



Article

Fractional-Order African Vulture Optimization-Based Beamforming for Planar Antenna Array

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Abstract

Beamforming plays a central role in enhancing the performance of communication systems; however, suppressing sidelobes in planar antenna arrays (PAAs) while maintaining a compact aperture remains a challenging nonlinear optimization problem. This article presents a two-dimensional (2D) beamforming synthesis framework for PAAs based on the Fractional-Order African Vulture Optimization Algorithm (FO-AVOA), with the objective of minimizing the peak sidelobe level (PSLL) through the joint optimization of amplitude excitations and element placements. The proposed method is benchmarked against established metaheuristic optimizers, including Particle Swarm Optimization (PSO), the Gravitational Search Algorithm (GSA), hybrid PSO–GSA (PSOGSA), the Runge–Kutta Optimizer (RUN), the Slime Mould Algorithm (SMA), Harris Hawks Optimization (HHO), and the baseline African Vulture Optimization Algorithm (AVOA). Simulation results demonstrate that the FO-AVOA, coupled with the proposed 2D formulation, yields superior sidelobe suppression relative to the competing approaches, achieving a lower PSLL with fewer radiating elements, thereby reducing array complexity and overall implementation cost. The obtained results validate the suitability of the FO-AVOA for solving PAA in the context of BFA beamforming and suggest the potential utility of the FO-AVOA for pattern synthesis for other array shapes in various communication systems.

Keywords: beamforming; fractional-order African vulture optimization algorithm (FO-AVOA); meta heuristic optimization techniques; planar antenna array (PAA); peak sidelobe level (PSLL)

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1. Introduction

The progress towards 6G is moving at a break-neck speed and is accompanied by increasing requirements for antenna designs by which antenna designers in the future will require precise directional capabilities and reduced stray lobes and beams in their designs. In the present wireless environment, one of the most prominent and impending technologies used in smart antenna systems is increasing coverage and increasing service quality in a particular region by increasing efficiency and power control using these systems [1,2]. Among the most influential technological advancements are those related to adaptive beamforming (ABF) and massive MIMO. These techniques are extensively employed to enhance the equivalent capacity and the link robustness. Their performances

remain strongly dependent on the design and configurations of the corresponding antenna array according to the considered task, and the main emphasis is cast on the control of the radiation pattern, including the reduction in the sidelobes and the realization of null steering performances [3–7].

Linear antenna arrays and circular antenna arrays are the two most common configurations for real-world wireless systems. Consequently, most of the literature focuses on beamforming optimization problems for LAAs and CAAs by including array parameters such as excitation amplitudes and positions of elements to make the radiation performance even higher [8–23]. LAAs are preferred due to their simplicity in technology and ease of implementation, while CAAs are preferred due to offering more flexibility in beam steering problems but with a penalty of complexity in design and construction. Pushing antenna directivity is a key lever to improve energy efficiency in communications, which has consequently shifted interest towards more directive array geometries. In this regard, PAAs are of particular interest as they can provide sharp and symmetric radiation patterns. PAAs are applied in remote sensing, target detection and tracking radars, as well as in satellite links. Most of the previous works related to the optimization of PAAs have considered the optimization of only one parameter, such as excitation amplitudes or element spacing [24–35]. In contrast, this work formulates the PAA beamforming problem as a two-dimensional optimization problem, jointly optimizing the amplitude excitations and the element positions to achieve improved sidelobe suppression and finer control of the beams.

Array beamforming in antenna arrays entails the aspect of choosing variables for designing a radiation pattern of a certain kind. However, realizing a target pattern as a real beam in the antenna array entails a nonlinear optimization challenge, since factors in the array show complex relations yet do not appear in a straightforward manner in the problem domain of nonlinear optimization. Hence, the optimization process aims to maximize the performance of the radiation pattern and minimize the PSLR value. In recent years, traditional sidelobe ratio reduction techniques have proved their merit. The mention may be made of techniques like Taylor synthesis, Chebyshev synthesis, or convex optimizations [36–38]. However, when one considers large-size arrays or when tight constraints are applied, one reaches their practical limits. For example, Chebyshev synthesis becomes computationally intensive when it involves large-size arrays, or convex optimization schemes require simplification or relaxation of constraints, thereby lowering their practical utility. In order to satisfy the increased performance requirements in modern communication systems, scientists have shifted their emphasis more towards the use of modern optimization approaches using more advanced forms of beamforming techniques. Among such optimization techniques, swarm intelligence and evolutionary optimization techniques appear to be more helpful and advantageous for designing antenna array systems [39]. Using these advantages, an optimization approach using a metaheuristic optimization technique is employed in the proposed work to optimize the beamforming for a two-dimensional antenna array system.

Optimization algorithms with a wide range of applications have been developed. To name a few examples in real applications using different algorithms: HHO has been successfully used for real-world applications in manufacturing process optimization and pattern recognition enhancement, optimizing power quality, and pharmaceutical molecule direction in references [40–42]. SMA was successfully developed in reference [43], and since then has been applied in a range of optimization methods such as optimizing schedules in reference [44]. PSO became widely established in metaheuristic search methods in references [45–48]. RUN applications have been discussed in references [49,50]. Apart from these algorithms, some combined search methods have been developed to increase

search efficiency; one such combined method is PSOGSA, successfully applied in several applications in references [51,52].

Of these metaheuristics, the AVOA has gained increasing attention because of its simplicity and high search ability [53,54]. Nevertheless, the problem associated with the AVOA is its potential deficiency in population diversity or the balance between the exploration and exploitation processes that might affect its ability to find the global optimum. To address these problems, different improvement methods have been introduced for enhancing its ability to find the global optimum [55–58]. Motivated by these findings, in this research, beamforming optimization in planar antenna arrays is conducted via an FO-AVOA, with the primary objective of reducing the PSLL, which is a key parameter in the optimization of antenna arrays in 6G communication technology. However, the beamforming optimization problem is formulated as a two-dimensional optimization problem to maximize the excitation amplitudes and locations. The efficacy of the FO-AVOA is determined in a simulation study that also includes comparisons between the FO-AVOA, PSO, GSA, PSOGSA, RUN, SMA, HHO, and standard AVOA.

1.1. Paper Contributions

The main contributions of this paper are summarized as follows:

- A beamforming synthesis-optimization problem is formulated for PAAs, where the fitness function is designed to achieve radiation-pattern shaping and effective PSLL suppression for wireless communication applications.
- To address the limitations of the conventional AVOA algorithm, the proposed framework employs the FO-AVOA algorithm to solve the formulated PSLL suppression problem and enhance global search capability and convergence behaviour.

Comprehensive simulations are conducted to validate the effectiveness and robustness of the proposed FO-AVOA-based approach for PSLL reduction in PAAs. The evaluation includes: (i) single-parameter optimization to assess the impact of individual design variables, and (ii) a 2D optimization strategy that jointly optimizes element amplitudes and positions to further verify the efficacy of the proposed method for radiation-pattern synthesis.

1.2. Roadmap

The organization of this paper is as follows. In Section 2, a discussion of the relevant literature for antenna array pattern synthesis based on optimization techniques for different array designs is presented. The design of a PAA and its formula for the array factor, as well as the function for the fitness used in the optimization procedure, are explained in more depth in Section 3. Then, the definition for an FO-AVOA is introduced in Section 4, coupled with a brief reference to its predecessor, the AVOA. The simulation results of this work are presented in Section 5, based on simulations for single-parameter optimization as well as a proposed 2D approach compared against a range of baseline algorithms. Finally, conclusions for this work can be found in Section 6.

2. Literature Review

A synthesis of the beam pattern of the antenna array has also been widely researched for various types of array geometries using evolutionary and swarm intelligence-based algorithms, mainly focusing on the minimization of the PSLL and sometimes null steering. For example, the synthesis of the non-uniform LAA using the GA has been introduced in [8,9]. In these studies, null steering and the minimization of the PSLL are taken into consideration at the same time. The synthesis of the array parameters using the CS algorithm is introduced for the minimization of the PSLL of the LAA and CAA in [10,11]. PSO has

also been used for the synthesis of the beam pattern of the array for the steering of the beam and the minimization of the PSLL, as introduced in [12–14]. The GSA is used for the synthesis of the dipole-type LAA, as introduced in [15], while the application of the GSA is extended for the synthesis of the array pattern for the adaptive beamforming problem, as introduced in [16]. The synthesis of the array pattern for the minimization of the PSLL of the LAA using the IBBO is introduced in [17]. Moreover, the synthesis of the array pattern using the Mayfly (MF) algorithm for the uniform and sparse array configurations of the LAA is introduced in [18], while the application of the Grasshopper Optimization Algorithm (GOA) is introduced for the synthesis of the broadband whip antenna array, as introduced in [19]. The synthesis of the array pattern using the SSA is introduced for the minimization of the PSLL of the LAA, as introduced in [20]. In addition, the synthesis of the array pattern using the GWO is introduced for array pattern synthesis, as introduced in [21,22], while the application of the Differential Search Algorithm (DSA) is introduced for CAA synthesis, as introduced in [23].

Although considerable progress has been made regarding linear and circular antenna arrays, the problem gets more complicated with a planar antenna array, due to its increased dimensionality and correlation between the variables. When the requirement is to nullify the sidelobes and simultaneously control the main beam of the radiation pattern, the correlations increase. PAAs are primarily employed for the generation of a highly directive and symmetric far-field radiation pattern. Therefore, PAAs are well suited for satellite communications, search and track radars, and remotely sensed platforms [24–26]. Recently, a lot of researchers have employed metaheuristic optimization techniques for planar array synthesis problems. For example, Biogeography-Based Optimization (BBO) techniques have been employed to optimize different planar array radiation patterns [27], and Improved Chicken Swarm Optimization (ICSO) techniques have been employed to minimize the PSLL [28]. Moth Flame Optimization (MFO) techniques have been employed in planar array pattern synthesis problems [29]. There are also sparse and thinned planar array design techniques using Teaching and Learning-Based Optimization (TLBO) techniques [32], and a modified Brain Storm Optimization (BSO) approach can be employed in planar thinned-array synthesis problems [33]. Moreover, Wind-Driven Optimization (WDO) techniques were employed to minimize the PSLL by adjusting amplitude and phase excitations [34], and Genetic Algorithms (GAs) were employed in uniform planar arrays with a focus on minimizing sidelobes [35].

Overall, the literature confirms the effectiveness of metaheuristic optimization for array pattern synthesis across multiple geometries. Nevertheless, the simultaneous optimization of multiple PAA design variables remains less explored compared with one-parameter formulations, particularly under a unified framework that jointly optimizes excitation and spatial configuration for enhanced sidelobe suppression.

3. Array Factor and Fitness Function

The radiation performance of an antenna array is determined by its geometry (e.g., linear, planar, or circular), the inter-element spacings (element locations), and the excitation coefficients (amplitudes/phases). These variables collectively shape the synthesized pattern, including the main lobe width and sidelobe behaviour. This study adopts a rectangular PAA with potentially non-uniform element positions and controllable excitations, and uses its array factor as the basis for beamforming optimization.

This section presents the PAA model adopted in this study and derives the corresponding array-factor expression used for beamforming optimization. In addition, the fitness function employed to guide the optimization process toward PSLL suppression and radiation-pattern synthesis is formulated and described.

3.1. Array Factor for PAA

Consider first an LAA composed of M elements located along the x -axis with symmetric excitations. The array factor of a symmetric LAA can be expressed as follows [59]:

$$AF_{LAA}(\theta, \varphi) = 2 \sum_{m=1}^M A_x e^{j(m-1)(kd_x \sin \theta \cos \varphi + \beta_x)}, \quad (1)$$

where M denotes the number of elements along the x -axis; A_x is the amplitude excitation of the m^{th} element, β_x represents the phase excitation; d_x is the element position along the x -axis; θ is the elevation angle; φ is the azimuth angle; and k is the wave number defined as $k = 2\pi/\lambda$, with λ being the wavelength.

Next, if N such linear arrays are placed adjacent to each other along the y -direction, a rectangular PAA is obtained, as illustrated in Figure 1. Assuming uniform unitary excitation, the overall array factor of the PAA can be written as follows:

$$AF_{LAA}(\theta, \varphi) = A_{xy} \sum_{m=1}^M A_x e^{j(m-1)(kd_x \sin \theta \cos \varphi + \beta_x)} \sum_{n=1}^N A_y e^{j(n-1)(kd_y \sin \theta \sin \varphi + \beta_y)} \quad (2)$$

with

$$A_{xy} = A_x A_y \quad (3)$$

where d_y and β_y denote the element position and phase excitation along the y -axis, respectively, and A_y is the amplitude excitation contribution along the y -axis.

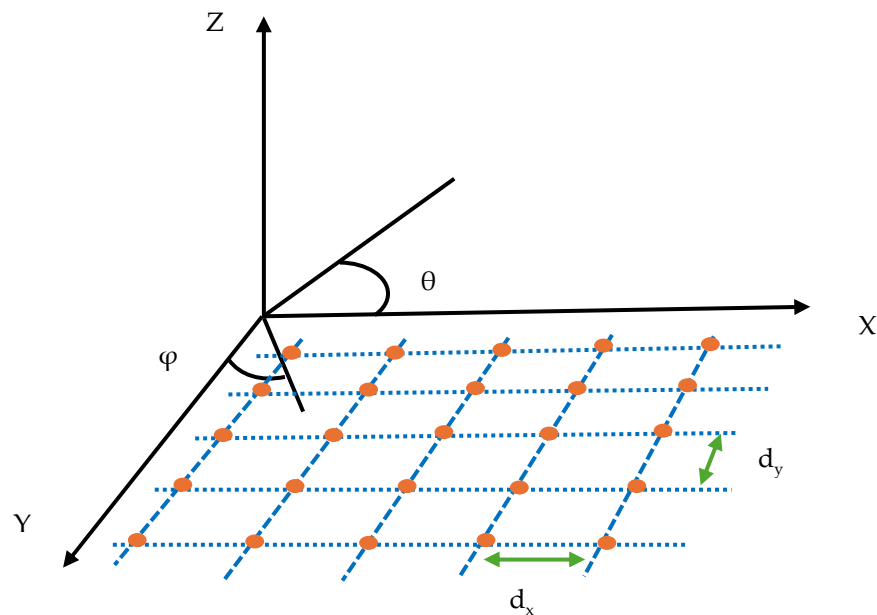


Figure 1. PAA with $M \times N$ elements for spacing d_x and d_y .

Accordingly, the beamforming synthesis problem considered in this work is to determine the set of amplitude excitations and element positions on the x - y plane that produce a radiation pattern with a minimum PSLL over specified angular regions. Using the adopted optimization algorithm, the parameters governing the array factor are iteratively adjusted to achieve the desired sidelobe suppression performance.

3.2. Fitness Function

Several performance metrics can be considered when defining the fitness function for antenna array synthesis, including directivity, gain, PSLL, aperture size, and implementation constraints such as weight, depending on the target application. In this research, the goal is to optimize the PAA shape in such a way that its PSLL is minimized, leading to efficient interference removal and mitigation and less noise in the received signal. This is made possible through efficient regulation of the parameters in the array factor (AF).

Accordingly, the optimization objective is defined through a normalized fitness function based on the peak sidelobe magnitude of the synthesized radiation pattern, expressed in decibels (dB), as follows:

$$\text{Fitness function} = \min(\max\{20\log |AF_{\text{PAA}}(\theta, \varphi)|\}), \quad (4)$$

This formulation seeks the design solution that minimizes the PSLL over the specified angular region.

4. Optimization Techniques

This section describes the optimization methods employed for PAA radiation-pattern synthesis. Specifically, the AVOA and its enhanced variant, the FO-AVOA, are presented in detail.

4.1. Original AVOA

The AVOA is a population-based metaheuristic that takes its inspiration from the foraging and social interactions of African vultures. The model proposes a starvation mechanism to drive the search process: candidate solutions follow the two strongest individuals in the group. Over time, their hunger changes, tuning the balance between global explorations of the search space and exploitation of promising local areas, and it also triggers different position update strategies. Adaptive switching enables the AVOA to roam complex, nonlinear optimization landscapes more effectively, hence its robustness in solving hard optimization problems.

4.1.1. Starvation Rate and Phase Control

The AVOA mimics the foraging behaviour of African vultures using a hunger (starvation) rate $H_i(k)$, which controls the transition between exploration (global search) and exploitation (local refinement). In general, large hunger values encourage exploration across the search space, while small hunger values intensify exploitation around promising solutions. At each iteration, the AVOA identifies two elite solutions (best and second-best) in Equation (5) and selects one of them as a leader to guide the other vultures:

$$L_i(k) = \begin{cases} \text{Elite}_1(k), & \text{if } p_0 \geq r \\ \text{Elite}_2(k), & \text{otherwise} \end{cases} \quad (5)$$

where $r \sim U[0,1]$ and p_0 is typically set close to 0.8.

4.1.2. Exploration Phase

When the hunger rate indicates exploration (commonly $H_i(k) \geq 1$), vultures diversify their search to discover new promising regions. Two exploration strategies are applied in Equations (7) and (8):

$$X_i(k+1) = \begin{cases} \text{Strategy 1}, & \text{if } p_1 \geq r_{p1} \\ \text{Strategy 2}, & \text{if } p_1 < r_{p1} \end{cases} \quad r_{p1} \sim U[0,1] \quad (6)$$

Strategy 1 (leader distance exploration):

$$X_i(k+1) = L_i(k) - D_i(k)H_i(k), D_i(k) = |\lambda L_i(k) - X_i(k)| \quad (7)$$

where $\lambda \in [-2, 2]$ is random.

Strategy 2 (random-range exploration):

$$X_i(k+1) = L_i(k) - H_i(k) + r((ub - lb)r + lb) \quad (8)$$

where ub and lb are the variable bounds.

4.1.3. Exploitation Phase

When the hunger rate decreases, vultures shift to exploitation and refine solutions around elite regions.

(a) Moderate exploitation: $0.5 \leq H_i(k) < 1$

$$X_i(k+1) = \begin{cases} \text{Strategy 3,} & \text{if } p_2 \geq r_{p2} \\ \text{Strategy 4,} & \text{if } p_2 < r_{p2} \end{cases} \quad r_{p2} \sim U[0,1] \quad (9)$$

Strategy 3 (protective/local correction):

$$X_i(k+1) = D_i(k) - (H_i(k) + r) - d_i(k), d_i(k) = L_i(k) - X_i(k) \quad (10)$$

Strategy 4 (spiral exploitation):

$$X_i(k+1) = L_i(k) - (S_1 - S_2) \quad (11)$$

with

$$S_1 = L_i(k) \left(\frac{rX_i(k)}{2\pi} \right) \cos(\alpha_s X_i(k)), S_2 = L_i(k) \left(\frac{rX_i(k)}{2\pi} \right) \sin(\alpha_s X_i(k)) \quad (12)$$

where α_s is a spiral-control parameter.

(b) Strong exploitation: $H_i(k) < 0.5$

$$X_i(k+1) = \begin{cases} \text{Strategy 5,} & \text{if } p_3 \geq r_{p3} \\ \text{Strategy 6,} & \text{if } p_3 < r_{p3} \end{cases} \quad r_{p3} \sim U[0,1] \quad (13)$$

Strategy 5 (elite averaging):

$$X_i(k+1) = \frac{Q_1 + Q_2}{2} \quad (14)$$

where Q_1 and Q_2 are candidate positions constructed around $Elite_1(k)$, $Elite_2(k)$, and $L_i(k)$ to pull solutions toward the best regions while maintaining diversity.

Strategy 6 (Lévy flight exploitation):

$$X_i(k+1) = L_i(k) - |\delta_i(k)|H_i(k)L(d)X_i(k+1) = \frac{Q_1 + Q_2}{2} \quad (15)$$

where $L(d)$ is a Lévy flight step and $\delta_i(k)$ controls the step direction and magnitude.

4.2. Limitations of the Original AVOA

Although the original AVOA has been proven to possess competitive optimization capabilities, several inherent limitations of the algorithm can be identified. First, the hunger rate regulation is controlled by a linear decay mechanism, which is not sufficient to model the nonlinear convergence behaviour that is usually encountered in complex, high-dimensional search spaces. As a consequence, the algorithm tends to prematurely transition into exploitation or remain stuck in an exploratory mode, thereby causing an unbalanced search process. Second, during the exploitation process, the algorithm is based on

predefined stochastic movement patterns, which have limited adaptability. As a consequence, the predefined update pattern might limit the algorithm's capability of performing accurate local optimization, thereby decreasing solution quality close to the optimum region. Third, there is no memory-based learning mechanism incorporated into the AVOA, which causes the search process of the algorithm to rely exclusively on the current iteration information. As a consequence, the algorithm is prone to oscillating search trajectories and convergence instability, especially when handling dynamic, nonlinear optimization problems.

4.3. FO-AVOA

To mitigate the limitations of the baseline AVOA, this study adopts a fractional-order enhanced variant, namely the FO-AVOA. The proposed enhancements are tailored to the PAA synthesis problem, where the search space is highly nonlinear and the design variables (element excitations and positions) are strongly coupled. In the FO-AVOA, three complementary mechanisms are incorporated to improve the balance between exploration and exploitation and to strengthen convergence reliability.

First, a nonlinear control strategy is introduced to regulate the transition between global exploration and local exploitation in an adaptive and smooth manner, thereby preventing abrupt behavioural shifts that may degrade search efficiency. Second, an adaptive Lévy flight exploitation strategy is incorporated in an attempt to promote local refinement within potentially optimal regions and concurrently preserve a degree of population diversity to prevent premature convergence. Third, fractional-order calculus is incorporated within the position update formula in an effort to provide a memory effect and thus allow for the exploitation of past search behaviour during the optimization process. The memory update approach can improve local optimal escape properties and is beneficial in antenna array pattern synthesis tasks.

In the FO-AVOA, the fractional-order parameter α , where $0 < \alpha \leq 1$, governs the degree of memory retained in the search dynamics. By incorporating a fractional-order difference operator into the vultures' position updates, the algorithm effectively combines non-local search behaviour with historical guidance, which leads to more robust global surveying and more reliable local intensification. As a result, the FO-AVOA provides improved search adaptability and convergence performance compared with the standard AVOA when applied to the joint optimization of PAA element excitations and placements for sidelobe suppression.

In this work, the fractional operator is formulated based on the Grünwald–Letnikov (G–L) definition of the fractional derivative, which is employed to realize the memory effect within the FO-AVOA update mechanism.

$$D^\alpha[f(t)] = \lim_{h \rightarrow 0} h^{-\alpha} \sum_{j=0}^{\infty} (-1)^j \binom{\alpha}{j} f(t - jh). \quad (16)$$

For a signal $x(t)$, the Grünwald–Letnikov fractional derivative of order $\alpha \in (0, 1]$ can be approximated numerically as

$$D_{GL}^\alpha x(t_k) \approx \frac{1}{(\Delta t)^\alpha} \sum_{j=0}^m (-1)^j \binom{\alpha}{j} x(t_{k-j}), \quad (17)$$

where $t_k = k\Delta t$, m is the memory length, and

$$\binom{\alpha}{j} = \frac{\Gamma(\alpha + 1)}{\Gamma(j + 1)\Gamma(\alpha - j + 1)}. \quad (18)$$

A fractional-order position update is used in the FO-AVOA: the fractional memory is implemented using a weighted history of past position increments,

$$\Delta X_i^{(\alpha)}(t) = \sum_{j=0}^m w_j^{(\alpha)} \Delta X_i(t-j), \quad (19)$$

where $w_j^{(\alpha)} = (-1)^j \binom{\alpha}{j}$, and the vulture position is updated as

$$X_i(t+1) = X_i(t) + \eta \Delta X_i^{(\alpha)}(t), \quad (20)$$

where η is a scaling factor (step-size) and controls the memory depth.

To reduce computational cost, the coefficients are computed recursively:

$$w_0^{(\alpha)} = 1, w_j^{(\alpha)} = \left(1 - \frac{\alpha+1}{j}\right) w_{j-1}^{(\alpha)}, j \geq 1. \quad (21)$$

These additions, provide an explicit calculation formula for the fractional derivative effect used in the proposed optimizer.

4.3.1. Starvation Rate and Phase Control

The FO-AVOA emulates the adaptive foraging behaviour of African vultures, where the search actions evolve according to a hunger-driven state variable, commonly referred to as the starvation (hunger) rate. It plays a controlling role in the process of exploration (or global search) vs. exploitation (or local search) in the optimization process itself, because a high level of starvation corresponds to a global search for promising regions of the search space, whereas a low level of starvation corresponds to the intensification of the search around quality solutions in the search space.

Through adaptively modifying the starvation rate in each iterative update, the FO-AVOA optimizes its search pressure according to the evolving status of the optimization process, thereby enhancing its capacity to deal with highly nonlinear and multi-dimensional objective functions typical of PAA synthesis problems. The control function concerning phases helps to promote population diversity, alleviate convergence to a local optimum prematurely, and increase convergence stability to better solutions exhibiting stronger sidelobe suppression performance. The procedural flow of the FO-AVOA is shown in Figure 2.

Figure 2 illustrates the whole process of the proposed FO-AVOA. Different from the traditional AVOA, the FO-AVOA has a clear main loop, in which the fractional-order update process is incorporated, and the past positions are used to contribute to the current search process. The memory function based on the fractional calculus method is used together with the starvation rate control and the Lévy flight exploration in the proposed FO-AVOA.

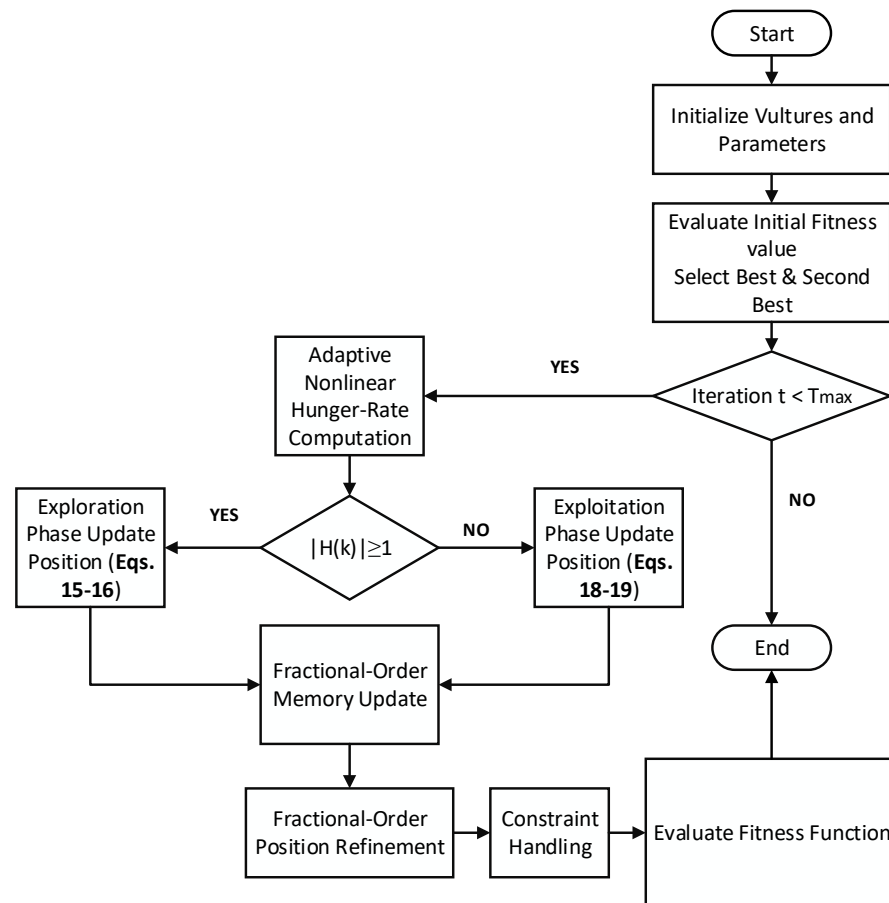


Figure 2. FO-AVOA flowchart.

In particular, the flowchart emphasizes that this includes a fractional-order memory update within the position update phase, an adaptive nonlinear starvation rate computation for effective phase regulation, and a fractional-order position refinement step applied prior to fitness evaluation. These integrated components collectively enhance convergence stability, preserve population diversity, and improve the robustness of the search process for planar antenna array beamforming optimization. The mathematical formulation of the starvation rate model is provided in Equation (22).

$$H_i^\alpha(k) = (2 \times r^\alpha + 1) \times \eta \times \left(1 - \left(\frac{k}{K}\right)^\beta\right) + \Delta_k^\alpha, \quad (22)$$

and

$$\Delta_k^\alpha = \xi \times \left[\sin^\omega \left(\frac{\pi}{2} \times \left(\frac{k}{K}\right)^\beta \right) + \cos \left(\frac{\pi}{2} \times \left(\frac{k}{K}\right)^\beta \right) - 1 \right] + \gamma \times \sum_{j=1}^M w_j \times H_i(k-j), \quad (23)$$

where $w_j = \left(1 - \frac{j}{M}\right)^\alpha$, (decaying memory weights), M is memory length, and γ is memory weight ($0 < \gamma < 1$).

In the FO-AVOA, the expression $H_i^\alpha(k)$ represents the fractional-order hunger rate of the i -th vulture at the k -th iteration and plays a pivotal role in controlling the adaptive process of the search agent. The value of Δ_k^α is a predetermined constant that needs to be defined before the start of the algorithm process, and k and T are the current and maximum number of iterations, respectively. The variable r takes the form of a uniformly distributed random number ranging between 0 and 1. Furthermore, the stochastic variables ξ and η are generated between the defined boundaries $[-2, 2]$ and $[-1, 1]$, respectively.

When the FO-AVOA is operated in the conventional setup, the value of the weighting coefficient w is maintained at 2.5.

A negative value of the hunger rate, i.e., $H_i^\alpha(k) < 0$, indicates that the vulture is starving and must search for new candidate solutions. As the value of η , denoted as “ η ”, tends to zero, the strategy transitions from exploration to exploitation, focusing the search process on the most favourable areas of the solution space identified so far. To represent the leadership exhibited by vultures as they move together, the strategy developed by the “AVOA” either picks the “best” candidate or the second “best” as the leader that directs the motion of the swarm formed by the vultures. This is defined by Equation (24).

$$L_i^\alpha(k) = \begin{cases} Elite_1^\alpha(k), & \text{if } p_0^\alpha \geq r^\alpha, \\ Elite_2^\alpha(k), & \text{otherwise,} \end{cases} \quad (24)$$

$$p_0^\alpha = p_0 + \mu \times D^\alpha[p_0].$$

Here, $L_i^\alpha(k)$ symbolizes a randomly chosen vulture from the existing population at the k -th step to add to the stochastic exploration phenomenon exhibited by the algorithm. Variables $Elite_1^\alpha(k)$ and $Elite_2^\alpha(k)$ are the representation of the most superior or best as well as the second-best vultures chosen from the existing pool depending upon their fitness values. These superior individuals help to direct the rest of the vultures towards the most apt regions of the search space to achieve a balance between exploration and exploitation processes. Here, control constant p_0^α takes a value of 0.8.

4.3.2. Exploration Phase

When $H_i^\alpha(k) \geq 1$, the vultures are diffused in the different parts of the search space to forage for food. This means that the FO-AVOA has entered into the exploration phase. During this process, the vultures exhibit dynamic movement patterns, especially to reinforce global search diversity and to avoid its early convergence. Inspired by these two modes of behaviour, two different exploration strategies were performed in the mathematical formulation of the FO-AVOA, expressed as follows:

$$X_i(k+1) = \begin{cases} \text{Strategy 1, if } p_1^\alpha \geq r_{p_1}^\alpha, \\ \text{Strategy 2, if } p_1^\alpha < r_{p_1}^\alpha, \end{cases} \quad p_1^\alpha = p_1 \times \exp\left(-\alpha \times \left(\frac{k}{K}\right)^\beta\right), \quad (25)$$

Strategy 1 (leader distance exploration with fractional coupling):

$$X_i(k+1) = L_i^\alpha(k) - D_i^\alpha(k) \times H_i^\alpha(k) + \lambda_\alpha \times D^\alpha[L_i^\alpha(k) - X_i(k)]$$

$$D_i^\alpha(k) = |\lambda \times L_i^\alpha(k) - X_i(k)|^\alpha, \quad (26)$$

where $\lambda \in [-2, 2]$ is random.

Strategy 2 (random-range exploration with fractional memory):

$$X_i(k+1) = L_i^\alpha(k) - H_i^\alpha(k) + r^\alpha \times ((ub - lb) \times r^\alpha + lb) + \zeta \times D^\alpha[X_i(k)]. \quad (27)$$

where ub and lb are the boundaries of the decision variables.

In Equation (25), $X_i(k+1)$ refers to the new location of the i -th vulture, whereas the control coefficient p_1^α is set to 0.6. The variable $r_{p_1}^\alpha$ in Equation (25) is a uniformly distributed random number between 0 and 1. In Equation (26), the variable $D_i^\alpha(k)$ refers to the Euclidean distance of the current vulture from the chosen leader vulture, which gives information regarding the direction of movement, whereas the random variable λ is generated in the range of $[-2, 2]$.

4.3.3. Exploitation Phase

When the fractional hunger rate is between 0.5 and 1, regarding the individual agent, the swarm enters the exploitation phase. During the exploitation phase, the vultures con-

dense their attention on the most favourable spots, improving the solutions. This is analogous to how real vultures protect food sources and compete with each other. To simulate the process of intensification, the phase of exploitation employs two adaptive search methods that control the local search. These equations are expressed as follows:

$$X_i(k+1) = \begin{cases} \text{Strategy 3, if } p_2^\alpha \geq r_{p_2}^\alpha, \\ \text{Strategy 4, if } p_2^\alpha < r_{p_2}^\alpha, \end{cases} \quad (28)$$

where p_2^α is typically 0.4 and $r_{p_2}^\alpha \sim \mathcal{U}[0,1]$.

Strategy 3 (protective behaviour with fractional correction):

$$\begin{aligned} X_i(k+1) &= D_i^\alpha(k) - (H_i^\alpha(k) + r^\alpha) - d_i^\alpha(k), \\ d_i^\alpha(k) &= L_i^\alpha(k) - X_i(k) + \delta \times D^\alpha[L_i^\alpha(k) - X_i(k)], \end{aligned} \quad (29)$$

where δ is a tuning coefficient.

Strategy 4 (spiral exploitation):

$$\begin{aligned} X_i(k+1) &= L_i^\alpha(k) - (S_1^\alpha - S_2^\alpha), \text{ with} \\ \begin{cases} S_1^\alpha = L_i^\alpha(k) \times \left(\frac{r^\alpha \times X_i(k)}{2\pi} \right) \times \cos(X_i(k) \times \alpha), \\ S_2^\alpha = L_i^\alpha(k) \times \left(\frac{r^\alpha \times X_i(k)}{2\pi} \right) \times \sin(X_i(k) \times \alpha). \end{cases} \end{aligned} \quad (30)$$

In Equation (28), the control coefficient, p_2^α , is set at 0.4, while the random variable, $r_{p_2}^\alpha$, is uniformly distributed between 0 and 1. In Equation (29), the superscript α refers to an elite vulture chosen as an agent for the search process exploited by $L_i^\alpha(k)$. The exploitation subphase is similar to vulture behaviour, which tends to group around a food source, which is then protected and competed for, eventually leading to vultures constricting their search patterns around a region of interest before optimization ends.

In the exploitation substage, where the hunger rate $H_i^\alpha(k) < 0.5$, the vultures demonstrate high competition and accumulation of food sources. This behaviour is mathematically represented as follows:

$$X_i(k+1) = \begin{cases} \text{Strategy 5, if } p_3^\alpha \geq r_{p_3}^\alpha, \\ \text{Strategy 6, if } p_3^\alpha < r_{p_3}^\alpha, \end{cases} \quad (31)$$

with $p_3^\alpha \approx 0.4$, $r_{p_3}^\alpha \sim \mathcal{U}[0,1]$.

Strategy 5 (elite averaging with fractional refinement):

where Q_1^α and Q_2^α are constructed from $Elite_1^\alpha$, $Elite_2^\alpha$, and $L_i^\alpha(k)$ to pull solutions toward the best regions while maintaining diversity (as in the previous formulation).

Strategy 6 (Lévy flight-based exploitation with fractional order):

$$\begin{aligned} X_i(k+1) &= \frac{Q_1^\alpha + Q_2^\alpha}{2} + \eta_\alpha \times D^\alpha \left[\frac{Q_1^\alpha + Q_2^\alpha}{2} \right], \\ \text{where } \begin{cases} Q_1^\alpha = Elite_1^\alpha(k) - \frac{Elite_1^\alpha(k) - X_i(k)^2}{Elite_1^\alpha(k)^2 - X_i(k)^2} \times L_i^\alpha(k), \\ Q_2^\alpha = Elite_2^\alpha(k) - \frac{Elite_2^\alpha(k) - X_i(k)^2}{Elite_2^\alpha(k)^2 - X_i(k)^2} \times L_i^\alpha(k), \end{cases} \end{aligned} \quad (32)$$

and

$$\begin{aligned} X_i(k+1) &= L_i^\alpha(k) - |\delta_i^\alpha(k)| \times H_i^\alpha(k) \times L^\alpha(d), \\ \delta_i^\alpha(k) &= L_i^\alpha(k) - X_i(k) + \rho \times D^\alpha[L_i^\alpha(k) - X_i(k)], \\ L^\alpha(d) &= 0.01 \times \frac{u}{|v|^{\beta_\alpha}}, \beta_\alpha = \beta \times \alpha. \end{aligned} \quad (33)$$

In Equation (31), p_3^α is set to 0.4, and $r_{p_3}^\alpha$ is a uniformly distributed random number within $[0, 1]$. The variables u and v in Equation (33) are random numbers that follow a Gaussian distribution.

5. Simulation Result

In this section, the performance of the proposed FO-AVOA for PAA optimization is evaluated. All simulations are carried out on a system equipped with an Intel® Core™ i9-14900 CPU operating at 2.0 GHz, with 32.00 GB of RAM, and a 64 bit operating system. The PAA radiation-pattern synthesis is performed using the FO-AVOA under two optimization scenarios. First, single-parameter optimization is conducted by independently optimizing the amplitude excitations A_{xy} and the element positions d_x and d_y . Subsequently, a 2D optimization framework is applied, in which both the amplitude excitations and the element positions are simultaneously optimized to achieve effective PSLL minimization.

To demonstrate the effectiveness and robustness of the FO-AVOA approach, its performance is compared with several well-established optimization algorithms, including the GSA, PSO, PSOGSA, AVOA, HHO, RUN, and SMA. The parameter settings of all comparative techniques are summarized in Table 1.

Table 1. Parameter settings for the adopted optimization algorithms.

Algorithm	Settings
PSO [13]	$c_1 = c_2 = 2$, $\omega = 0.4\text{--}0.9$, $r_1, r_2 = [0, 1]$
GSA [15]	$G_0 = 100$, $\alpha = 20$
GWO [21]	r decreased from 2 to 0, $s_1, s_2 = [0, 1]$
SMA [43]	$r [0, 1]$
RUN [49]	$a = 20$, $b = 12$
PSOGSA [51]	$C'_1 = 0.5$ and $c'_2 = 1.5$, $\omega = [0, 1]$
AVOA [53]	$p_0 = 0.8, p_1 = 0.6, p_2 = p_3 = 0.4r, r_{p1}, r_{p2}, r_{p3} \in [0, 1]$
FO-AVOA [58]	$\alpha = \{0.1, 0.2, \dots, 1.0\}$
I-GWO [60]	$\xi = [-2, 2]$, $w = 2.5$, $\eta = [-1, 1]$
	r is reduced from 2 to 0, $s_1, s_2 = [0, 1]$

In all cases, the parameters are selected according to the original studies or values commonly adopted in the literature. Specifically, for the AVOA and FO-AVOA, the main control parameters are adjusted following the guidelines reported in [53]. For the SMA [43], the control parameter r is randomly generated within the interval $[0, 1]$. In the PSOGSA [51], c'_1 and c'_2 denote the individual and social learning factors, respectively, while ω represents the inertia weight. In PSO [13], c_1 and c_2 are the cognitive and social learning factors, and ω is the inertia weight. For the GSA, the gravitational constant G_0 is set to 100 and the gradient constant α is fixed at 20, consistent with [15]. The RUN algorithm parameters a and B are set to 20 and 12, respectively, as suggested in [49].

5.1. PAA Amplitude Excitation Optimization

This subsection investigates PAA beamforming optimization for PSLL minimization by controlling the amplitude excitations (A_x, A_y) while assuming uniform phase excitation ($\beta_x = \beta_y = 0$) and fixed inter-element spacing of $\lambda/2$. The normalized fitness function defined in (4) is adopted to guide the optimization process. Under these assumptions, the array-factor formulation in Equation (23) is applied accordingly to evaluate the radiation characteristics.

$$AF_{PAA}(\theta, \varphi) = A_{xy} \sum_{m=1}^M e^{j(m-0.5)(\pi \sin \theta \cos \varphi)} \sum_{n=1}^N e^{j(n-0.5)(\pi \sin \theta \sin \varphi)} \quad (34)$$

The sidelobe region considered in the simulations is defined by $\varphi_{SL} \in [-80^\circ, -25^\circ]$. The lower and upper bounds of the decision variables are set to $XL = 0$ and $XU = 1$, respectively. The performance of the proposed FO-AVOA is benchmarked against several state-of-the-art algorithms, including PSO, GSA, PSOGSA, AVOA, SMA, RUN, and HHO.

5.1.1. 5×5 Element PAA Amplitude Excitation-Only Optimization

This subsection investigates PSLL suppression for a 5×5 PAA using amplitude excitation-only optimization, while maintaining a fixed FNBW. In this configuration, only the excitation amplitudes along the x- and y-directions are optimized, whereas the element positions remain uniformly spaced and the sidelobe region is $\varphi_{SL} \in [-80^\circ, -25^\circ]$. The optimized amplitude excitation distribution over the x–y plane obtained using the proposed FO-AVOA is illustrated in Figure 3, where the symmetry and smooth decay of excitation values across the array can be clearly observed. Such a distribution contributes directly to sidelobe suppression without distorting the main lobe structure.

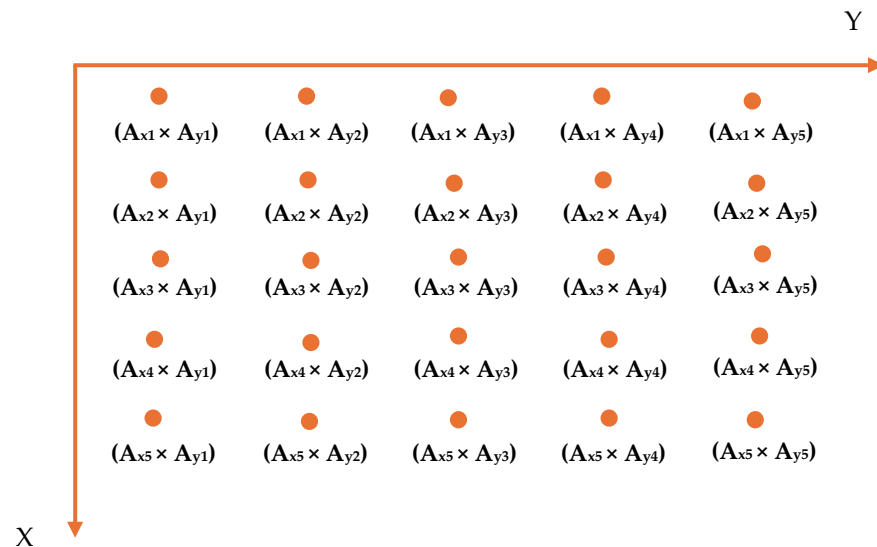


Figure 3. Elements amplitude distribution for PAAs in x-y coordinates.

Quantitative performance comparisons in terms of PSL and FNBW for all considered optimization algorithms are summarized in Table 2. As reported, all methods preserve an identical FNBW of 50° , ensuring a fair comparison focused solely on sidelobe suppression capability.

Table 2. Comparison of FNBW and PSL for the 5×5 planar antenna array obtained through amplitude-only optimization.

Method	Amplitude Excitation ($A_{x1}, A_{x2}, \dots, A_{x5}$)/($A_{y1}, A_{y2}, \dots, A_{y5}$)	PSLL (dB)	FNBW
I-GWO	(0.829, 0.849, 0.965, 0.859, 0.801)/ (0.271, 0.851, 0.329, 0.665, 0.943)	−13.74	50°
GWO	(0.843, 0.893, 0.998, 0.877, 0.843)/ (0.623, 0.306, 0.715, 0.321, 0.032)	−13.7	50°
PSOGSA	(0.852, 0.861, 0.991, 0.891, 0.821)/ (0.791, 1.000, 0.335, 0.276, 0.490)	−13.69	50°
GSA	(0.759, 0.837, 0.931, 0.807, 0.811)/ (0.510, 0.369, 0.439, 0.448, 0.761)	−13.7	50°
RUN	(0.695, 0.743, 0.846, 0.754, 0.734)/ (0.230, 0.066, 0.639, 0.070, 0.153)	−13.7	50°
SMA	(0.846, 1.000, 1.000, 0.821, 0.838)/ (1.000, 0.055, 0.300, 0.523, 0.331)	−12.56	50°
HHO	(0.257, 0.340, 0.372, 0.308, 0.366)/ (0.280, 0.299, 0.383, 0.347, 0.363)	−13.51	50°

PSO	(0.535, 0.604, 0.664, 0.567, 0.582)/ (0.021, 0.992, 0.752, 0.129, 0.949)	−13.69	50°
AVOA	(0.813, 0.856, 0.984, 0.859, 0.688)/ (0.299, 0.184, 0.910, 0.986, 0.621)	−14.78	50°
FO-AVOA	(0.454, 0.523, 0.602, 0.528, 0.462)/ (0.021, 0.105, 0.104, 0.565, 0.231)	−14.85	50°

From Table 2, the proposed FO-AVOA achieves the lowest PSLL of −14.85 dB, outperforming all benchmark techniques. The most similar solution, the AVOA, provides a PSLL of −14.78 dB, which is better than that of the FO-AVOA by 0.07 dB. The remaining techniques, such as I-GWO, GWO, GSA, RUN, PSOGSA, and PSO, provide values of the PSLL in the range of −13.7 dB, thus showing a poorer sidelobe level reduction capability than the proposed technique. Additionally, the SMA and HHO demonstrate relatively poorer performance with PSLL values of −12.56 dB and −13.51 dB, respectively.

Analytical benchmark tapers (Taylor and Dolph–Chebyshev) for the 5×5 array:

In addition to metaheuristic optimization, we evaluate two standard analytical amplitude tapers for the same 5×5 uniform array. The 2D excitation matrix is constructed via separable weighting $W(m,n) = w_x(m)w_y(n)$ using 1D Taylor and 1D Dolph–Chebyshev distributions (length five) along both axes, followed by normalization to a maximum amplitude of one.

In Table 3 As expected, analytical Chebyshev tapering offers a deterministic sidelobe floor close to its specified attenuation, at the expense of a slightly wider main lobe (larger FNBW). The proposed FO-AVOA achieves comparable sidelobe suppression while providing a flexible optimization framework that also extends to joint amplitude-and-position synthesis in subsequent subsections.

Table 3. Analytical (exact) baseline results for the 5×5 planar antenna arrays (uniform grid, amplitude tapering).

Method	Amplitude Excitation (Ax1, Ax2, ..., Ax5)/(Ay1, Ay2, ..., Ay5)	PSLL (dB)	FNBW
Uniform	(1.000, 1.000, 1.000, 1.000, 1.000)/ (1.000, 1.000, 1.000, 1.000, 1.000)	−12.04	47.11°
Taylor taper	(0.828, 0.848, 1.000, 0.848, 0.828)/ (0.828, 0.848, 1.000, 0.848, 0.828)	−13.18	49.25°
Dolph–Chebyshev taper	(0.761, 0.874, 1.000, 0.874, 0.761)/ (0.761, 0.874, 1.000, 0.874, 0.761)	−14.20	50.95°

The respective array-factor patterns for the 5×5 PAA under amplitude-only optimization for the FO-AVOA are illustrated in Figure 4. It is clear that the FO-AVOA provides the strongest sidelobe nulls. Moreover, the sidelobe envelopes obtained from the FO-AVOA lie below those obtained from the benchmark algorithms for the entire visible region of angles. This validates its stronger sidelobe-suppressing capabilities.

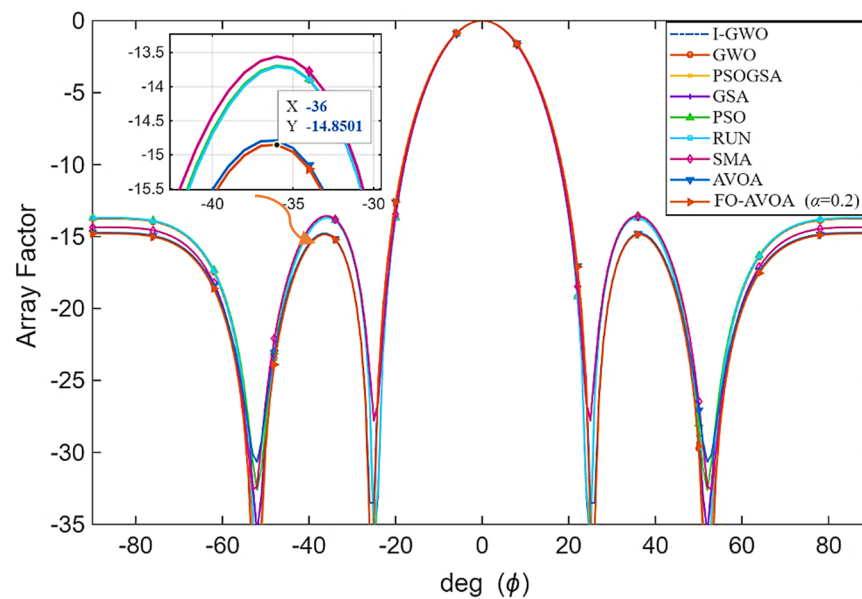


Figure 4. Normalized radiation patterns of the 5×5 planar antenna array obtained through amplitude-only optimization.

From the above results, it can be seen that the proposed FO-AVOA approach achieves better results compared to existing techniques under amplitude-only optimization in terms of sidelobe reduction without affecting the main lobe parameters. The efficacy of FOD dynamics in optimization helps in better exploration and convergence in beamforming problems with antenna arrays.

5.1.2. 10×10 Element PAA Amplitude Excitation-Only Optimization

This subsection examines sidelobe suppression for a larger 10×10 PAA using amplitude excitation-only optimization. In this configuration, the excitation amplitudes along the x- and y-directions are optimized while the inter-element spacing remains fixed. The sidelobe region is $\phi_{SL} \in [-80^\circ, -21^\circ]$. The comparative performance results are summarized in Table 4.

Table 4. Comparison of FNBW and PSLL for the 10×10 planar antenna arrays obtained through amplitude-only optimization.

Method	Amplitude Excitation ($A_{x1}, A_{x2}, \dots, A_{x10}$)/ ($A_{y1}, A_{y2}, \dots, A_{y10}$)	PSLL (dB)	FNBW
I-GWO	(0.1557, 0.3415, 0.6022, 0.8390, 0.9728, 0.9588, 0.7946, 0.5457, 0.3011, 0.1134)/ (0.6657, 0.9315, 0.9716, 0.6446, 0.6690, 0.2666, 0.2843, 0.4744, 0.8085, 0.5350)	−37.96	42°
GWO	(0.1494, 0.3442, 0.5956, 0.8457, 0.9632, 0.9642, 0.7801, 0.5409, 0.2952, 0.0985)/ (0.0308, 0.0030, 0.4711, 0.0048, 0.0484, 0.6339, 0.7361, 0.1396, 0.9774, 0.6611)	−36.99	42°
PSOGSA	(0.1861, 0.3881, 0.6566, 0.8846, 1.0000, 0.9570, 0.7717, 0.5121, 0.2703, 0.0994)/ (0.5238, 0.4942, 0.6641, 0.6584, 0.8686, 0.9718, 0.0634, 0.0014, 0.2621, 0.9574)	−37.48	42°
GSA	(0.0947, 0.2472, 0.4670, 0.7038, 0.8797, 0.9162, 0.8176, 0.6101, 0.3625, 0.1716)/ (0.2685, 0.2502, 0.3644, 0.3326, 0.3386, 0.5444, 0.2636, 0.5541, 0.3030, 0.4002)	−37.31	42°
RUN	(0.1940, 0.4110, 0.6744, 0.8960, 0.9965, 0.9393, 0.7519, 0.5013, 0.2637, 0.1087)/ (0.6684, 0.2532, 0.5079, 0.1688, 0.2938, 0.5184, 0.4778, 0.1510, 0.0832, 0.3413)	−36.82	42°
SMA	(0.0602, 0.1653, 0.3183, 0.4924, 0.6116, 0.6531, 0.5790, 0.4373, 0.2640, 0.1275)/ (0.7843, 0.0881, 0.7411, 0.0000, 0.3263, 0.0287, 1.0000, 0.1019, 0.0083, 0.1892)	−36.99	42°
HHO	(0.1113, 0.1113, 0.3065, 0.5496, 0.5962, 0.6265, 0.5120, 0.4020, 0.2217, 0.1113)/ (0.2593, 0.1113, 0.4795, 0.1113, 0.5046, 0.5814, 0.6138, 0.1113, 0.1113, 0.4700)	−26.93	42°

PSO	(0.1227, 0.2889, 0.5561, 0.8094, 1.0063, 1.0414, 0.9228, 0.6735, 0.4067, 0.1749)/ (0.7779, 0.1861, 0.8423, 0.2788, 0.7838, 0.7744, 0.4950, 1.4884, 0.1123, 0.2804)	-37.13	42°
AVOA	(0.0391, 0.1447, 0.3244, 0.5011, 0.6868, 0.7682, 0.7067, 0.5708, 0.3536, 0.1748)/ (0.0086, 0.9606, 0.3450, 0.0150, 0.3348, 0.9793, 0.6974, 0.9940, 0.9846, 0.8290)	-34.16	42°
FO-AVOA	(0.1695, 0.3812, 0.6385, 0.8705, 0.9993, 0.9413, 0.7646, 0.4906, 0.2518, 0.0827)/ (0.5070, 0.9399, 0.1785, 0.5336, 0.2950, 0.4421, 0.4296, 0.4826, 0.5572, 0.3926)	-37.79	42°

As reported in Table 4, all optimization methods maintain an identical FNBW of 42°, ensuring a consistent basis for PSLL comparison. The proposed FO-AVOA achieves a PSLL of -37.79 dB, which represents one of the lowest sidelobe levels among all considered techniques. Although I-GWO yields a slightly lower PSLL of -37.96 dB, the FO-AVOA provides a highly competitive result while maintaining smoother excitation distributions and stable convergence behaviour. Compared with the remaining benchmark algorithms, the FO-AVOA outperforms the PSOGSA (-37.48 dB), GSA (-37.31 dB), PSO (-37.13 dB), RUN (-36.82 dB), and SMA and GWO (both -36.99 dB). The AVOA exhibits comparatively inferior sidelobe suppression with a PSLL of -34.16 dB, while HHO shows the weakest performance, achieving only -26.93 dB. These results clearly indicate the robustness of the proposed FO-AVOA in handling higher-dimensional amplitude optimization problems.

The normalized array-factor patterns corresponding to the optimized amplitude distributions are illustrated in Figure 5. It can be observed that the FO-AVOA produces deep and well-controlled sidelobe nulls while preserving the main lobe shape and beamwidth. The sidelobe envelope obtained using the FO-AVOA remains consistently low across the defined angular range, demonstrating effective sidelobe suppression without introducing distortions in the main radiation pattern.

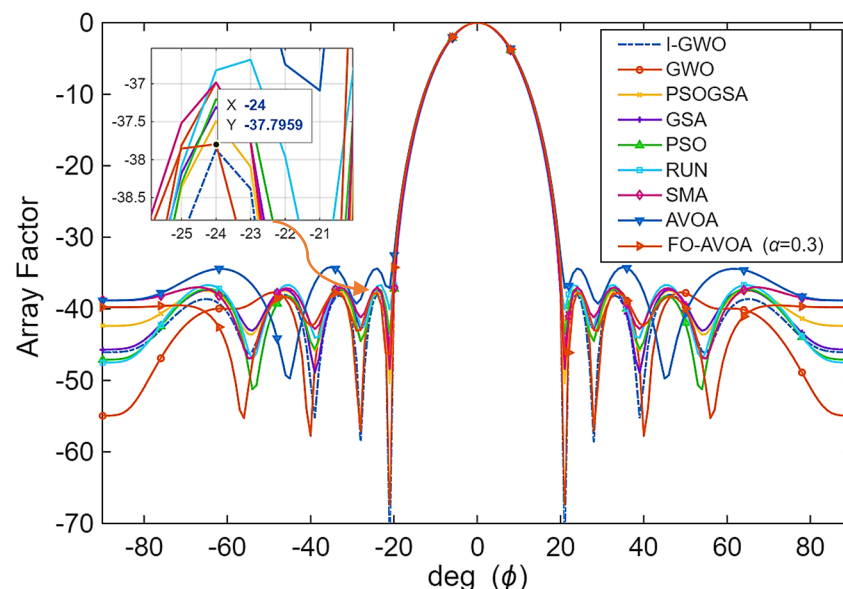


Figure 5. Normalized radiation patterns of the 10×10 planar antenna array obtained through amplitude-only optimization.

By comparing the results of Sections 5.1.1 and 5.1.2, it is evident that increasing the array size from 5×5 to 10×10 leads to a substantial reduction in PSLL for all optimization algorithms, accompanied by a decrease in FNBW. This behaviour highlights the well-known trade-off between sidelobe suppression and beamwidth narrowing in planar antenna array design. Moreover, the consistent performance of the FO-AVOA across both

array sizes confirms its effectiveness and scalability for amplitude-only beamforming optimization.

5.2. PAA Position-Only Optimization

This section is concerned with the minimization of the PSLL in array element position optimization, in which the values of the spacings between the elements along the x- and y-axes, d_x and d_y , are optimized, while considering equal amplitude excitation, $A_{xy} = 1$, and equal phase excitations, $\beta_x = \beta_y = 0$. The fitness function is the normalized fitness function, as described in Equation (4). In that case, the array factor of the planar antenna is given by

$$AF_{PAA}(\theta, \varphi) = A_{xy} \sum_{m=1}^M e^{j(m-1)(kd_x \sin \theta \cos \varphi)} \sum_{n=1}^N e^{j(n-1)(kd_y \sin \theta \sin \varphi)} \quad (35)$$

Two array configurations are investigated, namely 5×5 and 10×10 elements. Proper element placement is a critical aspect of array design: excessively small inter-element spacing leads to mutual coupling effects, whereas overly large spacing results in grating lobes. To avoid these undesirable phenomena, the element-spacing constraints are enforced such that the optimized positions satisfy practical spacing requirements. Accordingly, the lower and upper bounds of the decision variables are set to $XL = 0.5\lambda$ and $XU = 1\lambda$, respectively.

5.2.1. 5×5 Element PAA Position-Only Optimization

This subsection investigates the position-only optimization of a 5×5 PAA, where the element positions along the x- and y-directions are optimized while the excitation amplitudes remain uniform. The sidelobe region is defined as $\varphi_{SL} \in [-80^\circ, -20^\circ]$. The performance of the proposed FO-AVOA is compared with several benchmark optimization techniques, including PSO, GSA, PSOGSA, AVOA, SMA, RUN, HHO, GWO, and I-GWO. The optimized element position distribution in the x-y plane obtained using the FO-AVOA is illustrated in Figure 6. The resulting layout exhibits non-uniform inter-element spacing, which effectively disrupts periodicity in the array geometry and contributes to sidelobe level reduction.

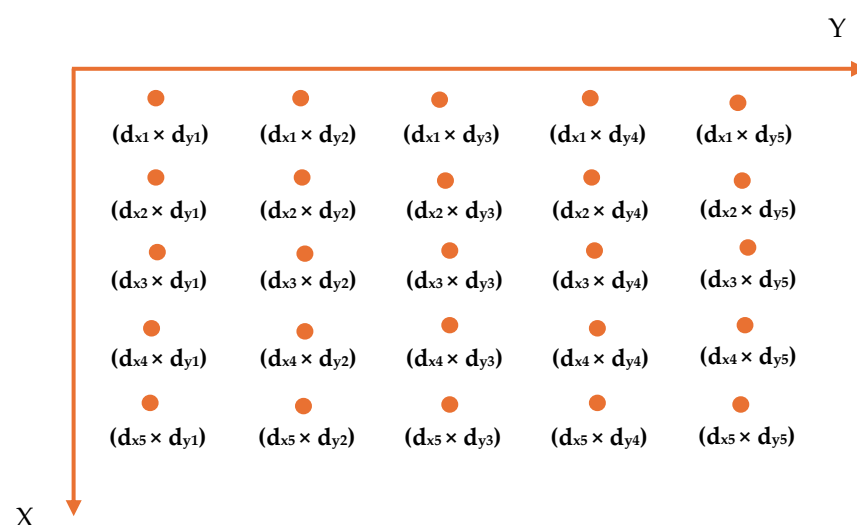


Figure 6. Element position distributions for PAA in x-y coordinates.

Quantitative performance comparisons in terms of PSLL and FNBW are summarized in Table 5.

Table 5. Comparison of FNBW and PSLL for the 5×5 planar antenna array obtained through position-only optimization.

Method	Element Position ($dx_1, dx_2, \dots, dx_5$)/ ($dy_1, dy_2, \dots, dy_5$)	PSLL (dB)	FNBW
I-GWO	(0.502, 1.222, 1.856, 2.356, 2.997)/ (0.999, 1.499, 2.000, 2.540, 3.040)	−15.65	28°
GWO	(0.803, 1.443, 1.943, 2.577, 3.300)/ (0.955, 1.470, 2.260, 2.883, 3.434)	−14.63	28°
PSOGSA	(0.857, 1.580, 2.213, 2.713, 3.351)/ (0.502, 1.007, 1.532, 2.531, 3.508)	−14.28	28°
GSA	(0.616, 1.306, 2.109, 2.820, 3.649)/ (0.727, 1.517, 2.231, 2.994, 3.692)	−12.25	30°
HHO	(0.553, 1.237, 1.854, 2.408, 3.058)/ (0.862, 1.860, 2.859, 3.543, 4.097)	−11.3	28°
SMA	(0.597, 1.319, 1.951, 2.452, 3.089)/ (0.512, 1.271, 1.771, 2.271, 3.271)	−13.5	30°
RUN	(0.500, 1.138, 1.638, 2.271, 2.994)/ (0.500, 1.002, 1.647, 2.157, 2.657)	−13.46	28°
PSO	(0.803, 1.487, 2.073, 2.576, 3.082)/ (1.357, 1.857, 2.457, 2.987, 3.647)	−13.59	28°
AVOA	(0.500, 1.498, 1.998, 2.633, 3.157)/ (0.500, 1.500, 2.356, 3.356, 4.356)	−23.32	28°
FO-AVOA	(0.500, 1.280, 1.829, 2.402, 3.173)/ (0.500, 1.500, 2.165, 2.927, 3.927)	−24.07	28°

As shown, the proposed FO-AVOA achieves the lowest PSLL of −24.07 dB while maintaining an FNBW of 28°, outperforming all competing methods. The closest competitor, the AVOA, attains a PSLL of −23.32 dB for the same FNBW, which is higher than that of the FO-AVOA by 0.75 dB. Other benchmark algorithms yield noticeably higher PSLL values, including I-GWO (−15.65 dB), GWO (−14.63 dB), PSOGSA (−14.28 dB), PSO (−13.59 dB), RUN (−13.46 dB), SMA (−13.50 dB), and GSA (−12.25 dB). The HHO algorithm exhibits the weakest performance, achieving a PSLL of −11.30 dB. These results clearly demonstrate the superior sidelobe suppression capability of the FO-AVOA under position-only optimization.

The corresponding normalized array-factor patterns for the 5×5 PAA are depicted in Figure 7. However, in general, it can be pointed out from these profiles that the FO-AVOA yields the most profound sidelobe attenuation in the defined angular region without degrading the shape of the main lobe or increasing the beamwidth. The envelope of the sidelobes for the FO-AVOA continuously stays below those obtained from the benchmark algorithms, thus confirming its efficacy in position-based optimization of beam-forming.

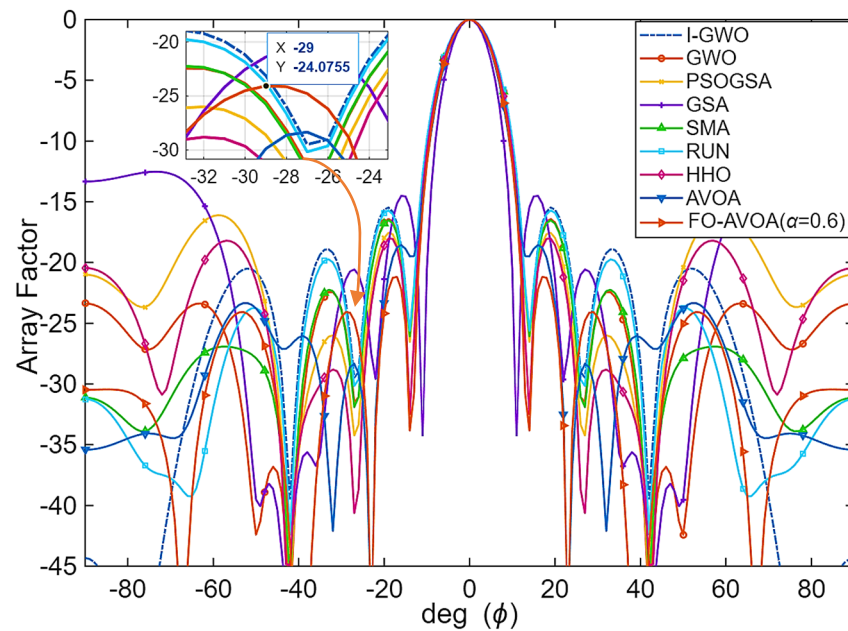


Figure 7. Normalized radiation patterns of the 5×5 planar antenna array obtained through position-only optimization.

From the above discussion and the results obtained and presented in Table 4 and Figure 7, the improvement in PSLL offered by the position-only optimization method over the conventional uniform spacing method is confirmed. Also, the improved PSLL offered by the FO-AVOA indicates the effectiveness of the fractional-order adaptation dynamics in the search process in achieving improved sidelobe suppression in the design of a planar antenna array.

5.2.2. 10×10 Element PAA: Position-Only Optimization

This subsection continues the position-only optimization analysis for a 10×10 PAA with element position optimization along the x-and y-coordinates given equal amplitudes for the excitations. The sidelobe area is specified as $\phi_{SL} \in [-80^\circ, -11^\circ]$, with a table summarizing the performance comparison results presented in Table 6.

Table 6. FNBW and PSLL results for the 10×10 element PAA using position-only optimization.

Method	Element Position ($dx_1, dx_2, \dots, dx_{10}$)/ ($dy_1, dy_2, \dots, dy_{10}$)	PSLL (dB)	FNBW
I-GWO	(0.9011, 1.6378, 2.3058, 2.8159, 3.3233, 3.8234, 4.3255, 4.8987, 5.5825, 6.3544)/ (0.6524, 1.2132, 2.0735, 3.0011, 3.5561, 4.1643, 4.6715, 5.3374, 5.9172, 6.6162)	-18.63	22°
GWO	(0.5106, 1.2608, 1.9289, 2.4806, 2.9806, 3.4806, 3.9950, 4.5282, 5.2093, 5.9630)/ (0.8632, 1.7099, 2.3933, 2.9959, 3.5062, 4.3404, 4.9482, 5.7468, 6.3447, 6.9099)	-18.59	22°
PSOGSA	(0.5293, 1.3583, 2.3583, 2.9583, 3.7583, 4.3583, 5.0083, 5.5283, 6.1083, 6.7883)/ (0.5145, 1.0145, 1.5245, 2.0745, 2.5945, 3.3945, 4.1745, 5.0545, 5.9345, 6.4945)	-16.7	22°
GSA	(0.3068, 1.0357, 1.6443, 2.1119, 2.6205, 3.0777, 3.5482, 4.0918, 4.7624, 5.5004)/ (0.5693, 1.0693, 1.6093, 2.1393, 2.6693, 3.1893, 3.7893, 4.4893, 5.0393, 5.5693)	-18.59	22°
HHO	(0.5001, 1.2706, 2.0475, 2.5476, 3.1232, 3.6233, 4.1234, 4.7002, 5.2397, 5.9420)/ (0.5001, 1.0002, 1.8412, 2.6396, 3.4527, 4.2658, 4.8293, 5.6425, 6.4552, 6.9554)	-17.03	20°
SMA	(0.5035, 1.2747, 1.9743, 2.5565, 3.0565, 3.5565, 4.0588, 4.5601, 5.2166, 5.9566)/ (0.5081, 1.0551, 1.6007, 2.1007, 2.8604, 3.5328, 4.4931, 5.3080, 5.9194, 6.4195)	-18.62	22°
RUN	(0.5183, 1.2914, 1.9499, 2.5275, 3.0282, 3.5282, 4.0404, 4.5478, 5.2388, 5.9725)/ (0.5050, 1.0099, 1.5207, 2.0236, 2.5240, 3.0351, 3.5432, 4.0480, 4.5540, 5.0773)	-18.59	22°

PSO	(0.7159, 1.5254, 2.1978, 2.7533, 3.2402, 3.7301, 4.2480, 4.7525, 5.4140, 6.1013)/ (0.5067, 1.0718, 1.6279, 2.6383, 3.1684, 3.7125, 4.2425, 4.7523, 5.2525, 5.7528)	-18.59	22°
AVOA	(0.5000, 1.2430, 1.9299, 2.4299, 2.9770, 3.4770, 3.9770, 4.5212, 5.1577, 5.9618)/ (0.5000, 1.2360, 1.8117, 2.8117, 3.5008, 4.0610, 4.8714, 5.4458, 6.0071, 6.9341)	-18.53	22°
FO-AVOA	(0.5000, 1.2924, 1.9635, 2.5476, 3.0630, 3.5636, 4.0914, 4.5978, 5.2970, 6.0534)/ (0.5000, 1.2891, 1.8692, 2.8253, 3.7806, 4.4812, 4.9914, 5.5227, 6.2889, 6.8058)	-18.76	22°

Table 5 reports that most optimization algorithms maintain a constant FNBW of 22° for a fair comparison in terms of sidelobe suppression performance. Among the listed optimization methods, the proposed FO-AVOA yields the lowest PSLL with a value of -18.76 dB under the same constraint on beamwidth. The second-best methods include I-GWO and the SMA, which follow with PSLL values of -18.63 dB and -18.62 dB respectively, which are higher than the PSLL of the proposed FO-AVOA by 0.13 dB and 0.14 dB, respectively. GWO, GSA, RUN, and PSO generate comparable PSLL values close to -18.59 dB, whereas the AVOA generates a value of -18.53 dB. However, the PSOGSA and HHO show poor PSLL values, -16.70 dB and -17.03 dB, respectively, which affirm that the FO-AVOA has the superior capability in sidelobe suppression for position-only optimized large-scale arrays.

The normalized array-factor patterns corresponding to the 10×10 PAA for the FO-AVOA and AVOA are shown in Figure 8. From the plots, it can be observed that the FO-AVOA approach guarantees a lower sidelobe envelope value within the specified angular sector without compromising the main lobe pattern and size. Furthermore, the greater sidelobe level suppression offered by the FO-AVOA proves its efficiency in leveraging non-uniform spatial patterns to reduce sidelobe levels.

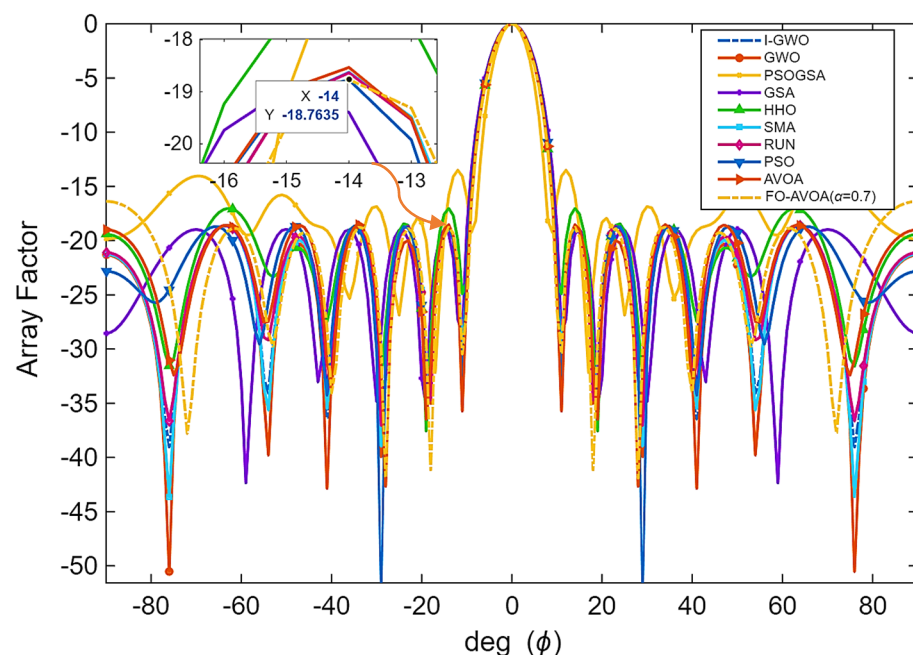


Figure 8. Normalized array factors of the 10×10 element PAA using position-only optimization.

By analyzing the comparisons between the results of Sections 5.2.1 and 5.2.2, it is noted that the increase in the array size from 5×5 to 10×10 results in enhanced sidelobe levels as well as reduced FNBWs for all optimization approaches, while at the same time, the FO-AVOA technique is clearly superior, irrespective of the dimension of the array, thus reinforcing the benefits of using FO approaches during metaheuristic optimization searches, as already outlined above.

5.3. PAA Amplitude and Position-Only Optimization

In this problem, a 2D optimization method is used by simultaneously optimizing two parameters in array design, excitation amplitudes and positions, with a constant phase excitation ($\beta_x = \beta_y = 0$). This joint optimization aims to further enhance PSLL suppression compared with single-parameter approaches. For this purpose, the PAA factor is expressed as

$$AF_{PAA}(\theta, \varphi) = A_{xy} \sum_{m=1}^M e^{j(m-1)(kd_x \sin \theta \cos \varphi)} \sum_{n=1}^N e^{j(n-1)(kd_y \sin \theta \sin \varphi)}. \quad (36)$$

Two array configurations are examined, namely 5×5 and 10×10 elements. For amplitude excitation optimization, the lower and upper bounds of the decision variables are set to $XL = 0$ and $XU = 1$, respectively. For element position optimization, the bounds are constrained to $XL = 0.5\lambda$ and $XU = 1\lambda$ to avoid mutual coupling and grating-lobe effects. The performance of the proposed FO-AVOA is compared with several benchmark techniques, including PSO, GSA, PSOGSA, AVOA, SMA, and RUN.

5.3.1. 5×5 Element PAA 2D Optimization

This subsection investigates the 2D optimization of a 5×5 PAA, where both the excitation amplitudes and the element positions along the x- and y-directions are simultaneously optimized. The sidelobe region is defined as $\varphi_{SL} \in [-80^\circ, -22^\circ]$. The quantitative performance comparison among the considered optimization algorithms is summarized in Table 7.

Table 7. FNBW and PSLL performance of the 5×5 PAA using 2D optimization.

Method	Amplitude Excitations ($A_{x1}, A_{x2}, \dots, A_{x5}$)/($A_{y1}, A_{y2}, \dots, A_{y5}$) Element Position ($dx1, dx2, \dots, dx5$)/($dy1, dy2, \dots, dy5$)	PSLL (dB)	FNBW
I-GWO	(0.3946, 0.7887, 0.9739, 0.7593, 0.3580)/(0.8185, 0.0445, 0.5506, 0.9623, 0.1222) (0.6923, 1.4384, 2.1850, 2.9271, 3.6692)/(0.6859, 1.2586, 1.9040, 2.4119, 3.2306)	−25.3	44°
GWO	(0.3440, 0.7681, 1.0000, 0.8222, 0.4305)/(0.0568, 0.00589, 0.3333, 0.1830, 0.3494) (0.5135, 1.2595, 2.0052, 2.7495, 3.4915)/(0.8765, 1.7842, 2.4365, 3.2625, 4.1659)	−25.11	44°
PSOGSA	(0.5880, 0.9913, 0.9952, 0.4632, 0.5641)/(0.9998, 0.5912, 0.5753, 0.1487, 0.8585) (0.5379, 1.2740, 1.8684, 2.5684, 3.1450)/(0.9210, 1.5210, 2.9508, 3.5619, 4.0542)	−18.42	46°
GSA	(0.2704, 0.6488, 0.8772, 0.7456, 0.4185)/(0.3352, 0.4893, 0.4142, 0.6118, 0.6448) (0.6533, 1.3963, 2.1378, 2.8839, 3.6301)/(0.7197, 1.5622, 2.2006, 2.8471, 3.6822)	−24.73	44°
RUN	(0.3484, 0.7649, 1.0000, 0.8289, 0.4282)/(0.2969, 0.1354, 0.4571, 0.2547, 0.3850) (0.5662, 1.2008, 1.7857, 2.3763, 3.0324)/(0.5817, 1.0961, 1.7063, 2.2996, 2.8780)	−19.3	46°
SMA	(0.6682, 0.7482, 0.9944, 1.0000, 0.7578)/(0.5000, 0.5000, 0.5000, 0.5943, 0.7462) (0.7001, 1.4002, 2.1004, 2.8005, 3.5006)/(0.7001, 1.4002, 2.1004, 2.8005, 3.5006)	−17.39	44°
PSO	(0.2612, 0.5370, 0.7097, 0.5931, 0.2938)/(0.0057, 0.1362, 0.2520, 0.5286, 0.1453) (0.6968, 1.4313, 2.1729, 2.9166, 3.6647)/(0.9912, 1.9334, 2.5388, 3.2228, 3.9250)	−25	44°
AVOA	(0.2156, 0.5058, 0.6379, 0.5161, 0.2559)/(0.4035, 0.0383, 0.3660, 0.3473, 0.4537) (0.5000, 1.2884, 2.0310, 2.8037, 3.5437)/(0.5000, 1.2975, 1.9249, 2.5842, 3.3883)	−24.90	44°
FO-AVOA	(0.3660, 0.8047, 1.0000, 0.7735, 0.3561)/(0.0000, 0.1518, 0.2126, 0.4033, 0.6759) (0.5000, 1.2524, 2.0004, 2.7553, 3.5067)/(0.5000, 1.4150, 2.3199, 2.8199, 3.5709)	−26.58	44°

As reported in Table 7, all methods maintain a comparable first null beamwidth (FNBW), allowing a fair assessment of sidelobe suppression capability. The proposed FO-AVOA achieves the lowest peak sidelobe level (PSLL) of −26.58 dB, with an FNBW of 44°, outperforming all benchmark techniques. In comparison, the AVOA and PSO yield PSLL values of −24.90 dB and −25.00 dB, respectively, which are higher than that of the FO-AVOA by 1.68 dB and 1.58 dB. Similarly, I-GWO and GWO achieve PSLL values of −25.30

dB and -25.11 dB, respectively, remaining inferior to the proposed method. Other algorithms exhibit noticeably weaker sidelobe suppression performance. Specifically, the GSA attains a PSL of -24.73 dB, while RUN, PSOGSA, and SMA yield PSL values of -19.30 dB, -18.42 dB, and -17.39 dB, respectively. These results evidently show that it is beneficial to optimize amplitude excitation and positions simultaneously rather than relying on optimization based on a single variable.

Figure 9 shows the normalized array-factor patterns of the 5×5 PAA under 2D optimization. It is clear that the maximum sidelobe attenuation is obtained by the FO-AVOA, followed by the other methods. The envelope of the sidelobes of the FO-AVOA is always lower than those of the other methods, thus demonstrating that the FO-AVOA is the best beamforming technique in the current scenario.

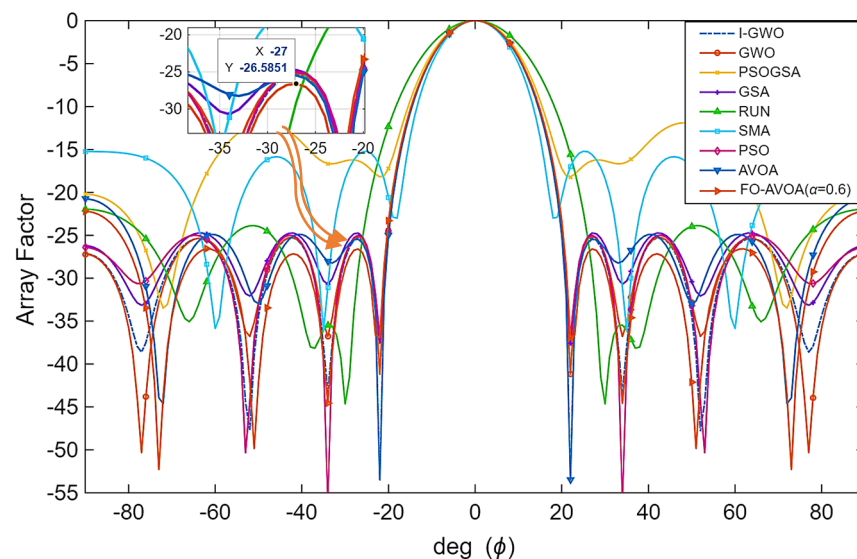


Figure 9. Normalized array factors of the 5×5 PAA using 2D optimization.

From Table 6 and Figure 9 above, the findings confirm that 2D optimization achieved a marked improvement in the reduction in sidelobes compared to those optimized in terms of amplitude alone or position alone. Furthermore, this superiority in performance demonstrated by the FO-AVOA not only underscores the success of using fractional dynamics in optimizing beamforming in planar antenna arrays but also reaffirms the superiority of this technique over other methods in terms of reduction in sidelobes.

5.3.2. 10×10 Element PAA 2D Optimization

This subsection extends the 2D optimization framework to a 10×10 PAA, where both the excitation amplitudes and the element positions along the x- and y-directions are simultaneously optimized. The sidelobe region is defined as $\phi_{SL} \in [-80^\circ, -21^\circ]$. The comparative performance results are summarized in Table 8.

Table 8. Comparison of FNBW and PSL for the 10×10 planar antenna array obtained through 2D optimization.

Method	Amplitude Excitations ($Ax_1, Ax_2, \dots, Ax_{10}$)/($Ay_1, Ay_2, \dots, Ay_{10}$) Element Position ($dx_1, dx_2, \dots, dx_{10}$)/($dy_1, dy_2, \dots, dy_{10}$)	PSLL (dB)	FNBW
I-GWO	(0.0662, 0.0402, 0.1225, 0.3469, 0.7187, 0.9964, 0.8833, 0.8173, 0.6741, 0.2865)/	-38.81	42°
	(0.0078, 0.0803, 0.7610, 0.0000, 0.6740, 0.0000, 0.4250, 0.0000, 0.0000, 0.5440)		
	(0.6227, 1.1834, 1.8206, 2.6009, 3.3688, 4.0829, 4.6970, 5.2234, 5.8458, 6.5510)/		
	(0.6186, 1.1775, 1.7817, 2.3395, 3.0152, 3.7484, 4.6377, 5.4853, 6.0092, 6.9355)		
GWO	(0.9003, 0.2179, 0.5401, 0.8049, 0.8064, 0.5253, 0.2030, 0.0002, 0.0007, 0.0000)/	-32.57	42°

PSOGSA	(0.8810, 0.9044, 0.0000, 0.0000, 0.0000, 1.0000, 0.0000, 1.0000, 0.0000, 0.4300)	−31.55	42°
	(0.9396, 1.7354, 2.4672, 3.2060, 3.9477, 4.6894, 5.4349, 6.3912, 7.3130, 8.1446)/		
	(0.7392, 1.4458, 2.0199, 2.7473, 3.3113, 3.9679, 4.5953, 5.1739, 5.9421, 6.6671)		
	(0.4619, 0.1004, 0.7552, 0.1140, 0.9966, 0.8099, 0.6795, 0.9216, 0.8769, 0.5465)/		
	(0.9003, 0.1538, 0.3105, 0.1487, 0.8641, 0.1329, 0.1047, 0.1253, 0.4466, 0.1360)		
GSA	(0.9661, 1.6057, 1.7616, 1.8974, 2.5526, 3.4037, 4.3484, 5.2068, 6.0207, 6.7849)/	−32.09	34°
	(0.5016, 1.0633, 1.7655, 2.3705, 2.9234, 3.5178, 4.1817, 4.7608, 5.4915, 6.7819)		
	(0.3221, 0.5791, 0.6909, 0.5787, 0.4562, 0.5768, 0.8271, 0.8965, 0.6335, 0.2533)/		
	(0.3609, 0.4653, 0.5075, 0.3633, 0.5765, 0.2570, 0.4813, 0.2664, 0.5012, 0.4558)		
	(0.8925, 1.5820, 2.2914, 3.0102, 3.7636, 4.5424, 5.2845, 6.0127, 6.7408, 7.4817)/		
SMA	(0.7496, 1.6369, 2.4278, 3.2124, 3.8115, 4.5372, 5.2732, 5.8676, 6.5914, 7.4808)	−31.62	44°
	(0.3743, 0.7211, 0.2163, 0.7574, 0.9210, 0.9206, 0.9967, 0.9229, 0.6552, 0.3284)/		
	(0.1001, 0.1462, 0.1044, 0.1000, 0.1012, 0.1011, 0.1073, 0.1139, 0.1000, 0.1112)		
	(0.5344, 1.4909, 2.8400, 3.6212, 4.2014, 4.6197, 5.4038, 5.9340, 6.6212, 7.5004)/		
	(0.5714, 0.4685, 1.1191, 1.2359, 1.7645, 1.8645, 1.9645, 2.2449, 3.1497, 3.9709)		
RUN	(0.7790, 0.4754, 0.7448, 1.0000, 0.6565, 0.5065, 0.4622, 0.4934, 0.7049, 0.9039)/	−7.321	24°
	(0.1001, 0.1000, 0.1000, 0.1004, 0.1000, 0.1274, 0.1498, 0.1854, 0.1221, 0.2219)		
	(0.5262, 1.3007, 2.2298, 1.9296, 3.6630, 4.2851, 4.9268, 5.5284, 6.2151, 6.8312)/		
	(0.5501, 1.2575, 1.7718, 2.5528, 3.7065, 4.5573, 5.3522, 6.0780, 6.7732, 7.5797)		
	(0.3221, 0.5791, 0.6909, 0.5787, 0.4562, 0.5768, 0.8271, 0.8965, 0.6335, 0.2533)/		
PSO	(0.3609, 0.4653, 0.5075, 0.3633, 0.5765, 0.2570, 0.4813, 0.2664, 0.5012, 0.4558)	−24	26°
	(0.8441, 1.8441, 2.8441, 3.6362, 4.6362, 5.6362, 6.5878, 7.0878, 8.0878, 8.6981)/		
	(0.6573, 1.6237, 2.6237, 3.2726, 4.2726, 5.1389, 5.8861, 6.8861, 7.8006, 8.4773)		
	(0.1636, 0.4412, 0.5800, 0.4378, 0.5212, 0.3406, 0.2402, 0.1320, 0.0858, 0.0954)/		
	(0.5162, 0.2247, 0.3776, 0.5287, 0.3020, 0.3025, 0.3248, 0.0280, 0.9371, 0.7444)		
AVOA	(0.5000, 1.2993, 1.9537, 2.4714, 2.9899, 3.4899, 3.9993, 4.8369, 5.8163, 6.6963)/	−36.68	42°
	(0.5000, 1.1993, 2.1159, 3.0652, 3.6888, 4.2157, 5.0157, 5.8199, 6.4226, 6.9472)		
	(0.0000, 0.1087, 0.3334, 0.8188, 0.9664, 1.0000, 0.7017, 0.7587, 0.6179, 0.0000)/		
	(1.0000, 0.1035, 0.0000, 0.0582, 0.8519, 1.0000, 0.4362, 0.0000, 0.4566, 1.0000)		
	(0.5000, 1.4724, 2.1562, 3.1562, 3.9294, 4.6462, 5.1462, 5.6639, 6.4878, 6.9878)/		
FO-AVOA	(0.5000, 1.0000, 1.8422, 2.7848, 3.4123, 4.3164, 5.0948, 6.0948, 6.8916, 7.8916)	−40.96	42°

From Table 8, it can also be seen that the proposed FO-AVOA outperforms all comparisons in terms of PSLL, with a value of −40.96 dB and FNBW of 42°. Compared with I-GWO and the AVOA, which have PSLLs of −38.81 and −36.68 dB, respectively, their PSLLs are larger by 2.15 and 4.28 dB for a fixed beamwidth. The PSLLs of other algorithms are obviously poorer than that of the proposed algorithm. The PSLLs of GWO, GSA, PSOGSA, and SMA are −32.57 dB, −32.09 dB, −31.55 dB, and −31.62 dB, respectively. The PSO algorithm has a PSLL of −24 dB, and the RUN algorithm is poorest, with a PSLL of −7.32 dB and an enlarged FNBW of 24° in 2D optimization.

What can be extracted from Table 7 is that in both the FO-AVOA and AVOA solutions, there are several array elements that have zero or close-to-zero values of excitation amplitudes. This feature is indicative of the fact that the number of excited array elements is lowered, leading to lower power consumption and hardware complexity, as well as simpler feeding networks, without affecting the radiation performance.

The respective array-factor patterns of the 10 × 10 PAA subjected to 2D optimization for the FO-AVOA and other algorithms are depicted in Figure 10. It is clear that the FO-AVOA outperforms others and provides the highest amount of sidelobe attenuation; furthermore, the main lobe pattern is maintained. The sidelobe envelopes of the FO-AVOA always remain lower compared to the envelopes of other algorithms. This proves that the FO-AVOA outperforms others regarding the ideal beamforming technique.

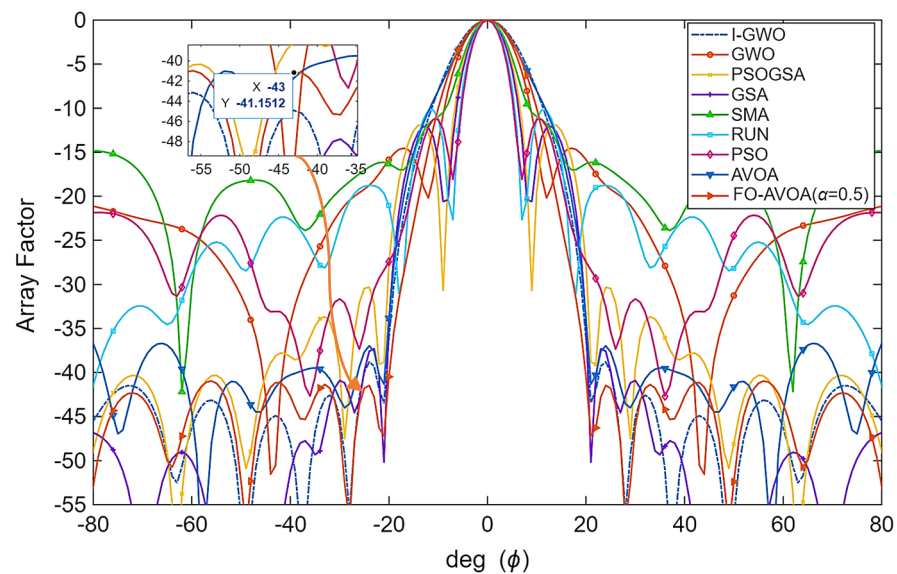


Figure 10. Normalized array factors of the 10×10 PAA using position-only optimization.

From the above analysis, the outcome shown in Table 7 and Figure 10 indicates that the FO-AVOA applied for 2D optimization in a 2D array achieves a remarkably effective sidelobe level reduction for a large-scale 2D planar array. The simultaneous design for array excitation and array element positions in the FO-AVOA allows for the design to feature improved radiation characteristics while employing less involved array excitation, thus improving the array gain and array efficiency and increasing the array's cost-effectiveness.

5.4. Comparative Analysis of Results

The analysis of the performance of the planar antenna array (PAA) optimization problem solved by the proposed FO-AVOA, compared with a number of different metaheuristics, is presented in this section. The analysis is carried out for three different optimization problems, amplitude-only, position-only, and 2D problems, for a size of 5×5 as well as 10×10 arrays. The results for the achieved peak sidelobe level (PSLL) are presented in Table 8 by means of Figure 11. For the case of amplitude-only optimization problems, the FO-AVOA demonstrates its competitive power for both array sizes. For a problem defined in a 5×5 array, the FO-AVOA demonstrates the best PSLL at a level of -14.85 dB among all algorithms. At a problem size of 10×10 , the FO-AVOA demonstrates a PSLL of -37.79 dB, thus outperforming the AVOA with a PSLL of -34.16 dB, the PSOGSA with a PSLL of -37.48 dB, and the GSA with a PSLL of -37.31 dB, but comparable to the results of the I-GWO algorithm at a level of -37.96 dB. The results for larger problem sizes clearly show that the proposed FO-AVOA preserves its power even for highly dimensional problems.

For the position-only optimization, the FO-AVOA overwhelmingly outperforms the other algorithms for the suppression of the PSLL. For the 5×5 array, the result of the proposed algorithm is -24.07 dB, which shows a noticeable improvement over the AVOA (-23.32 dB), while performing much better than classical methods like GWO and the PSOGSA. For the 10×10 array, the FO-AVOA results in a PSLL value of -18.76 dB, which is slightly better than that of the SMA (-18.62 dB), I-GWO (-18.63 dB), and PSO (-18.59 dB). Though the gap reduces for larger arrays, the FO-AVOA is still the best method for this optimization mode.

In 2D optimization, the highest improvement in performance is found in the 2D optimization problem, which involves both amplitude and position excitations being determined simultaneously. For an array size of 5×5 , the FO-AVOA provides the lowest value

of the PSL, which is -26.58 dB, performing better than all other compared algorithms. A great improvement is achieved in the 10×10 array problem, in which the FO-AVOA provides a value of the PSL of -40.96 dB, which is significantly lower compared to both I-GWO (-38.81 dB) and the AVOA (-36.68 dB). Compared with amplitude-only and position-only optimization for the same array size, the 2D strategy yields clear additional PSL reductions, demonstrating the effectiveness of jointly optimizing multiple array parameters.

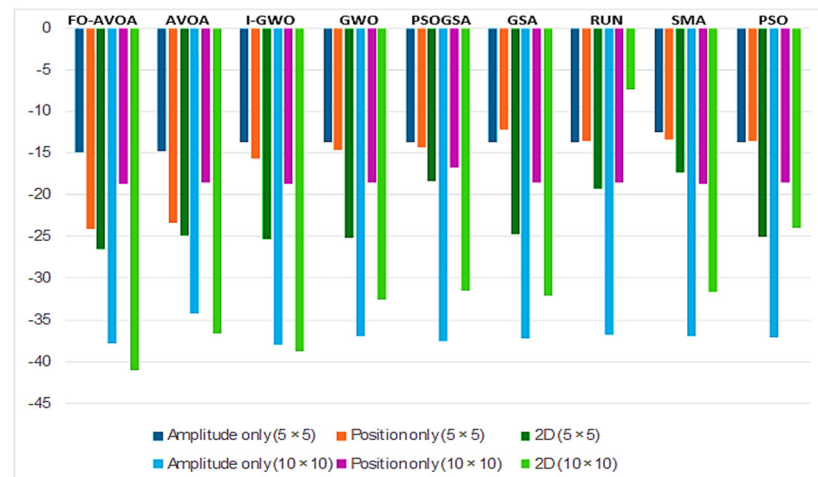


Figure 11. Comparative performance of the proposed and benchmark optimization algorithms for amplitude-only, position-only, and 2D optimizations using 5×5 and 10×10 array configurations.

Based on the results presented in Table 9 and Figure 11, the conclusion can be drawn that the proposed FO-AVOA performs better in all optimization problems compared to the existing approaches. As the size of the optimization problem increases, the performance gap among the algorithms also tends to be increased, which reflects the capabilities of the fractional-order dynamic approach to cope with the exploration/exploitation trade-off in the antenna array optimization problem.

Table 9. PSL (dB) performance comparison for amplitude-only, position-only, and 2D optimization using 5×5 and 10×10 arrays.

Method	Amplitude Only (5×5) PSL (dB)	Position Only (5×5) PSL (dB)	2D (5×5) PSL (dB)	Amplitude Only (10×10) PSL (dB)	Position Only (10×10) PSL (dB)	2D (10×10) PSL (dB)
FO-AVOA	-14.85	-24.07	-26.58	-37.79	-18.76	-40.96
AVOA	-14.78	-23.32	-24.90	-34.16	-18.53	-36.68
I-GWO	-13.74	-15.65	-25.3	-37.96	-18.63	-38.81
GWO	-13.7	-14.63	-25.11	-36.99	-18.59	-32.57
PSOGSA	-13.69	-14.28	-18.42	-37.48	-16.7	-31.55
GSA	-13.7	-12.25	-24.73	-37.31	-18.59	-32.09
RUN	-13.7	-13.5	-19.3	-36.82	-18.59	-7.321
SMA	-12.56	-13.46	-17.39	-36.99	-18.62	-31.62
PSO	-13.69	-13.59	-25	-37.13	-18.59	-24

5.5. Convergence Speed

This subsection examines the convergence behaviour of the proposed FO-AVOA in terms of comparison to the basic AVOA, in addition to the hybrid PSOGSA. Each algorithm was tested for 400 iterations.

As seen in Figure 12, the FO-AVOA has better convergence behaviour compared to the AVOA and PSOGSA. The FO-AVOA converges faster to better solution areas in comparison to the AVOA and PSOGSA. The AVOA has poor convergence properties in terms of speed of convergence, apart from having oscillations. On the other hand, the PSOGSA converges prematurely, and in the latter iterations, it has poor refinement properties.

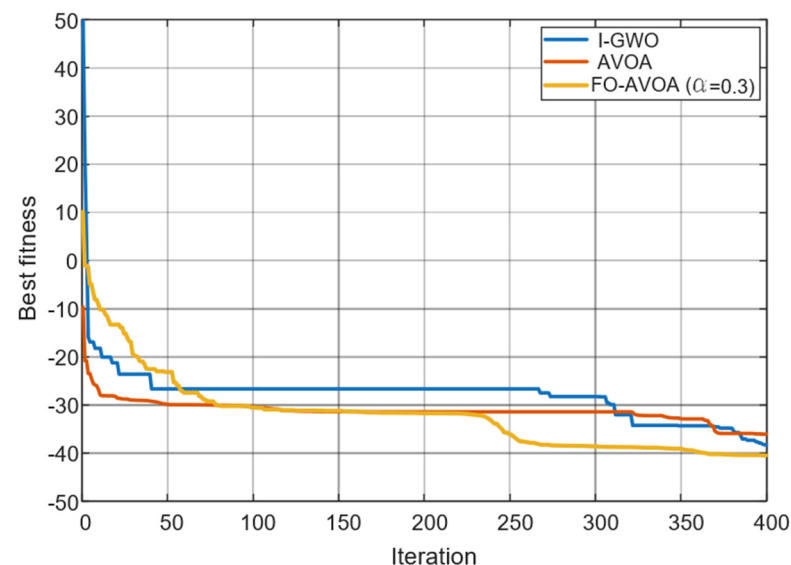


Figure 12. Convergence graph.

The reason for the enhanced convergence speed of the FO-AVOA can be traced to the incorporation of fractional-order memory terms for better exploitation of the results of previous iterations, as well as the adaptation of mechanisms for balancing the trade-off between exploitation and exploration. It can be visualized that the FO-AVOA can efficiently exploratively search the search space initially, increasing the degree of exploitation as it algorithmically advances. Probably, this helps in lessening the chances of premature convergence.

It can be seen from the results above that the convergence properties corroborate the robust optimization ability of the FO-AVOA, especially when it comes to large-scale beamforming problems. The consistent convergence pattern also clearly illustrates its adaptability in the optimization of large planar antenna arrays, where faster convergence rates and a stable result are of prime importance.

5.6. Computational Complexity

The computation time for the optimization of PAA beamforming using the FO-AVOA in the case of two-parameter optimization (amplitude and position) is calculated in Table 10, and the results are compared with other algorithms in Table 10: the results of PSO, GSA, PSOGSA, GWO, SMA, and RUN.

Table 10. Execution time for 2D optimization for 10×10 PAA.

Method	Time (s)	PSLL (dB)
FO-AVOA	2.610	−40.96
AVOA	2.990	−36.68

PSO	1.537549	−24
GWO	2.680383	−32.57
GSA	2.084569	−32.09
PSOGSA	3.316675	−31.55
SMA	3.425231	−31.62
RUN	4.054988	−7.321
I-GWO	6.043369	−38.81

The execution time is the measure of the computation time taken by the algorithm to converge to the optimum solution. The FO-AVOA's execution time is 2.610 s with a PSLL of −40.96 dB. The AVOA's execution time is 2.990 s; for I-GWO, GWO, PSOGSA, GSA, RUN, SMA, and PSO the execution time is 6.043369 s, 2.680383 s, 3.316675 s, 2.084569 s, 4.054988 s, 3.425231 s, and 1.537549 s, respectively. From the above analysis, it can be concluded that the FO-AVOA performed better than the other methods.

6. Conclusions

This work introduced a holistic beamforming design methodology for PAAs using the proposed FO-AVOA. The main focus was to minimize the PSLL of the array's far-field radiation pattern using amplitude-only, position-only, and 2D optimization techniques for 5×5 and 10×10 arrays. The simulation results clearly illustrate that the FO-AVOA always outperforms the best available metaheuristic algorithms for all tested scenarios. For amplitude-only designs, the FO-AVOA obtains a PSLL of −14.85 dB for the 5×5 array and −37.79 dB for the 10×10 array, offering up to 7.4% improvements over the best available alternatives under equal beamwidth constraints. For position-only designs, the FO-AVOA obtains a PSLL of −24.07 dB for the 5×5 array and −18.76 dB for the 10×10 array, offering up to 53% improvements over classical alternatives such as PSO and the GSA. The greatest improvements can be realized by employing the novel 2D design approach, which optimizes both the excitation amplitudes and positions concurrently. For this case, the FO-AVOA obtains a PSLL of −26.58 dB for the 5×5 array and −40.96 dB for the 10×10 array. Compared to the amplitude-only and position-only designs, these results offer additional sidelobe reductions of up to 22.2 dB and 20.2 dB, respectively, for the same array sizes. Notably, the FO-AVOA also outperforms the best available alternative for the 10×10 2D design problem by a margin of approximately 2.15–4.28 dB, thus clearly demonstrating its superior scalability for high-dimensional design problems. Another key practical benefit of the proposed design methodology is its ability to produce sparse excitation distributions, for which a number of antenna elements have zero or very low amplitudes. Such sparse distributions can reduce the number of excited elements, thus offering lower power consumption, easier feeding networks, and less expensive implementations with desirable beamwidth properties.

In general, the simulation outcomes verify that FO-AVOA is an efficient and scalable solution for PAA beamforming optimization, especially when used in the 2D optimization framework. The use of FO in the dynamic model allows for a good exploration–exploitation trade-off, leading to improved solution quality for the complex design of the antenna array. Future studies may include the use of the proposed approach in the design of larger arrays, massive MIMO, Reconfigurable Intelligent Surfaces, and designs beyond 5G and 6G communications, as well as the use of hybrid/multi-objective optimization problems.

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Abbreviations

The following abbreviations are used in this manuscript:

PAA	Planner Antenna Array
2D	Two Dimensional
FO-AVOA	Fractional-Order African Vulture Optimization Algorithm
PSLL	Peak Side Lobe Level
PSO	Particle Swarm Optimization
GSA	Gravitational Search Algorithm
PSOGSA	Hybrid PSO–GSA
RUN	Runge–Kutta Optimizer
SMA	Slime Mould Algorithm
HHO	Harris Hawks Optimization
AVOA	African Vulture Optimization Algorithm
MIMO	Multi-Input–Multi-Output
LAA	Linear Antenna Arrays
CAA	Circular Antenna Arrays
GA	Genetic Algorithm
CS	Cuckoo Search
IBBO	Improved Biogeography-Based Optimization
DSA	Differential Search Algorithm
MFO	Moth Flame Optimization
TLBO	Teaching–Learning-Based Optimization
BSO	Brain Storm Optimization
WDO	Wind-Driven Optimization

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