



Advances in Boundary Value Problems for Fractional Differential Equations—2nd Edition

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The Special Issue “Advances in Boundary Value Problems for Fractional Differential Equations—2nd Edition” presents recent advances in the theory and applications of fractional differential equations, fractional inclusions, and systems of fractional differential equations involving Riemann–Liouville, Caputo, Hadamard, Hilfer–Hadamard and other generalized fractional derivatives under a variety of boundary conditions. Fractional partial differential equations and systems, as well as fractional q -difference equations and inclusions, are also addressed in this Special Issue. The contributions encompass a broad spectrum of topics, including the existence, uniqueness, stability, and approximation of solutions. In what follows, these papers are briefly presented and organized according to their main research themes.

1. Fractional Differential Equations, Systems and Inclusions

Paper [1] is devoted to the multi-term implicit fractional differential equation supplemented with the boundary conditions

$$\begin{cases} \left(\sum_{i=1}^m \lambda_i {}^C D^{q_i} + \sum_{j=1}^m K_j {}^R I^{p_j} \right) u(t) = f\left(t, u(t), \left(\sum_{i=1}^m \lambda_i {}^C D^{q_i} + \sum_{j=1}^m K_j {}^R I^{p_j} \right) u(t)\right), & t \in [0, 1], \\ u(0) = 0, \quad {}^C D^k u(0) = 0, \quad k = 1, 2, \dots, n-2, \\ u(1) = {}^R I^\omega u(\eta). \end{cases} \quad (1)$$

Here $q_1 \in (n-1, n)$, $q_i \in (0, 1)$, $i = 2, \dots, m$, $p_j \in (0, 1)$, $j = 1, \dots, m$, $\omega \in (0, 1)$, $\eta \in [0, 1]$, λ_i and K_i , $i = 1, \dots, m$ are positive constants, ${}^C D^\kappa u$ is the Caputo fractional derivative of function u of order κ , for $\kappa \in \{q_1, \dots, q_m\}$, ${}^R I^\kappa u$ is the Riemann–Liouville fractional integral of function u of order κ , for $\kappa \in \{p_1, \dots, p_m\}$, and $f : [0, 1] \times \mathbb{R} \times \mathbb{R} \rightarrow \mathbb{R}$ satisfies some assumptions. The authors prove the existence and uniqueness of solutions to problem (1) by applying the Schauder fixed point theorem and the Banach fixed point theorem. The Ulam–Hyers and Ulam–Hyers–Rassias stability of solutions to (1) are also investigated.

Paper [2] examines the fractional boundary value problem

$$\begin{cases} D_{0+}^\alpha x(t) = \phi(t, x(t), I_{0+}^\alpha x(t)), & t \in [0, \zeta], \\ \gamma x(0) + \mu x(\zeta) = c, \end{cases} \quad (2)$$

where $\alpha \in (0, 1)$, the constants ζ, c, γ, μ are real with $\mu \neq 0$, $D_{0+}^\alpha x$ and $I_{0+}^\alpha x$ are the left Riemann–Liouville fractional derivative and integral of function x of order α , respectively, the function $\phi(\cdot, x(\cdot), I_{0+}^\alpha x(\cdot)) \in L^{p(\cdot)}([0, \zeta] \times \mathbb{R} \times \mathbb{R}, \mathbb{R})$, and $x \in L^{p(\cdot)}([0, \zeta], \mathbb{R})$ is the unknown function. Here $p : [0, \zeta] \rightarrow [1, \infty)$ is a step function given by $p(t) = \sum_{k=1}^n p_k I_k(t)$ with $p_k \in [1, \infty)$, $I_k(t) = \{1, \text{ for } t \in (\zeta_{k-1}, \zeta_k]; 0, \text{ elsewhere}\}$ for $k = 1, \dots, n$, and $\zeta_i \in [0, \zeta]$, $i = 0, \dots, n$, $\zeta_0 = 0$ and $\zeta_n = \zeta$. By using two fixed point theorems, the authors



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investigate the existence and uniqueness of solutions for problem (2). The Ulam-Hyers stability is also investigated in this paper.

Paper [3] is focused on the system of nonlinear Caputo fractional differential equations

$$\begin{cases} ({}^C D^\alpha + \lambda {}^C D^{\alpha-1})u(t) = f(t, u(t), v(t), I_{0+}^{p_1} u(t), I_{0+}^{p_2} v(t)), & t \in (0, 1), \\ ({}^C D^\beta + \mu {}^C D^{\beta-1})v(t) = g(t, u(t), v(t), I_{0+}^{q_1} u(t), I_{0+}^{q_2} v(t)), & t \in (0, 1), \end{cases} \quad (3)$$

subject to coupled integral boundary conditions

$$\begin{cases} u(0) = u'(0) = 0, & u'(1) = 0, & u(1) = \int_0^1 u(s) d\mathcal{H}_1(s) + \int_0^1 v(s) d\mathcal{H}_2(s), \\ v(0) = v'(0) = 0, & v'(1) = 0, & v(1) = \int_0^1 u(s) d\mathcal{K}_1(s) + \int_0^1 v(s) d\mathcal{K}_2(s). \end{cases} \quad (4)$$

Here $\alpha, \beta \in (3, 4]$, $p_i > 0$, $q_i > 0$ for $i = 1, 2$, $\lambda > 0$, $\mu > 0$, ${}^C D^\kappa$ denotes the Caputo fractional derivative of order κ (for $\kappa = \alpha, \alpha - 1, \beta, \beta - 1$), I_{0+}^κ is the Riemann-Liouville fractional integral of order κ (for $\kappa = p_i, q_i$, $i = 1, 2$); $f, g : [0, 1] \times \mathbb{R}^4 \rightarrow \mathbb{R}$ are continuous functions, and the integrals from (4) are Riemann-Stieltjes integrals with bounded variation functions $\mathcal{H}_i, \mathcal{K}_i$, $i = 1, 2$. Based on the Leray-Schauder alternative and the Banach fixed point theorem, the authors prove the existence and uniqueness of solutions to problem (3)–(4).

Paper [4] explores the system of Hadamard fractional differential equations on an infinite interval

$$\begin{cases} {}^H D_{1+}^\alpha u(t) + c(t)f(u(t), v(t)) = 0, & t \in (1, \infty), \\ {}^H D_{1+}^\beta v(t) + d(t)g(u(t), v(t)) = 0, & t \in (1, \infty), \end{cases} \quad (5)$$

with the nonlocal coupled boundary conditions

$$\begin{cases} u(1) = u'(1) = \dots = u^{(n-2)}(1) = 0, \\ {}^H D_{1+}^{\alpha-1} u(\infty) = \int_1^\infty u(s) d\mathfrak{H}_1(s) + \int_1^\infty v(s) d\mathfrak{H}_2(s), \\ v(1) = v'(1) = \dots = v^{(m-2)}(1) = 0, \\ {}^H D_{1+}^{\beta-1} v(\infty) = \int_1^\infty u(s) d\mathfrak{K}_1(s) + \int_1^\infty v(s) d\mathfrak{K}_2(s). \end{cases} \quad (6)$$

Here $\alpha \in (n-1, n]$, $\beta \in (m-1, m]$, $n, m \in \mathbb{N}$, $n, m \geq 2$, ${}^H D_{1+}^\kappa$ denotes the Hadamard fractional derivative of order κ (for $\kappa = \alpha, \beta, \alpha - 1, \beta - 1$), the functions $c, d : [1, \infty) \rightarrow \mathbb{R}_+$ and the functions $f, g : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ verify some assumptions, ($\mathbb{R}_+ = [0, \infty)$), and the functions $\mathfrak{H}_i, \mathfrak{K}_i : [1, \infty) \rightarrow \mathbb{R}$, $i = 1, 2$ from the Riemann-Stieltjes integrals of (6) have bounded variations. By using the Guo-Krasnosel'skii fixed point theorem and the Leggett-Williams fixed point theorem, the authors establish the existence of positive solutions for problem (5)–(6).

Paper [5] is concerned with the system of fractional differential equations

$$\begin{cases} {}^{HH} D_1^{\alpha, \beta} u(t) = f(t, u(t), v(t)), & t \in (1, T), \\ {}^{HH} D_1^{\gamma, \delta} v(t) = g(t, u(t), v(t)), & t \in (1, T), \end{cases} \quad (7)$$

supplemented with the coupled boundary conditions

$$\begin{cases} u(1) = 0, & {}^H D_1^\zeta u(T) = \sum_{i=1}^m \int_1^T {}^H D_1^{\varrho_i} u(s) d\mathcal{H}_i(s) + \sum_{i=1}^n \int_1^T {}^H D_1^{\sigma_i} v(s) d\mathcal{K}_i(s), \\ v(1) = 0, & {}^H D_1^\theta v(T) = \sum_{i=1}^p \int_1^T {}^H D_1^{\eta_i} u(s) d\mathcal{P}_i(s) + \sum_{i=1}^q \int_1^T {}^H D_1^{\theta_i} v(s) d\mathcal{Q}_i(s). \end{cases} \quad (8)$$

Here $T > 1$, $\alpha, \gamma \in (1, 2]$, $\beta, \delta \in [0, 1]$, $m, n, p, q \in \mathbb{N}$, $\zeta, \theta, \varrho_i, \sigma_i, \eta_i, \theta_i \in [0, 1]$, ${}^{HH} D_1^{\kappa_1, \kappa_2}$ is the Hilfer-Hadamard fractional derivative of order κ_1 and type κ_2 (for $\kappa_1 = \alpha, \gamma$ and $\kappa_2 = \beta, \delta$), ${}^H D_1^\kappa$ denotes the Hadamard fractional derivative of order κ (for $\kappa = \zeta, \theta, \varrho_i, \sigma_j, \eta_k, \theta_l$, $i =$

$1, \dots, m, j = 1, \dots, n, k = 1, \dots, p$ and $\iota = 1, \dots, q$), the integrals from the boundary conditions (8) are Riemann-Stieltjes integrals with $\mathcal{H}_i, \mathcal{K}_j, \mathcal{P}_k, \mathcal{Q}_\iota, i = 1, \dots, m, j = 1, \dots, n, k = 1, \dots, p$ and $\iota = 1, \dots, q$ functions of bounded variation, and $f, g : [1, T] \times \mathbb{R}^2 \rightarrow \mathbb{R}$ are continuous functions. The authors show the existence and uniqueness of solutions to problem (7)–(8) by applying varied fixed point theorems, such as: the Krasnosel'skii fixed point theorem for the sum of two operators, the Banach contraction mapping principle, the Leray-Schauder nonlinear alternative, and the Schaefer fixed point theorem.

Paper [6] deals with the nonlinear inverse problem

$$\begin{cases} D^{\sigma_n} \mathfrak{z}(t) = A\mathfrak{z}(t) + B(t, D^{\sigma_0} \mathfrak{z}(t), D^{\sigma_1} \mathfrak{z}(t), \dots, D^{\sigma_{n-1}} \mathfrak{z}(t), u(t)), & t \in [t_0, T], \\ D^{\sigma_k} \mathfrak{z}(t_0) = \mathfrak{z}_k, & k = 0, 1, \dots, n-1, \\ \Phi \mathfrak{z}(t) = \Psi(t), & t \in [t_0, T]. \end{cases} \quad (9)$$

Here A is a linear bounded operator on the Banach space $\mathcal{Z}$; Φ is a linear bounded mapping from $\mathcal{Z}$ to another Banach space $\mathcal{U}$; $B : [t_0, T] \times \mathcal{Z}^n \times \mathcal{U} \rightarrow \mathcal{Z}, \mathfrak{z}_k \in \mathcal{Z}$ for $k = 0, 1, \dots, n-1$, and $\Psi \in C[t_0, T; \mathcal{U}]$. $D^{\sigma_k} \mathfrak{z}(t)$ denotes the Dzhrbashyan-Nersesyan (D-N) fractional derivatives of orders $\sigma_k, k = 0, 1, \dots, n$, which correspond to the sequence $\{\alpha_0, \alpha_1, \dots, \alpha_n\}, \alpha_k \in (0, 1], k = 0, 1, \dots, n$ which are defined by

$$\begin{aligned} D^{\sigma_0} \mathfrak{z}(t) &= D_t^{\alpha_0-1} \mathfrak{z}(t), \\ D^{\sigma_k} \mathfrak{z}(t) &= D_t^{\alpha_k-1} D_t^{\alpha_{k-1}} D_t^{\alpha_{k-2}} \dots D_t^{\alpha_0} \mathfrak{z}(t), \quad k = 1, 2, \dots, n, \end{aligned}$$

where $D_t^\beta := J_t^{-\beta}$ is the Riemann-Liouville fractional integral of order $-\beta > 0$. For $\beta = 0$ then D_t^0 is the identical operator, and $D_t^\beta := D_t^m J_t^{m-\beta}$ is the Riemann-Liouville fractional derivative of order $\beta \in (m-1, m], m \in \mathbb{N}$. These D-N fractional derivatives generalize the Riemann-Liouville fractional derivative (for $\alpha_0 \in (0, 1), \alpha_k = 1, k = 1, 2, \dots, n$), and the Gerasimov-Caputo fractional derivative (for $\alpha_k = 1, k = 0, 1, \dots, n-1, \alpha_n \in (0, 1)$). By using the fixed point theorem for contraction mapping, the authors prove the existence of a local unique solution $(\mathfrak{z}, u) \in C([t_0, T_1]; \mathcal{Z}) \times C([t_0, T_1]; \mathcal{U})$ with $T_1 \in (t_0, T]$ of problem (9), and then they show the existence of a unique solution for (9) on $[t_0, T]$. Applications of the obtained results for nonlinear inverse problems for partial differential equations with D-N time-fractional derivatives and unknown coefficients (namely, for an integro-differential equation and for a linearized Kelvin-Voigh time-fractional system) are also presented.

In paper [7] the authors study the affine-periodic boundary value problem for the fractional differential inclusion

$$\begin{cases} {}^c D^{\alpha+1} z(t) + b {}^c D^\alpha z(t) \in \mathfrak{G}(t, z), & t \in [0, K], \\ z(K) = \lambda z(0), \quad z'(K) = \lambda z'(0), \end{cases} \quad (10)$$

where $\alpha \in (0, 1), b \in \mathbb{R}, \lambda \in \mathbb{R}, \lambda \neq 1, \lambda \neq e^{-bk}; \mathfrak{G} : [0, K] \times \mathbb{R} \rightarrow \mathcal{P}(\mathbb{R})$ is a multifunction, and ${}^c D^\kappa$ is the Caputo fractional derivative of order κ for $\kappa \in \{\alpha, \alpha+1\}$. By applying some results from the set-valued analysis theory, the Leray-Schauder alternative theorem, and the Covitz-Nadler fixed point theorem, they prove the existence of solutions for problem (10) when $\mathfrak{G}$ has convex values (the first case) or nonconvex values (the second case).

2. Fractional Partial Differential Equations and Systems

Paper [8] is devoted to the two-dimensional Riemann-Liouville distributed-order space fractional diffusion equation

$$\begin{aligned} \frac{\partial u}{\partial t}(x, y, t) &= K_x \int_1^2 \mathcal{P}(\alpha) \frac{\partial^\alpha u}{\partial x^\alpha}(x, y, t) d\alpha + K_y \int_1^2 \mathcal{Q}(\beta) \frac{\partial^\beta u}{\partial y^\beta}(x, y, t) d\beta + f(x, y, t), \\ t &\in (0, T], \quad (x, y) \in (a, b) \times (c, d), \end{aligned} \quad (11)$$

subject to the boundary and initial conditions

$$\begin{cases} u(x, y, t) = 0, & (x, y, t) \in (\mathbb{R}^2 \setminus \Omega) \times (0, T], \\ u(x, y, 0) = u_0(x, y), & (x, y) \in \bar{\Omega}. \end{cases} \quad (12)$$

Here K_x and K_y are the diffusion coefficients, and $\mathcal{P}, \mathcal{Q} : [1, 2] \rightarrow \mathbb{R}$ are weight functions which satisfy the conditions

$$\begin{aligned} \mathcal{P}(\alpha) &\geq 0, \quad \mathcal{P}(\alpha) \not\equiv 0, \quad \forall \alpha \in [1, 2], \quad \int_1^2 \mathcal{P}(\alpha) d\alpha < \infty, \\ \mathcal{Q}(\beta) &\geq 0, \quad \mathcal{Q}(\beta) \not\equiv 0, \quad \forall \beta \in [1, 2], \quad \int_1^2 \mathcal{Q}(\beta) d\beta < \infty. \end{aligned}$$

The authors present an alternating direction implicit scheme (ADIM) for problem (11)–(12), and prove the stability and convergent of this numerical scheme. They also implement an extrapolated technique for improving the convergence order of the scheme. A fast algorithm applied to the ADIM for accelerating the computation is also developed.

Paper [9] is focused on the system of time-fractional partial differential equations on disjoint domains

$$\begin{cases} \frac{\partial^{\delta_1} u_1}{\partial t^{\delta_1}}(x, t) - \frac{\partial}{\partial x} \left(p_1(x) \frac{\partial u_1}{\partial x}(x, t) \right) = f_1(x, t), & x \in \Omega_1 = (a_1, b_1), \quad t \in (0, T], \\ \frac{\partial^{\delta_2} u_2}{\partial t^{\delta_2}}(x, t) - \frac{\partial}{\partial x} \left(p_2(x) \frac{\partial u_2}{\partial x}(x, t) \right) = f_2(x, t), & x \in \Omega_2 = (a_2, b_2), \quad t \in (0, T], \end{cases} \quad (13)$$

with the external boundary conditions

$$u_1(a_1, t) = \varphi_1(t), \quad u_2(b_2, t) = \varphi_2(t), \quad t \in (0, T], \quad (14)$$

the interface mixed-type boundary conditions

$$\begin{cases} p_1(b_1) \frac{\partial u_1}{\partial x}(b_1, t) + \alpha_1 u_1(b_1, t) = \beta_1 u_2(a_2, t) + \gamma_1(t), & t \in (0, T], \\ -p_2(a_2) \frac{\partial u_2}{\partial x}(a_2, t) + \alpha_2 u_2(a_2, t) = \beta_2 u_1(b_1, t) + \gamma_2(t), & t \in (0, T], \end{cases} \quad (15)$$

and the initial conditions

$$u_1(x, 0) = u_1^0(x), \quad x \in \Omega_1; \quad u_2(x, 0) = u_2^0(x), \quad x \in \Omega_2. \quad (16)$$

Here $\delta_i \in (0, 1)$, $i = 1, 2$, $a_i, b_i \in \mathbb{R}$, $i = 1, 2$, $a_1 < b_1 < a_2 < b_2$; α_j, β_j , $j = 1, 2$ are given constants which satisfy the condition $\beta_1 \beta_2 \leq \alpha_2 \alpha_1$; $p_j \in L_\infty(\Omega_j)$ with $p_j(x) \geq \bar{p}_j > 0$, $x \in \Omega_j$, $j = 1, 2$, and the time-fractional derivatives from (13) are in the Caputo sense. The authors study the existence and uniqueness of solutions for the direct problem (13)–(16) at given coefficients, initial, boundary and interface conditions. Then they develop a decomposition technique for solving the direct problem and five inverse problems for (13)–(15) (with φ_1, φ_2 unknown) under some given observation data, and based on decoupling of the problem at each time level. A fully discrete numerical scheme for solving one inverse problem, and an algorithm for computational realization are then constructed. An extension to a two-dimensional problem is also investigated. Finally, the authors present numerical simulations for some examples.

In paper [10], the author presents some theorems related to the G-double Laplace transform. By using the multi-G-Laplace transform and the decomposition methods,

he studies the existence of solutions for the system of time-fractional partial differential equations with initial conditions

$$\begin{cases} D_{\vartheta}^{\beta_i} \psi_i(\zeta, \vartheta) = f(\zeta, \vartheta) + L_i \psi^* + N_i \psi^*, & i = 1, 2, \dots, \\ \frac{\partial^{m_i-1} \psi_i}{\partial \vartheta^{m_i-1}}(\zeta, 0) = f_{im_i}(\zeta), & m_i = 1, 2, \dots, n_i - 1, \quad i = 1, 2, \dots \end{cases} \quad (17)$$

Here $\beta_i \in (n_i - 1, n_i)$, $n_i \in \mathbb{N}$, for $i = 1, 2, \dots$, $\zeta^* = (\zeta_1, \zeta_2, \dots, \zeta_n) \in \mathbb{R}^n$, $\psi^* = \psi(\zeta^*, \vartheta) = (\psi_1(\zeta^*, \vartheta), \psi_2(\zeta^*, \vartheta), \dots, \psi_n(\zeta^*, \vartheta))$; $f(\zeta^*, \vartheta)$ is a given function, L_i and N_i are linear and nonlinear operators, respectively, and $D_{\vartheta}^{\beta_i}$ is the Caputo derivative of order β_i with respect to variable ϑ , for $i = 1, 2, \dots$. The convergence of this method for problem (17) is then investigated. Applications to homogeneous telegraphic equation and various examples of systems of time-fractional differential equations with numerical computations are finally presented.

In paper [11], the authors present firstly the shifted Fibonacci polynomials (SFPs) and some of their properties. Then they apply the collocation algorithm based on these polynomials for solving the time fractional Burgers equation with initial-boundary conditions

$$\begin{cases} \frac{\partial^\eta u}{\partial t^\eta}(\zeta, t) + u(\zeta, t) \frac{\partial u}{\partial \zeta}(\zeta, t) - \Psi \frac{\partial^2 u}{\partial \zeta^2}(\zeta, t) = S(\zeta, t), \\ u(\zeta, 0) = u_1(\zeta), \quad \zeta \in (0, 1], \\ u(0, t) = u_2(t), \quad u(1, t) = u_3(t), \quad t \in (0, 1], \end{cases} \quad (18)$$

where $\eta \in (0, 1)$, the time fractional derivative in (18) is of Caputo type, $S(\zeta, t)$ is the source term, and Ψ is the kinetic viscosity. The convergence of the shifted Fibonacci expansion is also investigated. Finally, numerical examples are provided to demonstrate the effectiveness and applicability of the proposed algorithm.

3. Fractional Q -Difference Equations and Inclusions

Paper [12] deals with the linear q -difference equations

$$({}^c\mathbb{D}_{(q)}^r \eta)(t) = h(t), \quad (19)$$

$$({}^c\mathbb{D}_{(q)}^r \eta)(t) - \alpha \eta(t) = h(t), \quad (20)$$

$$({}^c\mathbb{D}_{(q)}^{r_1} \eta)(t) - \alpha ({}^c\mathbb{D}_{(q)}^{r_2} \eta)(t) = h(t), \quad (21)$$

where $q \in (0, 1)$ is the quantum parameter, $r, r_1, r_2 \in (0, 1)$ with $r_2 < r_1$, and $\alpha \in \mathbb{R}$; ${}^c\mathbb{D}_{(q)}^\beta$ represents the q -derivative of the Caputo-like type of order β (for $\beta \in \{r, r_1, r_2\}$), and $h : [0, \infty) \rightarrow \mathbb{R}$ is a continuous function. Based on the quantum Laplace transform and the quantum Mittag-Leffler function, the authors prove the Ulam-Hyers stability of Equations (19)–(21). They also study the existence and the Ulam-Hyers-Rassias stability of solutions for the nonlinear q -difference equation with initial value

$$\begin{cases} ({}^c\mathbb{D}_{(q)}^r \eta)(t) = \Phi(t, \eta(t)), \\ \eta(0) = \eta_0, \end{cases}$$

where $r \in (0, 1)$.

Paper [13] is concerned with the sequential $(p; q)$ -model of the elastic beam with $(p; q)$ -Navier difference equation of the Caputo-like type

$$\begin{cases} {}^c\mathbb{D}_{(p;q)}^{r_1} ({}^c\mathbb{D}_{(p;q)}^{r_2} \eta)(t) = E(t, \eta(t), {}^c\mathbb{D}_{(p;q)}^{r_2} \eta(t)), \quad t \in \mathcal{I}_{(p;q)}^T = \left[0, \frac{T}{p}\right], \\ \beta \eta(0) = \lambda \eta\left(\frac{T}{p}\right) = \delta {}^c\mathbb{D}_{(p;q)}^{r_2} \eta(0) = \gamma {}^c\mathbb{D}_{(p;q)}^{r_2} \eta\left(\frac{T}{p}\right) = 0, \end{cases} \quad (22)$$

and the $(p; q)$ -Navier difference inclusion of the Caputo-like type

$$\begin{cases} {}^c\mathbb{D}_{(p;q)}^{r_1}({}^c\mathbb{D}_{(p;q)}^{r_2}\eta)(t) \in \mathcal{E}(t, \eta(t), {}^c\mathbb{D}_{(p;q)}^{r_2}\eta(t)), & t \in \mathcal{I}_{(p;q)}^T = \left[0, \frac{T}{p}\right], \\ \beta \eta(0) = \lambda \eta\left(\frac{T}{p}\right) = \delta {}^c\mathbb{D}_{(p;q)}^{r_2}\eta(0) = \gamma {}^c\mathbb{D}_{(p;q)}^{r_2}\eta\left(\frac{T}{p}\right) = 0, \end{cases} \quad (23)$$

where $0 < q < p \leq 1$, $T > 0$, $r_1, r_2 \in (1, 2]$; $\beta, \lambda, \delta, \gamma \in \mathbb{R}^+$, ${}^c\mathbb{D}_{(p;q)}^{(\cdot)}$ is the $(p; q)$ -derivative of the Caputo-like type, and the continuous function $E : [0, \frac{T}{p}] \times \mathbb{R}^2 \rightarrow \mathbb{R}$ and the multi-valued function $\mathcal{E} : [0, \frac{T}{p}] \times \mathbb{R}^2 \rightarrow \mathcal{P}(\mathbb{R})$ satisfy some assumptions. The authors prove the existence of solutions for problems (22) and (23), by using the properties of the $(p; q)$ -operators, the Krasnosel'skii fixed point theorem for the sum of two operators, a fixed point theorem for $\rho - \theta$ -contractions and ρ -admissible functions, its variant for multi-valued functions, and the approximate endpoint property for the multi-valued functions.

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