



Article

Fractional-Order Circuit Model-Based SOC Estimation for Lithium-Ion Batteries with LSTM Residual Correction

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Abstract

A fractional-order equivalent circuit model (FOECM) provides a compact and physically interpretable representation of the memory-dependent polarization behavior of lithium-ion batteries. Leveraging this property, a fractional order model-guided residual-correction framework is proposed for state-of-charge (SOC) estimation, in which the FOECM, unscented Kalman filter (UKF), and long short-term memory (LSTM) residual learner are integrated into a unified estimation chain rather than treated as separate modules. In this framework, the FOECM is parameterized using Dynamic Stress Test (DST) data and incorporated into the UKF to construct the FOECM + UKF estimator. The LSTM learns the history-dependent SOC residual from sequences of measured operating signals and FOECM + UKF SOC estimates, and its output is added to the UKF estimate without replacing the fractional-order physical model. The proposed hybrid estimator is trained and configured using the available DST, Supplemental Federal Test Procedure (US06), and Federal Urban Driving Schedule (FUDS) data, and is independently evaluated on the US06 and FUDS profiles of Cell 008. Compared with the FOECM + UKF estimator, the proposed hybrid estimator reduces the SOC root mean square error (RMSE) from 2.70% to 0.90% on US06 and from 2.93% to 1.20% on FUDS, with mean absolute error (MAE) values of 0.66% and 1.03%, respectively. These results demonstrate the effectiveness of coupling fractional-order memory modeling with sequence-based residual correction under the tested dynamic operating profiles.

Keywords: lithium-ion battery; state-of-charge estimation; fractional-order equivalent circuit model; unscented Kalman filter; LSTM residual correction



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1. Introduction

Lithium-ion batteries have been extensively used in electric vehicles, portable electronics, and large-scale energy storage systems due to their high energy density, long cycle life, and negligible memory effect [1]. Accurate state management is crucial for prolonging service life and enhancing system safety and efficiency [2–4]. Among various state metrics, state of charge (SOC) is a key variable in battery management systems (BMSs), as it indicates the remaining capacity and supports energy scheduling, range prediction, and protection strategies [5].

Lithium-ion batteries exhibit strong nonlinear behavior and parameter variations under operating conditions and aging. As a result, SOC cannot be directly measured and must be estimated using models or algorithms. Existing SOC estimation approaches can be broadly classified into Coulomb counting, open-circuit voltage (OCV)-based methods, equivalent circuit model (ECM)-based filters, and data-driven methods. Coulomb counting is easy to implement but suffers from accumulated drift and sensitivity to the initial SOC [6]. OCV-based methods reconstruct or adapt the OCV–SOC relationship for SOC inference, but their applicability is constrained by relaxation requirements and the flat voltage plateau of LiFePO₄ batteries [7,8]. Model-based filters, including extended Kalman filter (EKF), unscented Kalman filter (UKF), and particle filter (PF), embed an ECM or observer model into recursive estimators to improve online tracking under dynamic current profiles [9–11]. Data-driven methods, such as back-propagation neural network (BPNN) estimators, recurrent neural network (RNN)-based estimators, and other deep learning methods, can learn nonlinear mappings from measured signals to SOC, but their robustness may degrade when the testing condition differs from the training domain [12–14]. These limitations motivate model-based and model–data fusion methods that combine physical battery models with recursive filtering or neural network-based correction mechanisms [15,16].

Model-based SOC estimation methods combine an ECM with nonlinear recursive filters for online estimation. EKF is computationally efficient but requires Jacobian-based local linearization, while PF can represent non-Gaussian uncertainties and strong nonlinearities at the cost of a larger number of particles [11]. UKF avoids explicit Jacobian derivation by propagating sigma points through nonlinear state and measurement functions, and has been reported as a practical accuracy–complexity compromise for battery SOC estimation [10,16]. Recent enhanced ECM-based methods further incorporate diffusion-related mechanisms or online parameter adaptations to improve SOC estimation under dynamic conditions [10,16], but their accuracy still depends on model fidelity and parameter correctness.

Compared with conventional integer-order ECM-based models, fractional-order equivalent circuit model (FOECM) structures introduce constant phase elements (CPEs) and fractional orders to better describe memory-dependent polarization and diffusion behavior in lithium-ion batteries [17,18]. Fractional-order Kalman filtering has therefore been investigated for SOC estimation. For example, a fractional unscented Kalman filter was developed to combine fractional calculus with UKF-based SOC estimation [19]. Chen et al. further proposed the fuzzy fractional-order unscented Kalman filter (FFUKF), in which fuzzy inference is used to adapt the measurement-noise covariance under different operating conditions [20]. The fractional-order square-root unscented Kalman filter (FSR-UKF) was later developed to combine the numerical stability of square-root filtering with the memory-description capability of fractional calculus [21]. Recent fractional-order EKF and fractional-order fusion studies have also shown that fractional-order modeling remains an active direction for improving SOC estimation accuracy [22–24]. These studies demonstrate the effectiveness of combining fractional-order battery models with nonlinear Kalman filtering. However, their main focus is on improving the model or filtering structure itself, whereas the residual error remaining after fractional-order model-based filtering is less explicitly modeled by recurrent residual correction.

Hybrid model–data schemes have received increasing attention for integrating physical interpretability with data-driven adaptability [25,26]. Representative approaches include ECM–neural network fusion [27], neural network-assisted UKF error compensation [28], recurrent-network-assisted UKF estimation [29], and fractional-order-model-based fusion estimation for multi-chemistry lithium-ion batteries [24]. Temperature-dependent OCV analyses and impedance-based aging studies further provide experimental

evidence for assessing SOC estimation methods and interpreting model mismatch associated with aging or diffusion dynamics [30,31]. Representative methods are summarized in Table 1. Existing hybrid methods commonly rely on integer-order ECM backbones, train neural networks to estimate SOC directly, or primarily improve the filtering structure without explicitly characterizing the residual error. By contrast, the proposed hybrid estimator organizes the FOECM + UKF estimator and long short-term memory (LSTM) into a residual-correction chain, where the fractional-order estimator provides the primary SOC estimate and the LSTM residual learner captures the remaining history-dependent error.

Table 1. Representative fractional-order and model–data fusion SOC estimation studies and comparison with our work.

Model	Correction or Learning Role	Main Feature/Limitation
ECM+Neural Network [27]	Model–data fusion for SOC estimation	Improves estimation accuracy; fractional-order dynamics are not considered
Neural Network + UKF [28]	Neural network-based error cancelation	Combines filtering and learning; residual correction is not built on an FOECM + UKF estimator
Fuzzy Fractional-Order UKF (FFUKF) [20]	Fuzzy adaptation of measurement-noise covariance	Improves fractional-order UKF under different temperatures; recurrent residual learning is not considered
Fractional-Order Square-Root UKF (FSR-UKF) [21]	Numerically stable fractional-order filtering	Improves covariance stability; residual error beyond the filter is not explicitly learned
Bidirectional Gated Recurrent Unit (BGRU) + UKF [29]	Recurrent-network-assisted SOC estimation	Uses sequence learning; FOECM backbone is not included
Fractional-Order-Model-Based Fusion [23,24]	Neural or fusion-assisted SOC estimation	Captures fractional-order behavior; LSTM residual correction on an FOECM + UKF estimator is less emphasized
Our Work	LSTM-based SOC residual learning	Complements fractional-order filtering through physics-informed additive residual correction

Motivated by the above, a fractional-order model-guided SOC estimation framework with recurrent residual correction is developed. In this framework, an FOECM with a CPE is parameterized using Dynamic Stress Test (DST) data and embedded into the UKF to provide the primary FOECM + UKF SOC estimate. An LSTM residual learner is then trained to capture the temporal residual of the FOECM + UKF estimator and is used as an additive correction term. Independent tests on the Supplemental Federal Test Procedure (US06) and Federal Urban Driving Schedule (FUDS) profiles of Cell 008 show that the proposed hybrid estimator further reduces the residual SOC errors of the FOECM + UKF estimator.

The main contributions of this paper are summarized as follows:

- (1) An FOECM + UKF + LSTM estimator with residual correction is organized as a unified estimation chain for lithium-ion battery SOC estimation. The FOECM + UKF estimator provides the primary physics-based SOC estimate, and the LSTM residual learner supplies an additive correction to form the final SOC output.
- (2) A standard $R_1||CPE$ polarization branch is incorporated into the FOECM and discretized using a finite-memory Grünwald–Letnikov (GL) scheme. The resulting history-dependent polarization voltage enters the terminal-voltage prediction in the UKF measurement update, providing a physically interpretable fractional-order component for subsequent SOC estimation and residual correction.
- (3) An LSTM-based residual learning strategy is designed for FOECM + UKF estimation errors. During training, residual labels are constructed from the difference between the reference SOC and the FOECM + UKF SOC estimates. During online estimation, the SOC correction is computed from historical current, terminal voltage, measured temperature, and the FOECM + UKF SOC estimate. This shifts the network’s learning target from direct SOC mapping to estimation-error mapping, enabling compensation of systematic errors due to parameter mismatch, fractional-order discretization, and filter noise assumptions.

- (4) A structured evaluation framework under a consistent data-partition and evaluation protocol is established to compare the fractional-order model backbone and residual-correction configurations. An additional ECM + UKF + LSTM configuration and several FOECM + UKF-based residual learners are evaluated on the independent US06 and FUDS profiles of Cell 008.

The remainder of this paper is organized as follows: Section 2 describes the FOECM and its offline parameterization. Section 3 presents the FOECM + UKF estimator for SOC estimation. Section 4 details the LSTM-based residual learning strategy and the resulting FOECM + UKF + LSTM estimator. Section 5 reports the experimental setup and comparative results under multiple dynamic profiles. Finally, Section 6 concludes the paper and outlines future work.

2. FOECM Modeling

2.1. Experimental Data and Operating Profiles

The dataset used for validation is the publicly available A123 battery test data provided by the Center for Advanced Life Cycle Engineering (CALCE) battery group. The detailed specifications of the tested cell are given in Table 2 [32,33]. Two experimental datasets (denoted as Cell 007 and Cell 008) are used, and both correspond to the same nominal 25 °C ambient test condition. Each dataset consists of two parts: (1) a low-current OCV test and (2) a composite dynamic load test.

The OCV test is conducted under a sufficiently small current (with long rest periods) to approximate equilibrium conditions, and the measured voltage SOC pairs are later used to construct the OCV–SOC mapping. The dynamic load test comprises three standard operating profiles applied sequentially: DST, the Supplemental Federal Test Procedure US06 driving cycle, and FUDS. The measured current and voltage trajectories of the dynamic load test are illustrated in Figure 1, where the segments from left to right correspond to DST, US06, and FUDS, respectively; in subsequent experiments, these profiles are segmented according to their time intervals. In addition, the current and voltage responses obtained during the low-current OCV test are presented in Figure 2.

To ensure consistent sampling and numerical stability, all dynamic records are resampled to a uniform interval of $\Delta t = 1$ s, with discharge current defined as positive. For each cell, the longest low-current discharge and charge segments in the OCV test are processed separately. The SOC coordinates of the two segments are reconstructed by Coulomb integration, their voltage samples are aggregated over uniformly spaced SOC bins, and the resulting discharge and charge curves are averaged to obtain a cell-specific OCV–SOC curve. The effective capacity Q_{eff} is taken as the mean of the measured discharge and charge capacities, yielding 1.06183 Ah for Cell 007 and 1.08654 Ah for Cell 008. Throughout the SOC equations, Q_{eff} is expressed in Ah; factor 3600 converts it to A s when current is measured in A and time in s. For each dynamic record, the reference SOC trajectory is obtained by integrating the measured current using the corresponding cell-specific capacity. The integrated trajectory is shifted so that its minimum over the initial 10 min is zero and it is then limited to the physical interval $[0, 1]$. The OCV–SOC curve of Cell 007 is used in the FOECM voltage model and UKF measurement function. The Cell 008 OCV test is used only to determine its effective capacity and construct the corresponding reference SOC trajectory; it is not used for FOECM parameterization. Since the Coulomb-integrated reference may still be affected by current-sensor bias, capacity uncertainty, and initial alignment, it is treated as an experimental reference rather than an error-free ground truth.

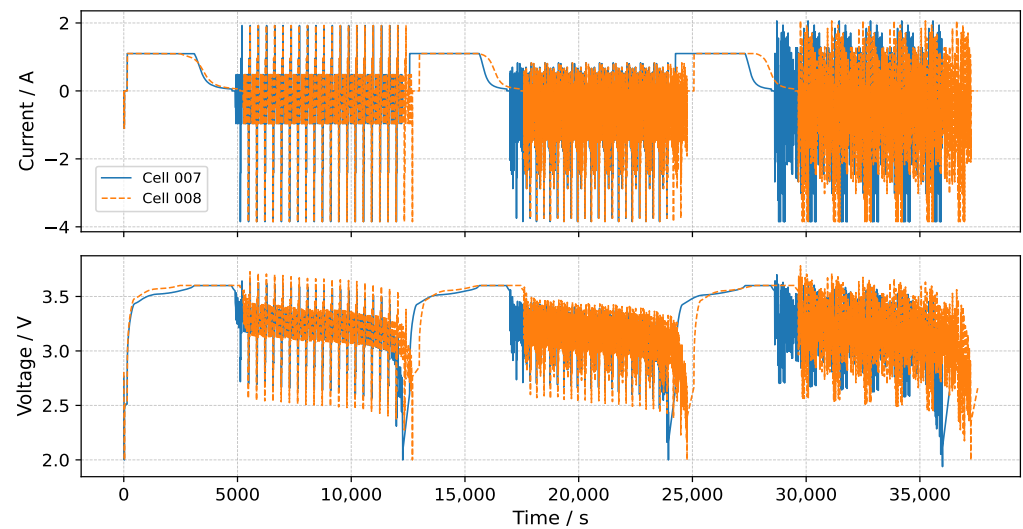


Figure 1. Measured current and voltage under the Dynamic Stress Test (DST), Supplemental Federal Test Procedure (US06), and Federal Urban Driving Schedule (FUDS) profiles.

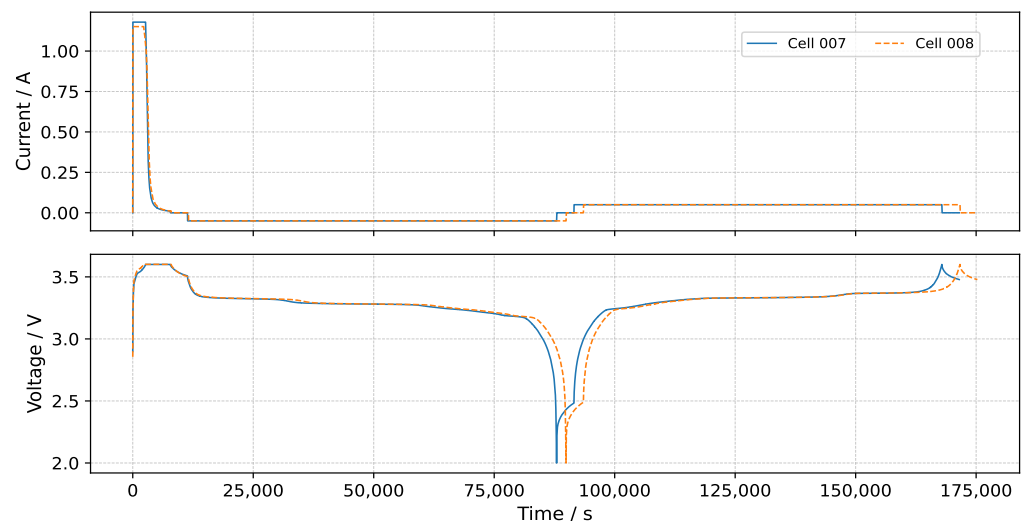


Figure 2. Measured current and voltage for the OCV test.

Table 2. A123 battery specifications.

Battery Parameter	Specification
Capacity rating	1100 mAh
Cell chemistry	LiFePO ₄
Diameter	25.4 mm
Length	65 mm
Special notes	Tab length not included in dimensions

The DST, US06, and FUDS profiles of Cell 007, together with the DST profile of Cell 008, form the training/validation corpus. The Cell 008 DST profile introduces cell-to-cell variability, whereas the Cell 008 US06 and FUDS profiles are reserved as independent test profiles. The FOECM is parameterized offline using the 4800–12,000 s interval of the Cell 007 DST profile.

2.2. Structure of the FOECM

ECMs are widely used in BMSs to approximate the electrochemical behavior of lithium-ion batteries. Classical integer-order ECMs typically consist of an OCV source in series with ohmic resistance and one or more first-order resistor–capacitor (RC) networks to represent

polarization. However, integer-order RC branches describe exponential relaxation with a single characteristic time constant, whereas practical lithium-ion batteries often exhibit distributed relaxation, diffusion-related polarization, and non-ideal frequency-dependent impedance. Electrochemical impedance spectroscopy (EIS) measurements reveal depressed semicircles and low-frequency power-law impedance behavior that cannot be accurately captured by integer-order components alone [34,35], motivating the use of fractional-order elements such as CPEs.

From the viewpoint of fractional calculus, the CPE introduces a power-law memory effect into the circuit model, allowing the polarization voltage to depend on historical current excitation rather than only on the instantaneous state. This feature is consistent with the distributed relaxation behavior of lithium-ion batteries and provides the physical basis for the finite-memory Grünwald–Letnikov discretization used later in the proposed hybrid estimator.

In this section, an FOECM with a single fractional polarization branch is adopted, as shown in Figure 3. The model consists of (1) an OCV source $V_{OCV}(SOC)$ representing the equilibrium potential as a function of SOC; (2) a series ohmic resistance R_0 accounting for ohmic losses in the electrodes, electrolyte, and current collectors; and (3) a fractional-order polarization branch composed of a resistor R_1 in parallel with a CPE characterized by (C_α, α) to capture charge-transfer and diffusion-related polarization effects.

The CPE represents a distributed capacitive behavior rather than an ideal capacitor, and the fractional order α controls the deviation from ideal first-order polarization. Therefore, the adopted FOECM can be interpreted as a compact approximation of distributed electrochemical relaxation using only one fractional polarization branch.

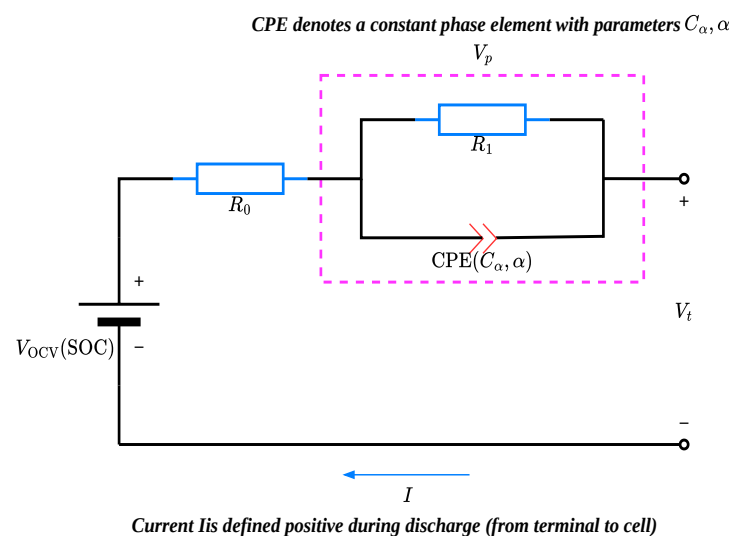


Figure 3. Fractional-order 1RC equivalent circuit model.

The FOECM used in the subsequent estimation experiments was parameterized using the 4800–12,000 s interval of the Cell 007 DST profile. The standard $R_1 \parallel$ CPE branch was discretized using a finite-memory Grünwald–Letnikov scheme, and the parameters $(R_0, R_1, C_\alpha, \alpha)$ were determined by minimizing the terminal-voltage residual. Candidate memory lengths of 300, 600, 1200, 2400, 3600, and 4800 samples were compared using the 4800–11,000 s interval for parameter fitting and the 11,000–12,000 s interval for validation. The 3600-sample memory produced the lowest validation voltage RMSE of 10.71 mV and was therefore selected. The model was subsequently refitted over the complete 4800–12,000 s interval with this memory length, and the resulting parameters were kept fixed for all evaluated profiles and cells. The final parameter values are summarized in Table 3.

Table 3. FOECM parameters used in the subsequent SOC estimation experiments.

Parameter	Value	Unit
R_0	0.143616	Ω
R_1	0.256468	Ω
C_α	77.44698	$\text{F s}^{\alpha-1}$
α	0.285525	–
Δt	1	s
N_{mem}	3600	samples
T_{mem}	3600	s

Note: N_{mem} denotes the number of retained samples in the finite-memory Grünwald–Letnikov approximation. With $\Delta t = 1$ s, $N_{\text{mem}} = 3600$ corresponds to a memory horizon of $T_{\text{mem}} = 3600$ s. The generalized capacitance C_α has the unit $\text{F s}^{\alpha-1}$, and the fractional order α controls the memory-dependent relaxation of the CPE branch.

The terminal current I is defined as positive during discharge, consistent with Figure 3. Under this sign convention, the terminal voltage is obtained by subtracting the ohmic voltage drop and the fractional polarization voltage from the OCV. The discrete implementation of this relationship is given later in (11).

In the frequency domain, the impedance of the CPE is given by

$$Z_{\text{CPE}}(j\omega) = \frac{1}{C_\alpha(j\omega)^\alpha}, \quad (1)$$

where C_α is the generalized capacitance with units $\text{F s}^{\alpha-1}$ so that $Z_{\text{CPE}}(j\omega)$ has units of Ω , and $\alpha \in (0, 1)$ is the fractional order. This expression follows the standard CPE definition, in which the ideal capacitive admittance $Cj\omega$ is generalized to $C_\alpha(j\omega)^\alpha$. Thus, the impedance takes the form $1/[C_\alpha(j\omega)^\alpha]$ rather than being an empirical transfer function chosen arbitrarily. The range $0 < \alpha < 1$ represents non-ideal capacitive behavior between an ideal resistor-like response and an ideal capacitor, which is consistent with diffusion-related and distributed polarization processes in lithium-ion batteries. In the time-domain interpretation, the CPE current is related to the polarization voltage through a fractional derivative, i.e.,

$$i_{\text{CPE}}(t) = C_\alpha D_t^\alpha v_p(t), \quad (2)$$

where $D_t^\alpha(\cdot)$ denotes the fractional-order derivative and $v_p(t)$ is the polarization voltage across the fractional branch. When $\alpha = 1$, the CPE degenerates into an ideal capacitor; when $0 < \alpha < 1$, the branch exhibits memory-dependent and non-exponential relaxation behavior. Since R_1 is connected in parallel with the CPE, the equivalent polarization impedance becomes

$$Z_p(j\omega) = \frac{R_1 Z_{\text{CPE}}(j\omega)}{R_1 + Z_{\text{CPE}}(j\omega)} = \frac{R_1}{1 + R_1 C_\alpha (j\omega)^\alpha}, \quad (3)$$

when $\alpha = 1$, (3) reduces to the impedance of a classical first-order R_1 –C RC network. For $0 < \alpha < 1$, the CPE introduces a non-ideal phase behavior and can reproduce the depressed semicircles and power-law frequency dependence observed in EIS. This frequency-domain interpretation motivates the finite-memory discretization of the CPE branch developed below.

2.3. SOC Dynamics and OCV–SOC Relationship

Neglecting side reactions and self-discharge over the time scale of interest, the SOC dynamics follow Coulomb counting:

$$\frac{d\text{SOC}(t)}{dt} = -\frac{\eta}{3600 Q_{\text{eff}}} I(t), \quad (4)$$

where $\eta \in (0, 1]$ is the Coulombic efficiency, Q_{eff} is the effective capacity in Ah, and $I(t)$ is defined positive during discharge, so that SOC decreases under positive current. The time variable t is expressed in seconds; hence, $3600Q_{\text{eff}}$ is the effective capacity in A s. With the sampling period Δt , (4) is discretized as

$$\text{SOC}_{k+1} = \text{SOC}_k - \frac{\eta \Delta t}{3600 Q_{\text{eff}}} I_k. \quad (5)$$

The OCV–SOC mapping $V_{\text{OCV}}(\text{SOC})$ used by the estimator is constructed from the Cell 007 low-current OCV test. As described above, the discharge and charge voltage curves are first obtained separately and then averaged at common SOC coordinates. The resulting discrete samples are interpolated using a shape-preserving piecewise cubic Hermite interpolating polynomial (PCHIP) to obtain a smooth function

$$V_{\text{OCV}}(\text{SOC}) \approx f_{\text{OCV}}(\text{SOC}), \quad (6)$$

which is used in the terminal-voltage model and the UKF measurement function. For LiFePO₄ cells, the relatively flat OCV plateau can reduce voltage sensitivity to SOC over part of the operating range. This reduced voltage sensitivity makes SOC estimation more dependent on the dynamic voltage response induced by current excitation. Therefore, the OCV mapping is not used as a standalone SOC estimator, but is combined with dynamic current excitation and the fractional-order polarization term in the UKF measurement update. The averaged OCV samples used for interpolation cover SOC coordinates from 0.01 to 0.99, and the corresponding full-range PCHIP interpolation used by the estimator is shown in Figure 4. During parameterization and filtering, if an SOC argument falls outside the sampled interval $[0.01, 0.99]$, the PCHIP function is evaluated by extrapolating the polynomial segment nearest to the corresponding endpoint. Consequently, SOC values in $[0, 0.01]$ appearing in the dynamic profiles are evaluated using the extrapolated first PCHIP segment. This rule is applied consistently during FOECM parameterization and UKF evaluation, without introducing OCV data from the independent Cell 008 test profiles. The effective SOC ranges of the dynamic profiles are reported separately in Section 5.1.

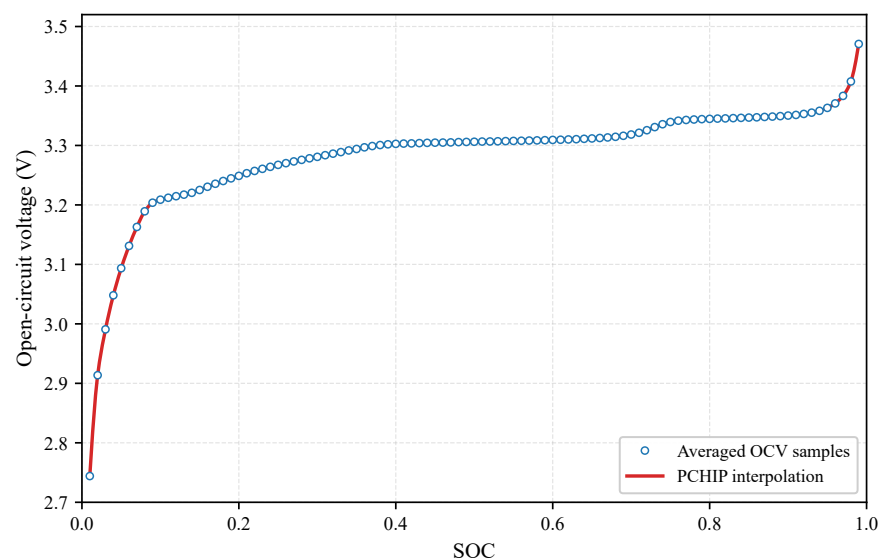


Figure 4. Cell 007 OCV–SOC mapping used by the estimator. Circles denote the charge–discharge-averaged OCV samples at common SOC coordinates, and the solid line represents the PCHIP interpolation $f_{\text{OCV}}(\text{SOC})$ over the SOC range 0.01–0.99.

2.4. Finite-Memory Grünwald–Letnikov Discretization

Combining the CPE relation in (2) with the current through the parallel resistor R_1 gives the continuous-time polarization-branch equation

$$C_\alpha D_t^\alpha v_p(t) + \frac{v_p(t)}{R_1} = I(t), \quad (7)$$

where $v_p(t)$ is the polarization voltage across the $R_1 \parallel$ CPE branch. At $t_k = k\Delta t$, the fractional derivative is approximated by the finite-memory Grünwald–Letnikov formula

$$D_t^\alpha v_p(t_k) \approx \Delta t^{-\alpha} \sum_{j=0}^{m_k} g_j^{(\alpha)} v_{p,k-j}, \quad m_k = \min(k, N_{\text{mem}}), \quad (8)$$

where $N_{\text{mem}} = T_{\text{mem}}/\Delta t$ is the retained memory length in samples. The Grünwald–Letnikov coefficients are generated recursively as

$$g_0^{(\alpha)} = 1, \quad g_j^{(\alpha)} = \left(1 - \frac{\alpha + 1}{j}\right) g_{j-1}^{(\alpha)}, \quad j \geq 1. \quad (9)$$

These coefficients are the generalized-binomial weights $g_j^{(\alpha)} = (-1)^j \binom{\alpha}{j}$ and therefore retain the alternating, power-law-decaying memory structure of the fractional derivative.

Substituting (8) into (7), separating the $j = 0$ term, and using $g_0^{(\alpha)} = 1$ yields the explicit polarization-voltage recursion

$$v_{p,k} = \frac{I_k - C_\alpha \Delta t^{-\alpha} \sum_{j=1}^{m_k} g_j^{(\alpha)} v_{p,k-j}}{R_1^{-1} + C_\alpha \Delta t^{-\alpha}}. \quad (10)$$

Zero polarization prehistory is used at the beginning of each processed current sequence. Because (10) is driven by the present current and the retained polarization-voltage history, the CPE memory enters the model explicitly rather than through an empirical current-weighting kernel.

The resulting discrete-time terminal-voltage equation is

$$V_{t,k} = V_{\text{OCV}}(\text{SOC}_k) - R_0 I_k - v_{p,k}. \quad (11)$$

Thus, the fractional-order contribution enters the voltage prediction through $v_{p,k}$, whose evolution is governed by the standard CPE equation and the Grünwald–Letnikov coefficients. This distinguishes the adopted FOECM from an integer-order ECM with an exponentially relaxing RC polarization voltage.

For a fixed candidate memory length, the parameter vector $\theta_{\text{FO}} = [R_0, R_1, C_\alpha, \alpha]^T$ is determined from the selected DST segment by solving

$$\min_{\substack{R_0, R_1, C_\alpha > 0 \\ 0 < \alpha < 1}} \sum_k \left[V_{\text{OCV}}(\text{SOC}_k^{\text{ref}}) - R_0 I_k - v_{p,k}(R_1, C_\alpha, \alpha) - V_{t,k}^{\text{meas}} \right]^2. \quad (12)$$

The candidate memory lengths are compared using the validation voltage RMSE described before Table 3; the selected $N_{\text{mem}} = 3600$ is then fixed while the model is refitted over the complete parameterization interval. The resulting coefficients are implementation-oriented model parameters for the tested operating conditions rather than invariant electrochemical constants.

Only the most recent N_{mem} polarization-voltage terms are retained in (10). With $\Delta t = 1$ s and $N_{\text{mem}} = 3600$, the memory horizon is $T_{\text{mem}} = 3600$ s. Increasing this horizon preserves a longer relaxation history but increases the recursion cost, whereas decreasing it reduces computation at the expense of stronger memory truncation.

For the subsequent online estimator, only SOC is selected as the UKF state, while $v_{p,k}$ is updated deterministically using (10). This scalar-state formulation avoids augmenting the UKF state with the complete polarization-history vector while retaining the fractional-order memory contribution in the voltage measurement model.

2.5. Measurement Model and Noise

In practice, the terminal-voltage measurement of the battery is affected by sensor noise and external disturbances. Based on the finite-memory Grünwald–Letnikov recursion in (10), the deterministic voltage drop at time step k is written as

$$u_{\text{drop},k} = R_0 I_k + v_{p,k}. \quad (13)$$

The measured terminal voltage is then modeled as

$$V_{\text{meas},k} = V_{\text{OCV}}(\text{SOC}_k) - u_{\text{drop},k} + n_{v,k}, \quad (14)$$

where $n_{v,k}$ denotes zero-mean Gaussian white voltage measurement noise with variance σ_v^2 ,

$$n_{v,k} \sim \mathcal{N}(0, \sigma_v^2). \quad (15)$$

The variance in $n_{v,k}$ contributes to the effective UKF measurement covariance R , which also accounts for residual FOECM voltage model mismatch, as described in Section 3.4.

3. UKF-Based SOC Estimation with the FOECM

This section embeds the FOECM in Section 2 into a UKF framework for online SOC estimation. The choice of UKF is motivated primarily by the nonlinear OCV–SOC measurement equation. Compared with EKF, UKF avoids explicit Jacobian derivation while retaining a compact scalar-state implementation. The FOECM provides the physics-based voltage observation model, and the UKF accounts for model mismatch, process disturbances, and measurement noise. The resulting FOECM + UKF SOC estimate is subsequently corrected by the LSTM residual learner in Section 4.

3.1. Overall Architecture of the Estimator

Figure 5 depicts the signal flow of the FOECM + UKF estimator. Given the measured current I_k and the prior SOC estimate $\hat{x}_{k|k-1}$, the FOECM generates the predicted terminal voltage $\hat{y}_{k|k-1} = h(\hat{x}_{k|k-1}, I_{0:k})$ using the fixed parameters $(R_0, R_1, C_\alpha, \alpha, N_{\text{mem}})$, where $I_{0:k} = \{I_0, I_1, \dots, I_k\}$ and $v_{p,k}$ is updated from this current history using (10). The innovation is formed as $v_k = y_k - \hat{y}_{k|k-1}$ with $y_k = V_{t,k}$, and is assimilated by the UKF to update the posterior SOC estimate $\hat{x}_{k|k} = \text{SOC}_{k|k}^{\text{UKF}}$. The updated FOECM + UKF SOC estimate is fed back as the prior for the next UKF step. The output of the FOECM + UKF + LSTM estimator introduced later is only used as the final output correction and is not fed back into the UKF state or covariance recursion. The FOECM parameters listed in Table 3 remain fixed during online operation.

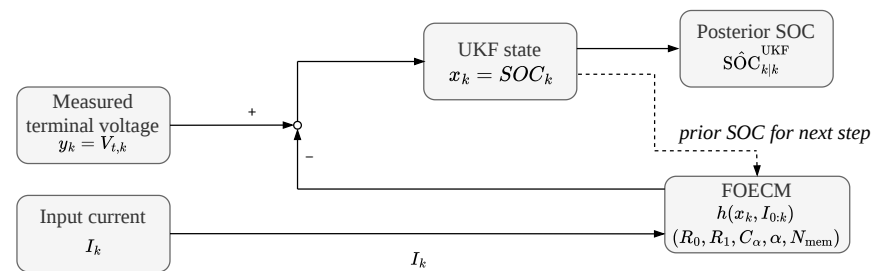


Figure 5. Block diagram of the FOECM + UKF estimator. The FOECM uses the prior SOC and current to generate a predicted terminal voltage, which is compared with the measured voltage to form the innovation driving the UKF update of the posterior SOC.

3.2. State–Space Formulation with the FOECM

With the FOECM parameterized offline, the SOC dynamics can be written in discrete time as

$$\text{SOC}_{k+1} = \text{SOC}_k - \frac{\eta \Delta t}{3600 Q_{\text{eff}}} I_k + w_k, \quad (16)$$

where I_k is the applied current (positive for discharge), η is the Coulombic efficiency, Q_{eff} is the effective capacity in Ah obtained in Section 2, and w_k denotes the process noise accounting for unmodeled dynamics and capacity mismatch. The UKF state is chosen as

$$x_k = \text{SOC}_k. \quad (17)$$

Thus, the online UKF state dimension is $L = 1$. In this formulation, the fractional-order dynamics enter the UKF through the voltage observation model rather than through an augmented state-transition equation. The UKF correction is therefore driven by the voltage innovation that contains the finite-memory fractional polarization term.

The FOECM predicts the terminal voltage at time step k as

$$V_{t,k} = V_{\text{OCV}}(\text{SOC}_k) - R_0 I_k - v_{p,k}, \quad (18)$$

where $V_{\text{OCV}}(\cdot)$ is the OCV–SOC mapping obtained from the OCV tests and $v_{p,k}$ is the fractional polarization voltage. As described in Section 2.4, $v_{p,k}$ is updated causally from I_k and the preceding polarization-voltage terms using the Grünwald–Letnikov recursion in (10). For fixed $(R_1, C_\alpha, \alpha, N_{\text{mem}})$ and the specified initial prehistory, $v_{p,k}$ is a deterministic function of the measured current history $I_{0:k}$ and is evaluated before the UKF voltage measurement update.

Although $v_{p,k}$ is not estimated as an online UKF state, its accuracy is affected by errors in the offline parameterization of R_0 , R_1 , C_α , and α , as well as by truncation at N_{mem} . These effects appear as voltage observation model mismatch and are represented in the effective measurement uncertainty of the UKF update, while the remaining systematic SOC error is further compensated by the residual learner in Section 4. State augmentation provides a valid alternative formulation. However, the present work intentionally retains SOC as the only online UKF state and treats $v_{p,k}$ as a deterministic finite-memory quantity obtained from the GL recursion, thereby avoiding a higher-dimensional representation of the polarization history and the associated covariance tuning. State-augmented fractional filtering will be investigated in future work.

The voltage measurement equation is written as

$$y_k = h(x_k, I_{0:k}) + v_k, \quad (19)$$

with

$$h(x_k, I_{0:k}) = V_{OCV}(x_k) - R_0 I_k - v_{p,k}, \quad (20)$$

where $I_{0:k} = \{I_0, I_1, \dots, I_k\}$ and $v_{p,k}$ contains the retained current-dependent polarization history through (10). The term v_k denotes the effective voltage-domain uncertainty, encompassing terminal-voltage sensor noise and residual FOECM observation-model mismatch. In the UKF design, w_k and v_k are modeled as zero-mean Gaussian noises with variances Q and R , respectively.

It should be emphasized that, for a known input current, the SOC state-transition equation in (16) is affine rather than nonlinear. The principal nonlinearity of the adopted state-space model arises from the OCV–SOC relationship in the voltage measurement function (19). The fractional polarization voltage also enters the measurement equation, but it is computed deterministically using the fixed-parameter Grünwald–Letnikov recursion rather than being estimated as an additional online state.

Consequently, the unscented transformation is not essential for propagating the state mean through the affine SOC transition. It is retained within the standard UKF recursion to perform the nonlinear voltage measurement update without explicit Jacobian derivation. A conventional linear Kalman filter cannot be applied directly to the nonlinear OCV–SOC measurement function without introducing a linearization or other approximation.

3.3. Unscented Kalman Filtering for SOC Estimation

The scalar-state UKF recursion used for SOC estimation is summarized below.

For the scalar state x_k ($L = 1$), the UKF employs $2L + 1 = 3$ sigma points; therefore, the sigma-point index satisfies $i \in \{0, 1, 2\}$ in the following derivation:

(1) Time update: Given the posterior estimate $\hat{x}_{k-1|k-1}$ and covariance $P_{k-1|k-1}$, the sigma points are generated as

$$\chi_{k-1}^{(0)} = \hat{x}_{k-1|k-1}, \quad \chi_{k-1}^{(1)} = \hat{x}_{k-1|k-1} + \sqrt{(L + \lambda)P_{k-1|k-1}}, \quad (21)$$

$$\chi_{k-1}^{(2)} = \hat{x}_{k-1|k-1} - \sqrt{(L + \lambda)P_{k-1|k-1}}, \quad (22)$$

where $\lambda = \alpha_u^2(L + \kappa) - L$ is the scaling parameter determined by α_u and κ . Here α_u is the UKF spread parameter and should not be confused with the fractional order α of the FOECM. Each sigma point is propagated through the SOC dynamics (16):

$$\chi_{k|k-1}^{(i)} = \chi_{k-1}^{(i)} - \frac{\eta \Delta t}{3600 Q_{\text{eff}}} I_{k-1}, \quad i = 0, 1, 2. \quad (23)$$

The prior mean and covariance are computed as follows:

$$\hat{x}_{k|k-1} = \sum_{i=0}^2 W_i^{(m)} \chi_{k|k-1}^{(i)}, \quad (24)$$

$$P_{k|k-1} = \sum_{i=0}^2 W_i^{(c)} (\chi_{k|k-1}^{(i)} - \hat{x}_{k|k-1})^2 + Q, \quad (25)$$

where $W_i^{(m)}$ and $W_i^{(c)}$ are the standard UKF weights for the mean and covariance, respectively.

(2) Measurement update: The propagated sigma points are mapped through the measurement function:

$$\gamma_k^{(i)} = h(\chi_{k|k-1}^{(i)}, I_{0:k}), \quad i = 0, 1, 2, \quad (26)$$

and the predicted measurement and innovation covariance are given by

$$\hat{y}_{k|k-1} = \sum_{i=0}^2 W_i^{(m)} \gamma_k^{(i)}, \quad (27)$$

$$S_k = \sum_{i=0}^2 W_i^{(c)} (\gamma_k^{(i)} - \hat{y}_{k|k-1})^2 + R. \quad (28)$$

The cross-covariance between the state and the measurement is

$$P_{xy,k} = \sum_{i=0}^2 W_i^{(c)} (\chi_{k|k-1}^{(i)} - \hat{x}_{k|k-1}) (\gamma_k^{(i)} - \hat{y}_{k|k-1}). \quad (29)$$

The Kalman gain and posterior estimates are updated as

$$K_k = \frac{P_{xy,k}}{S_k}, \quad (30)$$

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (y_k - \hat{y}_{k|k-1}), \quad (31)$$

$$P_{k|k} = P_{k|k-1} - K_k S_k K_k^T. \quad (32)$$

The role of the fractional polarization term in this recursion can now be stated explicitly. At time step k , (10) is evaluated first to obtain $v_{p,k}$. This value enters every sigma-point voltage prediction $\gamma_k^{(i)}$ through (20) and therefore changes the predicted terminal voltage $\hat{y}_{k|k-1}$ and the innovation $y_k - \hat{y}_{k|k-1}$. Because the same deterministic $v_{p,k}$ is applied to all sigma points, it cancels from the centered differences used to compute S_k and $P_{xy,k}$ and hence does not directly change the instantaneous Kalman gain K_k . Nevertheless, its effect on the innovation is multiplied by K_k in the posterior update, so the GL polarization dynamics directly affect $\hat{x}_{k|k}$ without being introduced as an additional UKF state.

The above recursion corresponds to the scalar-state UKF employed in the proposed hybrid estimator and follows the standard UKF formulation. It can be extended to higher-dimensional states if additional parameters are incorporated into the state vector for joint estimation. Here, only SOC is estimated online to maintain a compact estimator, since the FOECM is parameterized offline and its parameters are fixed during operation.

3.4. Practical Configuration and Tuning

In practice, the performance of the UKF depends on the choice of its tuning parameters. The initial SOC mean $\hat{x}_{0|0}$ is obtained from the reference SOC at the initial sampling instant, while the initial covariance $P_{0|0}$ reflects the uncertainty of this prior information. This reference-based initialization is applied consistently to all model-based estimators to provide a controlled comparison. The small value of $P_{0|0} = 10^{-6}$ reflects high confidence in this initialization; robustness to an unknown or deliberately biased initial SOC is not evaluated in the present experiments. Since only SOC is estimated online and the FOECM parameters are fixed after offline parameterization, the process noise variance Q represents uncertainty in SOC propagation, including current-integration error and effective-capacity mismatch. The effective measurement variance R represents the combined uncertainty of terminal-voltage sensing and the FOECM voltage observation model, including residual mismatch caused by fixed model parameters, finite-memory truncation, and unmodeled polarization dynamics. The sigma-point scaling parameters $(\alpha_u, \kappa, \beta)$ are chosen following common practice in the UKF literature, with a small α_u to keep the sigma points close to the mean and $\beta = 2$ for Gaussian priors. The final parameter values are summarized in Table 4. With this configuration, the FOECM + UKF estimator provides a stable and

reasonably accurate SOC estimate over the DST, US06, and FUDS profiles, which will be further improved by the LSTM-based residual learning in Section 4.

Table 4. Tuning parameters of the FOECM + UKF estimator.

Parameter	Symbol	Value
State dimension	L	1
Sigma-point spread	α_u	10^{-3}
Secondary scaling	κ	0
Distribution parameter	β	2
Process noise variance	Q	10^{-9}
Effective measurement var.	R	$(2 \times 10^{-2})^2$
Initial SOC mean	$\hat{x}_{0 0}$	$\text{SOC}_{\text{ref}}(0)$
Initial SOC variance	$P_{0 0}$	10^{-6}

4. LSTM-Based Residual Learning and FOECM + UKF + LSTM Estimator

Sections 2 and 3 established the FOECM and the FOECM + UKF estimator for on-line SOC estimation. Residual SOC errors may still arise from fixed offline parameters, finite-memory approximation, filter-noise tuning, measurement imperfections, and other unmodeled effects. This section develops an LSTM residual learner to model these history-dependent errors from measured operating signals and integrates it with the FOECM + UKF estimator to form the FOECM + UKF + LSTM estimator.

4.1. Motivation for Residual Learning

Let $\hat{\text{SOC}}_{k|k}^{\text{UKF}}$ denote the posterior SOC estimate produced by the FOECM + UKF estimator at sample k , and let $\text{SOC}_k^{\text{ref}}$ denote the reference SOC constructed by the offline Coulomb integration and alignment procedure described in Section 2. The instantaneous SOC error of the FOECM + UKF estimator is defined as

$$e_k^{\text{UKF}} = \text{SOC}_k^{\text{ref}} - \hat{\text{SOC}}_{k|k}^{\text{UKF}}. \quad (33)$$

For numerical training, the fractional-scale residual is converted to percentage-SOC units as

$$r_k = 100e_k^{\text{UKF}}, \quad (34)$$

where r_k is expressed in %SOC.

In practice, e_k^{UKF} is not purely random; it exhibits temporal correlation and varies with the operating trajectory, including current excitation, terminal-voltage response, measured operating signals, and past history embedded in the FOECM polarization dynamics. This suggests modeling r_k as the output of a nonlinear dynamical mapping driven by these measurable signals.

Rather than replacing the FOECM + UKF estimator with a black-box network, the proposed hybrid estimator retains the physics-based backbone and only learns the residual. Specifically, an LSTM residual learner parameterized by ϕ is trained to approximate the percentage-SOC residual

$$r_k \approx f_{\phi}(z_{k-L_{\text{seq}}+1}, \dots, z_k), \quad (35)$$

where z_k denotes the input feature vector at time k and L_{seq} is the sequence length. The hybrid SOC estimate is obtained by residual correction:

$$\hat{\text{SOC}}_{k|k}^{\text{hyb}} = \hat{\text{SOC}}_{k|k}^{\text{UKF}} + \frac{\hat{r}_k}{100}, \quad (36)$$

where $\hat{r}_k$ is the LSTM-predicted residual in %SOC. Factor 1/100 converts the predicted correction back to the fractional SOC scale before it is added to the FOECM + UKF SOC estimate. The LSTM correction is used as a post-filter output correction and is not fed back into the UKF recursion; the UKF continues to propagate its own posterior state and covariance. Therefore, the learned residual is tied to the adopted FOECM + UKF estimator configuration, and substantial changes in the FOECM parameters, UKF tuning, or preprocessing protocol may require retraining or recalibration of the residual learner.

The online computational load is determined by the scalar-state UKF recursion, the 3600 s finite-memory Grünwald–Letnikov update, and a single LSTM forward pass at each sampling instant, while embedded timing validation is reserved for future implementation.

4.2. LSTM Residual Learner

(1) Input features and sequence construction

For each dynamic profile, the FOECM + UKF estimator in Section 3 is first executed offline to generate the FOECM + UKF SOC trajectory $\hat{\text{SOC}}_{k|k}^{\text{UKF}}$. The feature vector at time k is defined as

$$\mathbf{z}_k = \begin{bmatrix} I_k & V_{t,k} & T_k & \hat{\text{SOC}}_{k|k}^{\text{UKF}} \end{bmatrix}^T, \quad (37)$$

where I_k and $V_{t,k}$ are the measured current and terminal voltage, T_k is the measured temperature signal recorded in the dataset, and $\hat{\text{SOC}}_{k|k}^{\text{UKF}}$ is the FOECM + UKF SOC estimate. Because the available data correspond to a single nominal 25 °C ambient test condition, T_k is retained only as a measured operating channel to keep the residual-learning framework compatible with multi-temperature datasets. The present experiments therefore do not evaluate or claim temperature generalization. The learning target is the percentage-SOC residual r_k defined in (34).

To exploit temporal dependencies, a sliding window of length L_{seq} is formed as

$$\mathbf{Z}_k = [\mathbf{z}_{k-L_{\text{seq}}+1}, \mathbf{z}_{k-L_{\text{seq}}+2}, \dots, \mathbf{z}_k], \quad (38)$$

and each window $\mathbf{Z}_k$ is paired with the scalar target r_k . In other words, the network predicts the current residual using a fixed-length history of measured signals and the FOECM + UKF SOC estimate.

The input windows and percentage-SOC residual targets are standardized using statistics computed only from the training subset:

$$\tilde{\mathbf{Z}}_k = \frac{\mathbf{Z}_k - \boldsymbol{\mu}_{\mathbf{Z}}}{\boldsymbol{\sigma}_{\mathbf{Z}}}, \quad \tilde{r}_k = \frac{r_k - \mu_r}{\sigma_r}. \quad (39)$$

The resulting statistics ($\boldsymbol{\mu}_{\mathbf{Z}}, \boldsymbol{\sigma}_{\mathbf{Z}}, \mu_r, \sigma_r$) are stored and reused for the validation subset and independent test set; no normalization statistics are recomputed from the independent Cell 008 US06/FUDS test profiles.

(2) Network architecture

A two-layer stacked LSTM is adopted to model the temporal evolution of the SOC residual. At each time step, the input feature vector is $\mathbf{z}_k \in \mathbb{R}^d$, where $d = 4$ for the adopted feature set. A fixed-length sequence window is constructed as $\mathbf{Z}_k = [\mathbf{z}_{k-L_{\text{seq}}+1}, \dots, \mathbf{z}_k]$. The first LSTM layer produces a hidden-state sequence, and dropout with rate p_{drop} is applied between the two layers to improve generalization. The second LSTM layer outputs the top-layer hidden state at the final time step, denoted by $\mathbf{h}_k \in \mathbb{R}^{n_h}$. A two-layer fully connected head then maps $\mathbf{h}_k$ to the predicted residual:

$$\mathbf{q}_k = \text{ReLU}(\mathbf{W}_1 \mathbf{h}_k + \mathbf{b}_1), \quad \hat{\hat{r}}_k = \mathbf{w}_2^T \mathbf{q}_k + b_2, \quad (40)$$

where $\hat{r}_k$ is the predicted standardized percentage-SOC residual, and $W_1 \in \mathbb{R}^{64 \times n_h}$, $b_1 \in \mathbb{R}^{64}$, $w_2 \in \mathbb{R}^{64}$, and $b_2 \in \mathbb{R}$ are trainable parameters. With $d = 4$, $n_h = 64$, and two stacked LSTM layers, the LSTM layers contain 51,200 parameters, while the $64 \rightarrow 64 \rightarrow 1$ output head contains 4225 parameters. The complete residual learner therefore has 55,425 trainable parameters. This relatively small parameter size is consistent with the role of the LSTM as a residual corrector rather than a direct replacement for the FOECM + UKF estimator.

Figure 6 illustrates the residual-learner architecture. The network parameters ϕ are optimized offline to minimize the mismatch between $\tilde{r}_k$ and $\hat{r}_k$ over the training subset.

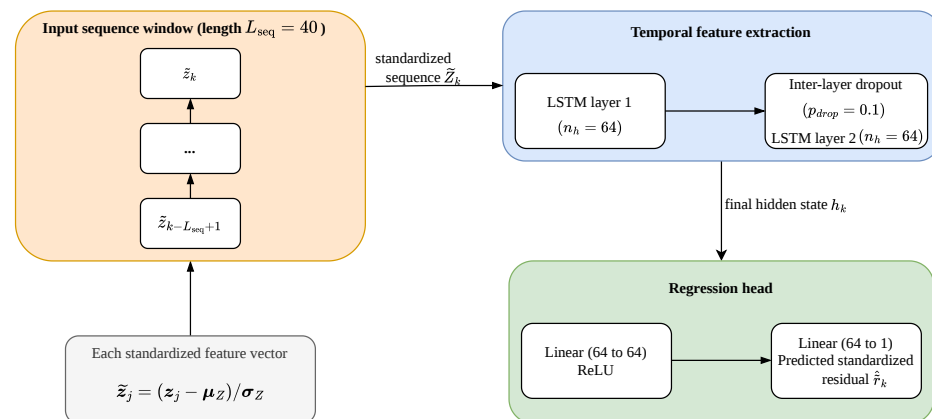


Figure 6. Architecture of the LSTM residual learner. The network takes a standardized window of measured operating signals and the FOECM + UKF SOC estimate as input and outputs the predicted standardized percentage-SOC residual.

4.3. Offline Training Strategy

The residual learner is trained using dynamic profiles that sufficiently excite battery dynamics under different current patterns. The DST, US06, and FUDS cycles of Cell 007, together with the DST cycle of Cell 008, are used to form the training/validation corpus. The US06 and FUDS cycles of Cell 008 are excluded from training, validation, normalization-statistics computation, and early-stopping selection. Each profile is processed by the FOECM + UKF estimator in Sections 2 and 3 to obtain the FOECM + UKF SOC trajectory $\hat{\text{SOC}}_{k|k}^{\text{UKF}}$, and the corresponding reference SOC $\text{SOC}_k^{\text{ref}}$ is constructed by the offline Coulomb integration and alignment procedure described in Section 2. The training target is then computed as

$$r_k = 100(\text{SOC}_k^{\text{ref}} - \hat{\text{SOC}}_{k|k}^{\text{UKF}}) = 100e_k^{\text{UKF}}.$$

The target is expressed in %SOC and is subsequently standardized according to (39) before being used for network training.

Sliding windows Z_k are generated separately within each dynamic profile according to (38). All windows from the DST, US06, and FUDS profiles of Cell 007 are assigned to the training subset. For the Cell 008 DST profile, the first 70% of the chronologically ordered windows are added to the training subset, whereas the final 30% are used as the validation subset for hyperparameter selection and early stopping. A boundary gap of $L_{\text{seq}} = 40$ window indices is inserted between these two portions so that no input window overlaps the training-validation boundary. Training mini-batches are shuffled only after this profile-aware split has been fixed. The Cell 008 US06 and FUDS profiles remain completely unseen during training, validation, normalization-statistics computation, and early-stopping selection and constitute the independent test set in Section 5.

The LSTM parameters are optimized by minimizing the Smooth L1 loss:

$$\mathcal{L}(\phi) = \frac{1}{N} \sum_{k=1}^N \ell_{\text{SLI}}(\tilde{r}_k - \hat{r}_k), \quad (41)$$

where N is the number of training samples. Both arguments of the loss are dimensionless standardized residuals. Optimization is performed using Adam with mini-batches. The main hyperparameters are summarized in Table 5.

No residual-target clipping is applied in the reported implementation. Instead, Smooth L1 loss is used to reduce sensitivity to occasional large residuals, and gradient clipping with a maximum norm of 5.0 is applied during training. After training, the model yielding the lowest validation loss is selected and exported together with the normalization statistics for online deployment.

Table 5. Main hyperparameters of the LSTM residual learner.

Item	Symbol	Value/Comment
Sequence length	L_{seq}	40 samples
Hidden units per layer	n_h	64
Number of LSTM layers	N_{layer}	2
Dropout rate	p_{drop}	0.1 between layers
Optimizer	–	Adam; learning rate 10^{-3} ; weight decay 10^{-5}
Batch size	–	512 training; 512 validation
Training loss	–	Smooth L1 loss ($\beta = 1$) of standardized percentage-SOC residual
Random seed	–	20260714(fixed)
Gradient clipping	–	Maximum norm 5.0
Postprocessing	M	11-sample causal moving average for reported test results
Trainable parameters	–	55,425
Evaluation metrics	–	Mean absolute error (MAE), root mean square error (RMSE), maximum absolute error (e_{max})
Training epochs	–	200 maximum; early-stopping patience 15 epochs

4.4. Online Hybrid SOC Estimation

Once the residual learner has been trained offline, it is deployed together with the fixed FOECM + UKF estimator to form the FOECM + UKF + LSTM estimator. At each sampling instant k , the online procedure consists of four steps:

(1) The FOECM receives the measured current I_k and generates the model-predicted voltage according to (18);

(2) The UKF fuses the predicted voltage with the measured terminal voltage $V_{t,k}$ and updates the FOECM + UKF SOC estimate $\hat{\text{SOC}}_{k|k}^{\text{UKF}}$ as described in Section 3;

(3) The most recent L_{seq} samples of $(I_k, V_{t,k}, T_k, \hat{\text{SOC}}_{k|k}^{\text{UKF}})$ are stacked to form a feature window, standardized using the stored normalization statistics, and fed into the LSTM residual learner to obtain $\hat{r}_k$. The percentage-SOC correction is then recovered as

$$\hat{r}_k = \sigma_r \hat{r}_k + \mu_r. \quad (42)$$

(4) The hybrid SOC estimate is computed using (36) and, if necessary, saturated to the physically admissible interval $[0, 1]$:

$$\hat{\text{SOC}}_{k|k}^{\text{hyb}} = \text{sat}\left(\hat{\text{SOC}}_{k|k}^{\text{UKF}} + \frac{\hat{r}_k}{100}; [0, 1]\right). \quad (43)$$

To suppress high-frequency fluctuations in the neural network correction while preserving the main dynamic trend, a light moving-average smoothing can be applied to the hybrid SOC:

$$M_k = \min(M, k), \quad \widetilde{\text{SOC}}_k = \frac{1}{M_k} \sum_{i=0}^{M_k-1} \text{SOC}_{k-i}^{\text{hyb}}, \quad (44)$$

where $M = 11$ is the maximum moving-average window length and M_k is the number of samples available at sampling instant k . Thus, only the current and preceding samples are used, and the denominator is reduced accordingly during the first $M - 1$ sampling instants. In the subsequent experimental results, $\widetilde{\text{SOC}}_k$ is reported as the final SOC estimate.

For implementation convenience, the complete offline–online procedure of the proposed hybrid estimator is summarized in Algorithm 1. Figure 7 summarizes the complete offline–online workflow.

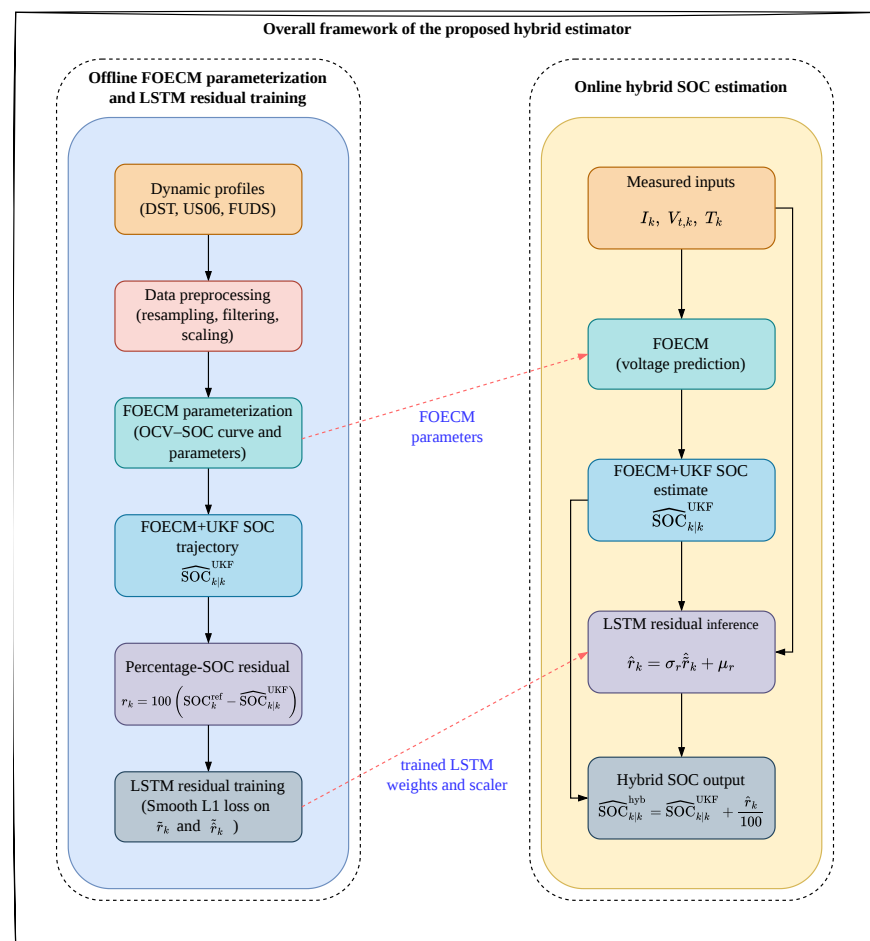


Figure 7. Overall workflow of the proposed hybrid estimator. The left part shows the offline stage, including data preprocessing, FOECM parameterization, FOECM + UKF SOC trajectory generation, residual-sequence construction, normalization, and LSTM residual training. The right part shows the online stage, including FOECM voltage prediction, FOECM + UKF SOC estimation, LSTM residual inference, and additive correction to obtain the final SOC estimate.

The FOECM + UKF estimator provides the physics-based component, while the LSTM residual learner captures systematic discrepancies under dynamic operating conditions. Consequently, the proposed hybrid estimator enhances SOC estimation accuracy compared with the FOECM + UKF estimator. Experimental validation under the available driving profiles is presented in Section 5.

Algorithm 1 Complete offline–online workflow of the proposed hybrid SOC estimator**Input:**

FOECM parameters θ_{FO} ; UKF parameters $(Q, R, \alpha_u, \kappa, \beta, \hat{x}_0, P_0)$;
 Labeled dataset $\mathcal{D} = \{(I_k, V_{t,k}, T_k, \text{SOC}_k^{\text{ref}})\}$; sequence length L_{seq} ;
 LSTM hyperparameters; causal smoothing length $M = 11$.

Output: Reported SOC sequence $\{\widehat{\text{SOC}}_k\}_{k=1}^N$.

Offline training stage

(1) For each trajectory in $\mathcal{D}$:

Run FOECM + UKF $\Rightarrow \text{SOC } \{\hat{\text{SOC}}_k^{\text{UKF}}\}$.
 Percentage-SOC residual: $r_k = 100(\text{SOC}_k^{\text{ref}} - \hat{\text{SOC}}_k^{\text{UKF}})$.
 Feature: $\mathbf{z}_k = [I_k, V_{t,k}, T_k, \hat{\text{SOC}}_k^{\text{UKF}}]^\top \in \mathbb{R}^4$.
 For $k = L_{\text{seq}}, \dots, N$: $\mathbf{Z}_k = [\mathbf{z}_{k-L_{\text{seq}}+1}, \dots, \mathbf{z}_k] \in \mathbb{R}^{L_{\text{seq}} \times 4}$.
 Collect pairs $\{(\mathbf{Z}_k, r_k)\}$.

(2) Form the training and validation subsets using the profile-aware split in Section 4.3.

(3) Compute normalization statistics from the training subset only:

$$\mu_Z = \mathbb{E}[\mathbf{Z}_k], \quad \sigma_Z = \sqrt{\mathbb{V}[\mathbf{Z}_k]}, \quad \mu_r = \mathbb{E}[r_k], \quad \sigma_r = \sqrt{\mathbb{V}[r_k]}.$$

(4) Normalize the training and validation subsets using the training statistics:

$$\tilde{\mathbf{Z}}_k = (\mathbf{Z}_k - \mu_Z) / \sigma_Z, \quad \tilde{r}_k = (r_k - \mu_r) / \sigma_r.$$

(5) Train the LSTM residual learner f_ϕ by minimizing the Smooth L1 loss

$$\mathcal{L}(\phi) = \frac{1}{|\mathcal{T}|} \sum_{k \in \mathcal{T}} \ell_{\text{SL1}}(\tilde{r}_k - f_\phi(\tilde{\mathbf{Z}}_k)),$$

with early stopping on a validation set.

(6) Store ϕ^* , μ_Z , σ_Z , μ_r , and σ_r .

Online estimation stage

(1) Initialize UKF state/covariance $(\hat{x}_0, P_0)$ and an empty feature buffer $\mathcal{B}$.

(2) For $k = 1, \dots, N$ do

(a) UKF prediction/update with FOECM:

$$(\hat{x}_{k|k}, P_{k|k}) \leftarrow \text{UKF}_{\theta_{\text{FO}}}(\hat{x}_{k-1|k-1}, P_{k-1|k-1}, I_k, V_{t,k}),$$

$$\hat{\text{SOC}}_k^{\text{UKF}} = \hat{x}_{k|k}.$$

(b) Residual correction with LSTM:

$$\mathbf{z}_k = [I_k, V_{t,k}, T_k, \hat{\text{SOC}}_k^{\text{UKF}}]^\top.$$

If $k = 1$, initialize $\mathcal{B} = [\mathbf{z}_1, \dots, \mathbf{z}_1]$ with L_{seq} copies;

otherwise, discard the oldest element of $\mathcal{B}$ and append $\mathbf{z}_k$. Set $\mathbf{Z}_k = \mathcal{B}$.

$$\tilde{\mathbf{Z}}_k = (\mathbf{Z}_k - \mu_Z) / \sigma_Z, \quad \hat{r}_k = f_{\phi^*}(\tilde{\mathbf{Z}}_k), \quad \hat{r}_k = \sigma_r \hat{r}_k + \mu_r.$$

(c) Hybrid SOC:

$$\text{SOC}_k^{\text{hyb}} = \text{sat}(\hat{\text{SOC}}_k^{\text{UKF}} + \hat{r}_k / 100; [0, 1]).$$

(d) Causal smoothing and reported output:

$$M_k = \min(M, k), \quad \widehat{\text{SOC}}_k = \frac{1}{M_k} \sum_{i=0}^{M_k-1} \text{SOC}_{k-i}^{\text{hyb}}.$$

(3) End for; return $\{\hat{\text{SOC}}_k^{\text{UKF}}\}$, $\{\text{SOC}_k^{\text{hyb}}\}$, and the reported sequence $\{\widehat{\text{SOC}}_k\}$.

5. Experimental Validation and Discussion

This section evaluates SOC estimation performance under dynamic current profiles. Two physics-based estimators are considered: an integer-order ECM combined with a UKF, and the proposed fractional-order ECM combined with a UKF. Building on the FOECM + UKF SOC estimate, four residual-learning models are constructed: a single-step BPNN, a multi-step BPNN with a short temporal window, a vanilla RNN, and the proposed LSTM residual learner. The objective is to assess whether residual learning can improve the FOECM + UKF SOC estimate under the available dynamic profiles and the selected cross-cell test setting.

5.1. Experimental Setup and Datasets

Section 2 describes the test bench, the A123 LiFePO₄ cells, and the driving profiles (DST, US06, and FUDS), with the measured current/voltage trajectories shown in Figure 1 and the cell specifications summarized in Table 2. This section details the data partitioning and organization for training, validation, and testing of the proposed SOC estimators.

To evaluate generalization across cells and operating profiles, a multi-cycle dataset is constructed from two nominally identical cells:

(1) Training/validation corpus: The DST, US06, and FUDS profiles of Cell 007 are assigned entirely to the training subset. The Cell 008 DST profile is divided chronologically into training and validation portions using the 70/30 protocol and the 40-window boundary gap described in Section 4.3. This setting provides multiple dynamic profiles from Cell 007 and introduces cell-to-cell variability through the DST profile of Cell 008.

(2) Independent test set: The US06 and FUDS profiles of Cell 008 are kept completely unseen during training, validation, normalization-statistics computation, and early-stopping selection, and are used only for quantitative performance evaluation and method comparison.

The resulting dataset organization is summarized in Table 6, including the cell ID, driving profile, data role, measured temperature range, sampling interval, complete resampled length, and SOC range.

Table 6. Dataset organization used for training, validation, and testing.

Cell	Profile	Role	Temp. Range	Sampling	Samples	SOC Range
007	DST-US06-FUDS	training subset	26.44–28.35 °C	1 s	36,293	0.000–0.980
008	DST	70/30 training-validation split	26.24–27.78 °C	1 s	11,997	0.000–0.970
008	US06	independent test set	26.55–27.77 °C	1 s	12,000	0.010–0.967
008	FUDS	independent test set	26.54–27.91 °C	1 s	13,543	0.000–0.961

The Samples column reports the complete profile length after 1 s resampling. To maintain temporal alignment with the sequence length $L_{\text{seq}} = 40$, the performance metrics of all compared methods are calculated from the 40th resampled sample onward. The reported results are therefore limited to the SOC intervals covered by the corresponding independent test profiles.

The FOECM is parameterized as described in Section 2, and its parameters are reported in Table 3. The UKF configuration is summarized in Table 4, and the main hyperparameters of the LSTM residual learner are given in Table 5. All settings are kept fixed and directly applied to the US06 and FUDS test profiles without retuning.

5.2. Performance Metrics

Let $\text{SOC}_k^{\text{ref}}$ denote the reference SOC constructed by the offline Coulomb integration and alignment procedure described in Section 2, and let $\hat{\text{SOC}}_k$ denote the corresponding online SOC estimate produced by a given estimator. The instantaneous estimation error is

$$e_k = \text{SOC}_k^{\text{ref}} - \hat{\text{SOC}}_k. \quad (45)$$

To quantitatively evaluate the performance, three standard error metrics are used: mean absolute error (MAE), root mean square error (RMSE), and maximum absolute error (e_{max}):

$$\text{MAE} = \frac{1}{N} \sum_{k=1}^N |e_k|, \quad (46)$$

$$\text{RMSE} = \sqrt{\frac{1}{N} \sum_{k=1}^N e_k^2}, \quad (47)$$

$$e_{\text{max}} = \max_{1 \leq k \leq N} |e_k|, \quad (48)$$

where N is the number of samples in the considered profile. MAE reflects the average accuracy, RMSE penalizes large errors more strongly, and e_{max} highlights the worst-case

deviation, which is especially relevant for safety-critical applications. The dimensionless values obtained from these equations are multiplied by 100 when reported in %SOC.

All learning-based configurations use the same training/validation corpus, preprocessing protocol, evaluation interval, and metric definitions, while architecture-specific hyperparameters are configured separately. Their reported SOC estimates are postprocessed using the same 11-sample causal moving average defined in (44). In contrast, the ECM+UKF and FOECM + UKF estimators are evaluated directly from their filter outputs without moving-average postprocessing.

5.3. Results on the US06 Profile

The first test is conducted on the US06 driving cycle of Cell 008, which is characterized by frequent and aggressive current transients. Figure 8a compares the reference SOC, SOC_k^{ref} , with the estimates obtained from the integer-order ECM+UKF baseline, the FOECM + UKF baseline, and four FOECM-based hybrid estimators. Figure 8b presents the corresponding SOC error trajectories, with a late-profile segment magnified for clarity.

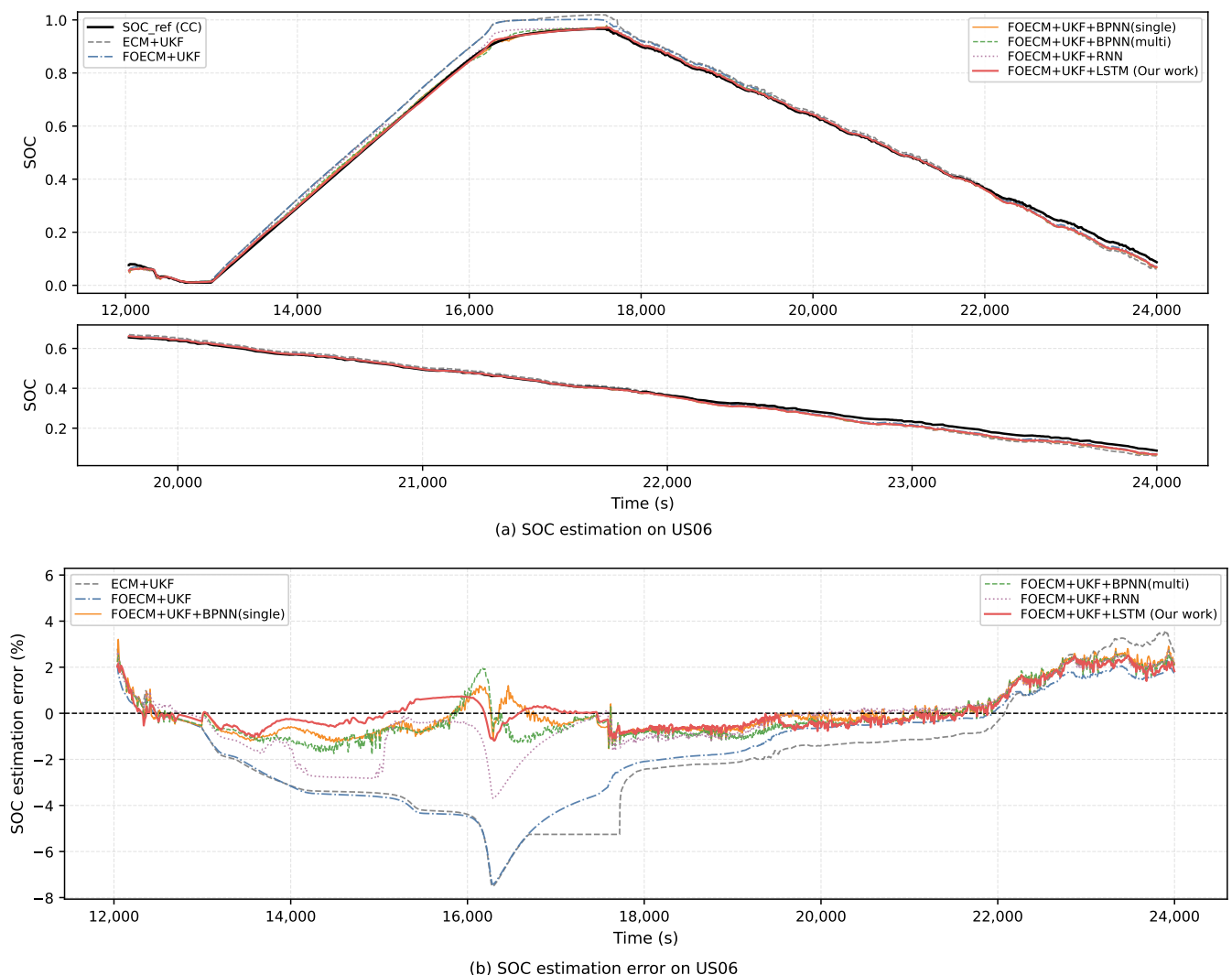


Figure 8. SOC estimation results on the US06 profile of Cell 008. (a) Reference SOC and estimates from the integer-order ECM+UKF baseline, the FOECM + UKF baseline, and four residual-learning variants (single-step BPNN, multi-step BPNN, RNN, and the FOECM + UKF + LSTM estimator). (b) Corresponding SOC estimation errors.

Compared with the ECM+UKF baseline, the FOECM + UKF estimator follows the reference SOC more closely, especially during the long acceleration and deceleration around the peak SOC region. This is consistent with Table 7, where the fractional-order model reduces the MAE on US06 from 2.60% to 2.14% and the RMSE from 3.05% to 2.70%. Nevertheless, the FOECM + UKF still exhibits noticeable low-frequency drifts in e_k , reflecting the mismatch between the simple fractional-order 1RC structure and the true cell dynamics.

Table 7. Comparison of SOC estimation performance for different estimation configurations and profiles.

Method	Learning/ Correction Role	US06 Profile			FUDS Profile		
		MAE (%)	RMSE (%)	Maximum Absolute Error ($e_{\max}$, %)	MAE (%)	RMSE (%)	Maximum Absolute Error ($e_{\max}$, %)
ECM+UKF	Physics-only filtering	2.60	3.05	7.51	2.78	3.28	7.92
FOECM + UKF	Physics-only filtering	2.14	2.70	7.42	2.31	2.93	7.83
ECM+UKF	LSTM residual correction	0.86	1.17	3.41	1.13	1.31	3.22
FOECM + UKF	BPNN (single-step)	0.78	1.03	3.21	1.11	1.23	2.92
FOECM + UKF	BPNN (multi-step)	0.90	1.10	2.74	1.20	1.39	3.50
FOECM + UKF	RNN residual correction	1.08	1.41	3.70	1.34	1.64	4.12
Direct LSTM	Direct SOC estimation	5.43	7.42	20.33	6.43	10.29	31.80
Our work	LSTM residual correction	0.66	0.90	2.52	1.03	1.20	3.40

The four hybrid estimators further correct these residual biases. Both BPNN variants and the RNN reduce the average error, but they exhibit different trade-offs between average accuracy and peak-error suppression. The FOECM + UKF + LSTM estimator achieves the lowest US06 MAE, RMSE, and maximum absolute error among the implemented residual learners, with values of 0.66%, 0.90%, and 2.52%, respectively. The corresponding RNN values are 1.08%, 1.41%, and 3.70%. This indicates that exploiting temporal structure in the UKF residuals via sequence modeling leads to substantial accuracy gains under aggressive driving transients.

5.4. Results on the FUDS Profile

The second test considers the FUDS profile of Cell 008, which features a milder, stop-and-go current pattern compared with US06. The resulting SOC trajectories and error curves are shown in Figure 9. Under this profile, the FOECM + UKF estimator still improves upon the integer-order ECM+UKF estimator, although its average errors are slightly higher than those on US06. The fractional-order model reduces the MAE from 2.78% to 2.31% and the RMSE from 3.28% to 2.93%, confirming again the benefit of introducing a fractional-order polarization branch.

Nevertheless, small drifts and local peaks of several percent in SOC can still be seen in the FOECM + UKF error, especially after long rest periods or sequences of mild current pulses. The four hybrid estimators provide additional refinement. The FOECM + UKF + LSTM estimator achieves the lowest FUDS MAE and RMSE among the implemented residual learners, with values of 1.03% and 1.20%, respectively, and reduces the maximum absolute error of the FOECM + UKF estimator from 7.83% to 3.40%. The single-step BPNN gives a slightly lower maximum absolute error of 2.92%, but its MAE and RMSE are higher at 1.11% and 1.23%. The corresponding RNN values are 1.34%, 1.64%, and 4.12%. These results show that residual correction remains effective under the milder urban-like FUDS profile.

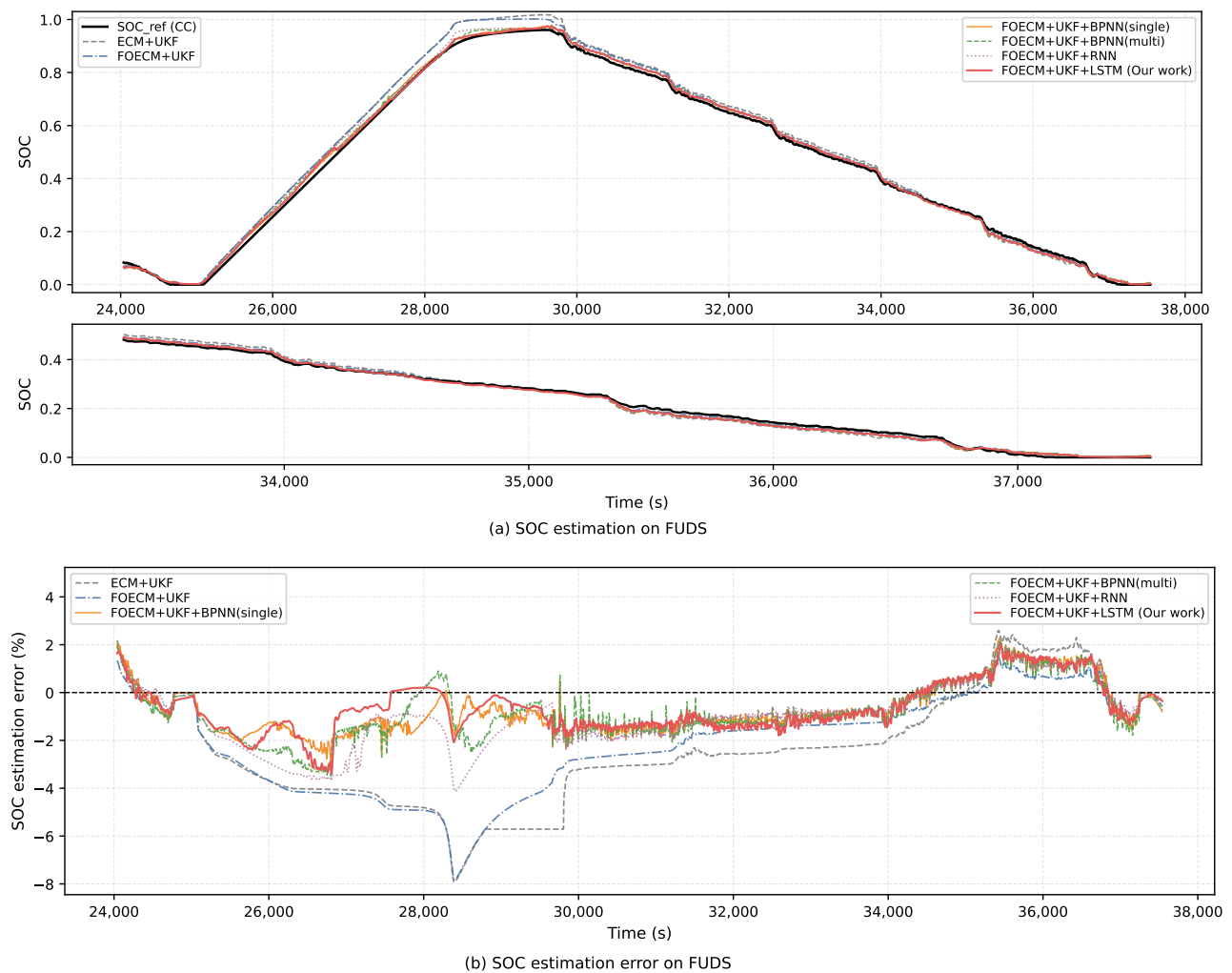


Figure 9. SOC estimation results on the FUDS profile of Cell 008. (a) Reference SOC and estimates from the integer-order ECM+UKF baseline, the FOECM + UKF baseline, and four residual-learning variants (single-step BPNN, multi-step BPNN, RNN, and the FOECM + UKF + LSTM estimator). (b) Corresponding SOC estimation errors.

5.5. Comparison with Alternative Residual Models

To evaluate the contributions of the fractional-order model and residual correction, the primary ablation analysis follows the staged estimation chain from ECM + UKF to FOECM + UKF and subsequently to the FOECM + UKF-based residual estimators. The comparison between ECM+UKF and FOECM + UKF examines the effect of the fractional-order model, whereas the comparison between FOECM + UKF and the residual-corrected estimators examines the contribution of residual learning. Within the residual-learning stage, the proposed LSTM residual learner is further compared with a single-step BPNN, a multi-step BPNN, and a vanilla RNN.

In addition, ECM + UKF + LSTM and Direct LSTM are included as supplementary comparison configurations. The former examines whether recurrent residual correction can also improve an integer-order estimator, whereas the latter distinguishes direct SOC learning from physics-guided residual correction. The primary ablation conclusions are drawn from the staged estimation chain, while these two additional configurations provide complementary evidence.

All learning-based configurations use the training/validation corpus and chronological partition described in Section 5.1. For a controlled comparison, the Direct LSTM uses the same sequence length, two-layer LSTM architecture, regression head, optimizer, loss

function, batch sizes, and early-stopping protocol as the proposed LSTM residual learner; only the input features and learning target are changed. The residual learners use the corresponding model-based SOC estimate as an input and learn its SOC residual, whereas the Direct LSTM excludes the model-based estimate and predicts SOC directly from the measured current, terminal voltage, and temperature. The FOECM parameters and the FOECM + UKF estimator configuration are kept unchanged.

The time-domain curves in Figures 8 and 9 focus on the FOECM + UKF-based residual-learning variants. The supplementary ECM + UKF + LSTM comparison is summarized quantitatively in Table 7 and Figure 10. The Direct LSTM is reported only in Table 7; its substantially larger errors would expand the vertical scale of Figure 10 and obscure the differences among the physics-guided configurations. The US06 and FUDS groups are reported together with MAE, RMSE, and maximum absolute error ($e_{\max}$).

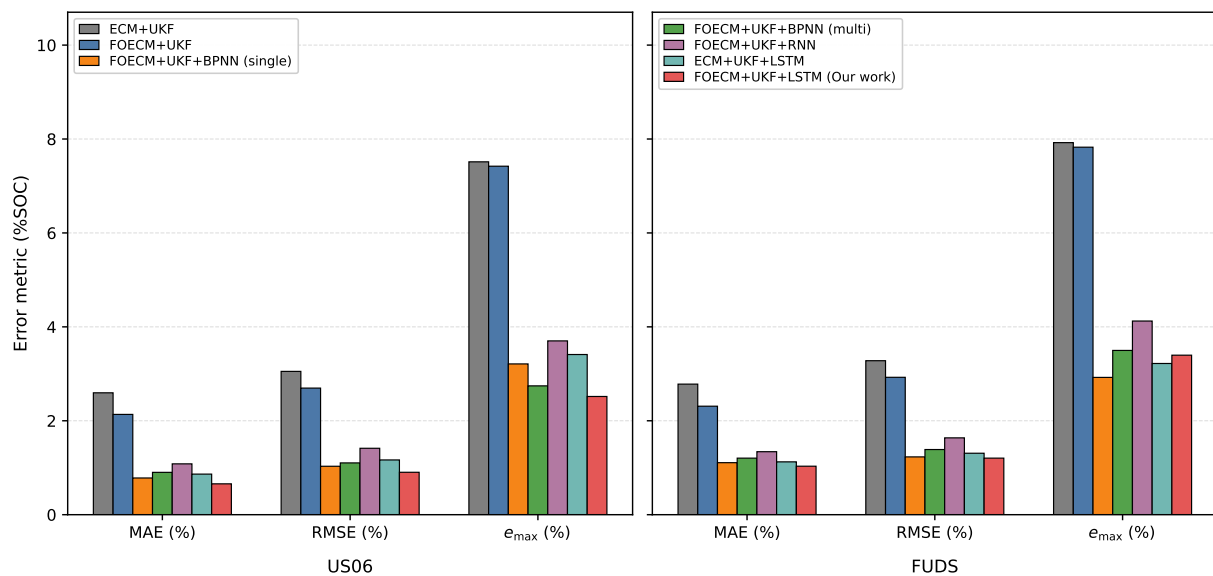


Figure 10. Comparison of SOC estimation errors on the US06 and FUDS profiles for different estimation configurations, including ECM+UKF, ECM + UKF + LSTM, FOECM + UKF, and FOECM + UKF with BPNN, RNN, and LSTM residual learners. The Direct LSTM result is reported only in Table 7 because of its substantially different error scale.

Under the same cross-profile data-partition protocol, the Direct LSTM exhibits considerably larger errors, with MAE/RMSE values of 5.43%/7.42% on US06 and 6.43%/10.29% on FUDS. This result should not be interpreted as evidence that LSTM networks are inherently unsuitable for SOC estimation. Instead, removing the model-based SOC estimate requires the network to learn the complete mapping from measured signals to SOC, which degrades its transfer to the independent Cell 008 profiles under the present data partition. The FOECM + UKF SOC estimate therefore acts as a structured physics-based prior, allowing the residual learner to focus on the remaining systematic estimation error.

The error curves and aggregate metrics were recalculated from the same finalized output files using the common evaluation interval beginning at the 40th resampled sample and the metric definitions in Section 5.2. MAE and RMSE characterize overall tracking accuracy over this interval, whereas Figures 8 and 9 also reveal local peak deviations during sharp current transients. Consequently, isolated graphical peaks do not directly represent the profile-level MAE or RMSE.

To further contextualize the obtained error level, Table 8 summarizes representative recently reported SOC estimation results from filtering, learning-based, and hybrid estimation studies.

Table 8. Reported error levels of representative recent SOC estimation studies.

Study	Method	Battery/Validation Condition	Reported Error	Main Difference/Limitation
Liu et al. [36]	DBN + EKF	Lithium-ion batteries; DST/US06; reported temperatures	MAE within 1.1%; RMSE within 1.2%	Learning model cascaded with EKF; no fractional-order circuit backbone
Zhao et al. [37]	LSTM-RNN + AEKF	LIB data; DST/UDDS/FUDS/NEDC; −10 to 45 °C	Maximum error below 2.5%; RMSE below 1.01%; MAE below 0.64%	LSTM-RNN SOC pre-estimation followed by AEKF; no fractional-order circuit backbone
Chen et al. [38]	LSTM-SA	Two LIB types; varied loading profiles, temperatures, and aging levels	With inaccurate initial SOC: average absolute error within 2%; average RMSE within 3%	Direct LSTM self-attention estimator; no physics-based filtering backbone
Wu et al. [39]	FVO-ECM + FVOO	LIB experiments; multiple cycles and temperatures; initial-SOC and noise tests A123 LiFePO ₄ cells; nominal	SOC error within $\pm 2\%$; RMSE within 1%	Variable-order FOECM and observer; no recurrent residual learning
Our work	FOECM + UKF + LSTM	25 °C ambient condition; independent Cell 008 US06/FUDS	MAE 0.66%/1.03%; RMSE 0.90%/1.20% on US06/FUDS	Fixed-order FOECM + UKF with additive LSTM residual correction; limited cells and temperature

Note: DBN, deep belief network; AEKF, adaptive extended Kalman filter; LIB, lithium-ion battery; LSTM-SA, long short-term memory with self-attention; FVO-ECM, fractional variable-order equivalent circuit model; FVOO, fractional variable-order observer.

Because the listed studies use different battery chemistries, datasets, temperature ranges, SOC intervals, drive profiles, and training/testing protocols, this comparison should not be interpreted as a strictly controlled numerical ranking. Instead, it is used to position the proposed hybrid estimator relative to recent SOC estimation studies with reported error levels.

5.6. Training Behavior of the LSTM Residual Learner

As shown in Figure 11, the training loss decreases rapidly during the first few epochs and then remains at a low level. The block-validation loss is higher and fluctuates because the validation subset is the chronologically held-out final 30% of the Cell 008 DST profile, whereas the training subset contains the three Cell 007 profiles and the first 70% of the Cell 008 DST. This separation reflects the stricter chronological block-validation protocol rather than random-window interpolation. The minimum validation loss of 0.6500 is obtained at epoch 16. Because no lower validation loss is observed during the following 15 epochs, training stops at epoch 31, and the checkpoint from epoch 16 is used for evaluation on the independent Cell 008 US06 and FUDS test profiles.

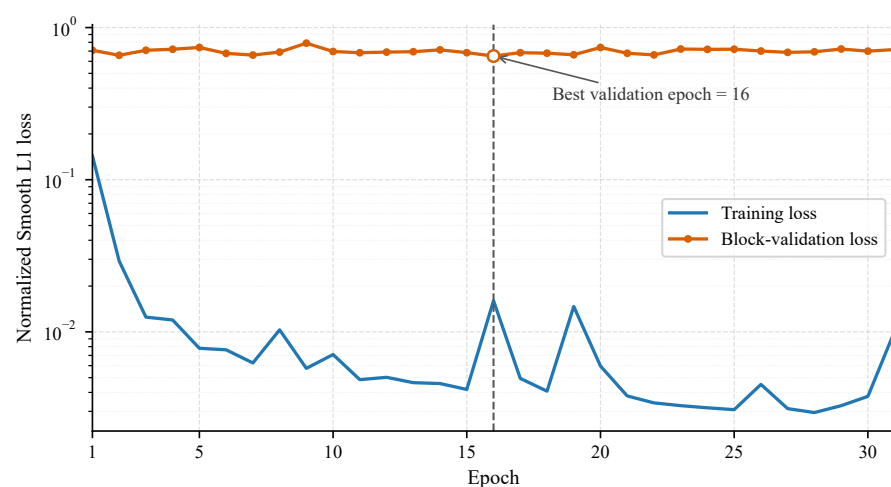


Figure 11. Training and block-validation loss curves of the LSTM residual learner. The loss is shown on a logarithmic scale. The vertical dashed line and open marker identify the minimum block-validation loss at epoch 16; early stopping terminates training at epoch 31.

5.7. Overall Discussion

The experimental results illustrate the complementary roles of the two components in the proposed hybrid estimator. The FOECM + UKF estimator captures the dominant current–voltage–SOC relationship and history-dependent polarization, whereas the LSTM residual learner compensates for errors associated with fixed parameters, finite-memory approximation, filter-noise tuning, and unmodeled effects. The Direct LSTM comparison further indicates that the model-based SOC estimate provides a structured physics-based prior for residual learning under the present cross-profile data partition. Nevertheless, several peak errors remain during sharp current transients because a fixed residual learner cannot instantaneously compensate for all transient mismatch. Accordingly, the reported improvements should be interpreted within the tested profiles rather than as a complete resolution of transient SOC-tracking limitations.

The primary comparison is designed to evaluate residual-learning variants built on the same FOECM + UKF estimator. The literature comparison in Table 8 places the obtained error level in the context of recently reported SOC estimation studies without directly ranking methods evaluated using heterogeneous datasets and validation protocols. More comprehensive comparisons with advanced fractional-order filtering structures, such as FFUKF and FSR-UKF, require a unified dataset, parameterization protocol, and filter-tuning procedure. The present validation is limited to two cells, one nominal 25 °C ambient test condition, and the available dynamic profiles.

6. Conclusions

The proposed hybrid estimator was developed by integrating a fractional-order equivalent circuit model, a UKF, and an LSTM residual learner. The FOECM with a single fractional polarization branch describes the memory-dependent polarization behavior of the LiFePO₄ cell and is embedded into the UKF to form the FOECM + UKF estimator. Rather than replacing the fractional-order estimator with a black-box neural network, the LSTM residual learner estimates the SOC residual from historical measured signals and FOECM + UKF SOC estimates.

Among the implemented configurations, the FOECM + UKF + LSTM estimator achieves the lowest values for five of the six reported metrics: MAE and RMSE on both independent test profiles and maximum absolute error on US06. Transient peak errors remain on FUDS, where some comparison configurations yield a lower maximum absolute error. The reported improvement is therefore limited to the tested cells and operating profiles and does not imply uniform superiority for every local peak-error metric.

The proposed hybrid estimator preserves the interpretability of the FOECM + UKF estimator, as the neural network acts only on the residual SOC error and its correction is explicitly superimposed on the FOECM + UKF SOC estimate. The current validation is limited to the available cells and operating profiles under a single nominal 25 °C ambient test condition. Future work will extend the proposed hybrid estimator to multi-temperature and aging-state validation, online parameter adaptation, joint SOC–state-of-health (SOH) estimation, embedded timing evaluation, alternative sequence models, and direct comparisons with advanced fractional-order filtering methods, such as FFUKF and FSR-UKF, under a unified evaluation protocol.

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Data Availability Statement: The raw battery data analyzed in this study are publicly available from the CALCE Battery Research Data repository at <https://calce.umd.edu/battery-data> (accessed on 16 July 2026) [32]. The processed datasets and implementation files are available from the corresponding author upon reasonable request.

Conflicts of Interest: The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Abbreviations

The following abbreviations are used in this manuscript:

SOC	State of charge
BMS	Battery management system
OCV	Open-circuit voltage
ECM	Equivalent circuit model
FOECM	Fractional-order equivalent circuit model
UKF	Unscented Kalman filter
LSTM	Long short-term memory
DST	Dynamic Stress Test
US06	Supplemental Federal Test Procedure
FUDS	Federal Urban Driving Schedule
MAE	Mean absolute error
RMSE	Root mean square error
BPNN	Back-propagation neural network
RNN	Recurrent neural network
CPE	Constant phase element
GL	Grünwald–Letnikov
EKF	Extended Kalman filter
PF	Particle filter
FFUKF	Fuzzy fractional-order unscented Kalman filter
FSR-UKF	Fractional-order square-root unscented Kalman filter

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