</td>
<td>5.38</td>
<td>9.3804</td>
</tr>
</tbody>
</table>

4.3. Trajectory Tracking Analysis

In this subsection, the third case study, the trajectory tracking analysis, is considered to examine the precision of the proposed MP-SEDA-FOPID controller in tracking the desired trajectory of the terminal voltage response. The trajectory tracking employed in this study differs from the previous studies conducted in [32], which utilized different step changes. In contrast, in this study, we focused on tracking the combination of sinusoidal and trapezoidal reference signals. As demonstrated, this reference input signal is formed in series between the sinusoidal and ramp signals with multiple set points. This new trajectory tracking intends to escalate the challenges of controlling the AVR system and evaluate the efficiency of the proposed MP-SEDA-FOPID controller. Since there are various set points and slopes, this trajectory tracking is practically exciting to implement in the AVR system. Moreover, the FOPID gains in Table 7 are maintained in analyzing the trajectory tracking assessment. The performance evaluation of each controller in comparison is evaluated in
terms of the performance indices stated in Equations (20)–(23). In this trajectory tracking analysis, the simulation time is set to $t_s = 15$ s.

The trajectory tracking responses by all controllers are displayed in Figure 11, while their magnified versions for different time intervals are shown in Figure 12a–f. The figures clearly show that the proposed MP-SEDA-FOPID controller provides an excellent trajectory tracking response compared to other algorithms. Also, it is noticeable that the MP-SEDA response successfully follows the reference input with high efficiency. Specifically, the MP-SEDA produces less overshoot and faster settling time at different set points, such as during time intervals $t = 4–5$ s, $t = 6–7$ s, $t = 9–10$ s, and $t = 11–15$ s in comparison with other controllers, especially the NSGA II [29] which produces 10 to 30 times higher overshoot and 1 to 7 times slower settling time than MP-SEDA. Meanwhile, we can say that the PSO [28] produces the worst trajectory tracking response, which is unable to follow the reference input accurately. Hence, it can be concluded that the MP-SEDA-FOPID is a better controller compared to others by producing a remarkable trajectory tracking capability, as shown by the voltage response profile that is very close to the reference input signal.

![Figure 11. Trajectory tracking responses obtained by FOPID controller from different algorithms.](image)

Merely observing the trajectory tracking response is insufficient to distinguish which controller is best for tracking the desired input. Thus, the numerical evaluation in terms of ISE, IAE, ITSE, and ITAE performance indices for trajectory tracking responses for all FOPID-based controllers was conducted, and the results are tabulated in Table 10. The best performance among the controllers in following the reference signal can be identified by observing the smaller values of performance indices. Table 10 illustrates that the proposed MP-SEDA produces the smallest values of ISE and ITSE compared to other algorithms, as highlighted in bold. Meanwhile, the JOA has scored the best values for IAE and ITAE. Thus, a detailed comparison between the MP-SEDA and the JOA is made to indicate which algorithm is best for trajectory tracking. Firstly, for the IAE performance index, that of the MP-SEDA is 1.014 times greater than that of JOA, ranking it third compared to other algorithms. Secondly, for the ISE performance index, that of the JOA is 1.059 times greater than that of MP-SEDA and is ranked fourth. Thirdly, for the ITAE performance index, that of the JOA is 1.115 times greater than that of MP-SEDA, placing it at no. 4. If we total up how much better the two algorithms were than the others, it is evident that the MP-SEDA produces a better result, with a score of 2.052 times better, compared to the JOA, with a score of 2.174 times better. Then, if we compare both algorithms in terms of rank, the total rank produced by MP-SEDA is considerably better, with a total rank of 8, compared to JOA, with a total rank of 10. Therefore, these numerical results of performance indices indicate that the proposed MP-SEDA-based
method possessed superior control efficacy in following the trajectory tracking compared to other algorithms. Besides that, the proposed MP-SEDA-FOPID controller has won all performance indices compared to the existing MPA-FOPID. It is justified that the MPA modification is worth implementing for tuning the FOPID controller of the AVR system.

Figure 12. Magnified view for trajectory tracking responses with different time intervals (a) for $t = 0–2$ s, (b) for $t = 2–4$ s, (c) for $t = 4–6$ s, (d) for $t = 6–8$ s, (e) for $t = 8–10$ s, and (f) for $t = 10–12$ s.

Next, the measurement noise $e(t)$ is added to the AVR system, as shown in Figure 13, in order to further evaluate the efficiency of each algorithm in the trajectory tracking analysis. In this study, $e(t)$ is a white noise with a noise power of 0.001. Meanwhile, the trajectory tracking response with the existence of measurement noise obtained by the MP-SEDA-FOPID controller is shown in Figure 14. From the response, it is evident that the proposed MP-SEDA response is able to follow the reference signal satisfactorily, especially
during the sinusoidal input. While there is a slight ripple during the trapezoid input, it follows the reference without failure. In the meantime, Table 11 shows the numerical results of all performance indices obtained by all the controllers. The table shows that the proposed MP-SEDA-FOPID controller has produced the lowest values for all performance indices compared to other controllers, as highlighted in bold. Specifically, the MP-SEDA has produced a value of IAE that is 0.002 times better than the second place, which is the existing MPA. Compared to the last place (PSO [28]), the IAE value of MP-SEDA is 4.874 times better. In the meantime, it is obvious that the MP-SEDA and the existing MPA are the only algorithms producing an ISE reading of less than 0.24. Furthermore, the ITAE and ITSE values obtained through the MP-SEDA are approximately 1.001 to 14.34 times better than those produced by other FOPID-based controllers. In essence, although the AVR system is subjected to measurement noise during the trajectory tracking simulation, the proposed MP-SEDA-FOPID controller is still able to cater to it and successfully proves that this method is the most efficient and robust.

Table 10. Trajectory tracking performances comparison of different FOPID-based controllers.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>IAE</th>
<th>ISE</th>
<th>ITAE</th>
<th>ITSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP-SEDA</td>
<td>0.8433</td>
<td>0.0973</td>
<td>3.643</td>
<td>0.2791</td>
</tr>
<tr>
<td>MPA</td>
<td>0.8451</td>
<td>0.0977</td>
<td>3.651</td>
<td>0.2803</td>
</tr>
<tr>
<td>SA-MRFO [21]</td>
<td>1.16</td>
<td>0.2027</td>
<td>4.756</td>
<td>0.5945</td>
</tr>
<tr>
<td>GBO [36]</td>
<td>1.042</td>
<td>0.1648</td>
<td>4.254</td>
<td>0.4729</td>
</tr>
<tr>
<td>ChBWO [37]</td>
<td>0.8423</td>
<td>0.1013</td>
<td>3.535</td>
<td>0.2902</td>
</tr>
<tr>
<td>JOA [36]</td>
<td>0.8314</td>
<td>0.1031</td>
<td>3.511</td>
<td>0.3112</td>
</tr>
<tr>
<td>C-YSGA [35]</td>
<td>1.268</td>
<td>0.2447</td>
<td>5.24</td>
<td>0.739</td>
</tr>
<tr>
<td>SSA [34]</td>
<td>1.063</td>
<td>0.1684</td>
<td>4.513</td>
<td>0.537</td>
</tr>
<tr>
<td>CS [33]</td>
<td>0.8991</td>
<td>0.1131</td>
<td>3.725</td>
<td>0.3136</td>
</tr>
<tr>
<td>SA [32]</td>
<td>2.277</td>
<td>0.7945</td>
<td>9.484</td>
<td>2.485</td>
</tr>
<tr>
<td>PSO1 [31]</td>
<td>1.765</td>
<td>0.4688</td>
<td>7.388</td>
<td>1.438</td>
</tr>
<tr>
<td>PSO2 [28]</td>
<td>6.146</td>
<td>3.683</td>
<td>35.74</td>
<td>17.85</td>
</tr>
<tr>
<td>CAS [30]</td>
<td>1.859</td>
<td>0.5041</td>
<td>7.932</td>
<td>1.53</td>
</tr>
<tr>
<td>NSGA II [29]</td>
<td>1.823</td>
<td>0.4715</td>
<td>7.649</td>
<td>1.385</td>
</tr>
</tbody>
</table>

Figure 13. AVR control system with the presence of measurement noise.

4.4. Disturbance Rejection Analysis

In this subsection, the fourth case study, which is the disturbance rejection analysis, is discussed. Notably, the capability of the system to withstand unpredicted disturbances is a significant property that has to be considered by the designer when developing a robust system. Consequently, the performances of the proposed MP-SEDA-FOPID and all compared FOPID-based controllers were evaluated in the presence of disturbances, as shown by the equivalent block diagram in Figure 15. In this analysis, a similar trajectory tracking input to that in the previous section was employed with additional disturbance signals $D(t)$, which were modeled as follows:
Figure 14. Trajectory tracking response for MP-SEDA-FOPID controller with the presence of measurement noise.

Table 11. Trajectory tracking performance comparison of different FOPID-based controllers with the presence of measurement noise.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>IAE</th>
<th>ISE</th>
<th>ITAE</th>
<th>ITSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP-SEDA</td>
<td>1.42</td>
<td>0.2372</td>
<td>9.194</td>
<td>1.344</td>
</tr>
<tr>
<td>MPA</td>
<td>1.422</td>
<td>0.2379</td>
<td>9.209</td>
<td>1.347</td>
</tr>
<tr>
<td>SA-MRFO [21]</td>
<td>1.822</td>
<td>0.3778</td>
<td>11.3</td>
<td>1.91</td>
</tr>
<tr>
<td>GBO [38]</td>
<td>1.792</td>
<td>0.3675</td>
<td>11.57</td>
<td>2.005</td>
</tr>
<tr>
<td>ChBWO [37]</td>
<td>1.497</td>
<td>0.2596</td>
<td>9.866</td>
<td>1.495</td>
</tr>
<tr>
<td>JOA [36]</td>
<td>1.638</td>
<td>0.3115</td>
<td>11.23</td>
<td>1.899</td>
</tr>
<tr>
<td>C-YSGA [35]</td>
<td>1.919</td>
<td>0.4227</td>
<td>11.73</td>
<td>2.073</td>
</tr>
<tr>
<td>SSA [34]</td>
<td>1.776</td>
<td>0.3675</td>
<td>11.59</td>
<td>2.049</td>
</tr>
<tr>
<td>CS [33]</td>
<td>1.52</td>
<td>0.2654</td>
<td>9.824</td>
<td>1.469</td>
</tr>
<tr>
<td>SA [32]</td>
<td>2.797</td>
<td>0.9932</td>
<td>15.11</td>
<td>3.9</td>
</tr>
<tr>
<td>PSO1 [31]</td>
<td>2.327</td>
<td>0.6492</td>
<td>13.18</td>
<td>2.761</td>
</tr>
<tr>
<td>PSO2 [28]</td>
<td>6.294</td>
<td>3.874</td>
<td>37.17</td>
<td>19.28</td>
</tr>
<tr>
<td>CAS [30]</td>
<td>2.401</td>
<td>0.7009</td>
<td>13.65</td>
<td>2.964</td>
</tr>
<tr>
<td>NSGA II [29]</td>
<td>2.564</td>
<td>0.7654</td>
<td>14.41</td>
<td>3.353</td>
</tr>
</tbody>
</table>

Note that the disturbance expression $D(t)$ in Equation (24) imitates the unexpected load changes with a persistently high or low voltage that happen in real power system operation due to the loss of important sources of reactive power support or loss of important transmission capability [63]. In contrast to existing studies reported in [23] that implemented two disturbance perturbations with a duration of 0.5 s and an amplitude of 20% of the set point, this study implemented four disturbance perturbations with a duration of 0.1 s and an amplitude of 50% of the set point. The reason for introducing these...
new disturbance signals is to further challenge the controller in recovering the reference signal. The same FOPID values listed in Table 7 are then used in this simulation test.

Figure 15. AVR block diagram with additional disturbance.

Figure 16 presents the system responses with disturbances obtained by the proposed MP-SEDA-FOPID and other compared FOPID-based controllers. Also, Figure 17a–d show the magnified version of Figure 16 to assist the reader in classifying the best response. It is noted that during the first disturbance, perturbations occur. As shown in Figure 17a, the MP-SEDA has the superior capability to reject the disturbance that occurs since it can regain the reference signal faster than other algorithms. At the same time, the responses obtained via SA-MRFO, GBO, ChBWO, JOA, C-YSGA, SSA, CS, SA, and CAS reveal fewer oscillations after the first disturbance perturbation. However, their responses fail to follow the desired response since their trajectories have diverged from the reference signal. The same pattern can be seen during the second, third, and fourth disturbance perturbations, as shown in Figure 17b, Figure 17c, and Figure 17d, respectively, where the MP-SEDA has shown the best performance by requiring less time to recover the reference signal after the disturbances.

Figure 16. System responses with disturbances.

Additionally, Table 12 tabulates the performance indices (i.e., IAE, ISE, ITAE, and ITSE) obtained using the proposed MP-SEDA-FOPID and other FOPID-based controllers. The table evidently shows that the proposed method obtains the lowest values for IAE, ISE, and ITAE, as highlighted in bold. Meanwhile, for ITSE, the MP-SEDA-FOPID produces the second lowest value, with only a 0.0033 difference from the existing MPA-FOPID. These results undoubtedly support the outcomes of system responses with disturbance, as shown.
Overall, these simulation results prove that the proposed MP-SEDA-FOPID controller is efficient in tracking the desired signal and provides better robustness compared to other FOPID-based controllers, even with the existence of disturbance perturbations.

![Image of Figure 17](image)

**Figure 17.** Magnified view of system responses with disturbances (a) for first disturbance perturbation, (b) for second disturbance perturbation, (c) for third disturbance perturbation, and (d) for fourth disturbance perturbation.

**Table 12.** Disturbance rejection comparison of different FOPID-based controllers.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>IAE</th>
<th>ISE</th>
<th>ITAE</th>
<th>ITSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP-SEDA</td>
<td>1.0340</td>
<td>0.1687</td>
<td>4.6910</td>
<td>0.7071</td>
</tr>
<tr>
<td>MPA</td>
<td>1.0360</td>
<td>0.1692</td>
<td>4.7010</td>
<td>0.7038</td>
</tr>
<tr>
<td>SA-MRFO [21]</td>
<td>1.3400</td>
<td>0.2877</td>
<td>5.7530</td>
<td>1.1130</td>
</tr>
<tr>
<td>GBO [38]</td>
<td>1.2940</td>
<td>0.2682</td>
<td>5.6680</td>
<td>1.0940</td>
</tr>
<tr>
<td>ChBWO [37]</td>
<td>1.0500</td>
<td>0.1827</td>
<td>4.6920</td>
<td>0.7711</td>
</tr>
<tr>
<td>JOA [36]</td>
<td>1.1140</td>
<td>0.2134</td>
<td>5.1230</td>
<td>0.9609</td>
</tr>
<tr>
<td>C-YSGA [35]</td>
<td>1.4340</td>
<td>0.3295</td>
<td>6.1470</td>
<td>1.2630</td>
</tr>
<tr>
<td>SSA [34]</td>
<td>1.3110</td>
<td>0.2703</td>
<td>5.8630</td>
<td>1.1480</td>
</tr>
<tr>
<td>CS [33]</td>
<td>1.1020</td>
<td>0.1903</td>
<td>4.8410</td>
<td>0.7719</td>
</tr>
<tr>
<td>SA [32]</td>
<td>2.3140</td>
<td>0.8626</td>
<td>9.5320</td>
<td>2.9890</td>
</tr>
<tr>
<td>PSO1 [31]</td>
<td>1.8630</td>
<td>0.5430</td>
<td>7.8160</td>
<td>1.9350</td>
</tr>
<tr>
<td>PSO2 [28]</td>
<td>6.1580</td>
<td>3.7080</td>
<td>36.3400</td>
<td>18.5300</td>
</tr>
<tr>
<td>CAS [30]</td>
<td>1.9600</td>
<td>0.5864</td>
<td>8.3540</td>
<td>2.0710</td>
</tr>
<tr>
<td>NSGA II [29]</td>
<td>1.9420</td>
<td>0.6128</td>
<td>8.6400</td>
<td>2.2120</td>
</tr>
</tbody>
</table>
4.5. Parameter Variation Analysis

The final case study, which is the parameter uncertainties analysis, is discussed within this subsection to assess the performance of the MP-SEDA-FOPID controller in dealing with the different conditions of the system’s parameters. In particular, two case studies are conducted: time-invariant and time-varying parameter uncertainties. For the prior case study, the parameters of the sensor \((T_S)\), generator \((T_G)\), exciter \((T_E)\), and amplifier \((T_A)\) were initially varied from −50% to +50% in an increment of 25%. Meanwhile, in the case of the latter, the time-varying uncertainties of the generator gain \(K_G\) were considered. Note that the time-varying uncertainty in the generator gain mimics the actual behavior of the armature winding, which was subjected to a varying magnetic flux that contributed to surface eddy current losses [1]. Thus, the new generator gain can be formulated by

\[
\tilde{K}_G(t) = K_G + \Delta_G(t),
\]

where \(\Delta_G(t)\) is the time-varying uncertainty in the generator that randomly changes throughout the time in the range of \([-0.3, 0.3]\).

The terminal voltage step responses of the AVR system produced by the MP-SEDA-FOPID controller under the time-invariant parameter uncertainties are displayed in Figure 18a–d. Meanwhile, the numerical results of the obtained time response specification are presented in Table 13. It can be observed that the all-time response specifications \((M_p, T_r, \text{ and } T_{set})\) differ in minimal ranges compared to the nominal value (see Table 8). Besides that, Table 14 shows the range of deviation (ROD) and the maximum deviation (MD) in times for all variations of the components in terms of \(M_p, T_r, \text{ and } T_{set}\). For instance, if we analyze the variation of component \(T_A\), the ROD of \(M_p\) is between +50% rate of change, which is the maximum value (4.03%), and −50% rate of change, which is the minimum value (0.75%) (refer to Table 13). Meanwhile, for the MD of \(M_p\), the calculation is performed by dividing the maximum value of \(M_p\) (4.03%) with the nominal value in Table 8 (0.56%). These calculations of ROD and MD are also applied to other time response specifications \((T_r \text{ and } T_{set})\). Overall, it clearly shows that the proposed MP-SEDA-FOPID controller has produced tolerable MD and ROD values for all time response specifications \((M_p, T_r, \text{ and } T_{set})\) within all variations of the AVR components.

**Table 13.** Time invariant parameter uncertainties analysis of the MP-SEDA-FOPID controller.

<table>
<thead>
<tr>
<th>Component of Parameters</th>
<th>Rate of Change (%)</th>
<th>(M_p) (%)</th>
<th>(T_r) (s)</th>
<th>(T_{set}) (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_A)</td>
<td>+50</td>
<td>4.0300</td>
<td>0.1085</td>
<td>0.1432</td>
</tr>
<tr>
<td></td>
<td>+25</td>
<td>2.3500</td>
<td>0.0979</td>
<td>0.1299</td>
</tr>
<tr>
<td></td>
<td>−25</td>
<td>0.7600</td>
<td>0.0737</td>
<td>0.3511</td>
</tr>
<tr>
<td></td>
<td>−50</td>
<td>0.7500</td>
<td>0.0596</td>
<td>0.3845</td>
</tr>
<tr>
<td>(T_E)</td>
<td>+50</td>
<td>3.2000</td>
<td>0.1335</td>
<td>0.1906</td>
</tr>
<tr>
<td></td>
<td>+25</td>
<td>1.8200</td>
<td>0.1090</td>
<td>0.1500</td>
</tr>
<tr>
<td></td>
<td>−25</td>
<td>4.4700</td>
<td>0.0653</td>
<td>0.3529</td>
</tr>
<tr>
<td></td>
<td>−50</td>
<td>11.1500</td>
<td>0.0442</td>
<td>0.2647</td>
</tr>
<tr>
<td>(T_G)</td>
<td>+50</td>
<td>2.7100</td>
<td>0.1479</td>
<td>0.3549</td>
</tr>
<tr>
<td></td>
<td>+25</td>
<td>1.6500</td>
<td>0.1135</td>
<td>0.3524</td>
</tr>
<tr>
<td></td>
<td>−25</td>
<td>6.4300</td>
<td>0.0637</td>
<td>0.3258</td>
</tr>
<tr>
<td></td>
<td>−50</td>
<td>16.1500</td>
<td>0.0423</td>
<td>0.2405</td>
</tr>
<tr>
<td>(T_S)</td>
<td>+50</td>
<td>4.3600</td>
<td>0.0796</td>
<td>0.3532</td>
</tr>
<tr>
<td></td>
<td>+25</td>
<td>2.3900</td>
<td>0.0828</td>
<td>0.3506</td>
</tr>
<tr>
<td></td>
<td>−25</td>
<td>0.7500</td>
<td>0.0908</td>
<td>0.3480</td>
</tr>
<tr>
<td></td>
<td>−50</td>
<td>0.7500</td>
<td>0.0961</td>
<td>0.3480</td>
</tr>
</tbody>
</table>
Figure 18. Terminal voltage responses for (a) $T_A$, (b) $T_E$, (c) $T_G$, and (d) $T_S$ varying from $+50\%$ to $-50\%$.

Table 14. The range of deviation (ROD) and maximum deviation (MD) in times for the MP-SEDA-FOPID controller.

<table>
<thead>
<tr>
<th>Component of Parameters</th>
<th>Time Response Specifications</th>
<th>ROD</th>
<th>MD (Times)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_A$</td>
<td>$M_p$</td>
<td>3.28</td>
<td>7.20</td>
</tr>
<tr>
<td></td>
<td>$T_r$</td>
<td>0.0489</td>
<td>1.31</td>
</tr>
<tr>
<td></td>
<td>$T_{set}$</td>
<td>0.2546</td>
<td>3.49</td>
</tr>
<tr>
<td>$T_E$</td>
<td>$M_p$</td>
<td>9.33</td>
<td>19.91</td>
</tr>
<tr>
<td></td>
<td>$T_r$</td>
<td>0.0893</td>
<td>1.61</td>
</tr>
<tr>
<td></td>
<td>$T_{set}$</td>
<td>0.2029</td>
<td>3.20</td>
</tr>
<tr>
<td>$T_G$</td>
<td>$M_p$</td>
<td>14.5</td>
<td>28.84</td>
</tr>
<tr>
<td></td>
<td>$T_r$</td>
<td>0.1056</td>
<td>1.78</td>
</tr>
<tr>
<td></td>
<td>$T_{set}$</td>
<td>0.1144</td>
<td>3.22</td>
</tr>
<tr>
<td>$T_S$</td>
<td>$M_p$</td>
<td>3.61</td>
<td>7.79</td>
</tr>
<tr>
<td></td>
<td>$T_r$</td>
<td>0.0165</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>$T_{set}$</td>
<td>0.0052</td>
<td>3.20</td>
</tr>
</tbody>
</table>

Meanwhile, to assess the effectiveness of the MP-SEDA-FOPID controller for time-varying parameter uncertainties, the same trajectory tracking input is used. Then, the numerical results in terms of IAE, ISE, ITAE, and ITSE performances are again recorded and shown in Table 15. Furthermore, the percentages of deviation between the nominal values of IAE, ISE, ITAE, and ITSE (see Table 10) and their corresponding values for time-varying cases are also presented in Table 15. As seen from the results, the percentage of deviation for all performance indicators is less than 4%, which indicates the dominance of the proposed MP-SEDA-FOPID controller in handling the time-varying parameter uncertainty in the generator. Overall, for this parameter variation analysis, those results confirmed the robustness of the proposed MP-SEDA-FOPID controller with the capability
to maintain the desired terminal voltage response despite the parameter uncertainties in the AVR system.

Table 15. Time-varying parameter uncertainties analysis of the MP-SEDA-FOPID controller.

<table>
<thead>
<tr>
<th>Component of Parameter</th>
<th>Performance Indicator</th>
<th>IAE</th>
<th>ISE</th>
<th>ITAE</th>
<th>ITSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{K}_C(t)$</td>
<td></td>
<td>0.8291</td>
<td>0.0943</td>
<td>3.619</td>
<td>0.2751</td>
</tr>
<tr>
<td>Percentage of deviation (%)</td>
<td></td>
<td>1.68</td>
<td>3.08</td>
<td>0.66</td>
<td>1.43</td>
</tr>
</tbody>
</table>

5. Conclusions

In this paper, a new optimization method for tuning the FOPID controller of the AVR system based on the MP-SEDA-based method with tunable $CF$ is presented. The simulation results revealed that the MP-SEDA-FOPID controller outperformed other FOPID-based controllers in terms of the convergence curve of the objective function, the statistical analysis of the objective function, Wilcoxon’s rank test, the step response analysis, stability analyses, and robustness analyses where the AVR system was subjected to noise, disturbance, and parameter uncertainties. Overall, the proposed method has good potential to be employed in several control applications because of its feasibility and applicability. Nevertheless, for the time being, the proposed method is only applicable to optimization problems with a single fitness function. Thus, for multiple fitness functions with conflicting objectives, this method may provide an unsatisfactory solution. Furthermore, the proposed MP-SEDA will introduce more predetermined coefficients that require preliminary investigation works. Therefore, for future work, consideration of the multi-objective MP-SEDA-based method with tunable $CF$ can be developed in order to enhance the performance accuracy of the system. Furthermore, the MP-SEDA-based method with tunable $CF$ can be further applied to other non-linear PID controllers to solve real application problems such as multi-input–multi-output (MIMO) gantry crane control systems, electric vehicles with induction motor drive, and MIMO twin-rotor system maneuvering control.

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