



Article

Fully Distributed Observer-Based Dynamic Double-Event-Triggered Bipartite Consensus Tracking of Fractional-Order Multi-Agent Systems with Input Saturation

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Abstract

This paper investigates the fully distributed observer-based dynamic double-event-triggered bipartite consensus tracking problem of fractional-order multi-agent systems (FOMASs) with input saturation under a directed graph. First, to address this complex challenge, a pull-based dynamic double-event-triggered mechanism (DDETM) with different event-triggered conditions and capable of operating independently is designed, which can effectively reduce communication costs and controller updates concurrently. Then, the low-gain feedback technique is used to solve the input saturation problem faced by FOMASs under a directed graph. Based on the estimated state information, a fully distributed control protocol with pull-based DDETM is proposed to ensure the achievement of bipartite consensus tracking for FOMASs. A noteworthy feature of this control protocol is its ability to achieve system stability without the need for global information. Correspondingly, the sufficient conditions for achieving bipartite consensus is obtained with the help of low gain feedback technology and Lyapunov stability theory. Moreover, the Zeno behavior is precluded. Finally, a simulation example is presented to illustrate the theoretical results.

Keywords: fractional-order multi-agent systems; dynamic double-event-triggered mechanism; fully distributed control; bipartite consensus tracking; input saturation; directed graph

1. Introduction

In the context of the rapid development of information technology, the problem of cooperative control of multi-agent systems (MASs) has attracted a great deal of attention, with a wide range of applications in the fields of multi-robot systems [1], sensor networks [2], and social networks [3] due to its characteristics of autonomy, distribution, and coordination. As one of the most fundamental and primary cooperative control issues for MASs, consensus control has also raised many concerns. To date, researchers [4–6] have provided numerous insights for the consensus, optimization and resource allocation of MASs, covering integer-order systems from first-order, second-order to higher-order dynamic systems as well as nonlinear fractional-order multiagent systems over unbalanced digraphs, where the pull-based event-triggered mechanism is further extended to fractional-order scenarios with time delay and nonlinearity to reduce communication congestion. However, with the advent of fractional calculus, fractional-order multi-agent systems are investigated by more and more people, offering a more comprehensive and flexible framework to address



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the complexities of cooperative and distributed systems, which have been investigated in [4,7–10]. In [4], a distributed event-triggered control law with time delay was proposed for nonlinear FOMASs. It is noticed that the papers mentioned above are based on an assumption that the information interactions among agents are cooperative, that is, the edge weights of the communication network graph among agents are all non-negative. However, in the real world, such as in social networks, cooperation and antagonism between agents exist simultaneously.

To address the issue of information interactions among agents highlighted above, the concept of bipartite consensus was first introduced by Altafini in [11], which means that the edge weights of the communication network graph among agents are both positive and negative. Further, some corresponding results about the bipartite consensus have been achieved in [12–14]. In [13], a straightforward algebraic graph condition was obtained for the bipartite consensus of matrix-weighted second-order MASs. The distributed bipartite control was investigated by proposing a novel distributed hybrid event-driven strategy in [14]. It is crucial to observe that controllers in several articles, such as [4,15,16], are intricately linked to the global information of the communication graph. Nevertheless, in practical scenarios, the introduction or removal of an agent or alterations in the graph's connectivity will lead to corresponding modifications in the Laplacian matrix of the communication topology. Consequently, addressing the global information issues of the communication topology graph poses a substantial challenge.

How to design an appropriate controller so that each agent can realize cooperative control without relying on global information? This question has garnered significant attention within the academic community. Through the efforts of researchers, some corresponding results have been obtained in [17–23], covering both integer-order and incommensurate heterogeneous fractional-order multi-agent systems. As mentioned above in [15,21,22], all of them only investigated the consensus of MASs (including incommensurate heterogeneous fractional-order MASs) under undirected communication topology graph without input saturation. It is essential to recognize that directed graph is capable of building complex topologies that represent in detail the dependencies between agents in real-world scenarios. Moreover, input saturation is an inevitable phenomenon in practical applications, stemming from the security requirements of real systems and the limitations of physical devices.

In recent years, consensus and consensus tracking problems of multi-agent systems (MASs) under practical constraints have attracted increasing attention. In particular, input saturation and state constraints are inevitable in real-world applications due to actuator limitations and safety requirements, and may significantly degrade system performance if not properly addressed. To this end, various control strategies have been developed to handle constrained consensus problems in both integer-order and fractional-order multi-agent systems. For integer-order MASs, consensus control under actuator saturation has been investigated using low-gain feedback, anti-windup techniques, and auxiliary dynamic systems [24,25]. In addition, constrained observer-based and event-triggered designs have been explored for systems with practical limitations [26]. Meanwhile, increasing attention has also been paid to fractional-order multi-agent systems (FOMASs) under actuator saturation and state constraints. For instance, semi-global leader-following output consensus of heterogeneous FOMASs with input saturation was studied via observer-based protocols in [27], and fuzzy adaptive tracking control combined with event-triggered strategies was developed for FOMASs with partial state constraints and input saturation in [28]. However, most existing studies are limited to undirected communication graphs or require global topology information. Moreover, they mainly adopt continuous communication schemes or static event-triggered mechanisms, which may lead to frequent controller updates and increased communication burden.

To tackle these challenges, researchers have explored the control protocols incorporating a static event-triggered mechanism (ETM) to alleviate communication congestion or reduce the frequency of controller updates. While the static ETM initially proves effective in reducing communication costs, its drawback lies in the increasing frequency of event-triggered as the threshold diminishes. In response to this issue, the dynamic event-triggered mechanism (DETM) was proposed in [29]. Subsequently, numerous scholars have focused on addressing consensus and formation control problems in fractional-order MASs utilizing dynamic event-triggered mechanisms (DETM), such as the dynamic-threshold DETM proposed for underactuated AUVs in [30–32]. Building upon DETM, the DDETM was examined in [33] with the aim of simultaneously decreasing communication costs and controller updates. Recently, DDETM have been proposed to further reduce communication and controller update frequency compared with traditional single event-triggered strategies. For instance, fixed-time cluster consensus of nonlinear multi-agent systems was investigated via DDETM in [34], where convergence speed and communication efficiency were simultaneously improved. Moreover, the bipartite leader-following consensus of linear multi-agent systems with unknown disturbances under directed graphs was studied using DDETM in [35], demonstrating the effectiveness of double triggering in complex networked environments. Although these studies have revealed the advantages of DDETM for integer-order multi-agent systems, they mainly focus on linear or nonlinear integer-order dynamics and do not simultaneously address fractional-order dynamics and actuator saturation. Motivated by this observation, the present paper develops a fully distributed observer-based dynamic double-event-triggered control framework for fractional-order multi-agent systems with input saturation under directed graphs. To the best of the authors' knowledge, results on fully distributed DDETM-based semiglobal bipartite consensus tracking for fractional-order multi-agent systems under directed graphs remain limited, which further highlights the significance of the proposed study.

Inspired by the preceding discussion, this paper addresses the fully distributed dynamic double-event-triggered SBCT problem for FOMASs with input saturation under a directed graph. The key contributions of this work include the design of a pull-based DDETM with different event-triggered conditions that can operate independently to simultaneously reduce communication costs and the frequency of controller updates. The challenge of input saturation in FOMASs is tackled through the application of low-gain feedback technology. Building on the estimated information, a fully distributed DDETM control protocol is proposed to address SBCT in FOMASs under directed graph, and a rigorous proof excluding Zeno behavior is given.

In comparison to the existing literature, the advantages of this article include the following. Unlike [30,36], which focus solely on communication resource consumption or the cost of controller updates for event-triggered strategies, we consider both factors and adopt a DDETM with different event-triggered conditions that can operate independently. As a result, our design provides a better balance between dynamic performance and computational cost by reducing the cost of controller updates while minimizing unnecessary communication resource consumption. The algebraic Riccati equations in some of the existing results [15,16] usually require global communication topology information; however, obtaining global information in practice is challenging, especially when the connectivity of the network topology graph changes and the Laplace matrix of the communication topology changes accordingly. To address this challenge, this paper proposes a fully distributed controller by employing an adaptive technique so that the control protocol does not depend on any global topology information. This approach helps to improve the feasibility of the controller in practical applications. The results in the existing research [15,21,22] typically do not take input saturation into account. However, in practical applications,

inherent security requirements and constraints imposed by physical devices have a significant impact on system behavior and performance. Based on this, we employ a low-gain feedback technique to address the input saturation problem faced by multi-agent systems under directed graph, thereby improving the stability and performance of multi-agent systems in the presence of input constraints. To the best of the authors' knowledge, there is a shortage of results on the fully distributed DDETMM bipartite consensus of FOMASs under a directed graph. Consequently, investigating this problem for FOMASs presents a significant challenge.

The remainder of this article can be summarized as follows. Section 2 presents the elementary preliminaries and problem formulation. Section 3 investigates the fully distributed observer-based SBCT problem under the DDETMM. Section 4 presents a simulation experiment to validate the theoretical results. Section 5 summarizes the article.

Notations: For any given real vector $x = [x_1, x_2, \dots, x_n]^T \in \mathbb{R}^n$, $\|x\|$ is defined as 2-norm. $\otimes$ denotes the Kronecker product. I_N stands for the $N \times N$ identity matrix. $\text{diag}(\dots)$ represents a block-diagonal matrix. A^T means the transpose of matrix A . $A > 0$ (or $A < 0$) signifies that A is positive definite (or negative definite). The notations $\mathbb{R}^n$ and $\mathbb{R}^{m \times n}$ are the spaces of n -dimensional real column vectors and $m \times n$ real matrices, respectively. $\text{sgn}(\cdot)$ represents the sign function.

2. Preliminaries and Problem Formulation

2.1. Graph Theory

We consider a signed communication graph $\mathcal{G} = (\mathcal{V}, \mathcal{R})$ that consists of one leader and N followers, where $\mathcal{V} = \{1, 2, \dots, N\}$ is the set of nodes, $\mathcal{R} \subseteq \mathcal{V} \times \mathcal{V}$ is the set of edges. Consider a diagonal matrix $\mathcal{H} = \text{diag}(a_{10}, a_{20}, \dots, a_{N0})$, where $a_{i0} = 1$ represents agent i that can receive the information from leader agent; otherwise, $a_{i0} = 0$. For two diverse nodes, $(i, j) \in \mathcal{R}$ indicates that agent j can get the information from the agent i . Denote the adjacency matrix $\mathcal{A} = [a_{ij}]_{N \times N}$; if an edge $(j, i) \in \mathcal{R}$, then $a_{ij} \neq 0$, where $a_{ij} > 0$ ($a_{ij} < 0$) represent the cooperative interaction (antagonistic interaction) between agent i and j , respectively. The Laplacian matrix $\mathcal{L}$ can be given as $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D} = \text{diag}(b_1, b_2, \dots, b_N)$, $b_i = \sum_{j=1}^N |a_{ij}|$ and $\mathcal{L}_H$ is defined as $\mathcal{L}_H = \mathcal{L} + \mathcal{H}$.

Definition 1 ([11]). A signed communication graph $\mathcal{G}$ is called a structurally balanced graph, if $\mathcal{V}$ can be classified into two disjoint subsets $\mathcal{V}_1, \mathcal{V}_2$, satisfying $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$, $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$, $a_{ij} \geq 0$, $i, j \in \mathcal{V}_q$ and $a_{ij} \leq 0$, $i \in \mathcal{V}_q, j \in \mathcal{V}_{3-q}$ ($q \in \{1, 2\}$).

Lemma 1 ([11]). A signed communication graph $\mathcal{G}$ is structurally balanced if, and only if, one can always find a matrix $\hat{D} \in \hat{\mathcal{D}} = \{\text{diag}(s_1, s_2, \dots, s_N)\}$, such that $\mathcal{H}^* = \hat{D} \mathcal{L}_H \hat{D}$ is positive definite, and $\mathcal{L}^* = \hat{D} \mathcal{L} \hat{D}$ is semipositive definite, where $s_i \in \{1, -1\}$. Moreover, the bipartition of $\mathcal{V}$ depends on $\hat{D}$, such as $\mathcal{V}_1 = \{i | s_i = 1\}$ and $\mathcal{V}_2 = \{i | s_i = -1\}$.

Assumption 1. The signed communication graph $\mathcal{G}$ includes a directed spanning tree rooted at the leader.

Lemma 2 ([37]). There exists a positive diagonal matrix G such that $G \mathcal{L}_H + \mathcal{L}_H^T G > 0$. One such positive diagonal matrix G can be chosen as $\text{diag}(g_1, g_2, \dots, g_N)$, where $[g_1, g_2, \dots, g_N]^T = (\mathcal{L}_H^T)^{-1} \cdot \mathbf{1}_N$.

Lemma 3 ([37]). Given any constants $\kappa_1 > 0$, $\kappa_2 \geq 0$ and $\kappa_3 \geq 0$, there exists a large enough constant $\kappa_4 > 0$ such that

$$-\kappa_1 \bar{C}^2(t) + \kappa_2 I + \kappa_3 G \bar{C}(t) - \kappa_4 G \leq -G \bar{C}(t) \quad (1)$$

holds for all $t \geq 0$, where $\bar{C}(t) = \text{diag}(\bar{c}_1(t), \bar{c}_2(t), \dots, \bar{c}_N(t))$.

2.2. Caputo Fractional Derivative and Its Properties

Definition 2 ([38]). The Caputo fractional derivative of function f with order $0 < \alpha < 1$ can be given as

$$D^\alpha f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\ell)^{-\alpha} f'(\ell) d\ell, \quad (2)$$

where $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt$.

Definition 3 ([38]). The Caputo fractional integral of function f with order $\alpha \in (0, 1)$ is defined as

$$I^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int_0^t (t-\ell)^{\alpha-1} f(\ell) d\ell. \quad (3)$$

Lemma 4 ([38]). Consider a continuous function $f(t)$; if $0 < \alpha < 1$, then the following equality holds

$$I^\alpha D^\alpha f(t) = f(t) - f(0). \quad (4)$$

Lemma 5 ([39]). For a continuous function $f(t)$, if $\forall t > t_0 \geq 0$, $\lim_{t \rightarrow \infty} t_0 I_t^\alpha |f(t)|^p \leq M$, then $\lim_{t \rightarrow \infty} f(t) = 0$, where $0 < \alpha < 1$, p and M are positive constants.

Definition 4 ([40]). For $c > 0$ and $d > 0$, the Mittag-Leffler function is given as follows

$$E_{c,d}(\vartheta) = \sum_{k=0}^{\infty} \frac{\vartheta^k}{\Gamma(kc + d)}, \quad (5)$$

where $\vartheta \in \mathbb{C}$ and $E_{1,1}(\vartheta) = e^\vartheta$. Denote $E_{c,1}(\vartheta) = E_c(\vartheta)$. Especially, note that if $c \in (0, 1]$, $d > 0$ and $t > 0$, then $0 < E_c(-dt^c) < 1$, $E_{c,c}(-dt^c) > 0$ and $0 < t^c E_{c,c+1}(-dt^c) < \frac{1}{d}$.

Lemma 6 ([40]). For $\alpha \in (0, 1]$ and constant $q \in \mathbb{R}$, considering a continuous function $h(t)$, the solution of $D_t^\alpha x(t) = qx(t) + h(t)$ can be described by

$$x(t) = x(t_0)E_\alpha(q(t-t_0)^\alpha) + \int_{t_0}^t (t-\ell)^{\alpha-1} E_{\alpha,\alpha}(q(t-\ell)^\alpha) h(\ell) d\ell. \quad (6)$$

Lemma 7 ([40]). Consider a differentiable and continuous function $y(t) \in \mathbb{R}^n$. For $\ell > 0$, $0 < \alpha < 1$, and matrix $Q > 0$, the following inequality holds:

$$D^\alpha(y^T(\ell)Qy(\ell)) \leq 2y^T(\ell)QD^\alpha y(\ell). \quad (7)$$

Lemma 8 ([41]). For any vectors $w_1, w_2 \in \mathbb{R}^n$, the following relationship holds:

$$2w_1^T E F w_2 \leq w_1^T E G E^T w_1 + w_2^T F^T G^{-1} F w_2, \quad (8)$$

where matrices $G > 0$, E and F are with appropriate dimensions.

2.3. Problem Formulation

Consider a group of FOMAS consisting of one leader and N followers. $\mathcal{F} = \{1, \dots, N\}$ represents the set that consists of N followers and 0 represents the leader. The dynamic model of follower i is given as follows:

$$\begin{aligned} D^\alpha x_i(t) &= Ax_i(t) + B\phi_s(u_i(t)), \\ y_i(t) &= Fx_i(t), \quad i \in \mathcal{F}, \end{aligned} \quad (9)$$

where $0 < \alpha < 1$, $x_i(t) \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}^m$, and $y_i(t) \in \mathbb{R}^p$ represent the state, control input, and output of follower i , respectively. $\phi_s : \mathbb{R}^m \rightarrow \mathbb{R}^m$ denotes a saturation function, and $\phi_s(u_i(t)) = (\phi_s(u_{i1}(t)), \phi_s(u_{i2}(t)), \dots, \phi_s(u_{im}(t)))^T$ with $\phi_s(u_{ij}(t)) = \text{sgn}(u_{ij}(t)) \min\{|u_{ij}(t)|, v\}$, where input saturation threshold $v > 0$, and $\text{sgn}(\cdot)$ represents the sign function.

The dynamic model of leader is shown as follows:

$$\begin{aligned} D^\alpha x_0(t) &= Ax_0(t), \\ y_0(t) &= Fx_0(t), \end{aligned} \quad (10)$$

where $0 < \alpha < 1$, $x_0(t) \in \mathbb{R}^n$, and $y_0(t) \in \mathbb{R}^p$ represent the state and output of leader 0, respectively.

Definition 5. The FOMAS (9) and (10) are said to reach SBCT; if for any initial conditions $x_i(0) \in \Omega$, $i = 0, 1, \dots, N$, the following condition satisfies:

$$\lim_{t \rightarrow \infty} \|x_i(t) - s_i x_0(t)\| = 0, \quad i \in \mathcal{F} \quad (11)$$

where $s_i = 1$ or -1 , Ω is a bounded set.

Assumption 2. The matrix pair (A, B) is stabilizable and detectable.

Remark 1. Assumption 1 is a sufficient condition for achieving consensus of MASs on a directed graph. Assumption 2 is a basic assumption, which is a prerequisite to achieve consensus for MASs in [15,16].

Under Assumptions 2, we construct the following state observer for each agent:

$$\begin{aligned} D^\alpha \varpi_i(t) &= A\varpi_i(t) + B\phi_s(u_i(t)) + R(F\varpi_i(t) - y_i(t)), \\ D^\alpha \varpi_0(t) &= A\varpi_0(t) + R(F\varpi_0(t) - y_0(t)), \end{aligned} \quad (12)$$

where $\varpi_i(t)$ is the estimation of $x_i(t)$, R is designed such that $A + RF$ is a fractional-order Hurwitz matrix.

3. Main Results

In the section, a fully distributed observer-based controller by utilizing pull-based DDETm will be proposed to realize the SBCT of FOMAS (9) with the leader (10) under a directed graph, and the Zeno behavior will be precluded.

Denote the combined measurement as

$$q_i(t) = \sum_{j=1}^N |a_{ij}|(\varpi_i(t) - \text{sgn}(a_{ij})\varpi_j(t)) + a_{i0}(\varpi_i(t) - s_i\varpi_0(t)). \quad (13)$$

Then, for FOMAS (9), the distributed observer-based pull-based event-triggered control protocol can be designed as

$$u_i(t) = Kc_i(t)q_i(t_k^i), \quad t \in [t_k^i, t_{k+1}^i), \quad (14)$$

where

$$c_i(t) = a_i p_i(t) + b_i \varphi_i(t_k^i), \quad (15)$$

$$\varphi_i(t) = q_i^T(t) P q_i(t), \quad (16)$$

$$D^\alpha p_i(t) = q_i^T(t_k^i) P B B^T P q_i(t_k^i), \quad t \in [t_k^i, t_{k+1}^i), \quad (17)$$

$K = -B^T P$ is the feedback gain matrix, a_i, b_i are positive constants, and t_k^i denotes the k -th triggering time sequence of the i -th agent.

Define the communication measurement error as

$$\Xi_{ri}(t) = q_i(t_r^i) - q_i(t), \quad t \in [t_r^i, t_{r+1}^i), \quad (18)$$

the event-triggered sequence t_r^i determines the r -th triggering time sequence of communicating with neighboring agents, which is determined by the following triggered function:

$$\begin{aligned} t_0^i &= 0, \\ t_{r+1}^i &= \inf \left\{ t > t_r^i \mid \mathcal{J}_{ri}(t) > 0 \right\}, \end{aligned} \quad (19)$$

where $\mathcal{J}_{ri}(t) = \delta_{1i}(\Delta_{ri} \Xi_{ri}^T(t) P B B^T P \Xi_{ri}(t) - \zeta \varepsilon q_i^T(t_r^i) P B B^T P q_i(t_r^i)) - \psi_{1i}(t)$, $\Delta_{ri} = (1 + \varrho_2)(\bar{c}_i^2(t) + \bar{c}_i(t))$, $\bar{c}_i(t) = a_i p_i(t) + b_i \varphi_i(t)$ and $\varepsilon \in (0, 1)$. The dynamics of $\psi_{1i}(t)$ is characterized by

$$D^\alpha \psi_{1i}(t) = -\epsilon_{1i} \psi_{1i}(t) - (\Delta_{ri} \Xi_{ri}^T(t) P B B^T P \Xi_{ri}(t) - \zeta \varepsilon q_i^T(t_r^i) P B B^T P q_i(t_r^i)), \quad (20)$$

where $\epsilon_{1i} > 0$ and $\psi_{1i}(0) > 0$. Then, define the control measurement error as

$$\Xi_{ki}(t) = q_i(t_k^i) - q_i(t_r^i), \quad t \in [t_k^i, t_{k+1}^i). \quad (21)$$

The event-triggered sequence t_k^i determines the k -th triggering time sequence of the updating controller, which is determined by the following triggered function:

$$\begin{aligned} t_0^i &= 0, \\ t_{k+1}^i &= \inf \left\{ t > t_k^i \mid \mathcal{J}_{ki}(t) > 0 \right\}, \end{aligned} \quad (22)$$

where $\mathcal{J}_{ki}(t) = \delta_{2i}(\Delta_{k1i} \Xi_{ki}^T(t) P B B^T P \Xi_{ki}(t) - \Delta_{k2i} q_i^T(t_r^i) P B B^T P q_i(t_r^i) - 2 \Xi_{ki}^T(t) P B B^T P q_i(t_r^i)) - \psi_{2i}(t)$, $\Delta_{k1i} = (1 + \frac{1}{\varrho_2})(\bar{c}_i^2(t) + \bar{c}_i(t)) + d_i^2(t)$, $\Delta_{k2i} = \varsigma(1 - \varepsilon) - d_i^2(t)$, and $d_i(t) = \varphi_i(t) - \varphi_i(t_k^i)$. The dynamics of $\psi_{2i}(t)$ is characterized by

$$\begin{aligned} D^\alpha \psi_{2i}(t) &= -\epsilon_{2i} \psi_{2i}(t) - (\Delta_{k1i} \Xi_{ki}^T(t) P B B^T P \Xi_{ki}(t) \\ &\quad - \Delta_{k2i} q_i^T(t_r^i) P B B^T P q_i(t_r^i) - 2 \Xi_{ki}^T(t) P B B^T P q_i(t_r^i)), \end{aligned} \quad (23)$$

where $\epsilon_{2i} > 0$ and $\psi_{2i}(0) > 0$. Meanwhile, the parameter ϱ_2 in inequalities (19) and (22) is a positive constant.

Remark 2. The literature [26] only focuses on static ETM, which, while initially efficient in resource minimization, encounter the issue of increasingly frequent event triggering as error decreases. In contrast, our article introduces dynamic variables $\psi_{1i}(t)$ and $\psi_{2i}(t)$ in inequalities (19) and (22). These variables serve as the key elements in dynamically adjusting triggering thresholds, thereby mitigating the aforementioned challenge. It is worth noting that the adaptive variables are updated in a self-regulated manner and do not introduce additional communication requirements, which helps maintain low computational complexity at each agent.

Remark 3. It is worth noting that the event-triggered strategy in [30,36] only considers the consumption of communication resources or the cost of controller updates, while in this paper, a DDETM—as shown in (19) and (22)—is introduced to take both of them into consideration, which not only adjusts the threshold dynamically according to the preset conditions, but also reduces controller updates and unnecessary communication resources simultaneously. In addition, the event-triggered control mechanism in this paper is pull-based, which means that each agent only updates the controller based on its own trigger instants.

Lemma 9. For the priori given parameters δ_{1i} , δ_{2i} , ϵ_{1i} , ϵ_{2i} , and $\psi_{1i}(0) > 0$, $\psi_{2i}(0) > 0$, the dynamic variables $\psi_{1i}(t)$, $\psi_{2i}(t)$ in event-triggered function satisfy

$$\psi_{1i}(t) > 0, \quad \psi_{2i}(t) > 0, \quad i \in \mathcal{F}. \quad (24)$$

Proof. For $t \in [t_r^i, t_{r+1}^i)$, referring to Equation (17), one obtains that

$$\Delta_{ri}\Xi_{ri}^T(t)PBB^TP\Xi_{ri}(t) - q_i^T(t_r^i)PBB^TPq_i(t_r^i) \leq \frac{\psi_{1i}(t)}{\delta_{1i}}. \quad (25)$$

Based on Equation (20), one gets that

$$D^\alpha \psi_{1i}(t) \geq -\left(\epsilon_{1i} + \frac{1}{\delta_{1i}}\right)\psi_{1i}(t). \quad (26)$$

According to Definition 4 and Lemma 6, it yields that $\psi_{1i}(t) \geq \psi_{1i}(t_r^i)E_\alpha(-(\epsilon_{1i} + \frac{1}{\delta_{1i}})(t - t_r^i)^\alpha)$. Then, for $t \in [t_{r-1}^i, t_r^i)$, the following inequality holds: $\psi_{1i}(t_r^i) \geq \psi_{1i}(t_{r-1}^i)E_\alpha(-(\epsilon_{1i} + \frac{1}{\delta_{1i}})(t_r^i - t_{r-1}^i)^\alpha)$. Thus, for $t \in [t_{r-1}^i, t_{r+1}^i)$, one has $\psi_{1i}(t) \geq \psi_{1i}(t_{r-1}^i)E_\alpha(-(\epsilon_{1i} + \frac{1}{\delta_{1i}})(t - t_{r-1}^i)^\alpha)$. Then, for $t \in [0, \infty) = [t_0^i, t_1^i) \cup [t_1^i, t_2^i) \cup \dots \cup [t_r^i, t_{r+1}^i)$, it can be further derived that

$$\psi_{1i}(t) \geq \psi_{1i}(0)E_\alpha\left(-\left(\epsilon_{1i} + \frac{1}{\delta_{1i}}\right)t^\alpha\right) > 0. \quad (27)$$

Using the similar method, one can also obtain that

$$\psi_{2i}(t) \geq \psi_{2i}(0)E_\alpha\left(-\left(\epsilon_{2i} + \frac{1}{\delta_{2i}}\right)t^\alpha\right) > 0. \quad (28)$$

□

Remark 4. The event-triggering thresholds are jointly regulated by the error-related terms and the dynamically evolving threshold variables. The parameters δ_{1i} and δ_{2i} adjust the sensitivity of the triggering conditions, thereby determining how measurement and control errors affect the triggering instants. The dynamic variables $\psi_{1i}(t)$ and $\psi_{2i}(t)$ serve as adaptive thresholds, which effectively prevent excessive triggering and play a key role in ensuring Zeno-free behavior. Meanwhile, the coefficients Δ_{ri} , Δ_{k1i} , and Δ_{k2i} explicitly incorporate the sampling-induced mismatch characterized by $d_i(t)$. This design guarantees that the event-triggered mechanism remains consistent with the Lyapunov-based stability analysis, and thus ensures the closed-loop stability of the overall system.

Theorem 1. Suppose that Assumption 2 is satisfied, for initial values $x_i(0) \in \Omega$, $i = 0, 1, \dots, N$, the FOMAS (9) with the leader (10) achieve SBCT under the state observer (12) and the observer-

based control protocol (14) driven by DDET (19) and (22), if there exists a symmetric positive definite matrix P satisfying:

$$PA + A^T P - PBB^T P + \nu I_n = 0. \quad (29)$$

Proof. Denote $\zeta_i(t) = x_i(t) - s_i x_0(t)$, $\hat{\zeta}_i(t) = \omega_i(t) - s_i \omega_0(t)$, and $\tilde{\zeta}_i(t) = \zeta_i(t) - \hat{\zeta}_i(t)$, $i \in \mathcal{F}$. Let $\zeta(t) = [\zeta_1^T(t), \zeta_2^T(t), \dots, \zeta_N^T(t)]^T$ and $\tilde{\zeta}(t) = [\tilde{\zeta}_1^T(t), \tilde{\zeta}_2^T(t), \dots, \tilde{\zeta}_N^T(t)]^T$. By the definition of $\zeta_i(t)$, we derive

$$\begin{aligned} D^\alpha \zeta_i(t) &= D^\alpha x_i(t) - s_i D^\alpha x_0(t) \\ &= A \zeta_i(t) + B \phi_s \left(c_i(t) K q_i(t_k^i) \right). \end{aligned} \quad (30)$$

By recalling the definition of $\tilde{\zeta}(t)$, we obtain

$$D^\alpha \tilde{\zeta}_i(t) = D^\alpha \zeta_i(t) - D^\alpha \hat{\zeta}_i(t) = (A + RF) \tilde{\zeta}_i(t). \quad (31)$$

Since the condition in Theorem 1, $A + RF$ is a fractional-order Hurwitz matrix, it is clear that $\lim_{t \rightarrow \infty} \tilde{\zeta}_i(t) = 0$, and this demonstrates that the state observer is available.

Let $V_{\tau 1}(t) = \sum_{i=1}^N g_i(a_i p_i(t) \phi_i(t) + \frac{1}{2} b_i \phi_i^2(t)) + \sum_{i=1}^N \frac{a_i^2}{2b_i} g_i(p_i(t) - \gamma)^2$, $V_{\tau 2}(t) = \sum_{i=1}^N \tilde{\zeta}_i^T(t) Q \tilde{\zeta}_i(t)$, $V_{\tau 3}(t) = \frac{\gamma}{\mu} \sum_{i=1}^N \psi_{1i}(t)$, $V_{\tau 4}(t) = \frac{\gamma}{\mu} \sum_{i=1}^N \psi_{2i}(t)$, where matrix P is defined in Theorem 1, γ and μ are positive constants, and Q satisfies $Q(A + RF)^T + (A + RF)Q + I_n = 0$.

Consider the Lyapunov function:

$$V_\tau(t) = V_{\tau 1}(t) + \frac{\sigma_3 \|PRF\|^2}{2} V_{\tau 2}(t) + V_{\tau 3}(t) + V_{\tau 4}(t), \quad (32)$$

where σ_3 is a positive constant. Evidently, $V_\tau(t)$ is a positive definite function in the light of Lemma 9. Define a constant $\wp > 0$ such that

$$\wp \geq \sup_{x_i(0) \in \Omega, i=1,2,\dots,N} V_\tau(0). \quad (33)$$

The constant $\wp$ can be guaranteed as a result of the existence of a bounded set Ω and $\lim_{\nu \rightarrow 0} P = 0$. Define $L_{v_\tau}(\wp) = \{p(t) \in \mathbb{R}^N, \phi(t) \in \mathbb{R}^N, \psi_1(t) \in \mathbb{R}^N, \psi_2(t) \in \mathbb{R}^N : V_\tau(t) \leq \wp\}$. Then, $\exists v^* \in (0, 1]$ s.t. $p(t) \in L_{v_\tau}(\wp)$, $\phi(t) \in L_{v_\tau}(\wp)$, $\psi_1(t) \in L_{v_\tau}(\wp)$ and $\psi_2(t) \in L_{v_\tau}(\wp)$ for any $\nu \in (0, v^*]$. The existence of the constant v^* is by reason of the fact that $\lim_{\nu \rightarrow 0} P = 0$. Moreover, due to $A + RF$ is a fractional-order Hurwitz matrix, we can obtain that $\tilde{\zeta}(t)$ is bounded. Therefore, combining $\lim_{\nu \rightarrow 0} P = 0$, one obtains that

$$\|u_i(t)\|_\infty \leq v. \quad (34)$$

Thus, referring to inequality (34), one gets that $\phi_s(u_i(t))$ becomes $u_i(t)$, then Equation (30) can be transformed as

$$D^\alpha \zeta_i(t) = A \zeta_i(t) + B c_i(t) K q_i(t_k^i). \quad (35)$$

Furthermore, by the definition of $q_i(t)$, we derive

$$D^\alpha q(t) = (I_N \otimes A)q(t) + (\mathcal{L}_H C \otimes BK)(q(t) + \Xi_k(t) + \Xi_r(t)) - (\mathcal{L}_H \otimes RF)\tilde{\zeta}(t), \quad (36)$$

where $C = \text{diag}\{c_1(t), c_2(t), \dots, c_N(t)\}$, $q(t) = [q_1^T(t), q_2^T(t), \dots, q_N^T(t)]^T$, $\Xi_k(t) = [\Xi_{k1}^T(t), \Xi_{k2}^T(t), \dots, \Xi_{kN}^T(t)]^T$, and $\Xi_r(t) = [\Xi_{r1}^T(t), \Xi_{r2}^T(t), \dots, \Xi_{rN}^T(t)]^T$. Taking the α -order derivative of $V_{\tau 1}(t)$ gives

$$\begin{aligned} D^\alpha V_{\tau 1}(t) &= \sum_{i=1}^N g_i \bar{c}_i(t) D^\alpha \varphi_i(t) + \sum_{i=1}^N \frac{a_i}{b_i} g_i (\bar{c}_i(t) - a_i \gamma) D^\alpha p_i(t) \\ &\leq q^T(t) (\bar{C} G \otimes PA + A^T P) q(t) - 2q^T(t) (\bar{C} G \mathcal{L}_H C \otimes PBB^T P) (q(t) + \Xi_k(t) + \Xi_r(t)) \\ &\quad - 2q^T(t) (\bar{C} G \mathcal{L}_H \otimes PRF) \tilde{\zeta}(t) + \sum_{i=1}^N \frac{a_i}{b_i} g_i (\bar{c}_i(t) - a_i \gamma) D^\alpha p_i(t), \end{aligned} \quad (37)$$

where $\bar{C} \triangleq \bar{C}(t) = \text{diag}\{\bar{c}_1(t), \bar{c}_2(t), \dots, \bar{c}_N(t)\}$. In light of Lemma 2, one has

$$\begin{aligned} -2q^T(t) (\bar{C} G \mathcal{L}_H \bar{C} \otimes PBB^T P) q(t) &= -q^T(t) (\bar{C} (G \mathcal{L}_H + \mathcal{L}_H^T G) \bar{C} \otimes PBB^T P) q(t) \\ &\leq -\hat{\lambda}_0 q^T(t) (\bar{C} \bar{C} \otimes PBB^T P) q(t), \end{aligned} \quad (38)$$

where $\hat{\lambda}_0 = \lambda_2(G \mathcal{L}_H + \mathcal{L}_H^T G)$. According to Equations (18) and (21), and Lemma 8, we obtain

$$\begin{aligned} -2q^T(t) (\bar{C} G \mathcal{L}_H \bar{C} \otimes PBB^T P) (\Xi_k(t) + \Xi_r(t)) \\ \leq \frac{1}{\sigma_1} q^T(t) (\bar{C} G \mathcal{L}_H \mathcal{L}_H^T G \bar{C} \otimes PBB^T P) q(t) \\ + \sigma_1 (\Xi_k(t) + \Xi_r(t))^T (\bar{C} \bar{C} \otimes PBB^T P) (\Xi_k(t) + \Xi_r(t)) \end{aligned} \quad (39)$$

and

$$\begin{aligned} 2q^T(t) [\bar{C} G \mathcal{L}_H (\bar{C} - C) \otimes PBB^T P] (q(t) + \Xi_k(t) + \Xi_r(t)) \\ \leq \frac{1}{\sigma_2} q^T(t) (\bar{C} G \mathcal{L}_H \mathcal{L}_H^T G \bar{C} \otimes PBB^T P) q(t) + \sigma_2 \sum_{i=1}^N b_i^2 d_i^2(t) q_i^T(t_k^i) PBB^T P q_i(t_k^i), \end{aligned} \quad (40)$$

where $b_i d_i(t) = \bar{c}_i(t) - c_i(t)$, σ_1 and σ_2 are positive constants. Combining $\|q_i(t_k^i)\|^2 \leq 2\|\Xi_{ki}(t)\|^2 + 2\|q_i(t_r^i)\|^2$, one gets that

$$\begin{aligned} \sigma_2 \sum_{i=1}^N b_i^2 d_i^2(t) q_i^T(t_k^i) PBB^T P q_i(t_k^i) \\ \leq 2\sigma_2 \sum_{i=1}^N b_i^2 d_i^2(t) \Xi_{ki}^T(t) PBB^T P \Xi_{ki}(t) + 2\sigma_2 \sum_{i=1}^N b_i^2 d_i^2(t) q_i^T(t_r^i) PBB^T P q_i(t_r^i). \end{aligned} \quad (41)$$

In view of Lemma 8, it is easy to obtain that

$$-2q^T(t) (\bar{C} G \mathcal{L}_H \otimes PRF) \tilde{\zeta}(t) \leq \frac{1}{\sigma_3 \lambda} q^T(t) (\bar{C} G \mathcal{L}_H \mathcal{L}_H^T G \bar{C} \otimes PBB^T P) q(t) + \sigma_3 \|\text{PRF}\|^2 \tilde{\zeta}^T(t) \tilde{\zeta}(t), \quad (42)$$

where σ_3 is a positive constant, and $\lambda = \lambda_{\min}(PBB^T P)$. Based on Equation (17), we obtain

$$\begin{aligned} \sum_{i=1}^N \frac{a_i}{b_i} g_i (\bar{c}_i(t) - a_i \gamma) D^\alpha p_i(t) &= \sum_{i=1}^N \frac{a_i}{b_i} g_i (\bar{c}_i(t) - a_i \gamma) q_i^T(t_k^i) PBB^T P q_i(t_k^i) \\ &= \sum_{i=1}^N \frac{a_i}{b_i} g_i (\bar{c}_i(t) - a_i \gamma (1 - \varsigma)) q_i^T(t_k^i) PBB^T P q_i(t_k^i) \\ &\quad - \varsigma \gamma \sum_{i=1}^N \frac{a_i^2}{b_i} g_i q_i^T(t_k^i) PBB^T P q_i(t_k^i), \end{aligned} \quad (43)$$

where $\varsigma \in (0, 1)$. By employing Young's inequality, Equations (18), and (21), it derives that

$$\begin{aligned} & \sum_{i=1}^N \frac{a_i}{b_i} g_i(\bar{c}_i(t) - a_i \gamma(1 - \varsigma)) q_i^T(t_k^i) P B B^T P q_i(t_k^i) \\ & \leq \sum_{i=1}^N (\Pi_1 \bar{c}_i(t) g_i - \gamma \Pi_2 g_i) q_i^T(t) P B B^T P q_i(t) \\ & \quad + (1 + \varrho_1) \sum_{i=1}^N \frac{a_i}{b_i} g_i(\bar{c}_i(t) - a_i \gamma(1 - \varsigma)) (\Xi_{ki}(t) + \Xi_{ri}(t))^T P B B^T P (\Xi_{ki}(t) + \Xi_{ri}(t)), \quad (44) \end{aligned}$$

where $\Pi_1 = \max\{\frac{a_i}{b_i}(1 + \varrho_1)\}$ and $\Pi_2 = \min\{\frac{a_i^2}{b_i}(1 - \varsigma)(1 + \varrho_1)\}$. According to the definition of $q_i(t)$ and Young's inequality, we obtain

$$\begin{aligned} \sum_{i=1}^N (\Xi_{ki}(t) + \Xi_{ri}(t))^T P B B^T P (\Xi_{ki}(t) + \Xi_{ri}(t)) & \leq (1 + \frac{1}{\varrho_2}) \sum_{i=1}^N \Xi_{ki}^T(t) P B B^T P \Xi_{ki}(t) \\ & \quad + (1 + \varrho_2) \sum_{i=1}^N \Xi_{ri}^T(t) P B B^T P \Xi_{ri}(t), \quad (45) \end{aligned}$$

where ϱ_2 is a positive constant.

Taking the α -order derivative of $V_{\tau 2}(t)$ gives

$$\begin{aligned} D^\alpha V_{\tau 2}(t) & \leq 2 \sum_{i=1}^N \tilde{\zeta}_i^T(t) Q D^\alpha \tilde{\zeta}_i(t) \\ & = \sum_{i=1}^N \tilde{\zeta}_i^T(t) (Q(A + RF)^T + (A + RF)Q) \tilde{\zeta}_i(t) = - \sum_{i=1}^N \tilde{\zeta}_i^T(t) \tilde{\zeta}_i(t). \quad (46) \end{aligned}$$

Then, combining inequalities (37)–(46) yields the following estimate. Specifically, substituting the bounds in (38)–(45) into (37), and then adding (46), we obtain

$$\begin{aligned} D^\alpha V_{r1}(t) + D^\alpha V_{r2}(t) & \leq q^T(t) (\bar{C}G \otimes PA + A^T P) q(t) - q^T(t) (\bar{C}G \mathcal{L}_H \bar{C} \otimes P B B^T P) q(t) \\ & \quad - 2q^T(t) (\bar{C}G \mathcal{L}_H \otimes P B B^T P) (\Xi_k(t) + \Xi_r(t)) \\ & \quad - 2q^T(t) (\bar{C}G \mathcal{L}_H \otimes PRF) \tilde{\zeta}(t) \\ & \quad + \sum_{i=1}^N \frac{a_i}{b_i} g_i(\bar{c}_i(t) - a_i \gamma) D^\alpha p_i(t) - \sum_{i=1}^N \zeta_i^T(t) \zeta_i(t) \\ & \leq q^T(t) (CG \otimes PA + A^T P) q(t) \\ & \quad + q^T(t) (-\sigma_4 \bar{C}^2 + \Pi_1 G \bar{C} - \gamma \Pi_2 G \otimes) P B B^T P q(t) \\ & \quad + \frac{\gamma}{\mu} \sum_{i=1}^N \left[\frac{\sigma_1 \mu c_i^2(t)}{\gamma} + \left(1 + \frac{1}{\varrho_1}\right) \frac{a_i \mu}{b_i^2 \gamma_i} g_i \bar{c}_i(t) \right] (1 + \varrho_2) \Xi_{ri}^T(t) P B B^T P \Xi_{ri}(t) \\ & \quad + \frac{\gamma}{\mu} \sum_{i=1}^N \left[\frac{\sigma_1 \mu c_i^2(t)}{\gamma} + \left(1 + \frac{1}{\varrho_1}\right) \frac{a_i \mu}{b_i^2 \gamma_i} g_i \bar{c}_i(t) \right] \left(1 + \frac{1}{\varrho_2}\right) \Xi_{ki}^T(t) P B B^T P \Xi_{ki}(t) \\ & \quad + 2\sigma_2 \sum_{i=1}^N b_i^2 d_i^2(t) \Xi_{ki}^T(t) P B B^T P \Xi_{ki}(t) + 2\sigma_2 \sum_{i=1}^N b_i^2 d_i^2(t) q_i^T(t_k^i) P B B^T P q_i(t_k^i) \\ & \quad - \zeta \gamma \sum_{i=1}^N \frac{a_i^2}{b_i} g_i q_i^T(t_k^i) P B B^T P q_i(t_k^i), \quad (47) \end{aligned}$$

where $\sigma_4 = \lambda_2(G\mathcal{L}_H + \mathcal{L}_H^T G) - \left(\frac{1}{\sigma_1} + \frac{1}{\sigma_2} + \frac{1}{\sigma_3}\right)\lambda_{\max}(G\mathcal{L}_H + \mathcal{L}_H^T G)$. Specifically, the constant γ can be chosen as

$$\gamma > \max \left\{ \sigma_1 \mu, 2\sigma_2 b_i^2 \mu, \left(1 + \frac{a_i \mu}{\varrho_1 b_i^2 \gamma_i} g_i \right) \right\}, \quad \text{where } \mu > \frac{b_i}{a_i^2 g_i}.$$

According to Lemma 3, one can take a sufficiently large γ such that

$$-\sigma_4 C^2(t) + \Pi_1 G C(t) - \gamma \Pi_2 G \leq -G C(t). \quad (48)$$

Finally, combining (18) and (21), we have

$$\Xi_{ri}(t) = q_i(t_r^i) - q_i(t), \quad \Xi_{ki}(t) = q_i(t_k^i) - q_i(t_r^i),$$

which implies $q_i(t_k^i) = \Xi_{ki}(t) + q_i(t_r^i)$. Hence,

$$\begin{aligned} q_i^T(t_k^i) P B B^T P q_i(t_k^i) &= (\Xi_{ki}(t) + q_i(t_r^i))^T P B B^T P (\Xi_{ki}(t) + q_i(t_r^i)) \\ &= \Xi_{ki}^T(t) P B B^T P \Xi_{ki}(t) + q_i^T(t_r^i) P B B^T P q_i(t_r^i) + 2 \Xi_{ki}^T(t) P B B^T P q_i(t_r^i). \end{aligned}$$

Multiplying the above identity by $-\zeta \gamma \frac{a_i^2}{b_i} g_i$ and summing over $i = 1, \dots, N$ yields

$$\begin{aligned} -\zeta \gamma \sum_{i=1}^N \frac{a_i^2}{b_i} g_i q_i^T(t_k^i) P B B^T P q_i(t_k^i) &= -\zeta \gamma \sum_{i=1}^N \frac{a_i^2}{b_i} g_i \Xi_{ki}^T(t) P B B^T P \Xi_{ki}(t) \\ &\quad - \zeta \gamma \sum_{i=1}^N \frac{a_i^2}{b_i} g_i q_i^T(t_r^i) P B B^T P q_i(t_r^i) \\ &\quad - 2 \zeta \gamma \sum_{i=1}^N \frac{a_i^2}{b_i} g_i \Xi_{ki}^T(t) P B B^T P q_i(t_r^i). \end{aligned} \quad (49)$$

Based on inequalities (48) and (49), inequality (47) can be further rewritten as

$$\begin{aligned} D^\alpha V_{\tau 1}(t) + D^\alpha V_{\tau 2}(t) &\leq q^T(t) (\bar{C} G \otimes P A + A^T P - P B B^T P) q(t) \\ &\quad + \frac{\gamma}{\mu} \sum_{i=1}^N (\bar{c}_i^2(t) + \bar{c}_i(t)) (1 + \varrho_2) \Xi_{ri}(t)^T P B B^T P \Xi_{ri}(t) \\ &\quad + \frac{\gamma}{\mu} \sum_{i=1}^N \left[(\bar{c}_i^2(t) + \bar{c}_i(t)) \left(1 + \frac{1}{\varrho_2} \right) + d_i^2(t) \right] \Xi_{ki}(t)^T P B B^T P \Xi_{ki}(t) \\ &\quad + \frac{\gamma}{\mu} \sum_{i=1}^N d_i^2(t) q_i^T(t_r^i) P B B^T P q_i(t_r^i) - \frac{\gamma \varsigma}{\mu} \sum_{i=1}^N \Xi_{ki}^T(t) P B B^T P \Xi_{ki}(t) \\ &\quad - \frac{\gamma \varsigma}{\mu} \sum_{i=1}^N q_i^T(t_r^i) P B B^T P q_i(t_r^i) - 2 \frac{\gamma \varsigma}{\mu} \sum_{i=1}^N \Xi_{ki}^T(t) P B B^T P q_i(t_r^i). \end{aligned} \quad (50)$$

Taking the α -order derivative of $V_{\tau 3}(t)$ gives

$$\begin{aligned} D^\alpha V_{\tau 3}(t) &= \frac{\gamma}{\mu} \sum_{i=1}^N D^\alpha \psi_{i1}(t) \\ &= -\frac{\gamma}{\mu} \sum_{i=1}^N \epsilon_{1i} \psi_{1i} + \frac{\gamma \varsigma \epsilon}{\mu} \sum_{i=1}^N q_i^T(t_r^i) P B B^T P q_i(t_r^i) \\ &\quad - \frac{\gamma}{\mu} \sum_{i=1}^N (\bar{c}_i^2(t) + \bar{c}_i(t)) (1 + \varrho_2) \Xi_{ri}^T(t) P B B^T P \Xi_{ri}(t). \end{aligned} \quad (51)$$

Taking the α -order derivative of $V_{\tau 4}(t)$ gives

$$\begin{aligned} D^\alpha V_{\tau 4}(t) &= \frac{\gamma}{\mu} \sum_{i=1}^N D^\alpha \psi_{i2}(t) \\ &= -\frac{\gamma}{\mu} \sum_{i=1}^N \epsilon_{2i} \psi_{2i} - \frac{\gamma}{\mu} \sum_{i=1}^N \left[(\bar{c}_i^2(t) + \bar{c}_i(t)) \left(1 + \frac{1}{\varrho_2}\right) + d_i^2(t) \right] \Xi_{ki}^T(t) PBB^T P \Xi_{ki}(t) \\ &\quad + \frac{\gamma}{\mu} \sum_{i=1}^N [\zeta(1 - \epsilon) - d_i^2(t)] q_i^T(t_r^i) PBB^T P q_i(t_r^i) + 2 \frac{\gamma}{\mu} \sum_{i=1}^N \Xi_{ki}^T(t) PBB^T P q_i(t_r^i). \end{aligned} \quad (52)$$

Furthermore, it is easy from inequalities (45)–(48) to obtain

$$D^\alpha V_\tau(t) \leq q^T(t) (\bar{C}G \otimes PA + A^T P - PBB^T P) q(t) - \frac{\gamma}{\mu} \sum_{i=1}^N \epsilon_{1i} \psi_{1i}(t) - \frac{\gamma}{\mu} \sum_{i=1}^N \epsilon_{2i} \psi_{2i}(t). \quad (53)$$

Let $\bar{q}(t) = (\sqrt{\bar{C}G} \otimes I_n) q(t)$, from inequality (53), it can be further obtained

$$\begin{aligned} D^\alpha V_\tau(t) &\leq \bar{q}^T(t) (I_N \otimes PA + A^T P - PBB^T P) \bar{q}(t) \\ &\leq -\nu \|\bar{q}(t)\|^2. \end{aligned} \quad (54)$$

From inequality (54), we can derive that $V_\tau(t)$ is bounded. Furthermore, combining Definition 3 and Lemma 5, one has

$$\frac{1}{\Gamma(\alpha)} \int_0^t (t - \ell)^{\alpha-1} \|\bar{q}(\ell)\|^2 d\ell \leq \frac{1}{\nu} (V_\tau(0) - V_\tau(t)). \quad (55)$$

In light of Lemma 5, $\bar{q}(t)$ can converge to 0 as $t \rightarrow \infty$. Therefore, combining $\lim_{t \rightarrow \infty} \tilde{\zeta}(t) = 0$ and the definition of $q_i(t)$, we further have $\lim_{t \rightarrow \infty} \zeta(t) = 0$, that is, the FOMAS (9) and (10) with fully distributed observer-based control protocol (14) realize the SBCT by employing DDETM. This proof is completed. $\square$

Remark 5. Under Assumption 2, the pair (A, B) is stabilizable. It is well known that, for a sufficiently small design parameter $\nu > 0$, the algebraic Riccati Equation (29) admits a symmetric positive definite solution P . This property is commonly exploited in low-gain control design and provides the theoretical foundation for the stability analysis in Theorem 1.

Remark 6. In contrast to [15,16], which require global communication topology information, this paper proposes fully distributed controller (14) that makes the control protocol independent of global information. Indeed, it is well known that obtaining global information faces significant challenges for large networks. Moreover, when the connectivity of the topology graph changes, the Laplace matrix of the communication topology changes accordingly. Therefore, the design of a fully distributed controller facilitates its application in practice.

Next, we will show that agents are free from Zeno behavior under the corresponding DETM. Specifically, by establishing the boundedness of the fractional-order derivative $D^\alpha q_i(t)$ and deriving explicit strictly positive lower bounds on the inter-event times, the Zeno behavior can be rigorously excluded.

For any $t \in [t_r^i, t_{r+1}^i)$, based on the Caputo fractional integral and Equation (18), one gets

$$\begin{aligned}
\|\Xi_{ri}(t)\| &= \|q_i(t) - q_i(t_r^i)\| \\
&= \|I^\alpha(D^\alpha q_i(t)) - I^\alpha(D^\alpha q_i(t_r^i))\| \\
&= \left\| \frac{1}{\Gamma(\alpha)} \left(\int_0^t \frac{D^\alpha q_i(\ell)}{(t-\ell)^{1-\alpha}} d\ell - \int_0^{t_r^i} \frac{D^\alpha q_i(\ell)}{(t_r^i-\ell)^{1-\alpha}} d\ell \right) \right\| \\
&\leq \frac{1}{\Gamma(\alpha)} \left\| \int_0^{t_r^i} \left(\frac{D^\alpha q_i(\ell)}{(t-\ell)^{1-\alpha}} - \frac{D^\alpha q_i(\ell)}{(t_r^i-\ell)^{1-\alpha}} \right) d\ell \right\| + \frac{1}{\Gamma(\alpha)} \left\| \int_{t_r^i}^t \frac{D^\alpha q_i(\ell)}{(t-\ell)^{1-\alpha}} d\ell \right\| \quad (56)
\end{aligned}$$

According to Theorem 1, we can derive that $D^\alpha \zeta_i(t)$ and $D^\alpha \tilde{\zeta}_i(t)$ are bounded. Thus, we can further obtain that $D^\alpha q_i(t)$ is bounded, it is clear that there exists a positive constant $\mathcal{M} > 0$ such that $D^\alpha q_i(t) \leq \mathcal{M}$. Then,

$$\begin{aligned}
\|\Xi_{ri}(t)\| &\leq \frac{\mathcal{M}}{\Gamma(\alpha)} \left| \int_0^{t_r^i} \left(\frac{1}{(t-\ell)^{1-\alpha}} - \frac{1}{(t_r^i-\ell)^{1-\alpha}} \right) d\ell \right| + \frac{\mathcal{M}}{\Gamma(\alpha)} \left| \int_{t_r^i}^t \frac{1}{(t-\ell)^{1-\alpha}} d\ell \right| \\
&= \frac{\mathcal{M}}{\Gamma(\alpha+1)} \left(\left| (t-t_r^i)^\alpha - t^\alpha + (t_r^i)^\alpha \right| + (t-t_r^i)^\alpha \right) \\
&\leq \frac{2\mathcal{M}(t-t_r^i)^\alpha}{\Gamma(\alpha+1)}. \quad (57)
\end{aligned}$$

It can be seen from the corresponding DETM (19) that $\mathcal{J}_{ri}(t) \leq 0$, and we obtain the next event, which will not be reached before

$$\delta_{1i} \left(\Delta_{ri} \|\Xi_{ri}^T(t_1^*) PB\|^2 - \varsigma \varepsilon \|q_i^T(t_r^i) PB\|^2 \right) = \psi_{1i}(t_1^*). \quad (58)$$

Denote $\Lambda_1 = t_1^* - t_r^i$; it follows from inequality (57) and Equation (58) that

$$\begin{aligned}
\frac{\psi_{1i}(\Lambda_1 + t_r^i)}{\delta_{1i}} + \varsigma \varepsilon \|q_i^T(t_r^i) PB\|^2 &= \Delta_{ri} \|\Xi_{ri}^T(\Lambda_1 + t_r^i) PB\|^2 \\
&\leq \Delta_{ri} \|PB\|^2 \frac{4\mathcal{M}^2 \Lambda_1^{2\alpha}}{\Gamma^2(\alpha+1)}. \quad (59)
\end{aligned}$$

Thus, we can obviously get that

$$t_{r+1}^i - t_r^i \geq \Lambda_1 \geq \left[\frac{\Gamma^2(\alpha+1)}{4\mathcal{M}^2 \Delta_{ri} \|PB\|^2} \left(\frac{\psi_{1i}(\Lambda_1 + t_r^i)}{\delta_{1i}} + \varsigma \varepsilon \|q_i^T(t_r^i) PB\|^2 \right) \right]^{\frac{1}{2\alpha}} > 0. \quad (60)$$

Following this, we obtain that the positive lower bounds are obtained for interevent intervals of communication among agents, that is to say, the Zeno behavior is precluded under the corresponding DETM (19).

Next, for any $t \in [t_k^i, t_{k+1}^i)$, based on Equation (21) and $\Xi_{ki}(t_k^i) = 0$, it yields that

$$\begin{aligned}
\|\Xi_{ki}(t)\| &= \|q_i(t) - q_i(t_k^i) + \Xi_{ri}(t) + \Xi_{ki}(t_k^i)\| \\
&= \|q_i(t) - q_i(t_k^i) + \Xi_{ri}(t) - \Xi_{ri}(t_k^i)\| \\
&\leq \|q_i(t) - q_i(t_k^i)\| + \|\Xi_{ri}(t) - \Xi_{ri}(t_k^i)\|. \quad (61)
\end{aligned}$$

According to the Caputo fractional integral in Equations (18) and (21), inequality (61) can be transformed as

$$\begin{aligned}\|\Xi_{ki}(t)\| &= \|I^\alpha(D^\alpha q_i(t)) - I^\alpha(D^\alpha q_i(t_k^i))\| + \|I^\alpha(D^\alpha \Xi_{ri}(t)) - I^\alpha(D^\alpha \Xi_{ri}(t_k^i))\| \\ &= \left\| \frac{1}{\Gamma(\alpha)} \left(\int_0^t \frac{D^\alpha q_i(\ell)}{(t-\ell)^{1-\alpha}} d\ell - \int_0^{t_k^i} \frac{D^\alpha q_i(\ell)}{(t_k^i-\ell)^{1-\alpha}} d\ell \right) \right\| \\ &\quad + \left\| \frac{1}{\Gamma(\alpha)} \left(\int_0^t \frac{D^\alpha \Xi_{ri}(\ell)}{(t-\ell)^{1-\alpha}} d\ell - \int_0^{t_k^i} \frac{D^\alpha \Xi_{ri}(\ell)}{(t_k^i-\ell)^{1-\alpha}} d\ell \right) \right\| \\ &\leq \frac{2}{\Gamma(\alpha)} \left\| \int_0^{t_k^i} \left(\frac{D^\alpha q_i(\ell)}{(t-\ell)^{1-\alpha}} - \frac{D^\alpha q_i(\ell)}{(t_k^i-\ell)^{1-\alpha}} \right) d\ell \right\| + \frac{2}{\Gamma(\alpha)} \left\| \int_{t_k^i}^t \frac{D^\alpha q_i(\ell)}{(t-\ell)^{1-\alpha}} d\ell \right\|. \quad (62)\end{aligned}$$

Since $D^\alpha q_i(t)$ is bounded, inequality (62) can be further derived as

$$\begin{aligned}\|\Xi_{ki}(t)\| &\leq \frac{2\mathcal{M}}{\Gamma(\alpha)} \left| \int_0^{t_k^i} \left(\frac{1}{(t-\ell)^{1-\alpha}} - \frac{1}{(t_k^i-\ell)^{1-\alpha}} \right) d\ell \right| + \frac{2\mathcal{M}}{\Gamma(\alpha)} \left| \int_{t_k^i}^t \frac{1}{(t-\ell)^{1-\alpha}} d\ell \right| \\ &= \frac{2\mathcal{M}}{\Gamma(\alpha+1)} \left(\left| (t-t_k^i)^\alpha - t^\alpha + (t_k^i)^\alpha \right| + (t-t_k^i)^\alpha \right) \\ &\leq \frac{4\mathcal{M}(t-t_k^i)^\alpha}{\Gamma(\alpha+1)}. \quad (63)\end{aligned}$$

Note that $\Delta_{k1i}\Xi_{ki}^T(t)PBB^TP\Xi_{ki}(t) - \Delta_{k2i}q_i^T(t_r^i)PBB^TPq_i(t_r^i) - 2\Xi_{ki}^T(t)PBB^TPq_i(t_r^i) \geq (\Delta_{k1i} - \frac{1}{2\rho})\Xi_{ki}^T(t)PBB^TP\Xi_{ki}(t) - (\frac{\rho}{2} + \Delta_{k2i})q_i^T(t_r^i)PBB^TPq_i(t_r^i)$, where ρ is a positive design parameter to guarantee $\Delta_{k1i} - \frac{1}{2\rho} > 0$, $\frac{\rho}{2} + \Delta_{k2i} > 0$, and it can be seen from the corresponding DETM (22) that $\mathcal{J}_{ki}(t) \leq 0$. Thus, we obtain the next event, which will not be reached before

$$\delta_{2i} \left(\left(\Delta_{k1i} - \frac{1}{2\rho} \right) \|\Xi_{ki}^T(t_2^*)PB\|^2 - \left(\frac{\rho}{2} + \Delta_{k2i} \right) \|q_i^T(t_r^i)PB\|^2 \right) = \psi_{2i}(t_2^*). \quad (64)$$

Denote $\Lambda_2 = t_2^* - t_k^i$; it follows from inequality (63) and Equation (64) that

$$\begin{aligned}\frac{\psi_{2i}(\Lambda_2 + t_k^i)}{\delta_{2i}} + \left(\frac{\rho}{2} + \Delta_{k2i} \right) \|q_i^T(t_r^i)PB\|^2 &= \left(\Delta_{k1i} - \frac{1}{2\rho} \right) \|\Xi_{ki}^T(\Lambda_2 + t_k^i)PB\|^2 \\ &\leq \left(\Delta_{k1i} - \frac{1}{2\rho} \right) \|PB\|^2 \frac{16\mathcal{M}^2\Lambda_2^{2\alpha}}{\Gamma^2(\alpha+1)}. \quad (65)\end{aligned}$$

Thus, we can obviously get that

$$\begin{aligned}t_{k+1}^i - t_k^i &\geq \Lambda_2 \\ &\geq \left[\frac{\Gamma^2(\alpha+1)}{16\mathcal{M}^2(\Delta_{k1i} - \frac{1}{2\rho})\|PB\|^2} \left(\frac{\psi_{2i}(\Lambda_2 + t_k^i)}{\delta_{2i}} + \left(\frac{\rho}{2} + \Delta_{k2i} \right) \|q_i^T(t_r^i)PB\|^2 \right) \right]^{\frac{1}{2\alpha}} \\ &> 0. \quad (66)\end{aligned}$$

Following this, we obtain that the positive lower bounds are obtained for interevent intervals of the controller update, that is to say, the Zeno behavior is precluded under the corresponding DETM (22). Therefore, both the communication-triggering sequence and the controller-update sequence admit strictly positive minimum inter-event intervals.

4. Simulation

All numerical simulations in this paper are performed using MATLAB R2023a. The simulation results are generated and implemented in the MATLAB environment.

In this section, a simulation example is used to validate the above theoretical result. Consider a FOMAS consisting of one leader and four followers (labeled as 0, 1, 2, 3, and 4). The directed communication graph among agents is depicted in Figure 1. The system matrices of FOMAS (9) with the leader (10) under the fully distributed control law (14) are presented as

$$A = \begin{bmatrix} 0 & -2.6 \\ 3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad F = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad R = \begin{bmatrix} -4 \\ 1 \end{bmatrix}.$$

According to Figure 1, the Laplacian matrix among agents is represented as

$$\mathcal{L} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}.$$

Solving (29) with $\nu = 0.5$, we obtain

$$P = \begin{bmatrix} 0.6756 & -0.0093 \\ -0.0093 & 0.7500 \end{bmatrix}.$$

Moreover, the matrix K is given as

$$K = \begin{bmatrix} -0.6663 & -0.7407 \end{bmatrix}.$$

Assume that $a_i = 1$, $b_i = 0.02$, $\varsigma = 0.1$, $\varepsilon = 0.05$, $\delta_{11} = \delta_{21} = \delta_{12} = \delta_{22} = \delta_{13} = \delta_{23} = \delta_{14} = \delta_{24} = 30$, $\epsilon_{1i} = \epsilon_{2i} = 1.2$, and $\varrho_2 = 10$. Suppose the initial values of dynamic parameters are $\psi_{11}(0) = 3.4$, $\psi_{12}(0) = 5$, $\psi_{13}(0) = 6$, $\psi_{14}(0) = 4$, $\psi_{21}(0) = 8$, $\psi_{22}(0) = 12$, $\psi_{23}(0) = 6$, and $\psi_{24}(0) = 7$.

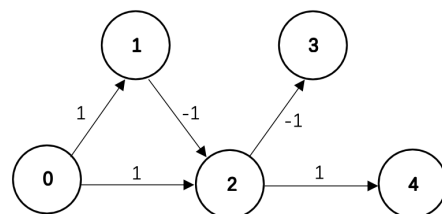


Figure 1. Communication topology graph.

Figure 2 illustrates the evolution of the observation errors of all followers. It can be observed that the observation errors converge to zero, which verifies the effectiveness of the proposed observer-based control design. Figure 3 depicts the temporal evolution of the dynamic variables $\psi_{1i}(t)$ and $\psi_{2i}(t)$. It is observed that these variables remain strictly positive, which provides convincing evidence for the validity of Lemma 9 and the feasibility of the proposed dynamic event-triggering mechanism.

Figure 4 shows the evolution of the adaptive gains $c_i(t)$ for all followers. It is noteworthy that these gains converge to stable positive constants, indicating the effectiveness of the adaptive adjustment mechanism. Figure 5 presents the control inputs of all agents. It can be seen that the control inputs remain bounded throughout the simulation, which confirms the practicality of the proposed low-gain control strategy in the presence of input saturation.

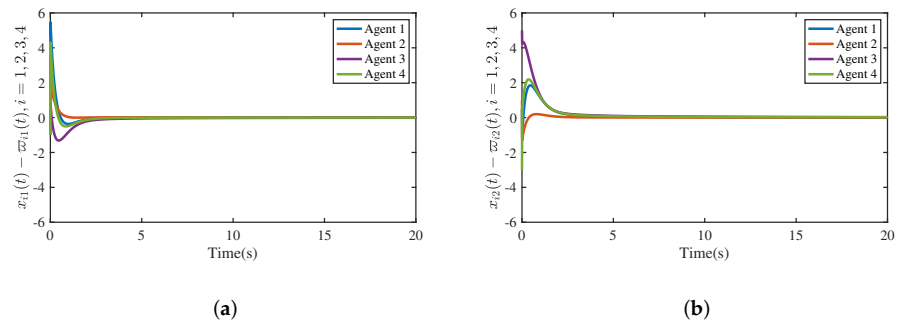


Figure 2. Observation errors of (a) $x_{i1}(t) - \omega_{01}(t)$, and (b) $x_{i2}(t) - \omega_{02}(t)$.

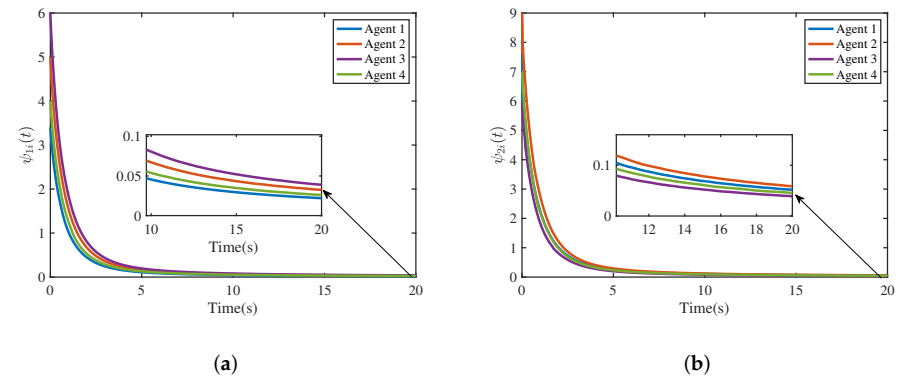


Figure 3. Evolution of each dynamic variable (a) $\psi_{1i}(t)$, and (b) $\psi_{2i}(t)$.

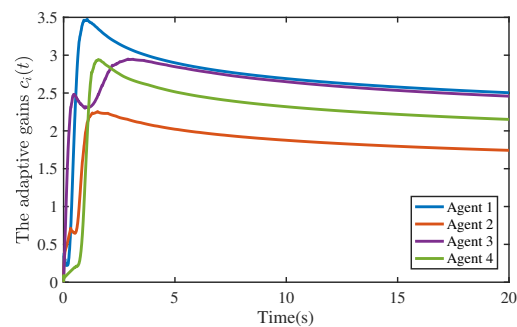


Figure 4. Evolution of the adaptive gains $c_i(t)$.

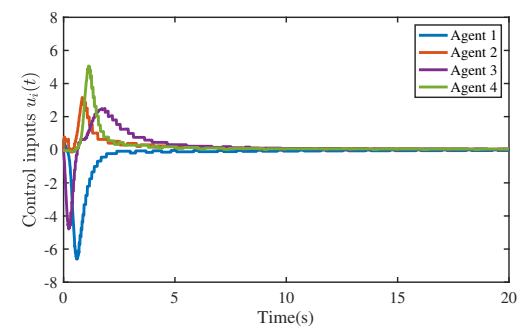


Figure 5. Evolution of the control inputs $u_i(t)$.

Figure 6 illustrates the trajectories of the leader and followers in the state space. It can be observed that all followers asymptotically track the leader's trajectory with identical magnitudes but opposite signs, which demonstrates the achievement of semiglobal bipartite consensus tracking. Figure 7 further depicts the tracking errors of the followers. The convergence of these errors verifies the theoretical result established in Theorem 1.

Figure 8 illustrates the triggering instants generated by the proposed dynamic double-event-triggered mechanism. It can be clearly seen that the triggering instants do not accumulate in finite time, which indicates that the Zeno behavior is precluded, in agreement with the theoretical analysis.

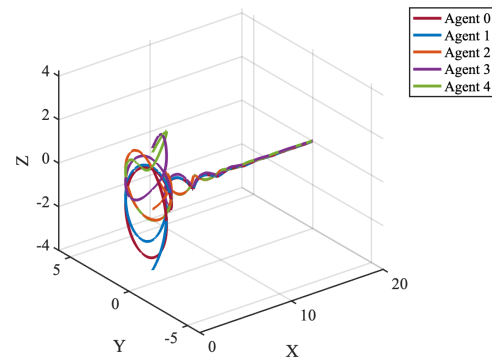


Figure 6. Trajectories of agents.

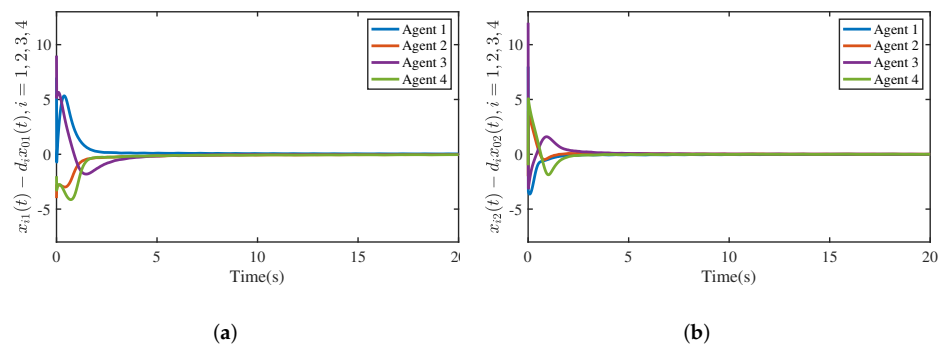


Figure 7. Tracking errors of (a) $x_{i1}(t) - d_i x_{01}(t)$, and (b) $x_{i2}(t) - d_i x_{02}(t)$.

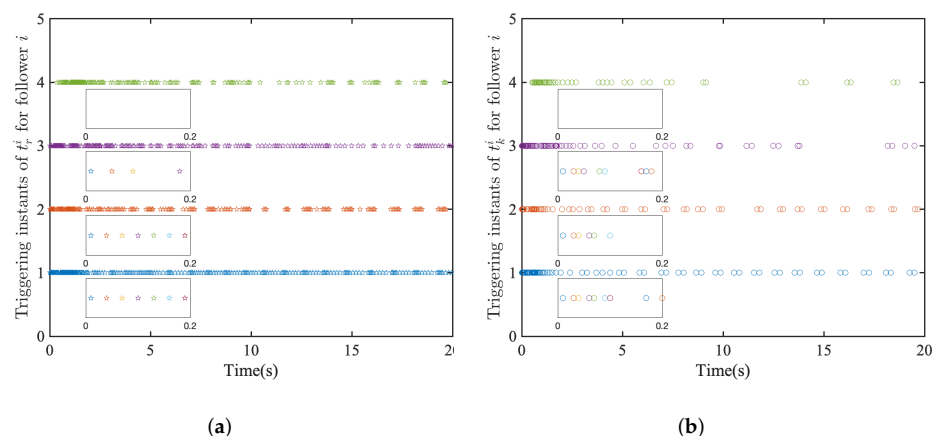


Figure 8. Triggering time of (a) triggered function $\mathcal{J}_{r_i}(t)$, and (b) triggered function $\mathcal{J}_{k_i}(t)$.

Furthermore, Table 1 compares the total number of triggering events under the static event-triggered mechanism and the proposed DDET. The results clearly show that the proposed DDET significantly reduces the triggering frequency while maintaining satisfactory control performance, thereby demonstrating its superiority in balancing communication efficiency and control effectiveness.

Table 1. Comparison with static situation.

Agent <i>i</i>	1	2	3	4	Total
ETM	478	536	575	473	2062
ETM	97	102	105	98	402
DETM- (19)	228	180	182	156	744
DETM- (22)	77	60	72	48	255

5. Conclusions

This paper has investigated the fully distributed observer-based dynamic double-event-triggered SBCT of FOMASs with input saturation under directed graph. First, an observer has been designed to estimate the state information of each agent. Then, the pull-based DDETM has been proposed in order to decrease the communication costs and the controller updates, simultaneously. Furthermore, the fully distributed observer-based control protocol with input saturation has been investigated by utilizing pull-based DDETM. Correspondingly, the designed control laws have been proved to be effective by using low-gain feedback technology and choosing suitable Lyapunov candidate functions. Finally, Zeno behavior has been precluded. As for future work, it is of interest to extend the proposed fully distributed DDETM framework to discrete-time multi-agent systems. Moreover, incorporating communication delays and model predictive control into fractional-order multi-agent systems is also a meaningful direction, which would require more advanced observer-based designs and refined stability analysis.

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