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Fuzzy Subordination Results for Meromorphic Functions Connected with a Linear Operator

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Abstract: The concept of subordination is expanded in this study from the fuzzy sets theory to the geometry theory of analytic functions with a single complex variable. This work aims to clarify fuzzy subordination as a notion and demonstrate its primary attributes. With this work's assistance, new fuzzy differential subordinations will be presented. The first theorems lead to intriguing corollaries for specific aspects chosen to exhibit fuzzy best dominance. The work introduces a new integral operator for meromorphic functions and uses the newly developed integral operator, which is starlike and convex, respectively, to obtain conclusions on fuzzy differential subordination.

Keywords: fuzzy differential subordination; fuzzy best dominant; meromorphic function; convolution; linear operator

MSC: 30C45; 26A33; 30C50; 30C80

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1. Introduction

In the field of complex analysis, fuzzy set theory was added into the research on Geometric Function Theory (GFT) in 2011, when the first article presenting the idea of subordination in fuzzy set theory [1] was published. Miller and Mocanu's classic aspects of subordination [2,3] served as an inspiration for this concept. The subsequent publications, which included concepts from the previously established theory of differential subordination, adopted the research path outlined by Miller and Mocanu and discussed fuzzy differential subordination [4–9]. The idea was swiftly embraced by GFT scholars, and all of the conventional research paths in this area were changed to account for the additional fuzzy elements. Research involving operators is a key area of study in GFT. Shortly after the idea was presented, in 2013 [10], such investigations to acquire new fuzzy subordination results were published. They continued in the following years [11–14] and then added superordination results [15–17]. We simply highlight a small number of the numerous publications that have been published in recent years to demonstrate how the research on this subject is constantly evolving [18–20].

Recent years have seen a significant advancement in fuzzy differential subordination incorporating fractional calculus, which has been shown to have applications in many research area.

Consider that $\mathfrak{U} = \{\zeta : \zeta \in \mathbb{C} \text{and } |\zeta| < 1\}$ represent the unit disk in the complex plane, and let $\mathcal{H}(\mathfrak{U})$ be the space of analytic functions representing $\mathfrak{U}$,

$$\mathcal{A}_n = \left\{\mathfrak{f} \in \mathcal{H}(\mathfrak{U}) : \mathfrak{f}(\zeta) = \zeta + a_{n+1}\zeta^{n+1} + \cdots \quad \zeta \in \mathfrak{U}\right\},$$

and

$$\mathcal{H}[a,n]=\left\{\mathfrak{f}\in\mathcal{H}(\mathfrak{U}):\mathfrak{f}(\zeta)=a+a_n\zeta^n+a_{n+1}\zeta^{n+1}+\cdots\quad \zeta\in\mathfrak{U}\right\},$$

for $a\in\mathbb{C}$ and $n\in\mathbb{N}=\{1,2,3,\cdots\}$. The normalized convex function class in $\mathfrak{U}$ is represented by

$$\mathfrak{K}=\left\{\mathfrak{f}\in\mathcal{A}_n\ :\mathrm{Re}\left(1+\frac{\zeta\mathfrak{f}''(\zeta)}{\mathfrak{f}'(\zeta)}\right)>0\ \ \zeta\in\mathfrak{U}\right\},$$

and $\mathcal{H}[1,1]=\mathcal{H}$. The class of meromorphic function indicated by Σ is given by

$$\mathfrak{f}(\zeta)=\frac{1}{\zeta}+\sum_{n=0}^{\infty}a_n\zeta^n \qquad (\zeta\in\mathfrak{U}^*=\mathfrak{U}\setminus\{0\}), \tag{1}$$

where $\mathfrak{U}^*$ is the punctured unit disc defined by $\mathfrak{U}^*=\{\zeta:\zeta\in\mathbb{C}\,\text{and}\ 0<|\zeta|<1\}$.

If $\mathfrak{f}\in\Sigma$, as in (1), and $\mathfrak{g}$ is given by

$$\mathfrak{g}(\zeta)=\frac{1}{\zeta}+\sum_{n=0}^{\infty}b_n\zeta^n,$$

the Hadamard product of $\mathfrak{f}$ and $\mathfrak{g}$ is represented by

$$(\mathfrak{f}*\mathfrak{g})(\zeta)=\frac{1}{\zeta}+\sum_{n=0}^{\infty}a_nb_n\zeta^n.$$

Let $\mathcal{S}^*_{\Sigma}$ and $\mathcal{C}_{\Sigma}$ be the subclasses of Σ, which are meromorphic starlike and meromorphic convex in $\mathfrak{U}^*$, respectively, and stated by

$$\mathcal{S}^*_{\Sigma}=\left\{\mathfrak{f}:\mathfrak{f}\in\Sigma and-\mathrm{Re}\left(\frac{\zeta\mathfrak{f}'(\zeta)}{\mathfrak{f}(\zeta)}\right)>0\quad(\zeta\in\mathfrak{U}^*)\right\},$$

and

$$\mathcal{C}_{\Sigma}=\left\{\mathfrak{f}:\mathfrak{f}\in\Sigma and-\mathrm{Re}\left(1+\frac{\zeta\mathfrak{f}''(\zeta)}{\mathfrak{f}'(\zeta)}\right)>0\quad(\zeta\in\mathfrak{U}^*)\right\}.$$

Let, for $\kappa\in\mathbb{Z}=\{\ldots,-2,-1,0,1,2,\ldots\}$ and for $\mathfrak{i}>0,\mathfrak{j}>0$, a linear operator $\mathcal{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}:\Sigma\to\Sigma$ be defined by

$$\begin{aligned}\mathcal{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}\mathfrak{f}(\zeta)&=\mathfrak{f}(\zeta),\kappa=0,\\&=\tfrac{\mathfrak{i}}{\mathfrak{j}}\zeta^{-1-\frac{\mathfrak{i}}{\mathfrak{j}}}\int_0^{\zeta}\mathfrak{t}^{\frac{\mathfrak{i}}{\mathfrak{j}}}\mathcal{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}\mathfrak{f}(\zeta)d\mathfrak{t},\quad(\kappa=-1,-2,\ldots;\zeta\in\mathbb{C})\\&=\tfrac{\mathfrak{j}}{\mathfrak{i}}\zeta^{-\frac{\mathfrak{i}}{\mathfrak{j}}}\tfrac{d}{d\zeta}\left(\zeta^{\frac{\mathfrak{i}}{\mathfrak{j}}+1}\mathcal{L}^{\kappa-1}_{\mathfrak{i},\mathfrak{j}}\mathfrak{f}(\zeta)\right),\quad(\kappa=1,2,\ldots;\zeta\in\mathfrak{U}^*).\end{aligned}$$

The fractional integral operator of order $\alpha\in\mathbb{C}\ \ (\Re(\alpha)>0)$ in the Riemann–Liouville is one of the most frequently used operators, as demonstrated by the definitions in, for instance, [21,22]; see also [23],

$$\left(I^{\alpha}_{0+}\,\mathfrak{f}\right)(x)=\frac{1}{\Gamma(\alpha)}\int_0^x(x-\tau)^{\alpha-1}\,\mathfrak{f}(\tau)\,d\tau\quad(x>0;\ \Re(\alpha)>0) \tag{2}$$

in terms of the Gamma function $\Gamma(\alpha)$ of Euler. The Erdélyi–Kober fractional integral operator of order $\alpha\in\mathbb{C}\ \ (\Re(\alpha)>0)$ is an intriguing variation of the Riemann–Liouville operator I^{α}_{0+}, as specified by

$$\left(I^{\alpha}_{0+;\sigma,\eta}\,\mathfrak{f}\right)(x)=\frac{\sigma x^{-\sigma(\alpha+\eta)}}{\Gamma(\alpha)}\int_0^x\tau^{\sigma(\eta+1)-1}\,(x^{\sigma}-\tau^{\sigma})^{\alpha-1}\,\mathfrak{f}(\tau)\,d\tau \tag{3}$$

$$(x>0;\ \Re(\alpha)>0),$$

which basically translates to (2) when $\sigma - 1 = \eta = 0$, as

$$\left(I^{\alpha}_{0+;1,0}\,\mathfrak{f}\right)(x) = x^{-\alpha}\,\left(I^{\alpha}_{0+}\,\mathfrak{f}\right)(x)(x > 0;\ \Re(\alpha) > 0).$$

Let $\mathfrak{t} > 0;\ \mathfrak{r}, \mathfrak{s} \in \mathbb{C}$, satisfing $\Re(\mathfrak{s} - \mathfrak{r}) \geq 0$ and $\Re(\mathfrak{r}) > \mathfrak{t}$ modifiying an Erdélyi–Kober fractional integral operator (3); in this case, a linear integral operator is examined $\mathcal{I}^{\mathfrak{r},\mathfrak{s}}_{\mathfrak{t}} : \Sigma \to \Sigma$ defined for a function $\mathfrak{f} \in \Sigma$ by

$$\mathcal{I}^{\mathfrak{r},\mathfrak{s}}_{\mathfrak{t}}\mathfrak{f}(\zeta) = \frac{\Gamma(\mathfrak{s}-\mathfrak{t})}{\Gamma(\mathfrak{r}-\mathfrak{t})\Gamma(\mathfrak{s}-\mathfrak{r})}\int_0^1 \tau^{\mathfrak{r}-1}\,(1-\tau)^{\mathfrak{s}-r-1}\,\mathfrak{f}(\zeta\tau^{\mathfrak{t}})d\tau, \tag{4}$$

$$(\mathfrak{t} > 0;\ \mathfrak{r}, \mathfrak{s} \in R;\ \mathfrak{s} > \mathfrak{r}).$$

When analyzed using the integral of the Eulerian beta-function:

$$\mathrm{B}(\alpha,\beta) := \begin{cases} \int_0^1 \tau^{\alpha-1}\,(1-\tau)^{\beta-1}\,d\tau & (\min\{\Re(\alpha),\Re(\beta)\} > 0) \\ \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)} & (\alpha,\beta \in \mathbb{C} \setminus \mathbb{Z}_0^-), \end{cases}$$

we readily find that

$$\mathcal{I}^{\mathfrak{r},\mathfrak{s}}_{\mathfrak{t}}\mathfrak{f}(\zeta) = \begin{cases} \frac{1}{\zeta} + \frac{\Gamma(\mathfrak{s}-\mathfrak{t})}{\Gamma(\mathfrak{r}-\mathfrak{t})}\sum\limits_{n=0}^{\infty}\frac{\Gamma(\mathfrak{r}+\mathfrak{t}n)}{\Gamma(\mathfrak{s}+\mathfrak{t}n)}\,a_n\zeta^n & (\mathfrak{s} > \mathfrak{r}) \\ \mathfrak{f}(\zeta) & (\mathfrak{s} = \mathfrak{r}). \end{cases}$$

By iterations of the linear operators (defined above), a class of operators $\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}$(r,s,t) : $\Sigma \to \Sigma$ is given by

$$\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)) = \mathcal{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathcal{I}^{\mathfrak{r},\mathfrak{s}}_{\mathfrak{t}}\mathfrak{f}(\zeta)) = \mathcal{I}^{\mathfrak{r},\mathfrak{s}}_{\mathfrak{t}}\left(\mathcal{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}\mathfrak{f}(\zeta)\right),$$

whose series expansion for $\kappa \in \mathbb{Z}, \mathfrak{i}, \mathfrak{j} > 0, \mathfrak{t} > 0, \Re(\mathfrak{s}-\mathfrak{r}) \geq 0; \Re(\mathfrak{r}) > \mathfrak{t}$ and for $\mathfrak{f}$, as in (1), is given by

$$\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)) = \frac{1}{\zeta} + \frac{\Gamma(\mathfrak{s}-\mathfrak{t})}{\Gamma(\mathfrak{r}-\mathfrak{t})}\sum_{n=0}^{\infty}\left(\frac{\mathfrak{i}+\mathfrak{j}(n+1)}{\mathfrak{i}}\right)^{\kappa}\frac{\Gamma(\mathfrak{r}+\mathfrak{t}n)}{\Gamma(\mathfrak{s}+\mathfrak{t}n)}\,a_n\zeta^n. \tag{5}$$

We note that this new class of operators $\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}$(r,s,t) was introduced in [24].

From (5), we obtain

$$\zeta\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)' = \frac{\mathfrak{i}}{\mathfrak{j}}\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta) - \left(1+\frac{\mathfrak{i}}{\mathfrak{j}}\right)\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta). \tag{6}$$

$$\zeta\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)' = \left(\frac{\mathfrak{r}-\mathfrak{t}}{\mathfrak{t}}\right)\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r}+1,\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta) - \frac{\mathfrak{r}}{\mathfrak{t}}\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta). \tag{7}$$

We note that

(i) $\mathfrak{L}^{\kappa}_{1,1}(\mathfrak{r},\mathfrak{r},\mathfrak{t})\mathfrak{f}(\zeta) = I^{\kappa}\mathfrak{f}(\zeta)$ (see Aqlan et al. [25], with $p = 1$);
(ii) $\mathfrak{L}^{\kappa}_{1,\mathfrak{j}}(\mathfrak{r},\mathfrak{r},\mathfrak{t})\mathfrak{f}(\zeta) = I^{\kappa}_{\mathfrak{j}}\mathfrak{f}(\zeta)$ (Lashin [26]);
(ii) $\mathfrak{L}^{\kappa}_{\mathfrak{i},1}(\mathfrak{r},\mathfrak{r},\mathfrak{t})\mathfrak{f}(\zeta) = I(\kappa,\mathfrak{i})\mathfrak{f}(\zeta)$ ($\kappa > 0$, (Cho et al. [27,28]));
(iv) $\mathfrak{L}^{\kappa}_{1,\mathfrak{j}}(\mathfrak{r},\mathfrak{r},\mathfrak{t})\mathfrak{f}(\zeta) = D^{\kappa}_{\mathfrak{j}}\,\mathfrak{f}(\zeta))$ ($\kappa > 0$, (Al-Oboudi and Al-Zkeri [29], with $p = 1$));
(v) $\mathfrak{L}^{\kappa}_{1,1}(\mathfrak{r},\mathfrak{r},\mathfrak{t})\mathfrak{f}(\zeta) = I^{\kappa}\,\mathfrak{f}(\zeta))$ ($\kappa > 0$, (Uralegaddi and Somanatha [30]);
(vi) $\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{r},\mathfrak{t})\mathfrak{f}(\zeta) = I^{\kappa}_{\mathfrak{i},\mathfrak{j}}\,\mathfrak{f}(\zeta))$ (see El-Ashwah [31], with $p = 1$).

2. Definitions and Preliminaries

The following lemma will be used as a tool for proving the new results included in the section that follows.

Definition 1 ([32]). *Allow $\lambda \neq \phi$ F to be defined as a fuzzy subset of λ that maps from λ to $[0,1]$.*

Definition 2 ([32]). *A Fuzzy subset of λ is a pair ($(\mathfrak{I}, F_{\mathfrak{I}})$, where $F_{\mathfrak{I}} : \lambda \to [0,1]$ is known as the membership function of the fuzzy set $(\mathfrak{I}, F_{\mathfrak{I}})$, and $\mathfrak{I} = \{x \in \lambda : 0 < F_{\mathfrak{I}}(x) \leq 1\} = \sup(\mathfrak{I}, F_{\mathfrak{I}})$ is called a fuzzy subset.*

Definition 3 ([1]). *Let there be two fuzzy subsets of η,$\left(\mathfrak{I}_1, F_{\mathfrak{I}_1}\right)$ and $\left(\mathfrak{I}_2, F_{\mathfrak{I}_2}\right)$. We say that the fuzzy subsets $\mathfrak{I}_1$ and $\mathfrak{I}_2$ are equal if and only if $F_{\mathfrak{I}_1}(\eta) = F_{\mathfrak{I}_2}(\eta)$, $\eta \in \lambda$, and we denote them by $\left(\mathfrak{I}_1, F_{\mathfrak{I}_1}\right) = \left(\mathfrak{I}_2, F_{\mathfrak{I}_2}\right)$. The fuzzy subset $\left(\mathfrak{I}_1, F_{\mathfrak{I}_1}\right)$ is contained in the fuzzy subset $\left(\mathfrak{I}_2, F_{\mathfrak{I}_2}\right)$ if and only if $F_{\mathfrak{I}_1}(\eta) \leq F_{\mathfrak{I}_2}(\eta)$, $\eta \in S$, and we denote this by $\left(\mathfrak{I}_1, F_{\mathfrak{I}_1}\right) \subseteq \left(\mathfrak{I}_2, F_{\mathfrak{I}_2}\right)$.*

Definition 4 ([11]). *Assume that $F : \mathbb{C} \to \mathbb{R}_+$ is a function satisfying $F_{\mathbb{C}}(\zeta) = |F(\zeta)|, \zeta \in \mathbb{C}$. We define as*

$$F_{\mathbb{C}}(\mathbb{C}) = \{\zeta : \zeta \in \mathbb{C} \text{and} 0 < |F(\zeta)| \leq 1\} = Supp(\mathbb{C}, F_{\mathbb{C}}(\zeta)),$$

the fuzzy subset of $\mathbb{C}$. The fuzzy unit disk is known as

$$F_{\mathbb{C}}(\mathbb{C}) = \{\zeta : \zeta \in \mathbb{C} \text{and} 0 < |F(\zeta)| \leq 1\} = \mathfrak{U}_F(0,1).$$

It has been noted that $(\mathbb{C}, F_{\mathbb{C}}(\zeta))$ is the same as its fuzzy unit disk $\mathfrak{U}_F(0,1)$.

Proposition 1 ([1]). *(i) If $(\mathfrak{I}, F_{\mathfrak{I}}) = (\mathfrak{U}, F_{\mathfrak{U}})$, then $\mathfrak{I} = \mathfrak{U}$, where $\mathfrak{I} = \sup(\mathfrak{I}, F_{\mathfrak{I}})$, and $\mathfrak{U} = \sup(\mathfrak{U}, F_{\mathfrak{U}})$.*

(ii) If $(\mathfrak{I}, F_{\mathfrak{I}}) \subseteq (\mathfrak{U}, F_{\mathfrak{U}})$, then $I \subseteq \mathfrak{U}$ where $I = \sup(I, F_{\mathfrak{I}})$ and $\mathfrak{U} = \sup(\mathfrak{U}, F_{\mathfrak{U}})$.

Let $\mathfrak{f}, \mathfrak{h} \in \mathcal{H}(\mathfrak{U})$.We say

$$\mathfrak{f}(\mathfrak{U}) = \sup\left(\mathfrak{f}(\mathfrak{U}), F_{\mathfrak{f}(\mathfrak{U})}\right) = \{\mathfrak{f}(\zeta) : 0 < \left|F_{\mathfrak{f}(\mathfrak{U})}(\mathfrak{f}(\zeta))\right| \leq 1, \zeta \in \mathfrak{U}, \tag{8}$$

and

$$\mathfrak{h}(\mathfrak{U}) = \sup\left(\mathfrak{h}(\mathfrak{U}), F_{\mathfrak{h}(\mathfrak{U})}\right) = \{\mathfrak{h}(\zeta) : 0 < \left|F_{\mathfrak{h}(\mathfrak{U})}(\mathfrak{h}(\zeta))\right| \leq 1, \zeta \in \mathfrak{U}. \tag{9}$$

Definition 5 ([1]). *Let $\zeta_0 \in \mathfrak{U}$ and $\mathfrak{f}, \mathfrak{h} \in \mathcal{H}(\mathfrak{U})$. Let $\mathfrak{f}$ be fuzzy subordinate to $\mathfrak{h}$ and be written as $\mathfrak{f} \prec_F \mathfrak{h}$ or $\mathfrak{f}(\zeta) \prec_F \mathfrak{h}(\zeta)$ if each of the subsequent requirements is satisfied*

$$\mathfrak{f}(\zeta_0) = \mathfrak{h}(\zeta_0) \text{and} \left|F_{\mathfrak{f}(\mathfrak{U})}(\mathfrak{f}(\zeta))\right| \leq \left|F_{\mathfrak{h}(\mathfrak{U})}(\mathfrak{h}(\zeta))\right|, \zeta \in \mathfrak{U}.$$

Proposition 2 ([1]). *Let $\zeta_0 \in \mathfrak{U}$, and $\mathfrak{f}, \mathfrak{h} \in \mathcal{H}(\mathfrak{U})$. If $\mathfrak{f}(\zeta) \prec_F \mathfrak{h}(\zeta)$,$\zeta \in \mathfrak{U}$, then*

$$\begin{aligned} (\mathrm{i})\mathfrak{f}(\zeta_0) &= \mathfrak{h}(\zeta_0), \\ (\mathrm{ii})\mathfrak{f}(\mathfrak{U}) &\subseteq \mathfrak{h}(\mathfrak{U}), \text{and} \left|F_{\mathfrak{f}(\mathfrak{U})}(\mathfrak{f}(\zeta))\right| \leq \left|F_{\mathfrak{h}(\mathfrak{U})}(\mathfrak{h}(\zeta))\right|, \zeta \in \mathfrak{U}, \end{aligned}$$

where $\mathfrak{f}(\mathfrak{U})$ and $\mathfrak{h}(\mathfrak{U})$ are given by (8) and (9), respectively.

Definition 6 ([4]). *Let $\psi : \mathbb{C}^3 \times U \to \mathbb{C}$, and let $\mathfrak{h}$ be an analytic function with $\psi(a,0,0,0) = \mathcal{H}(0) = a$. If ω is analytic in $\mathfrak{U}$ with $\omega(0) = a$ and satisfies the (second-order) fuzzy differential subordination:*

$$F_{\psi(\mathbb{C}^3 \times \mathfrak{U})}\left(\psi\left(\omega(\zeta), \zeta\omega'(\zeta), \zeta^2\omega''(\zeta); \zeta\right)\right) \leq \left|F_{\mathcal{H}(\mathfrak{U})}(\mathcal{H}(\mathfrak{U}))\right|, \tag{10}$$

i.e.

$$\psi\left(\left(\omega(\zeta), \zeta\omega'(\zeta), \zeta^2\omega''(\zeta); \zeta\right)\right) \leq_F (\mathcal{H}(\zeta)), \zeta \in \mathfrak{U},$$

then ω is a fuzzy solution, and ω is a fuzzy dominant if

$$\left|F_{\omega(\mathfrak{U})}\omega(\zeta)\right| \leq \left|F_{\chi(\mathfrak{U})}\chi(\zeta)\right|, i.e., \omega(\zeta) \prec_F \chi(\zeta)\zeta \in \mathfrak{U},$$

for all ω satisfying (8). A fuzzy dominant $\widetilde{\chi}$ that satisfies

$$\left|F_{\widetilde{\chi}(\mathfrak{U})}\widetilde{\chi}(\zeta)\right| \leq \left|F_{\chi(\mathfrak{U})}\chi(\zeta)\right| i.e., \widetilde{\chi}(\zeta) \prec_F \chi(\zeta)\zeta \in \mathfrak{U},$$

for all fuzzy dominant χ of (10), is the fuzzy best dominant of (10).

Lemma 1 ([33] Corollary 2.6g.2, p. 66)**.** *Assume that $X \in \mathcal{H}(\mathfrak{U})$ and*

$$\rho(\zeta) = \frac{1}{n\zeta^{\frac{1}{n}}}\int_0^\zeta t^{\frac{1}{n}-1}X(t)dt, (\zeta \in \mathfrak{U}).$$

If

$$\mathrm{Re}\left(\frac{\zeta X''(\zeta)}{X'(\zeta)} + 1\right) > -\frac{1}{2}(\zeta \in \mathfrak{U}),$$

then $\rho \in \mathfrak{K}$.

Lemma 2 ([34])**.** *Assume X is a convex function that satisfies $X(0) = a$, and let $\varepsilon \in \mathbb{C}^* = \mathbb{C} \setminus \{0\}$ such that $Re(\varepsilon) \geq 0$. If $\omega \in \mathcal{H}[a, n]$ with $\omega(0) = \mathfrak{a}$, and $\psi : (\mathbb{C}^2 \times \mathfrak{U}) \to \mathbb{C}$, $\psi(\omega(\zeta), \zeta\omega'(\zeta)) = \omega(\zeta) + \frac{1}{\varepsilon}\zeta\omega'(\zeta)$ is analytic in $\mathfrak{U}$, then*

$$\begin{array}{rcl} \left|F_{\psi(\mathbb{C}^3\times\mathfrak{U})}\left(\psi\left(\omega(\zeta) + \frac{1}{\varepsilon}\zeta\omega'(\zeta)\right)\right)\right| & \leq & \left|F_{X(\mathfrak{U})}(X(\zeta))\right| \Rightarrow \omega(\zeta) + \frac{1}{\varepsilon}\zeta\omega'(\zeta) \\ & \prec & {}_F X(\zeta)(\zeta \in \mathfrak{U}) \end{array}$$

implies

$$F_{\omega(\mathfrak{U})}(\omega(\zeta)) \leq F_{\chi(\mathfrak{U})}(\chi(\zeta)) \leq F_{X(\mathfrak{U})}(X(\zeta)) i.e., \omega(\zeta) \prec_F \chi(\zeta),$$

where

$$\chi(\zeta) = \frac{\varepsilon}{n\zeta^{\frac{\varepsilon}{n}}}\int_0^\zeta t^{\frac{\varepsilon}{n}-1}X(t)dt$$

is the best dominant and convex.

Lemma 3 ([34])**.** *Assume that χ is a convex function in $\mathfrak{U}$, and let*

$$X(\zeta) = \chi(\zeta) + n\upsilon\zeta\chi'(\zeta),$$

$\upsilon > 0$, and $n \in \mathbb{N}$. If $\omega \in \mathcal{H}[\chi(0), n]$, and $\psi : \mathbb{C}^2 \times \mathfrak{U} \to \mathbb{C}$,

$$\psi(\omega(\zeta), \zeta\omega'(\zeta)) = \omega(\zeta) + n\upsilon\zeta\omega'(\zeta)$$

is analytic in $\mathfrak{U}$, then

$$\begin{array}{rcl} \left|F_{\psi(\mathbb{C}^2\times\mathfrak{U})}(\omega(\zeta) + n\upsilon\zeta\omega'(\zeta))\right| & \leq & \left|F_{X(\mathfrak{U})}(X(\zeta))\right| \Rightarrow \omega(\zeta) + n\upsilon\zeta\omega'(\zeta) \\ & \prec & {}_F X(\zeta)(\zeta \in \mathfrak{U}) \end{array}$$

implies

$$\left|F_{\omega(\mathfrak{U})}(\omega(\zeta))\right| \leq \left|F_{\chi(\mathfrak{U})}(\chi(\zeta))\right| i.e., \omega(\zeta) \prec_F \chi(\zeta)$$

is the fuzzy best dominant.

This study finds adequate requirements for a class of fuzzy differential subordinations that are connected with an Erdélyi–Kober type integral operator $\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})$ and that are meromorphic analytic and univalent functions. The fuzzy best dominants are determined by obtaining fuzzy differential subordinations.

3. Main Results

Theorem 1. *Let ρ be a convex function in $\mathfrak{U}$, $\rho(0)=1$, and*

$$X(\zeta)=\rho(\zeta)+\zeta\rho'(\zeta)\zeta\in\mathfrak{U}.$$

If, for $\mathfrak{f}\in\Sigma$, satisfying

$$\begin{aligned}&\left|F_{\psi(\mathbb{C}^2\times\mathfrak{U}^*)}\left[\frac{\mathfrak{i}}{\mathfrak{j}}\left[\zeta\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)-\zeta\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right]+\zeta\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)\right]\right| \\ \leq\;&\left|F_{X(\mathfrak{U})}(X(\zeta))\right|,\end{aligned}\tag{11}$$

implies

$$\left[\frac{\mathfrak{i}}{\mathfrak{j}}\left[\zeta\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)-\zeta\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right]+\zeta\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)\right]\prec_F X(\zeta),$$

then

$$\begin{aligned}\left|F_{\zeta(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta))}\left(\zeta\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)\right)\right| &\leq \left|F_{X(\mathfrak{U})}(X(\zeta))\right|\Rightarrow \\ \zeta\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right) &\prec {}_F\rho(\zeta).\end{aligned}$$

Proof. Assume that

$$\omega(\zeta)=\zeta\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right).\tag{12}$$

From (12) and (6), we have

$$\begin{aligned}\omega(\zeta)+\zeta\omega'(\zeta) &= \zeta\left[\frac{\mathfrak{i}}{\mathfrak{j}}\left(\frac{1}{\zeta}+\frac{\Gamma(\mathfrak{s}-\mathfrak{t})}{\Gamma(\mathfrak{r}-\mathfrak{t})}\sum_{n=0}^{\infty}\left(\frac{\mathfrak{i}+\mathfrak{j}(n+1)}{\mathfrak{i}}\right)^{\kappa+1}\frac{\Gamma(\mathfrak{r}+\mathfrak{t}n)}{\Gamma(\mathfrak{s}+\mathfrak{t}n)}a_n\zeta^n\right)-\right. \\ &\left.(1+\frac{\mathfrak{i}}{\mathfrak{j}})\left(\frac{1}{\zeta}+\frac{\Gamma(\mathfrak{s}-\mathfrak{t})}{\Gamma(\mathfrak{r}-\mathfrak{t})}\sum_{n=0}^{\infty}\left(\frac{\mathfrak{i}+\mathfrak{j}(n+1)}{\mathfrak{i}}\right)^{\kappa}\frac{\Gamma(\mathfrak{r}+\mathfrak{t}n)}{\Gamma(\mathfrak{s}+\mathfrak{t}n)}a_n\zeta^n\right)\right] \\ &+2\zeta\left(\frac{1}{\zeta}+\frac{\Gamma(\mathfrak{s}-\mathfrak{t})}{\Gamma(\mathfrak{r}-\mathfrak{t})}\sum_{n=0}^{\infty}\left(\frac{\mathfrak{i}+\mathfrak{j}(n+1)}{\mathfrak{i}}\right)^{\kappa}\frac{\Gamma(\mathfrak{r}+\mathfrak{t}n)}{\Gamma(\mathfrak{s}+\mathfrak{t}n)}a_n\zeta^n\right) \\ &= \zeta\left[\frac{\mathfrak{i}}{\mathfrak{j}}\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)-(1+\frac{\mathfrak{i}}{\mathfrak{j}})\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right]+2\zeta\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right). \\ &= \frac{\mathfrak{i}}{\mathfrak{j}}\left[\zeta\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)-\zeta\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right]+\zeta\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right).\end{aligned}\tag{13}$$

We observe from (11) and (13) that

$$\left|F_{\zeta(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta))}\left(\zeta\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)\right)\right|\leq\left|F_{X(\mathfrak{U})}(X(\zeta))\right|,$$

which implies

$$\begin{aligned}\left|F_{\psi(\mathbb{C}^2\times\mathfrak{U}^*)}(\omega(\zeta)+\zeta\omega'(\zeta))\right| &\leq \left|F_{X(\mathfrak{U})}(X(\zeta))\right|\leq \\ &\quad\left|F_{\rho(\mathfrak{U})}(\rho(\zeta)+\zeta\rho'(\zeta))\right|.\end{aligned}$$

Consequently, using Lemma 2 with $\varepsilon=1$ yields

$$\left|F_{\omega(\mathfrak{U})}(\omega(\zeta))\right|\leq\left|F_{\rho(\mathfrak{U})}(\rho(\zeta))\right|$$

$$\Rightarrow\left|F_{\zeta(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta))}\left(\zeta\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)\right)\right|\leq\left|F_{\rho(\mathfrak{U})}(\rho(\zeta))\right|,$$

i.e.,

$$\zeta\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right) \prec_F \rho(\zeta).$$

This completes the proof. □

Theorem 2. *Let ρ be a convex function in $\mathfrak{U}$, $\rho(0) = 1$, and*

$$X(\zeta) = \rho(\zeta) + \zeta\rho'(\zeta)(\zeta \in \mathfrak{U}).$$

If, for $\mathfrak{f} \in \Sigma$, satisfying

$$\begin{aligned} &\left| F_{\psi(\mathbb{C}^2\times\mathfrak{U}^*)}\left[\left(\tfrac{\mathfrak{r}-\mathfrak{t}}{\mathfrak{t}}\right)\left[\zeta\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r}+1,\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta) - \zeta\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right] + \zeta\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)\right]\right| \\ \leq &\left| F_{X(\mathfrak{U})}(X(\zeta))\right|, \end{aligned}$$

which implies

$$\left[\left(\frac{\mathfrak{r}-\mathfrak{t}}{\mathfrak{t}}\right)\left[\zeta\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r}+1,\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta) - \zeta\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right] + \zeta\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)\right] \prec_F X(\zeta),$$

then

$$\begin{aligned} \left| F_{\zeta(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta))}\left(\zeta\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)\right)\right| &\leq \left| F_{X(\mathfrak{U})}(X(\zeta))\right| \Rightarrow \\ \zeta\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right) &\prec {}_F\rho(\zeta). \end{aligned}$$

Proof. The proof is comparable to the proof of Theorem 1 using (7); therefore, we omitted it. □

Theorem 3. *Let ρ be a convex function in $\mathfrak{U}$, $\rho(0) = 1$, and*

$$X(\zeta) = \rho(\zeta) + \zeta\rho^{'}(\zeta)\zeta \in \mathfrak{U}.$$

If, for $\mathfrak{f} \in \Sigma$, satisfying

$$\begin{aligned} &\left| F_{(\zeta^2\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta))'}\left(\zeta^2\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)'\right| \leq \left| F_{X(\mathfrak{U})}(X(\zeta))\right| \\ \Rightarrow \ &\left(\zeta^2\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)^{'} \prec_F X(\zeta), \end{aligned} \tag{14}$$

then

$$\begin{aligned} \left| F_{(\zeta^2\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)'}\left(\zeta^2\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta\right)^{'}\right| &\leq \left| F_{X(\mathfrak{U})}(X(\zeta))\right| \\ &\Rightarrow \zeta\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta) \prec_F \rho(\zeta). \end{aligned}$$

Proof. Let

$$\omega(\zeta) = \zeta\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta). \tag{15}$$

From (15) and (5), we have

$$\begin{aligned} \omega(\zeta) + \zeta\omega'(\zeta) &= 2\left(1 + \frac{\Gamma(\mathfrak{s}-\mathfrak{t})}{\Gamma(\mathfrak{r}-\mathfrak{t})}\sum_{n=0}^{\infty}\left(\frac{\mathfrak{i}+\mathfrak{j}(n+1)}{\mathfrak{i}}\right)^{\kappa}\frac{\Gamma(\mathfrak{r}+\mathfrak{t}n)}{\Gamma(\mathfrak{s}+\mathfrak{t}n)}\,a_n\zeta^n\right) \\ &\quad + \left(-1 + \frac{\Gamma(\mathfrak{s}-\mathfrak{t})}{\Gamma(\mathfrak{r}-\mathfrak{t})}\sum_{n=0}^{\infty}n\left(\frac{\mathfrak{i}+\mathfrak{j}(n+1)}{\mathfrak{i}}\right)^{\kappa}\frac{\Gamma(\mathfrak{r}+\mathfrak{t}n)}{\Gamma(\mathfrak{s}+\mathfrak{t}n)}\,a_n\zeta^n\right) \\ &= 1 + \sum_{n=0}^{\infty}(n+2)\left(\frac{\mathfrak{i}+\mathfrak{j}(n+1)}{\mathfrak{i}}\right)^{\kappa}\frac{\Gamma(\mathfrak{r}+\mathfrak{t}n)}{\Gamma(\mathfrak{s}+\mathfrak{t}n)}\,a_n\zeta^{n+1}. \end{aligned}$$

We find that

$$\omega(\zeta) + \zeta\omega'(\zeta) = \left(\zeta^2\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)^{'}.$$

We have

$$\left| F_{(\zeta^2 \mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta))'} \left(\left(\zeta^2 \mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta) \right)' \right) \right| \leq \left| F_{X(\mathfrak{U})}(X(\zeta)) \right|,$$

which implies

$$\begin{aligned} \left| F_{\psi(\mathbb{C}^2 \times \mathfrak{U}^*)}(\omega(\zeta) + \zeta\omega'(\zeta)) \right| \quad &\leq \quad \left| F_{X(\mathfrak{U})}(X(\zeta)) \right| \leq \\ & \quad\; \left| F_{\rho(\mathfrak{U})}\left(\rho(\zeta) + \zeta\rho'(\zeta) \right) \right|. \end{aligned}$$

Consequently, using Lemma 3 with $n = v = 1$, we obtain

$$\begin{aligned} \left| F_{\omega(\mathfrak{U})}(\omega(\zeta)) \right| \quad &\leq \quad \left| F_{\rho(\mathfrak{U})}(\rho(\zeta)) \right| \Rightarrow \left| F_{\zeta \mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)} \left(\zeta \mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta) \right) \right| \\ &\leq \quad \left| F_{\rho(\mathfrak{U})}(\rho(\zeta)) \right|, \end{aligned}$$

which implies that

$$\zeta \mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta) \prec_F \rho(\zeta).$$

This completes the proof. □

Theorem 4. *Assume that* $X \in \mathcal{H}(\mathfrak{U})$ *with* $X(0) = 1$, *and*

$$\operatorname{Re}\left(1 + \frac{\zeta X''(\zeta)}{X'(\zeta)} \right) > -\frac{1}{2} (\zeta \in \mathfrak{U}),$$

If $\mathfrak{f} \in \Sigma$, *and the fuzzy differential subordination stated below is valid,*

$$\begin{aligned} & \left| F_{(\zeta \mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta))} \left(\zeta \mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta) \right) \right| \leq \left| F_{X(\mathfrak{U})}(X(\zeta)) \right| \\ \Rightarrow \; & \left(\zeta \mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta) \right) \prec_F X(\zeta), \end{aligned} \tag{16}$$

then

$$\begin{aligned} \left| F_{\zeta \mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)} \left(\zeta \mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta) \right) \right| \quad &\leq \quad \left| F_{X(\mathfrak{U})}(X(\zeta)) \right| \\ &\Rightarrow \quad \zeta \mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta) \prec_F \rho(\zeta), \end{aligned}$$

where $\rho(\zeta)$, *defined as*

$$\rho(\zeta) = \frac{1}{\zeta} \int_0^{\zeta} X(t)dt,$$

is the fuzzy best dominant and convex.

Proof. Suppose

$$\omega(\zeta) = \zeta \mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta). \tag{17}$$

We have $\omega(\zeta) \in \mathcal{H}[1,1]$. Let $X \in \mathcal{H}(\mathfrak{U})$ with $X(0) = 1$, and

$$\operatorname{Re}\left(1 + \frac{\zeta X''(\zeta)}{X'(\zeta)} \right) > -\frac{1}{2} (\zeta \in \mathfrak{U}).$$

Lemma 1 gives

$$\rho(\zeta) = \frac{1}{\zeta} \int_0^{\zeta} X(t)dt,$$

which is convex and satisfies (16), and

$$X(\zeta) = \rho(\zeta) + \zeta\rho'(\zeta)(\zeta \in \mathfrak{U}),$$

is the fuzzy best dominant.

Next,

$$\begin{aligned} \omega(\zeta)+\zeta\omega'(\zeta) &= 1+\sum_{n=0}^{\infty}(n+2)\left(\frac{\mathfrak{i}+\mathfrak{j}(n+1)}{\mathfrak{i}}\right)^{\kappa}\frac{\Gamma(\mathfrak{r}+\mathfrak{t}n)}{\Gamma(\mathfrak{s}+\mathfrak{t}n)}\,a_n\zeta^{n+1} \\ &= \left(\zeta^2\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)'. \end{aligned} \tag{18}$$

From (18), the fuzzy differential subordination (16) involves

$$\left|F_{\omega(\mathfrak{U})}(\omega(\zeta)+\zeta\omega'(\zeta))\right| \leq \left|F_{X(\mathfrak{U})}X(\zeta)\right|.$$

Consequently, using Lemma 3 with $v=1$, we find

$$\left|F_{\omega(\mathfrak{U})}(\omega(\zeta))\right| \leq \left|F_{\rho(\mathfrak{U})}(\rho(\zeta))\right|.$$

This completes the proof. □

Putting $1\leq B<A\leq -1$, and

$$X(\zeta)=\frac{1+A\zeta}{1+B\zeta}(\zeta\in\mathfrak{U}),$$

the next result can be deduced from Theorem 4.

Corollary 1. *Assume* $1\leq B<A\leq -1$, *and*

$$X(\zeta)=\frac{1+A\zeta}{1+B\zeta}(\zeta\in\mathfrak{U}),$$

with $X(0)=1$ *if, for* $\mathfrak{f}\in\Sigma$, *it satisfies*

$$\begin{aligned} &\left|F_{(\zeta\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta))}\left(\zeta\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)\right| \leq \left|F_{X(\mathfrak{U})}(X(\zeta))\right| \\ \Rightarrow\ &\left(\zeta\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)\prec_F \rho(\zeta), \end{aligned} \tag{19}$$

where $\rho(\zeta)$ *is defined as*

$$\rho(\zeta)=\frac{A}{B}+\frac{1-\frac{A}{B}}{B\zeta}\log(1+B\zeta),$$

it is the fuzzy best dominant and convex.

Example 1. *Assume* $(i)A=2\delta-1, 0\leq\delta<1, B=1$

$$X(\zeta)=\frac{1+(2\delta-1)\zeta}{1+\zeta}(\zeta\in\mathfrak{U}),$$

with $X(0)=1$, *if, for* $\mathfrak{f}\in\Sigma$, *it satisfies*

$$\begin{aligned} &\left|F_{(\zeta\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta))}\left(\zeta\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)\right| \leq \left|F_{X(\mathfrak{U})}(X(\zeta))\right| \\ \Rightarrow\ &\left(\zeta\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)\prec_F \rho(\zeta), \end{aligned}$$

where $\rho(\zeta)$ *is defined as*

$$\rho(\zeta)=2\delta-1+\frac{2(1-\delta)}{\zeta}\log(1+\zeta),$$

it is the fuzzy best dominant and convex. $(ii)\delta = 0$; *then*

$$X(\zeta) = \frac{1-\zeta}{1+\zeta}(\zeta \in \mathfrak{U}),$$

where

$$\rho(\zeta) = -1 + \frac{2}{\zeta}\log(1+\zeta),$$

is the fuzzy best dominant and convex.

Theorem 5. *Let ρ be convex in $\mathfrak{U}$,$\rho(0) = 1$, and*

$$X(\zeta) = \rho(\zeta) + \zeta\rho'(\zeta).$$

Let $\mathfrak{f} \in \Sigma$, and $\left(\frac{\zeta\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}\right)'$ be analytic in $\mathfrak{U}$. When

$$\begin{aligned} &\left| F_{\left(\frac{\zeta\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}\right)'}\left(\frac{\zeta\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}\right)' \right| \leq \left|F_{X(\mathfrak{U})}(X(\zeta))\right| \\ \Rightarrow\ &\left(\frac{\zeta\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}\right)' \prec_F X(\zeta), \end{aligned} \tag{20}$$

then

$$\left| F_{\left(\frac{\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}\right)}\left(\frac{\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}\right) \right| \leq \left|F_{\rho(\mathfrak{U})}(\rho(\zeta))\right|,$$

i.e.,

$$\frac{\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)} \prec_F \rho(\zeta).$$

Proof. Let

$$\omega(\zeta) = \frac{\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}. \tag{21}$$

Then, $\omega(\zeta) \in \mathcal{H}[1,1]$. Differentiating (21),

$$\omega'(\zeta) = \frac{\left(\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)'}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)} - \omega(\zeta)\frac{\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)'}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}.$$

Then,

$$\begin{aligned} &\omega(\zeta) + \zeta\omega'(\zeta) \\ =\ &\frac{\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)\left[\zeta\left(\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)' + \left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)\right] -}{\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)^2} \\ &\frac{\left(\zeta\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)'}{\left(\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)\right)^2} \\ =\ &\left(\frac{\zeta\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}\right)'. \end{aligned} \tag{22}$$

Utilizing (22) in (20), we can obtain

$$\left| F_{\left(\frac{\zeta\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}\right)'}\left(\frac{\zeta\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}\right)' \right| \leq \left|F_{X(\mathfrak{U})}(X(\zeta))\right|,$$

which implies

$$\left|F_{\omega(\mathfrak{U})}(\omega(\zeta)+\zeta\omega'(\zeta))\right| \leq \left|F_{X(\mathfrak{U}^*)}(X(\zeta))\right| \leq \left|F_{\rho(\mathfrak{U}^*)}\left(\rho(\zeta)+\zeta\rho'(\zeta)\right)\right|.$$

Consequently, using Lemma 3 with $\upsilon = 1$, we obtain

$$\left|F_{\left(\frac{\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}\right)}\left(\frac{\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}\right)\right| \leq \left|F_{\rho(\mathfrak{U})}(\rho(\zeta))\right|,$$

i.e.,

$$\frac{\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)} \prec_F \rho(\zeta).$$

This completes the proof. □

Theorem 6. *Let ρ be convex in $\mathfrak{U}$, such that $\rho(0) = 1$, and*

$$X(\zeta) = \rho(\zeta) + \zeta\rho'(\zeta).$$

Let $\mathfrak{f} \in \Sigma$, and $\left(\frac{\zeta\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r}+1,\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}\right)'$ be holomorphic in $\mathfrak{U}$. If

$$\left|F_{\left(\frac{\zeta\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r}+1,\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}\right)'}\left(\frac{\zeta\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r}+1,\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}\right)'\right| \leq \left|F_{X(\mathfrak{U})}(X(\zeta))\right|$$
$$\Rightarrow \left(\frac{\zeta\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r}+1,\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}\right)' \prec_F X(\zeta),$$

then

$$\left|F_{\left(\frac{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r}+1,\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}\right)}\left(\frac{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r}+1,\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}\right)\right| \leq \left|F_{\rho(\mathfrak{U})}(\rho(\zeta))\right|,$$

i.e.,

$$\frac{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r}+1,\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)} \prec_F \rho(\zeta).$$

Proof. The proof is comparable to the proof of Theorem 5, so we omitted it. □

4. Conclusions

We have successfully used the integral operator $\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}$(r,s,t) given by relation (5), using the Erdélyi-Kober fractional integral operator, for the meromorphic function of the operator studied by El-Ashwah [24], in our current study of applications of fuzzy differential subordination in GFT. The fact that there are differential subordinations and superordinations offers another direction for future research on this topic, see [33,35]. We exclusively analyzed and examined first-order differential subordinations and differential superordinations in this presentation. In the first three theorems, certain fuzzy differential subordinations were given. In theorem four, we obtained integral representation for the best domimant and followed the theorem by a corollary and example. In theorems five and six, we obtained fuzzy differential subordination for $\frac{\mathfrak{L}^{\kappa+1}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}$ and $\frac{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r}+1,\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}{\mathfrak{L}^{\kappa}_{\mathfrak{i},\mathfrak{j}}(\mathfrak{r},\mathfrak{s},\mathfrak{t})\mathfrak{f}(\zeta)}$.

The different findings presented in this work are novel and would spur additional investigation into the area of GFT.

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