Article

Remaining Useful Life Prediction of a Planetary Gearbox Based on Meta Representation Learning and Adaptive Fractional Generalized Pareto Motion

Hongqing Zheng 1*, Wujin Deng 2, Wanjing Song 1*, Wei Cheng 1, Piercarlo Cattani 3 and Francesco Villecco 4

1 School of Electronic and Electrical Engineering, Minnan University of Science and Technology, Quanzhou 362700, China; zhqmnust@126.com (H.Z.); ck2004_02@163.com (W.C.)
2 School of Electronic & Electrical Engineering, Shanghai University of Engineering Science, Shanghai 201620, China; dwj0588@126.com
3 Department of Computer, Control and Management Engineering, University of Rome La Sapienza, Via Ariosto 25, 00185 Roma, Italy; cattani.1642554@studenti.uniroma1.it
4 Department of Industrial Engineering, University of Salerno, Via Giovanni Paolo II 132, 84084 Fisciano, Italy; fvillecco@unisa.it
* Correspondence: swqls@126.com

Abstract: The remaining useful life (RUL) prediction of wind turbine planetary gearboxes is crucial for the reliable operation of new energy power systems. However, the interpretability of the current RUL prediction models is not satisfactory. To this end, a multi-stage RUL prediction model is proposed in this work, with an interpretable metric-based feature selection algorithm. In the proposed model, the advantages of neural networks and long-range-dependent stochastic processes are combined. In the offline training stage, a general representation of the degradation trend is learned with the meta-long short-term memory neural network (meta-LSTM) model. The inevitable measurement error in the sensor reading is modelled by white Gaussian noise. During the online RUL prediction stage, fractional generalized Pareto motion (fGPm) with an adaptive diffusion is employed to model the stochasticity of the planetary gearbox degradation. In the case study, real planetary gearbox degradation data are used for the model validation.

Keywords: adaptive fractional generalized Pareto motion; meta representation learning; planetary gearbox remaining useful life prediction; measurement error; metric-based feature selection algorithm

1. Introduction

As a type of green energy, wind power is playing an increasingly important role in the power grid. The failure of wind turbines dramatically decreases the power output of the new energy power system (see Figure 1) [1]. The typical structure of a wind turbine is plotted in Figure 2 [2], in which the planetary gearbox is one of the most vulnerable components [3]. As pointed out in [4], the failure of planetary gearboxes causes the longest downtime of wind power.

A schematic diagram of a planetary gearbox can be seen in Figure 3 [5]. In the planetary gearbox, there are multiple planet gears simultaneously meshing with the sun and ring gears. The common cause for a planetary gearbox failure is a gear tooth fatigue crack caused by a cyclic load on the gear tooth [6]. The remaining useful life (RUL) prediction of the planetary gearbox can reduce an unnecessary economic loss by replacing the faulty components in advance [7].

Vibration-based RUL prediction methods are the most reliable and suitable approaches for rotary machinery [8]. Currently, deep neural networks and stochastic processes have been proposed to model degradation patterns based on vibration signals. Vibration analysis is based on the assumption that if failure or damage happens, the vibration response will be altered.
Previous literatures assume that a degradation trend can be described analytically, which is not realistic. Deep learning is a common tool for representation learning, which utilizes deep neural networks for feature extraction [9,10]. A deep-learning-based RUL prediction model has been proposed for planetary gearboxes [11]. In [12], a deep brief network and the particle filter are combined for the RUL prediction of a planetary gearbox. However, the RUL prediction models based on deep learning lack an uncertainty representation [13]. To this end, the long short-term memory neural network (LSTM) and Brownian motion are combined to propose a non-linear RUL prediction model (BM-LSTM) [14]. The degradation characteristics for the same kind of equipment are not the same due to tiny differences in the construction procedures. Considering the unit-to-unit variability and measurement error, [15] proposed a Brownian-motion-based RUL prediction model.

![Figure 1. Burnout accident of a wind turbine.](image1)

![Figure 2. Typical structure of a wind turbine.](image2)

Brownian motion is Markovian, whereas the actual planetary gearbox degradation data often present long-range dependence [16]. The temporal correlation for the long-range-dependent time series is strong, which is beneficial for RUL prediction [17]. Fractional Brownian motion (FBM) has been proposed for RUL prediction due to its non-Markovian characteristics [18]. The FBM model requires a Gaussian assumption for the vibration response, which is not always valid. In this paper, we propose the fractional generalized Pareto motion (fGPm) to model the degradation uncertainty with long-range dependence, which follows a generalized double Pareto assumption [19].

As the environmental noise changes, the variational speed of the degradation process also changes, which is neglected by previous literatures [20]. Thus, the diffusion coefficient should be adaptively updated [21]. In this work, the diffusion coefficient of the degradation model is adaptively updated with the evolution of random walk.

The multiple simultaneous vibration sources render the features in the frequency domain and time-frequency domain unsuitable to be a health indicator (HI) [22]. Therefore, only temporal features are extracted from the vibration signals to express the degradation path. Dimensionality reduction algorithms, such as principal components analysis and linear discriminant analysis, are based on a specific objective function, and thus lose vital information in the feature compression process [23,24]. To this end, a multi-stage feature selection and reduction algorithm is proposed in [25]. First, a fisher score feature selection algorithm is proposed to select the most representative features. Second, the resulted features are compressed with linear discriminant analysis. The proposed algorithm effectively decreases the loss of information during the feature compression. However, this algorithm lacks interpretability. In this paper, a metric-based feature selection algorithm is proposed as the data processing algorithm. The significance of the features can be visualized from the metric values.

Deep learning can reach an accurate data representation. However, it is not applicable in the few-shot learning case [26]. Meta-learning is proposed to improve the performance of deep learning with fewer training samples. Based on the stochastic gradient descent, the model-agnostic meta-learning model can reach a general representation with a single iteration step [27]. The meta-long short-term memory neural network (meta-LSTM) is proposed to avoid a high-order gradient calculation in gradient-descent-based meta-learning models [28]. Meta-representation-learning has already been used for the RUL prediction of a rocket engine bearing with proper pruning techniques (meta-pruning) [29]. In this
paper, the meta-LSTM model is employed to model the non-linear and non-analytical degradation trend.

The time-varying vibration transmission paths in the planetary gearboxes introduce an unavoidable measurement error to the sensor readings, and thus a measurement error term should be added in the degradation model [30]. To this end, the variational pattern is extracted from the degradation path and then used to train the Gaussian white noise, simulating the overall effects of measurement error.

An adaptive RUL prediction model is proposed for the planetary gearbox, which contains four stages: temporal feature selection and processing stage, offline model training stage, statistical criterion validation stage and online RUL prediction stage. In summary, the contribution of this paper can be summarized as follows:

1. An \( fGPm \) with an adaptive diffusion is proposed to model the uncertainty of the degradation. The statistical requirements for the engineering application are also analyzed.
2. Only temporal features are reliable for the planetary gearbox degradation. Therefore, multiple temporal features are extracted from the vibration signals. A metric-based feature selection algorithm is proposed in this paper to illustrate the significance of the degradation features.
3. In this paper, the meta-LSTM model is proposed to learn a general representation of the non-analytical planetary gearbox degradation trend, featuring the unit-to-unit variability.
4. The measurement error is not negligible for the planetary gearbox degradation due to the time-varying vibration transmission paths. In this work, a white Gaussian noise term is used to represent the measurement error.

The remainder of the paper is arranged as follows: in Section 2, the adaptive \( fGPm \) is proposed to model the uncertainty of the degradation; and in Section 3, an adaptive RUL prediction model is proposed for the planetary gearboxes, which contains four different stages. In the case study, multiple planetary gearbox vibration signals are utilized for the model validation. The conclusions are provided in Section 5. In the Appendix A, the acronyms used in this paper are summarized for the reader’s convenience.

2. The Adaptive \( fGPm \) for the Modelling of Degradation Uncertainty

2.1. Simulation of the \( fGPm \)

The probability density function (pdf) of the generalized double Pareto distribution is:

\[
f(x|\mu, \delta, \alpha) = \frac{1}{\delta} \left[1 + \frac{1}{\alpha \delta} |x - \mu|\right]^{-1-\alpha},
\]

where \( \mu \) is the location parameter, \( \delta \) is the scale parameter, and \( \alpha \) is the shape parameter [31]. The generalized double Pareto motion (\( GDPm \)) is the corresponding stochastic process.

The \( fGPm \) is defined with the Riemann–Liouville integral and explained with a convolution operation:

\[
fGPm(t) = \frac{1}{\Gamma(H+0.5)} \int_0^t (t-u)^{H-0.5} dGDPm(u)
\]

\[
= \frac{1}{\Gamma(H+0.5)} \int_0^t (t-u)^{H-0.5} \frac{dGDPm(u)}{du} du
\]

\[
= (t-u)^{H-0.5} \frac{dGDPm(u)}{du}
\]

where \( \Gamma \) is the Gamma function, \( H \) is the Hurst exponent, and \( * \) is the convolution operator. A simulated path for the \( fGPm \) is plotted in Figure 4.
2. The Adaptive fGPm for the Modelling of Degradation Uncertainty

In this paper, an adaptive fGPm is proposed to model the degradation uncertainty of a wind turbine planetary gearbox. An adaptive fGPm is an fGPm with an adaptive diffusion. In the real engineering scene, environmental noise is unavoidable, which means that the stochasticity of the degradation varies. The adaptation of the diffusion coefficient makes the fGPm more robust for degradation uncertainty modelling. The adaptive diffusion coefficient is:

$$\sigma_H(t) = \sigma_H(t - 1) + \epsilon,$$

where $\epsilon \sim N(0, \sigma^2)$. The variance of $\epsilon$ is estimated from the vibration signal. The initial value of the diffusion coefficient is one, indicating the historical variational tendency.

The simulation of the fGPm with an adaptive diffusion is shown in Figure 5.

**Figure 4.** Time series of the fGPm.

**Figure 5.** The fGPm with an adaptive diffusion.
2.3. Statistical Requirements of the fGPm with an Adaptive Diffusion

The stochastic process usually follows a certain distributional assumption, e.g., Brownian motion and fractional Brownian motion follow the Gaussian assumption. The application of adaptive fGPm requires the original vibration signal to follow the generalized double Pareto distribution. Thus, the parameters of GPDm are estimated from the vibration signal with the maximum likelihood method [32].

In reality, the degradation trend is long-range-dependent, with its Hurst exponent belonging to (0.5, 1). To model the stochasticity of the degradation, the fGPm should also be long-range-dependent. Long-range dependence of the fGPm requires the Hurst exponent to reside in the interval of \((1/\alpha, 1)\).

3. Adaptive RUL Prediction Model for the Wind Turbine Planetary Gearboxes

3.1. Multi-Stage Procedures for the Proposed RUL Prediction Model

In this paper, an adaptive RUL prediction model is proposed for the wind turbine planetary gearbox. The whole RUL prediction model contains four stages: temporal features selection and processing, offline model training, statistical criterion validation and online RUL prediction. The flow chart of the wind turbine planetary gearbox is depicted in Figure 6.

Stage 1: Temporal features selection and processing

Vibration signals → Temporal features extraction → Metric-based feature selection → Degradation trend and variability separation

Stage 2: Offline model training

Generalized double Pareto assumption → Long-range dependence criterion

Measurement error term training → Drift term training with meta-LSTM model

Stage 3: Statistical criterion validation

fGPm generation → Diffusion coefficient adaptation → Monte Carlo simulation → RUL prediction

Figure 6. Flowchart of the adaptive RUL prediction model.

In the stage of temporal features selection and processing temporal features are extracted from the vibration signals and then evaluated with statistical metrics, such as monotonicity, robustness and tradability. The selected feature is considered as the degradation path, from which we can separate the degradation trend and degradation variation.

In the offline model training process, the degradation trends from different planetary gearboxes are used to train a general representation using the meta-LSTM model. The
degradation variations from different planetary gearboxes are employed to train Gaussian white noise, simulating the inevitable measurement error.

In the stage of statistical criterion validation, the distributional assumption of the test vibration signal and the long-range dependence of the degradation trend are validated, so that the adaptive fGpm is applicable.

In the online RUL prediction stage, the fGpm with an adaptive diffusion is employed to model the stochasticity of the planetary gearbox degradation. With the Monte Carlo simulation algorithm, the pdf of the predicted RUL can be calculated. The point prediction of the RUL can be obtained as the mode value.

### 3.2. Metric-Based Feature Selection Algorithm

Common temporal features are extracted from the vibration signal, including the range, variance, mean absolute value (MA) and root mean square (RMS). Monotonicity, robustness and tradability are used for the feature selection [33]. The higher these values, the better the feature.

The formulas of these evaluation metrics are provided below:

\[
\text{Monotonicity} = |\frac{\# \frac{dx}{dt} > 0}{K-1} - \frac{\# \frac{dx}{dt} < 0}{K-1}|, \tag{4}
\]

where \(K\) represents the number of data points.

\[
\text{Robustness}(X) = \frac{1}{K} \sum_{k=1}^{K} \exp(-|\frac{x_k - \overline{x}_k}{x_k}|), \tag{5}
\]

where \(\overline{x}_k\) is the trend value, which is usually acquired from the smoothing method.

\[
\text{Trendability} = \frac{K(\sum_{k=1}^{K} x_k t_k) - (\sum_{k=1}^{K} x_k)(\sum_{k=1}^{K} t_k)}{[K \sum_{k=1}^{K} x_k^2 - (\sum_{k=1}^{K} x_k)^2][K \sum_{k=1}^{K} t_k^2 - (\sum_{k=1}^{K} t_k)^2]^{0.5}}, \tag{6}
\]

After the feature selection, the degradation path of the planetary gearbox is constructed. In this paper, variational mode decomposition (VMD) is proposed to separate the degradation trend and the degradation variation using the constructed degradation path [34].

### 3.3. Drift Term Considering the Unit-to-Unit Variability

The actual degradation trend is non-linear and not analytical. Thus, it is not appropriate to utilize a non-linear mathematical function as the drift term. Deep neural networks can reach a good representation for the non-linear and non-analytical degradation trend. However, degradation trends are different among the same kind of equipment, which is referred to as unit-to-unit variability.

In this work, we incorporate unit-to-unit variability into the training of the drift term with the meta-LSTM model. In the meta-LSTM model, the base LSTM learners can learn the individual degradation trend features. With the losses of the different base LSTM learners, the meta LSTM learner can reach a general representation of the degradation trends with unit-to-unit variability. The computational structure of the meta-LSTM model is provided in Figure 7. Each of the learners in Figure 7 is an LSTM model.
3.4. Adaptive RUL Prediction Model with a Measurement Error Term

The adaptive degradation model for the planetary gearbox in the wind turbine is:

\[ X(t) = X(0) + \eta(t) + \sigma_H(t) f_{GPm}(t), \]  

where \( \eta(t) \) is the drift term trained with the meta-LSTM model. The initial value of the degradation is normally zero.

The vibration transmission path in the planetary gearbox degradation is time-varying, which renders the sensor reading unreliable. In this work, we propose the Gaussian white noise \( \nu \) to simulate the overall influence of the measurement error. The mean and variance of the Gaussian noise are estimated from the variational components of all the training planetary gearbox degradation data. The adaptive degradation model with a measurement error term is:

\[
\begin{cases}
Y(t) = X(t) + \nu \\
\nu \sim N(\mu_\nu, \sigma_\nu^2)
\end{cases}
\]

The end of life (EOL) is defined as the first passage time for the degradation simulation to exceed the failure threshold (FT):

\[ l(t) = \inf\{t|Y(t) \geq \omega|Y(t-1) < \omega\}, \]

where \( \omega \) is the value of FT.

When the degradation trend passes the FT, we can conclude that the incipient failure happened and the planetary gearbox should be replaced. Empirically, the FT of a planetary gearbox in a wind turbine is 0.7 p. u. [35].

4. Case Study

4.1. Planetary Gearbox Degradation Experiments

The planetary gearbox vibration data used in the validation are provided by the University of Pretoria [36]. The test bench can be found in Figure 8. The vibration signals are acquired from accelerometers mounted on the planetary gearboxes. The sample frequency of the vibration signal is 38, 400 HZ.
4.2. Feature Selection and Incipient Failure Detection

One exemplary vibration response of the planetary gearbox is plotted in Figure 9, which is labeled as “G2_P0_38400Hz”. This dataset is used as the test dataset, and other datasets are employed as the training datasets. In this section, the initial failure of the planetary gearbox test dataset is detected, which serves as the ground truth to validate the proposed model.

Table 1. Metrics for the feature selection.

<table>
<thead>
<tr>
<th>Feature</th>
<th>Monotonicity</th>
<th>Robustness</th>
<th>Tradability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Variance</td>
<td>0.0456</td>
<td>0.8142</td>
<td>0.4332</td>
</tr>
<tr>
<td>VMA</td>
<td>0.1245</td>
<td>0.9835</td>
<td>0.6727</td>
</tr>
<tr>
<td>RMS</td>
<td>0.0623</td>
<td>0.7928</td>
<td>0.5216</td>
</tr>
<tr>
<td>MA</td>
<td>0.0923</td>
<td>0.8512</td>
<td>0.5641</td>
</tr>
</tbody>
</table>

Figure 8. Test bench for planetary gearbox degradation.

In the aforementioned planetary gearbox degradation experiments, different planetary gearboxes were installed in the test bench. Thus, multiple vibration signals were collected from the start of degradation to the moment just before the tooth crack.

In the aforementioned planetary gearbox degradation experiments, different planetary gearboxes were installed in the test bench. Thus, multiple vibration signals were collected from the start of degradation to the moment just before the tooth crack.

Figure 9. Exemplary vibration response of the planetary gearbox.

Monotonicity, robustness and tradability values are calculated for these four features, and the results are deposited in Table 1. As we can see from Table 1, the variance is the best feature. Thus, we choose the variance as the degradation path. The variance degradation path is plotted in Figure 10. The VMD algorithm is used to separate the degradation trend.
and the degradation variation in the degradation path. In Figure 11, the upper subplot is the degradation trend and the lower subplot is the degradation variation.

Table 1. Metrics for the feature selection.

<table>
<thead>
<tr>
<th></th>
<th>Range</th>
<th>Variance</th>
<th>MA</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monotonicity</td>
<td>0.0456</td>
<td>0.1245</td>
<td>0.0623</td>
<td>0.0923</td>
</tr>
<tr>
<td>Robustness</td>
<td>0.8142</td>
<td>0.9835</td>
<td>0.7928</td>
<td>0.8512</td>
</tr>
<tr>
<td>Tradability</td>
<td>0.4332</td>
<td>0.6727</td>
<td>0.5216</td>
<td>0.5641</td>
</tr>
</tbody>
</table>

Figure 10. Degradation path of the planetary gearbox.

Figure 11. Separation of the degradation trend and the degradation variation. (a) Separated degradation trend; (b) separated degradation variation.

4.3. Statistical Requirements Validation for the Adaptive fGPm

Prior to the RUL prediction, two statistical requirements of the adaptive fGPm need to be verified. First, the vibration signal should follow the generalized double Pareto
distribution. Second, the long-range-dependence criterion should be satisfied for the degradation trend.

The statistical fitting for the test vibration dataset is plotted in Figure 12. As we can see from Figure 12, the generalized double Pareto distribution is more appropriate than the Gaussian distribution for fitting the vibration signal.

![Figure 12. Statistical fittings of the vibration signal.](image)

The estimation method for the Hurst exponent depends on the stationarity of the time series [37]. For the stationary time series, the rescaled range algorithm should be used [38]. As for the non-stationary time series, the wavelet variance approach is needed [39].

The augmented Dickey–Fuller test confirms the non-stationarity of the degradation trend for the test vibration dataset [40]. Therefore, the wavelet variance approach is used to calculate the Hurst exponent in Figure 13.

![Figure 13. Calculation plot for the Hurst exponent.](image)

The estimated shape parameter is 1.982, and the calculated Hurst exponent is 0.8484. The Hurst exponent is larger than the reciprocal of the shape parameter. Thus, the long-range-dependence criterion for the RUL prediction model is satisfied.
4.4. Performance Evaluation of the Proposed Model

The RUL prediction results based on $f_{GPm}$ are provided in Figure 14, and the model comparison is depicted in Figure 15. The point predictions of different models are plotted in Figure 16. The root mean squared error (RMSE), mean absolute error (MAE) and score of accuracy (SOA) are used to evaluate the prediction accuracy. The standard deviation (std) of the prediction error is also calculated. The evaluation metrics are compiled in Table 2.

Without the stochasticity modelling, the meta-pruning model can be easily struck in the local extremum, which decreases the prediction accuracy. The planetary gearbox vibration response is more suitably fitted with the generalized Pareto double distribution. As a result, the prediction accuracy of the FBM model is not satisfactory. The considered BM–LSTM algorithm does not consider the inevitable measurement error and the long-range dependence of the degradation trend.

![Figure 14. RUL prediction results based on $f_{GPm}$.](image)

![Figure 15. RUL prediction results for the model comparison.](image)
5. Conclusions

In this paper, a multi-stage RUL prediction model is proposed for planetary gearboxes. In the temporal features selection and processing stage, temporal features are extracted from the vibration response. A metric-based feature selection algorithm is proposed to construct the degradation path. In the offline model training process, the meta-LSTM model is used to establish a general fitting for the degradation trend, and the measurement error is represented by a white Gaussian noise term. In the online RUL prediction stage, the adaptive fGPm is proposed to model the stochasticity of the planetary gearbox degradation.

There are three main advantages to our model. First, the special characteristics of the planetary gearbox degradation are considered in the algorithm. Due to the interference between multiple vibration sources, only temporal features are considered in the proposed model. This paper proposes the use of white Gaussian noise to simulate the measurement error caused by time-varying vibration transmission paths. Second, a metric-based feature selection algorithm is proposed in our model. The significance of the feature is reflected in the metric values, which improves the model interpretability. Third, the meta-LSTM model can provide a drift term with unit-to-unit variability. There is also a limitation to the proposed model, which is the statistical criterion for the adaptive fGPm. In the future, we
will extend the model to the RUL prediction of lithium batteries in energy storage facilities, where the non-Gaussian assumption and long memory effects are common [41].

**Supplementary Materials:** The following supporting information can be downloaded at: https://www.mdpi.com/article/10.3390/fractalfract8010014/s1.

**Author Contributions:** Conceptualization, H.Z., W.D. and W.S.; methodology, H.Z. and W.D.; software, W.S. and W.C.; validation, W.D. and W.S.; formal analysis, W.C. and P.C.; investigation, H.Z.; resources, W.S.; data curation, P.C.; writing—original draft preparation, W.D. and W.C.; writing—review and editing, P.C. and F.V.; visualization, F.V.; supervision, H.Z.; project administration, Wei Chen; funding acquisition, H.Z. and W.C. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was funded by the Science and Technology Project of Fujian Province, grant number 2023H6026, by the Science and Technology Project of Quanzhou City, grant number 2022N041, and by the Technology Innovation Project of Minnan University of Science and Technology (Grant No. 23XTD113).

**Data Availability Statement:** The planetary gearbox data used in this paper can be found in https://figshare.com/articles/dataset/Planetary_gearbox_vibration_data/13476525/2 (accessed on 27 August 2023).

**Acknowledgments:** We thank the experimental assistance provided by the Minnan University of Science and Technology and the experimental data from the University of Pretoria.

**Conflicts of Interest:** The authors declare no conflict of interest.

**Appendix A**

<table>
<thead>
<tr>
<th>RUL</th>
<th>remaining useful life</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSTM</td>
<td>long short-term memory neural network</td>
</tr>
<tr>
<td>BM–LSTM</td>
<td>Brownian motion–long short-term neural network</td>
</tr>
<tr>
<td>FBM</td>
<td>fractional Brownian motion</td>
</tr>
<tr>
<td>fGPm</td>
<td>fractional Generalized Pareto motion</td>
</tr>
<tr>
<td>HI</td>
<td>health indicator</td>
</tr>
<tr>
<td>meta-LSTM</td>
<td>meta-long short-term memory neural network</td>
</tr>
<tr>
<td>meta-pruning</td>
<td>meta-learning with pruning technique</td>
</tr>
<tr>
<td>pdf</td>
<td>probability density function</td>
</tr>
<tr>
<td>GDPm</td>
<td>generalized double Pareto motion</td>
</tr>
<tr>
<td>MA</td>
<td>mean absolute value</td>
</tr>
<tr>
<td>RMS</td>
<td>root mean square</td>
</tr>
<tr>
<td>VMD</td>
<td>variational mode decomposition</td>
</tr>
<tr>
<td>EOL</td>
<td>end of life</td>
</tr>
<tr>
<td>FT</td>
<td>failure threshold</td>
</tr>
<tr>
<td>RMSE</td>
<td>root mean squared error</td>
</tr>
<tr>
<td>MAE</td>
<td>mean absolute error</td>
</tr>
<tr>
<td>SOA</td>
<td>score of accuracy</td>
</tr>
<tr>
<td>std</td>
<td>standard deviation</td>
</tr>
</tbody>
</table>

**References**


18. Li, X.; Ma, Y. Remaining useful life prediction for lithium-ion battery using dynamic fractional brownian motion degradation model with long-term dependence. *J. Power Electron* 2022, 22, 2069–2080. [CrossRef]

19. Song, W.Q.; Duan, S.W.; Chen, D.D.; Zio, E.; Yan, W.D.; Cai, F. Finite Iterative Forecasting Model Based on Fractional Generalized Pareto Motion. *Fractal. Fract.* 2022, 6, 471. [CrossRef]


30. Sun, B.; Li, Y.; Wang, Z.L.; Ren, Y.; Feng, Q.; Yang, D.Z. An improved inverse Gaussian process with random effects and measurement errors for RUL prediction of hydraulic piston pump. *Measurement* 2021, 173, 108604. [CrossRef]


34. Hu, L.W.; Wang, W.B.; Ding, G.R. RUL prediction for lithium-ion batteries based on variational mode decomposition and hybrid network model. *Signal Image Video Process.* 2023, 17, 3109–3117. [CrossRef]


Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.