



## Article

# Adaptive Event-Triggered Projective Synchronization for FOCVNNs Subject to Aperiodic DoS Attacks

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## Abstract

This article focuses on the projective synchronization control problem for a class of fractional-order complex-valued neural networks (FOCVNNs) with additive time-varying delays (ATVDs) subject to aperiodic denial-of-service (DoS) attacks. Initially, a switching-delay FOCVNN model is designed to address the intermittent and unpredictable nature of aperiodic DoS attacks. Secondly, an adaptive event-triggering mechanism (AETM) with a sampled-data-based switching-adjusted threshold parameter is proposed, which can save network bandwidth effectively and strengthen the system's defense against aperiodic DoS attacks. Third, leveraging event-triggered information, a switching controller for projective synchronization is designed to counteract the effects of DoS attacks and maintain system stability during intermittent data transmission failures. Subsequently, a novel common Lyapunov–Krasovskii function (LKF) is constructed for both the attack and dormant phases of DoS attacks, and by utilizing the fractional-order inequality method, the sufficient conditions for the projective synchronization are presented as linear matrix inequalities (LMIs). Finally, numerical examples confirm the practicality and effectiveness of the theoretical results, and demonstrate its capacity to sustain robust synchronization performance in complex attack scenarios.

**Keywords:** adaptive event-triggered mechanism; fractional-order complex-valued neural networks; aperiodic denial of service attacks; projective synchronization

## 1. Introduction

Complex-valued neural networks (CVNNs) can be regarded as a generalization of real-valued neural networks (NNs), where inputs, states, weights, and activation functions are both complex-valued type parameters, and can handle both phase and amplitude information. This feature allows CVNNs to address some special tasks that are difficult for real-valued NNs, including the XOR problem, symmetry detection, and nonlinear electromagnetic backscattering. So, CVNNs have been widely used in signal processing, channel equalization, satellite communications, and adaptive beamforming [1–3]. Fractional-order calculus has recently been applied to CVNNs for its distinct capacity to capture memory and hereditary characteristics [4]. Consequently, FOCVNNs have been developed and employed to optimize network structures and performance while enhancing the capacity to model intricate nonlinear processes. Therefore, FOCVNNs combine the memory representation capability of fractional-order systems with the phase-description capability of



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complex-valued neural networks. This makes them suitable for modeling complex neural dynamic systems with memory effects, phase characteristics, and nonlinear interactions, and gives them promising practical applications in signal processing, pattern recognition, associative memory, image processing, and secure communication [5]. Moreover, time delays, which are inevitable in practical systems, may induce dynamic effects like instability and chaos. Therefore, the study of stability in FOCVNNs with delays has attracted considerable interest over the years [6–8].

It is crucial to observe that the majority of the previously mentioned studies have presumed a constant delay, whereas in real-world scenarios, signal transmission often traverses multiple network segments; ATVDs with varying characteristics caused by differences in network transmission environments and conditions are common. For instance, in a closed-loop control system with distributed physical devices, controllers, sensors, and actuators, two distinct ATVDs occur during the process of signal transmission: one from sensors to controllers and another from controllers to actuators [9]. Thus, modeling and analyzing these delays individually, rather than treating them as a single overall delays, is more reasonable. In recent years, significant attention has been directed toward the dynamic analysis of systems with ATVDs. Ref. [10] examined the stability of T-S fuzzy systems with two ATVDs, ref. [9] explored the stochastic stability of neutral-type NNs influenced by ATVDs and semi-Markovian jumps. Additionally, the acceptability and stability issues of descriptor T-S fuzzy systems with two ATVDs are addressed in [11]. However, the stability analysis and control synthesis of FOCVNNs with ATVDs have not been given enough attention, which is the first motivation of this paper.

On the one hand, network control systems (NCSs) are particularly susceptible to cyber attacks because of the openness of their communication links. Common attacks include false data injection attacks [12], DoS attacks [13], deception attacks [14], and hybrid attacks [15]. Among these, DoS attacks are particularly prevalent because they are easy to execute and highly destructive, overwhelming the network with irrelevant data to cause system paralysis [13]. In recent years, the vulnerability of NCSs to DoS attacks and defense techniques has become a major research focus. For example, in [16], the event-triggered  $H_\infty$  control problem for Markovian jump network-based physical systems under DoS attacks was examined, whereas [17] investigated the  $H_\infty$  control problem in network cascade control systems subjected to periodic DoS attacks, and ref. [18] introduced a switched event-triggered NN adaptive control method for switched nonlinear systems under DoS attacks. Most current research focuses on periodic or constant-strength DoS attacks, which follow fixed patterns and are easier to detect. However, in real-world scenarios, attackers often prefer unpredictable, aperiodic DoS attacks, which are increasingly hazardous due to their lack of regularity and their optimization for energy constraints and covert operations [19]. Research on aperiodic DoS attacks has gained attention recently. For example, ref. [13] addressed resilient load frequency control in networked power systems under aperiodic DoS attacks. Similarly, ref. [19] proposed a new PID load frequency control strategy for multi-regional interconnected wind power systems under aperiodic DoS attacks, using intermittent control. According to the author's knowledge, few studies have addressed the resilient control of FOCVNNs with ATVDs under aperiodic DoS attacks, which is the key factor that motivates this work.

Synchronization is essential in analyzing the dynamic behavior of NNs and is widely used in secure communication, image processing, artificial intelligence, and optics [20]. As research progressed, various synchronization theorems emerged, including lag synchronization [21], complete synchronization [22], anti-synchronization [23], projective synchronization [22], quasi-synchronization [24]. Among them, projective synchronization, achieved by proportionally synchronizing the slave system to the master system

using a scaling factor, is particularly effective in image processing, secure communication, and improving communication speed [25]. Achieving synchronization control has led to the development of various strategies, including sampled data control [26], intermittent control [27], sliding mode control [28], impulse control [29], quantized control [24], and event-triggered control (ETC) [30]. While time-triggered mechanisms (TTMs) remain prevalent, they often generate redundant computations and compromise efficiency due to bandwidth and resource limitations. Conversely, ETC transmits information only when predefined criteria are satisfied, effectively reducing trigger frequency and enhancing performance [31]. However, the conventional ETC method uses static thresholds, which cannot adjust to fluctuating network conditions, resulting in either redundant triggering or missed events, thus failing to effectively address the demands of complex network environments. In response, AETMs dynamically adjust thresholds according to system state variations, reducing communication frequency and gaining attention, as shown in [19]. Additionally, to mitigate DoS attack impacts, AETMs under DoS attacks have been explored. For example, ref. [13] proposed an AETM based on a switch observer, while [32] introduced a new AETM with adaptive laws linked to innovation errors. Although current AETMs reduce computational resource consumption and improve efficiency, further optimization is achievable. At present, the challenge of merging AETMs under DoS attacks with FOCVNNs for projective synchronization control remains underexplored, particularly with the inclusion of ATVDs, driving the focus of our work.

Synthesizing ideas from the prior discussion, this paper addresses the projective synchronization control of FOCVNNs with ATVDs, incorporating an AETM to mitigate aperiodic DoS attacks. The principal contributions are divided into three parts:

- (1) By focusing on the FOCVNNs model with ATVDs, a novel projective synchronization resilient control framework is established by leveraging the switching delay model to address the projective synchronization control problem under aperiodic DoS attacks, targeting their intermittent and unpredictable nature.
- (2) An innovative discrete AETM is designed using a sampled-data-based switching-adjusted threshold parameter (ATP) scheme to efficiently conserve network bandwidth, minimize data transmissions, and enhance the system's resilience to aperiodic DoS attacks. The mechanism responds more quickly, detects state changes earlier, and stabilizes the system, thereby mitigating the negative impacts of DoS attacks. As opposed to [13,33,34], the AETM proposed herein attains the lowest event-triggering frequency.
- (3) A new common LKFs, distinct from the switching LKFs method in [13,15,18], is formulated alongside the incorporation of fractional-order inequalities to establish the projective synchronization criterion. This approach effectively addresses aperiodic DoS attacks with reduced conservatism, fewer constraints, and lighter computational complexity.

Notations:  $\mathbb{N}$  refers to positive integers, and  $\mathbb{R}^n$  to  $n$ -dimensional vectors.  $\mathbb{C}^n$  and  $\mathbb{R}_+^{n \times m}$  correspond to  $n$ -dimensional complex vector space and  $n \times m$  positive-definite matrix.  $T$  is the transpose operator,  $i = \sqrt{-1}$  is the imaginary unit, and for  $x = x_p + ix_q$ , the modulus is  $|x| = \sqrt{x_p^2 + x_q^2}$  with  $\|x\| = \sqrt{x \cdot x^T}$ .  $\text{diag}\{\cdot\}$  is a diagonal matrix,  $\text{col}\{\cdot\}$  forms a column vector, and  $\text{sym}\{A\} = A + A^T$ .  $I$  and  $O$  are identity and zero matrices of corresponding dimensions.

## 2. Preliminaries

The model of FOCVNNs with ATVDs, aperiodic DoS attack model, AETM design, foundational definitions and lemmas are introduced in this section.

## 2.1. System Description

The FOCVNNs with ATVDs under consideration are as follows:

$$\begin{aligned} {}^C_{t_0}\mathfrak{D}_t^\alpha x(t) &= -Cx(t) + Af(x(t)) + Bf(x(t - b_1(t) - b_2(t))), \\ x(\theta) &= \phi(\theta), \forall \theta \in [t_0 - b, t_0]. \end{aligned} \quad (1)$$

where  $\alpha \in (0, 1)$ , and  $x(t) \in \mathbb{C}^n$  denotes the complex-valued neuron vector. The matrix  $C \in \mathbb{R}^{n \times n}$  represents a diagonal matrix. Meanwhile,  $A, B \in \mathbb{C}^{n \times n}$  correspond to the connection and delayed connection weight matrices.  $f(x(t)) \in \mathbb{C}^n$  describes the neuron function. The time-varying continuous functions  $b_1(t)$  and  $b_2(t)$  are constrained by  $0 \leq b_1(t) \leq b_1$ ,  $0 \leq b_2(t) \leq b_2$ , and  $b(t) = b_1(t) + b_2(t)$ , where  $b = b_1 + b_2$ , with  $b_1$  and  $b_2$  being positive constants.

Next, let us define the following variables:  $x(t) = x_p(t) + x_q(t)i$ ,  $f(x(\cdot)) = f_p(x_p(\cdot)) + f_q(x_q(\cdot))i$ ,  $\phi(\theta) = \phi_p(\theta) + \phi_q(\theta)i$ ,  $A = A_p + A_qi$ ,  $B = B_p + B_qi$ . Thus, the system (1) is reformulated as two equations:

$$\begin{aligned} {}^C_{t_0}\mathfrak{D}_t^\alpha x_p(t) &= -Cx_p(t) + A_p f_p(x_p(t)) - A_q f_q(x_q(t)) + B_p f_p(x_p(t - b(t))) \\ &\quad - B_q f_q(x_q(t - b(t))). \end{aligned} \quad (2)$$

$$\begin{aligned} {}^C_{t_0}\mathfrak{D}_t^\alpha x_q(t) &= -Cx_q(t) + A_q f_p(x_p(t)) + A_p f_q(x_q(t)) + B_q f_p(x_p(t - b(t))) \\ &\quad + B_p f_q(x_q(t - b(t))). \end{aligned} \quad (3)$$

Define  $\bar{x}(t) = \text{col}\{x_p(t), x_q(t)\}$ ,  $\bar{\phi}(\theta) = \text{col}\{\phi_p(\theta), \phi_q(\theta)\}$ ,  $\bar{C} = \text{diag}\{C, C\}$ ,  $\bar{\mathfrak{F}}(\bar{x}(\cdot)) = \text{col}\{f_p(x_p(\cdot)), f_q(x_q(\cdot))\}$ ,  $\bar{A} = \begin{bmatrix} A_p & -A_q \\ A_q & A_p \end{bmatrix}$ ,  $\bar{B} = \begin{bmatrix} B_p & -B_q \\ B_q & B_p \end{bmatrix}$ . As a result of (2) and (3), we obtain that

$$\begin{aligned} {}^C_{t_0}\mathfrak{D}_t^\alpha \bar{x}(t) &= -\bar{C}\bar{x}(t) + \bar{A}\bar{\mathfrak{F}}(\bar{x}(t)) + \bar{B}\bar{\mathfrak{F}}(\bar{x}(t - b(t))), \\ \bar{x}(\theta) &= \bar{\phi}(\theta), \forall \theta \in [t_0 - b, t_0]. \end{aligned} \quad (4)$$

Then the slave system corresponding to model (4) can be formulated as

$$\begin{aligned} {}^C_{t_0}\mathfrak{D}_t^\alpha \hat{x}(t) &= -\bar{C}\hat{x}(t) + \bar{A}\bar{\mathfrak{F}}(\hat{x}(t)) + \bar{B}\bar{\mathfrak{F}}(\hat{x}(t - b(t))) + u(t), \\ \hat{x}(\theta) &= \hat{\phi}(\theta), \forall \theta \in [t_0 - b, t_0], \end{aligned} \quad (5)$$

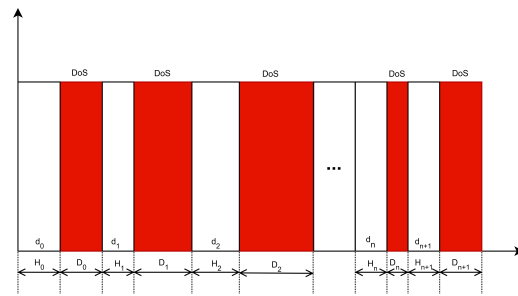
where  $u(t) = \text{col}\{u_p(t), u_q(t)\} \in \mathbb{R}^n$  represents the control input.

## 2.2. Aperiodic DoS Attacks Model

In Figure 1, the aperiodic DoS attacks are described as

$$\mathcal{I}_{\text{DoS}}(t) = \begin{cases} 0, & t \in \mathcal{H}_n, \\ 1, & t \in \mathcal{D}_n, \end{cases} \quad (6)$$

where  $\mathcal{H}_n \triangleq [h_n, h_n + d_n)$ ,  $(n \in \mathbb{N}, h_n > 0, h_0 \triangleq 0)$  denotes the  $n$ -th DoS attack sleep interval.  $\mathcal{D}_n \triangleq [h_n + d_n, h_{n+1})$  the corresponding attack interval. Let  $h_n$  be the start time and  $d_n$  the duration of the  $n$ -th sleep period. Furthermore,  $h_n$  and  $d_n$  fulfill the inequality:  $0 \leq h_n < h_n + d_n < h_{n+1} < \infty$ .



**Figure 1.** Aperiodic DoS attack signal.

**Assumption 1** ([13]). The DoS attack (6) is characterized by  $n \in \mathbb{R}$  and  $\Gamma_D \in \mathbb{R}$  such that  $n(t) \leq n + \frac{t}{\Gamma_D}$  for all  $t \in \mathbb{R}$ .

**Assumption 2** ([35]). For the time interval  $\mathcal{H}_n$ , a positive scalar  $b_{\max}$  exists such that

$$\sup_{n \in \mathbb{N}} \{h_n - h_{n-1} - d_{n-1}\} \leq b_{\max}.$$

For the time interval  $\mathcal{D}_n$ , a positive scalar  $d_{\min}$  satisfies the condition:

$$\inf_{n \in \mathbb{N}} \{d_n\} \geq d_{\min}$$

**Remark 1.** In practical scenarios, attackers often exploit the randomness and unpredictability of DoS attacks, rather than relying on simple periodic patterns, to maximize their destructive impact. Unlike periodic DoS attacks, aperiodic DoS attacks feature irregular time intervals and an absence of a fixed ratio between duration and interval, making them more complex and unpredictable, which poses challenges for traditional detection and defense mechanisms. Consequently, the modeling of aperiodic DoS attacks uses mathematical constraints to quantify the time intervals and durations of attacks, enabling system designers to predict their maximum impact and proactively develop strategies for communication recovery and data retransmission. This method is highly adaptable to scenarios with significant uncertainty in attack timing and frequency, offering a solid theoretical basis and practical response strategy. Therefore, studying aperiodic DoS attacks holds significant practical value and is a central focus of this paper.

**Remark 2.** The validity and importance of Assumptions 1 and 2 have been well demonstrated in many existing studies on networked control systems subject to DoS attacks, such as [13,17,35,36]. Assumption 1 imposes an upper bound on the frequency of aperiodic DoS attacks. This condition allows the attacks to occur irregularly and unpredictably, while excluding the unrealistic case where infinitely many attacks occur within a finite time interval. In practice, this is reasonable since malicious attackers are usually constrained by limited energy, bandwidth, and computational resources. Assumption 2 further imposes boundedness conditions on the attack-related time intervals. Specifically, the condition  $\inf_{n \in \mathbb{N}} d_n \geq d_{\min}$  guarantees that the duration of each attack-related interval is not smaller than a prescribed positive constant, while  $\sup_{n \in \mathbb{N}} h_n - h_{n-1} - d_{n-1} \leq b_{\max}$  ensures that the interval between two consecutive attack-related instants is uniformly bounded. These conditions preserve the intermittent and aperiodic characteristics of DoS attacks, while avoiding pathological cases such as infinitely fast switching or ill-defined attack sequences. Therefore, Assumptions 1 and 2 provide a reasonable and standard basis for constructing the switching delay model and for carrying out the subsequent Lyapunov–Krasovskii stability analysis.

### 2.3. Event-Triggered Controller Design

Achieving rapid recovery of resilient control performance post-DoS attack is a core challenge in ETC design. To address this, the following novel AETM is proposed:

$$t_{k,n+1}h = \{j_k h \text{ satisfying (8)} | j_k h \in \mathcal{H}_n\} \cup \{h_{n+1}\}, \quad (7)$$

with

$$\psi^T(j_k h) \Phi \psi(j_k h) \geq \varphi(t_{k,n}h) e^T(j_k h) \Phi e(j_k h). \quad (8)$$

In this context,  $t_{k,n}h$  refers to the  $k$ -th triggering time within the  $n$ -th sleeping interval, while  $j_k h = t_{k,n}h + jh$  for  $j = 0, 1, \dots$ . The function  $\psi(j_k h) = e(j_k h) - e(t_{k,n}h)$ , where  $e(j_k h)$  represents the current sampling state and  $e(t_{k,n}h)$  denotes the previous trigger state.  $\Phi$  is the weighting matrix to be determined later.

The ATP  $\varphi(t_{k,n}h) \in [\varphi_m, \varphi_M]$ ,  $\varphi(0) = \varphi_0$ , is updated in accordance with the bias of the error states.

$$\varphi(j_k h) = \varphi(t_{k,n}h) \begin{cases} 1 + \frac{2\rho_1}{\pi} \arctan\left(\frac{\rho_2 \mathcal{J}(j_k h)}{\mathcal{L}(j_k h)}\right), & \mathcal{T}(j_k h) \leq 0, \\ 1 - \frac{2\rho_1}{\pi} \arctan\left(\frac{\rho_2 \mathcal{J}(j_k h)}{\mathcal{L}(j_k h)}\right), & \mathcal{T}(j_k h) > 0, \end{cases} \quad (9)$$

with  $0 < \rho_1 < 2, \rho_2 > 0$ , and

$$\begin{aligned} \mathcal{J}(j_k h) &= \|e(j_k h) - e(t_{k,n}h)\|, \mathcal{L}(j_k h) = \|e(j_k h) + e(t_{k,n}h)\|, \\ \mathcal{T}(j_k h) &= \|e(j_k h)\| - \|e(t_{k,n}h)\|, \end{aligned}$$

The sequence  $t_{k,n}h$  can be obtained through an iterative process based on the AETM (7). It is important to observe that  $t_{1,1}h = h_1$ , and  $k \in \{1, \dots, k(n)\}$ , where  $k(n)$  is defined as  $\sup\{k \in \mathbb{N}^+ \mid t_{k,n}h \leq h_n + d_n\}$ .

**Remark 3.** The adjustment of the adaptive threshold parameter  $\varphi(j_k h)$  depends on two factors: the variation trend of the state two-norm between the current and previous triggering instants, and the state error measured in the Euclidean metric. If  $\mathcal{T}(j_k h) \leq 0$ , the system state may exhibit a tendency toward instability. In this case,  $\varphi(j_k h)$  is decreased to increase the frequency of data transmissions and strengthen the control action, thereby accelerating system convergence, i.e.,  $\varphi(j_k h) < \varphi(t_{k,n}h)$ . Conversely, when the system state becomes stable,  $\varphi(j_k h)$  is increased to reduce the frequency of data transmissions and conserve network bandwidth.

Therefore, the two-way regulation mechanism in (9) is constructed to balance control performance and communication burden. Compared with the fixed-threshold event-triggered mechanisms in [6,17,35], the proposed adaptive event-triggered mechanism (7) offers better adaptability. In addition, it achieves a lower triggering frequency than the adaptive event-triggered mechanisms in [13,33,34], leading to significant savings in network resources. A detailed comparison is provided in Section 4. Notably, when  $\varphi(t_{k,n}h) = 0$ , the scheme reduces to a time-triggered mechanism; when  $\varphi(t_{k,n}h)$  is constant, it operates as a standard event-triggered mechanism.

**Remark 4.** Optimizing event-triggered thresholds also hinges on parameter tuning. Compared with the AETM presented in [13], AETM (7) incorporates two adjustable parameters that enhance control flexibility:  $\rho_1$ , which regulates fluctuation amplitude (a larger  $\rho_1$  results in smaller fluctuations), and  $\rho_2$ , which governs convergence speed (a larger  $\rho_2$  leads to faster convergence). Additionally, these parameters allow for precise control of trigger frequency, contributing to enhanced control performance. Although [33,37] restricted  $\rho_1$  to positive values, excessively large  $\rho_1$  can push

$\varphi(t_{k,n}h)$  below zero, breaching the allowable interval  $(0, 1)$ . As a result, ensuring  $0 < \rho_1 < 2$  becomes necessary.

**Remark 5.** The AETM in [13,33] updates  $\delta(t_k h)$  based on the state modulus at consecutive triggering instants, continuing until the next triggering moment. While this method reduces network resource usage compared with static ETM, it lacks flexibility. To address this, this paper introduces a sampled-data-based adaptive thresholding method that updates the threshold and evaluates the trigger condition in real time, achieving faster convergence and fewer triggers.

**Remark 6.** It is essential to emphasize that the mechanism (9) presented in this paper is a sampled-data-based AETM for countering aperiodic DoS attacks. Specifically, ETC occurs only at sampling instants within each sleep interval. Consequently, the event-triggered interval not only exceeds or equals one sampling period  $h$  but also satisfies  $t_{k,n}h - t_{k,n-1}h \geq h$ , which effectively prevents the Zeno phenomenon.

The projective synchronization error signal is set as  $e(t) = \text{col}\{e_p(t), e_q(t)\} = \hat{x}(t) - \bar{\delta}\bar{x}(t) \in \mathbb{R}^{2n}$ , where  $\bar{\delta}$  is the projective coefficient. Then, one can get

$${}^C_{t_0} \mathcal{D}_t^\alpha e(t) = -\bar{C}e(t) + \bar{A}\mathfrak{F}(t) + \bar{B}\mathfrak{F}(e(t - \mathfrak{b}(t))) + u(t), \quad (10)$$

where  $\mathfrak{F}(e(\cdot)) = \mathfrak{F}(\hat{x}(t)) + \mathfrak{F}(\bar{x}(\cdot))$ , satisfying:

$$l_i^- \leq \frac{\mathfrak{F}_i(e_1) - \mathfrak{F}_i(e_2)}{e_1 - e_2} \leq l_i^+, \quad (11)$$

for  $e_1 \neq e_2$ , and  $l_i^-, l_i^+ \in \mathbb{R}$  for  $i = 1, 2, \dots, n$ .

Define  $\mathcal{U}_{k,n} = [t_{k,n}h, t_{k+1,n}h)$  as the hold time interval. It is assumed that the switching controller is

$$u(t) = \begin{cases} \mathcal{K}_1 e(t_{k,n}h), & t \in \mathcal{U}_{k,n} \cap \mathcal{H}_n \\ 0, & t \in \mathcal{D}_n \end{cases} \quad (12)$$

with  $\mathcal{K}_1 = \begin{bmatrix} K_{1p} & -K_{1q} \\ K_{1q} & K_{1p} \end{bmatrix}$ ,  $K_{1p} \in \mathbb{R}^{n \times n}$  and  $K_{1q} \in \mathbb{R}^{n \times n}$ . Split  $\mathcal{U}_{k,n}$  into subintervals

$$\mathcal{U}_{k,n} = \bigcup_{l=0}^{l_M} \mathcal{U}_l,$$

where  $\mathcal{U}_l = [i_l h + \gamma_{i_l}, i_l h + h + \gamma_{i_{l+1}})$ , ( $l = 0, 1, \dots, l_M$ ), with  $i_l h = t_{k,n}h + lh$ ,  $l_M = t_{k+1,n} - t_{k,n} - 1$ , and

$$\gamma_{i_l} = \begin{cases} d_{t_{k,n}}, & l = 0, 1, \dots, l_M - 1, \\ d_{t_{k+1,n}}, & l = l_M. \end{cases}$$

Defining  $\mu(t) = t - i_l h, \forall t \in \mathcal{U}_l$ , one has

$$\dot{\mu}(t) = 1, 0 \leq d_{t_{k,n}} \leq \mu(t) \leq \mu = h + \bar{d} < 2h.$$

According to the definition of  $\psi(j_k h)$ , there exist

$$e(t_{k,n}h) = e(t - \mu(t)) - \psi(i_l h), t \in \mathcal{U}_l \cap \mathcal{H}_n. \quad (13)$$

Hence, (8) can be reformulated as:

$$\psi(i_l h)^T \Phi \psi(i_l h) < \varphi_M e(t - \mu(t))^T \Phi e(t - \mu(t)). \quad (14)$$

Synthesizing (12), (13) and system (10) leads to

$${}^C_{t_0}\mathfrak{D}_t^\alpha e(t) = \begin{cases} -\bar{C}e(t) + \bar{A}\mathfrak{F}(e(t)) + \bar{B}\mathfrak{F}(e(t-b(t))) \\ +\mathcal{K}_1(e(t-\mu(t)) - \psi(i_l h)), & t \in \mathcal{U}_{k,n} \cap \mathcal{H}_n, \\ -\bar{C}e(t) + \bar{A}\mathfrak{F}(e(t)) + \bar{B}\mathfrak{F}(e(t-b(t))), & t \in \mathcal{D}_n. \end{cases} \quad (15)$$

To support the discussion that follows, important definitions are given below, supplemented with the necessary lemmas.

**Definition 1** ([38]). The fractional-order integration of  $x(t) \in \mathbb{R}^n$  is expressed as:

$${}_{t_0}\mathfrak{J}_t^\alpha x(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^t (t-s)^{\alpha-1} x(s) ds, t > 0,$$

with  $0 < \alpha < 1$ .  $\Gamma(\alpha) = \int_0^\infty e^{-t} t^{\alpha-1} dt$  is the gamma function.

**Definition 2** ([38]). The Caputo fractional-order derivative of  $x(t) \in \mathbb{R}^n$  is defined as:

$${}^C_{t_0}\mathfrak{D}_t^\alpha x(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^t (t-s)^{n-\alpha-1} x^n(s) ds,$$

where  $(n-1) < \alpha \leq n, n \in [\alpha] + 1$ . When  $\alpha \in (0, 1)$ ,

$${}^C_{t_0}\mathfrak{D}_t^\alpha x(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^t (t-s)^{-\alpha} x'(s) ds.$$

**Definition 3** ([39]). The definition of the two-parameter Mittag-Leffler function is:

$$E_{\alpha,\beta}(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(\alpha k + \beta)},$$

where  $\alpha > 0, \beta > 0$ , and  $x \in \mathbb{C}$ . For  $\beta = 1$ , its one-parameter form is shown by

$$E_\alpha(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(\alpha k + 1)} = E_{\alpha,1}(x).$$

**Lemma 1** ([40]). For  $t \geq t_0$ , let  $x(t) \in \mathbb{R}^n$  be a differentiable function, and  $P \in \mathbb{R}_+^{n \times n}$ . The inequality given below holds:

$${}^C_{t_0}\mathfrak{D}_t^\alpha (x^T(t) P x(t)) \leq 2x^T(t) P ({}^C_{t_0}\mathfrak{D}_t^\alpha x(t)).$$

**Lemma 2** ([39]). Let  $0 < \alpha < 1$ , and  $V(t)$  is a continuously differentiable function defined in  $[t_0, t_1]$ , if there exists a constant  $\beta$  such that

$${}^C_{t_0}\mathfrak{D}_t^\alpha V(t) \leq \beta V(t)$$

then

$$V(t) \leq V(t_0) E_\alpha(\beta(t-t_0)^\alpha).$$

**Lemma 3** ([39]). For  $0 < \alpha < 1$ ,  $E_\alpha[\mu(t-t_0)^\alpha]$  remains nonnegative and satisfies the following properties:

- (1)  $E_\alpha[\mu(t-t_0)^\alpha]$  is monotonically nonincreasing and  $E_\alpha[\mu(t-t_0)^\alpha] \in [0, 1]$  for  $t \geq t_0$  when  $\mu \leq 0$ .

- (2)  $E_\alpha[\mu(t-t_0)^\alpha]$  is monotonically nondecreasing and  $E_\alpha[\mu(t-t_0)^\alpha] \geq 1$  for  $t \geq t_0$  when  $\mu \geq 0$ .

**Lemma 4** ([41]). For  $0 < \alpha < 1, t \in \mathbb{R}_+$ , we have

$$\lim_{t \rightarrow +\infty} E_\alpha(t) \leq \lim_{t \rightarrow +\infty} \frac{1}{\alpha} e^{t^{\frac{1}{\alpha}}}.$$

**Lemma 5** ([42]). Suppose that  $Q \in \mathbb{R}_+^{n \times n}$ , for any  $x(t) \in \mathbb{R}^n$ . Then the following inequalities hold:

$${}_{t_0}\mathcal{I}_t^\alpha \left( \left( {}^C_{t_0}\mathfrak{D}_t^\alpha x(t) \right)^T Q \left( {}^C_{t_0}\mathfrak{D}_t^\alpha x(t) \right) \right) \geq \frac{\Gamma(\alpha+1)}{(t-t_0)^\alpha} \Lambda_1^T Q \Lambda_1 + \frac{3}{\Gamma(\alpha+1)(t-t_0)^\alpha} \Lambda_2^T Q \Lambda_2,$$

where

$$\begin{aligned} \Lambda_1 &= x(t) - x(t_0), \\ \Lambda_2 &= \frac{\Gamma(2\alpha+1)}{(t-t_0)^\alpha} ({}_{t_0}\mathcal{I}_t^\alpha x(t)) - \Gamma(\alpha+1)x(t) - \left( \frac{\Gamma(2\alpha+1)}{\Gamma(\alpha+1)} - \Gamma(\alpha+1) \right) x(t_0). \end{aligned}$$

**Lemma 6** ([6]). Consider scalars  $b > 0$  and  $h > 0$  such that  $b > h, R \in \mathbb{R}_+^{n \times n}$ , and  $x(t) \in \mathbb{R}^n$ , with  $t_k$  being the trigger time and  $0 < t_{k+1} - t_k = h(t) \leq h$ , and  $\lim_{k \rightarrow \infty} t_k = \infty$ . Then, the following inequality is obtained:

$$\begin{aligned} & {}^C_{t_0}\mathfrak{D}_t^\alpha \left\{ (b - (t - t_k)) \int_t^{t_{k+1}} \left( {}^C_{r-h(r)}\mathfrak{D}_r^{1-\alpha} x(r) \right)^T R \left( {}^C_{r-h(r)}\mathfrak{D}_r^{1-\alpha} x(r) \right) dr \right\} \\ & \leq -\frac{(b-h)\Gamma(2-\alpha)}{h^{1-\alpha}} \Theta_1^T Q \Theta_1 - \frac{3}{\Gamma(2-\alpha)h^{1-\alpha}} \Theta_2^T Q \Theta_2, \end{aligned}$$

where  $\Theta_1 = x(t) - x(t-h(t))$  and

$$\Theta_2 = \frac{\Gamma(3-2\alpha)}{h(t)^{1-\alpha}} \left( {}_{t-h(t)}\mathcal{I}_t^{1-\alpha} x(t) \right) - \Gamma(2-\alpha)x(t) - \left( \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)} - \Gamma(2-\alpha) \right) x(t-h(t)).$$

**Definition 4** ([43]). The master system (4) and slave system (5) achieve projective synchronization if

$$\lim_{t \rightarrow \infty} \|\hat{x}(t) - \tilde{\partial} \bar{x}(t)\| = 0,$$

where  $\tilde{\partial} \in \mathbb{R}$  is the projective coefficient.

### 3. Main Results

This section establishes the main synchronization criteria for the error system (15), building upon the preliminaries and system transformation introduced in the previous section. Theorem 1 derives sufficient LMI-based conditions under which the considered FOCVNNs achieve projective synchronization via the proposed adaptive event-triggered mechanism and switching controller. Furthermore, Corollary 1 presents the corresponding anti-synchronization criterion, which can be regarded as a special case of the projective synchronization result.

To simplify the subsequent analysis and streamline the notation in the remainder of the paper, the following vectors are introduced, and Appendix A is provided, where  ${}^C_{t_0}\mathfrak{D}_t^\alpha$  is abbreviated as  $\mathfrak{D}^\alpha$ .

$$\begin{aligned}
E_{1i} &= \text{col}\{0, \dots, 0, I, 0, \dots, 0\}, (i = 1, 2, \dots, 27), \\
E_{2i} &= \text{col}\{0, \dots, 0, I, 0, \dots, 0\}, (i = 1, 2, \dots, 26), \\
\tilde{\xi}_1(t) &= \text{col}\{\tilde{\xi}_2(t), \psi(i_1 h)\}, \tilde{\xi}_2(t) = \text{col}\{\tilde{\xi}_{11}(t), \tilde{\xi}_{12}(t), \tilde{\xi}_{13}(t)\}, \\
\tilde{\xi}_{11}(t) &= \text{col}\{e(t), f(e(t)), e(t - b(t)), f(e(t - b(t))), e(t - b), f(e(t - b)), e(t - b_1(t)), e(t - b_1), \\
&\quad e(t - b_2(t)), e(t - b_2), e(t - b_2 - b_1(t)), e(t - b_1 - b_2(t)), e(t - \mu(t)), e(t - \mu)\}, \\
\tilde{\xi}_{12}(t) &= \text{col}\left\{\frac{1}{b_1(t)^{1-\alpha}} \left(t - b_1(t)\right) \mathcal{I}_t^{1-\alpha} e(t), \frac{1}{b_1^{1-\alpha}} \left(t - b_1\right) \mathcal{I}_t^{1-\alpha} e(t), \frac{1}{b_2(t)^{1-\alpha}} \left(t - b_2(t)\right) \mathcal{I}_t^{1-\alpha} e(t), \right. \\
&\quad \frac{1}{b_2^{1-\alpha}} \left(t - b_2\right) \mathcal{I}_t^{1-\alpha} e(t), \frac{1}{(b_1 + b_2(t))^{1-\alpha}} \left(t - b_1 - b_2(t)\right) \mathcal{I}_t^{1-\alpha} e(t), \\
&\quad \frac{1}{(b_2 + b_1(t))^{1-\alpha}} \left(t - b_2 - b_1(t)\right) \mathcal{I}_t^{1-\alpha} e(t), \frac{1}{b(t)^{1-\alpha}} \left(t - b(t)\right) \mathcal{I}_t^{1-\alpha} e(t), \\
&\quad \frac{1}{b(t)^{1-\alpha}} \left(t - b(t)\right) \mathcal{I}_t^{1-\alpha} \mathfrak{F}(e(t)), \frac{1}{b^{1-\alpha}} \left(t - b\right) \mathcal{I}_t^{1-\alpha} e(t), \frac{1}{b^{1-\alpha}} \left(t - b\right) \mathcal{I}_t^{1-\alpha} \mathfrak{F}(e(t)), \\
&\quad \left. \frac{1}{\mu(t)^{1-\alpha}} \left(t - \mu(t)\right) \mathcal{I}_t^{1-\alpha} e(t), \frac{1}{\mu^{1-\alpha}} \left(t - \mu\right) \mathcal{I}_t^{1-\alpha} e(t)\right\}, \\
\tilde{\xi}_{13}(t) &= \text{col}\left\{\int_{t-b_1(t)}^t \int_s^t G_1(r) dr ds, \int_{t-b_1}^t \int_s^t G_2(r) dr ds, \int_{t-b_2(t)}^t \int_s^t G_3(r) dr ds, \right. \\
&\quad \int_{t-b_2}^t \int_s^t G_4(r) dr ds, \int_{t-b_1-b_2(t)}^t \int_s^t G_5(r) dr ds, \int_{t-b_2-b_1(t)}^t \int_s^t G_6(r) dr ds, \\
&\quad \int_{t-b(t)}^t \int_s^t G_7(r) dr ds, \int_{t-b(t)}^t \int_s^t G_7(r) dr ds, \int_{t-b}^t \int_s^t G_8(r) dr ds, \\
&\quad \left. \int_{t-b}^t \int_s^t G_8(r) dr ds, \int_{t-\mu(t)}^t \int_s^t G_9(r) dr ds, \int_{t-\mu}^t \int_s^t G_{10}(r) dr ds\right\}.
\end{aligned}$$

**Theorem 1.** Consider the event-triggered parameter  $\varphi_M$ , fractional order  $\alpha$ , and DoS attack parameters  $\beta_1 > 0$ ,  $\beta_2 > 0$ ,  $\Gamma \in \mathbb{R}$ ,  $d_{\min} \in \mathbb{R}$ ,  $b_{\max} \in \mathbb{R}$ , and scalars  $b_1 > 0$ ,  $b_2 > 0$ ,  $\mu > 0$ ,  $\mathfrak{D} \in \mathbb{R}$ , if there exist some matrices  $P \in \mathbb{R}_+^{n \times n}$ ,  $Q_i \in \mathbb{R}_+^{n \times n}$  ( $i = 1, 2, \dots, 4$ ),  $R_1 \in \mathbb{R}_+^{n \times n}$ ,  $R_2 \in \mathbb{R}_+^{n \times n}$ ,  $S_1 \in \mathbb{R}_+^{2n \times 2n}$ ,  $S_2 \in \mathbb{R}_+^{2n \times 2n}$ ,  $T_1 \in \mathbb{R}_+^{n \times n}$ ,  $T_2 \in \mathbb{R}_+^{n \times n}$ , diagonal matrices  $M_1 \in \mathbb{R}_+^{n \times n}$ ,  $M_2 \in \mathbb{R}_+^{n \times n}$ ,  $M_3 \in \mathbb{R}_+^{n \times n}$ , such that the following LMIs holds:

$$\Xi_1 < 0, \quad (16)$$

$$\Xi_2 < 0, \quad (17)$$

$$\lambda = \frac{-1}{\Gamma_D} \left( \ln E_\alpha(-\beta_1 d_{\min}^\alpha) - \ln \alpha + (\beta_2 b_{\max}^\alpha)^{\frac{1}{\alpha}} \right) > 0 \quad (18)$$

Then, the error system (15) is GES under aperiodic DoS attacks, with  $\mathcal{K}_1 = P^{-1}F$ .

**Proof.** See Appendix B.  $\square$

**Remark 7.** To reduce the conservatism of the LKFs, two methods are typically used: one is to deflate the LKFs using an appropriate integral inequality, and this paper adopts the improved WBFOII method from [44]; the other is to construct an LKFs that incorporates more system state information, thereby reducing conservatism in the theoretical results. In this paper, LKFs in  $V_2(t)$ ,  $V_3(t)$ ,  $V_4(t)$  are constructed by adapting the method from [6,9] and fully considering the ATVD information. For example,  $e(t - b_1(t))$ ,  $e(t - b_1)$ ,  $e(t - b_2(t))$ ,  $e(t - b_2)$ ,  $e(t - b_1 - b_2(t))$ ,  $e(t - b_2 - b_1(t))$ ,  $e(t - b(t))$ , and  $e(t - b)$ .

**Remark 8.** For the DoS attack (6), most existing studies, such as [13,32,36], use switched LKFs to analyze the system's dynamic behavior, assuming certain inequality relationships between the matrices in the LKFs, such as  $P_1 \leq w_1 P_2$ ,  $P_1 \leq w_1 e^{(\alpha_1 + \alpha_2)\tau} P_2$ . However, this approach introduces strong constraints, which become more pronounced when additional system state information is incorporated into the LKFs, leading to increased conservatism. Therefore, this paper constructs a common LKFs to reduce these constraints and minimize the conservatism of the theoretical results.

**Remark 9.** For the projective synchronization error signal  $e(t) = \hat{x}(t) - \bar{\partial}\bar{x}(t)$ :

- (1) When  $\bar{\partial} = 1$ , the synchronization between system (4) and system (5) corresponds to complete synchronization.
- (2) When  $\bar{\partial} = -1$ , the synchronization becomes anti-synchronization.
- (3) When  $\bar{\partial}$  is defined as a matrix, the synchronization transitions to generalized synchronization.

Next, assuming that  $\bar{\partial} = -1$ , the projection synchronization error signal can be changed to an anti-synchronization error signal, i.e.,  $e(t) = \text{col}\{e_p(t), e_q(t)\} = \hat{x}(t) + \bar{x}(t) \in \mathbb{R}^{2n}$ . Consequently, we can obtain the following results.

**Corollary 1.** Consider the event-triggered parameter  $\varphi_M$ , fractional order  $\alpha$ , and DoS attack parameters  $\beta_1 > 0$ ,  $\beta_2 > 0$ ,  $\Gamma \in \mathbb{R}$ ,  $d_{\min} \in \mathbb{R}$ ,  $b_{\max} \in \mathbb{R}$ , and scalars  $b_1 > 0$ ,  $b_2 > 0$ ,  $\mu > 0$ , if there exist some matrices  $P \in \mathbb{R}_+^{n \times n}$ ,  $Q_i \in \mathbb{R}_+^{n \times n}$  ( $i = 1, 2, \dots, 4$ ),  $R_1 \in \mathbb{R}_+^{n \times n}$ ,  $R_2 \in \mathbb{R}_+^{n \times n}$ ,  $S_1 \in \mathbb{R}_+^{2n \times 2n}$ ,  $S_2 \in \mathbb{R}_+^{2n \times 2n}$ ,  $T_1 \in \mathbb{R}_+^{n \times n}$ ,  $T_2 \in \mathbb{R}_+^{n \times n}$ , diagonal matrices  $M_1 \in \mathbb{R}_+^{n \times n}$ ,  $M_2 \in \mathbb{R}_+^{n \times n}$ ,  $M_3 \in \mathbb{R}_+^{n \times n}$ , such that the following LMIs holds:

$$\Xi_1 < 0, \quad (19)$$

$$\Xi_2 < 0, \quad (20)$$

$$\lambda = \frac{-1}{\Gamma_D} \left( \ln E_\alpha(-\beta_1 d_{\min}^\alpha) - \ln \alpha + (\beta_2 b_{\max}^\alpha)^{\frac{1}{\alpha}} \right) > 0 \quad (21)$$

Then, the error system (15) with  $K_1 = P^{-1}F$  is GES under aperiodic DoS attacks.

**Proof.** As the LKFs and proof of this corollary are similar to Theorem 1, the proof is omitted for brevity.  $\square$

## 4. Numerical Examples

This section presents numerical simulations of the FOCVNNs through several examples to verify the accuracy and validity of the theoretical results.

**Example 1.** Consider the following FOCVNNs with ATVDS,

$$C = \begin{bmatrix} 0.94 & 0 \\ 0 & 0.92 \end{bmatrix}, A = \begin{bmatrix} 0.9 + 0.9i & -0.9 + 0.8i \\ 0.8 + 0.4i & 0.2 + 0.7i \end{bmatrix}, B = \begin{bmatrix} -0.7 + 0.4i & -0.5 - 0.6i \\ -0.6 - 0.5i & 0.5 + 0.6i \end{bmatrix}.$$

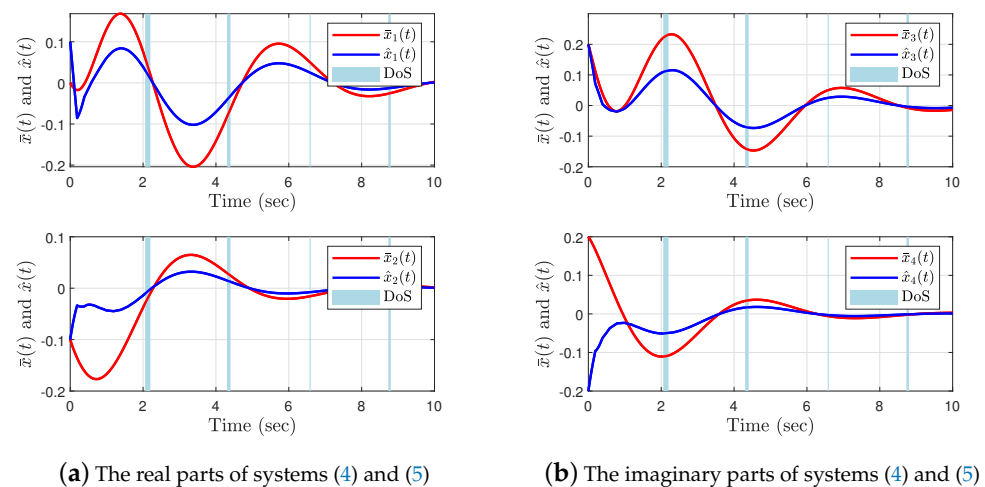
The activation function is  $f_j(x_j(t)) = \tanh((x_j(t)))$  ( $j = 1, 2$ ).  $L_1 = \text{diag}\{1, 1\}$ ,  $L_2 = \text{diag}\{0.5, 0.5\}$ . Specify the following parameters:  $h = 0.2$ ,  $\varphi_M = 0.1$ .  $\mu = 0.05$ ,  $\tau = 0.15$ ,  $\beta_1 = 0.75$ ,  $\beta_2 = 0.5$ .  $b_2 = 0.2$ ,  $b_1 = 2$ ,  $\rho_1 = 0.2$ ,  $\rho_2 = 0.8$ ,  $\bar{\partial} = 0.5$ . The ATVDS functions are taken as  $b_1(t) = 0.1|\sin(0.1t)|$ ,  $b_2(t) = 0.2|\cos(0.2t)|$  with  $b_1 = 0.1$ ,  $b_2 = 0.2$ .

Upon solving the LMIs in Theorem 1, the matrix of controller gains and event-triggered weights is determined as follows:

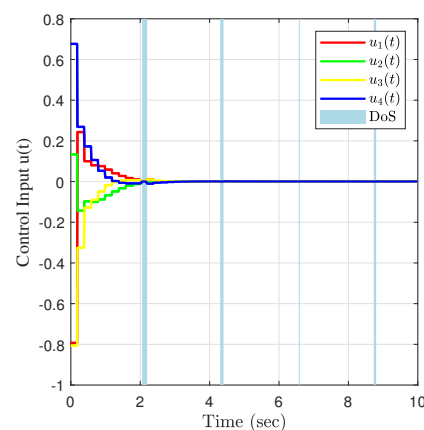
$$\mathcal{K}_1 = \begin{bmatrix} -4.6596 & 1.2731 & 0.0243 & 0.8863 \\ 1.0541 & -1.6893 & -0.6595 & -0.0320 \\ -0.0243 & -0.8862 & -4.6596 & 1.2731 \\ 0.6595 & 0.0320 & 1.0541 & -1.6893 \end{bmatrix}$$

$$\Phi = \begin{bmatrix} 48.4639 & -1.5081 & -0.0000 & -2.0473 \\ -1.5081 & 55.5144 & 2.0473 & 0.0000 \\ -0.0000 & 2.0473 & 48.4639 & -1.5081 \\ -2.0473 & 0.0000 & -1.5081 & 55.5144 \end{bmatrix}$$

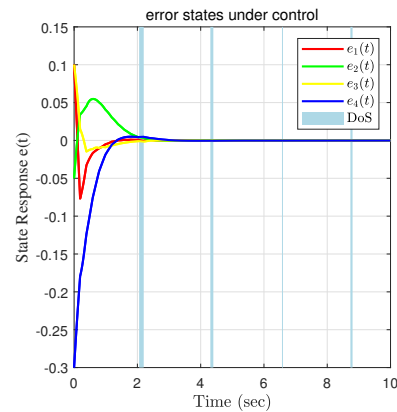
Figure 2 displays the trajectories of the real and imaginary parts of the projective synchronization states for the FOCVNNs under AETMs and an aperiodic DoS attack, with initial values given as  $\bar{x}(t_0) = [0; -0.1; 0.2; 0.2]^T$  and  $\hat{x}(t_0) = [0.1; -0.1; -0.2; -0.2]^T$ . The state trajectories of the control input  $u(t)$ , comprising real and imaginary components, are shown in Figure 3 under the impact of the aperiodic DoS attack. Figure 4 illustrates that the state responses of the projective synchronization error model (15) converge to zero, confirming the projective synchronization of systems (4) and (5). Using the improved AETM (7) along with the defined weighting and gain matrices, the trends of release moments, release intervals, and the adaptive tuning threshold parameter  $\varphi(t_{k,n}h)$  are illustrated in Figure 5. These findings highlight the effectiveness of the proposed AETM in addressing aperiodic DoS attacks and achieving master-slave system synchronization. It is apparent from Figure 5 and Table 1 that the proposed AETM greatly reduces the trigger frequency, validating its superior performance.



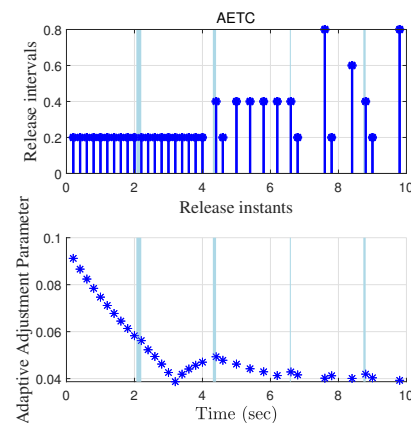
**Figure 2.** State responses of systems (4) and (5) for Example 1.



**Figure 3.** Control input under aperiodic DoS attacks for Example 1.



**Figure 4.** State responses of the error system for Example 1.



**Figure 5.** Release instants and intervals. The evolution of triggering threshold  $\varphi(t_{k,n}h)$ . The shaded area represents the DoS attacks, and the symbols \* denote the triggering threshold at each discrete time instant.

**Table 1.** Signal Volume with Various AETMs.

| AETM    | [33] | [13] | [34] | (7) |
|---------|------|------|------|-----|
| Amounts | 40   | 49   | 33   | 31  |

**Example 2.** Consider the four-dimensional FOCVNNs with ATVD specified below,

$$C = \text{diag}(0.5, 0.2, 0.4, 0.3),$$

$$A = \begin{bmatrix} -0.1 - 0.5i & -0.4 + 0.2i & -0.1 & -0.1 - 0.1i \\ -0.1 + 0.1i & -0.1 + 0.1i & 0.1 & 0.1i \\ 0.1i & 0.1 + 0.2i & 0.2 & -0.1 - 0.1i \\ 0.1i & 0.2i & 0.1 + 0.1i & -0.1 - 0.1i \end{bmatrix}$$

$$B = \begin{bmatrix} -0.1 - 0.4i & -0.4 - 0.1i & -0.1 + 0.2i & 0.2 - 0.1i \\ -0.1 - 0.1i & -0.1 & 0.2 + 0.2i & 0.1 + 0.1i \\ 0.1 & -0.1 + 0.1i & 0.1 & -0.1i \\ 0.1 - 0.1i & -0.1 + 0.2i & 0.1 + 0.3i & 0.1i \end{bmatrix}.$$

The activation function is viewed as  $f_j(x_j(t)) = 0.5(|x_j(t) + 1| - |x_j(t) - 1|)$  ( $j = 1, 2$ ),  $\rho_1 = 0.8, \rho_2 = 1.5, \vartheta = 0.5$ .  $L_1 = \text{diag}\{1, 1, 1, 1\}$ ,  $L_2 = \text{diag}\{0.5, 0.5, 0.5, 0.5\}$ . The ATVDs functions are taken as  $b_1(t) = 0.1 + 0.1\sin(0.1t)$ ,  $b_2(t) = 0.1 + 0.2\cos(0.2t)$  with  $b_1 = 0.2, b_2 = 0.3$ . For the other parameters, the values are kept consistent with Example 1.

Through the solution of the LMIs in Theorem 1, the following matrix of controller gains and event-triggered weights is derived.

$$\mathcal{K}_1 = \begin{bmatrix} -1.1592 & 0.2539 & -0.1196 & -0.2556 & -0.0133 & 0.3531 & 0.0770 & -0.0061 \\ 0.1933 & -0.7586 & -0.0256 & -0.3737 & -0.2968 & 0.0366 & 0.2400 & -0.1855 \\ -0.1016 & -0.0074 & -0.8944 & -0.1690 & -0.0152 & -0.1914 & -0.0108 & -0.4496 \\ -0.2048 & -0.3637 & -0.2319 & -1.0826 & -0.0078 & 0.2292 & 0.5356 & -0.0120 \\ 0.0133 & -0.3531 & -0.0770 & 0.0061 & -1.1592 & 0.2539 & -0.1196 & -0.2556 \\ 0.2968 & -0.0366 & -0.2400 & 0.1855 & 0.1933 & -0.7586 & -0.0256 & -0.3737 \\ 0.0152 & 0.1914 & 0.0108 & 0.4496 & -0.1016 & -0.0074 & -0.8944 & -0.1690 \\ 0.0078 & -0.2292 & -0.5356 & 0.0120 & -0.2048 & -0.3637 & -0.2319 & -1.0826 \end{bmatrix},$$

$$\Phi = \begin{bmatrix} 68.8318 & -6.5564 & 2.3921 & 3.9588 & -0.0000 & -3.9009 & -1.6889 & 1.3252 \\ -6.5564 & 72.4632 & 4.8296 & 3.6473 & 3.9009 & 0.0000 & -0.8767 & 3.4118 \\ 2.3921 & 4.8296 & 85.1305 & 4.5125 & 1.6889 & 0.8767 & -0.0000 & 11.8019 \\ 3.9588 & 3.6473 & 4.5125 & 75.9367 & -1.3252 & -3.4118 & -11.8019 & 0.0000 \\ -0.0000 & 3.9009 & 1.6889 & -1.3252 & 68.8318 & -6.5564 & 2.3921 & 3.9588 \\ -3.9009 & 0.0000 & 0.8767 & -3.4118 & -6.5564 & 72.4632 & 4.8296 & 3.6473 \\ -1.6889 & -0.8767 & -0.0000 & -11.8019 & 2.3921 & 4.8296 & 85.1305 & 4.5125 \\ 1.3252 & 3.4118 & 11.8019 & 0.0000 & 3.9588 & 3.6473 & 4.5125 & 75.9367 \end{bmatrix}.$$

The projective synchronization state trajectories for the real and imaginary parts of FOCVNNs (4) and (5) are shown in Figure 6 with the initial values of  $\bar{x}(t) = [0; -0.1; 0.2; 0.2; 0.3; -0.1; 0.2; 0.2]^T$  and  $\hat{x}(t) = [0.1; -0.1; -0.2; -0.2; -0.1; 0.1; -0.2; -0.2]^T$ . It is evident that the master-slave system of FOCVNNs successfully achieves projective synchronization under the proposed AETM and aperiodic DoS attacks. At the same time, Figure 7 illustrates the real and imaginary parts of the control input  $u(t)$ . Upon the occurrence of the aperiodic DoS attacks, the control channel is blocked, and signal transmission ceases, causing the control input to become zero. Figure 8 then presents the real and imaginary parts of the state response of the projective synchronization error model (15), where it is clear that all state trajectories converge to zero, indicating that systems (4) and (5) synchronize.

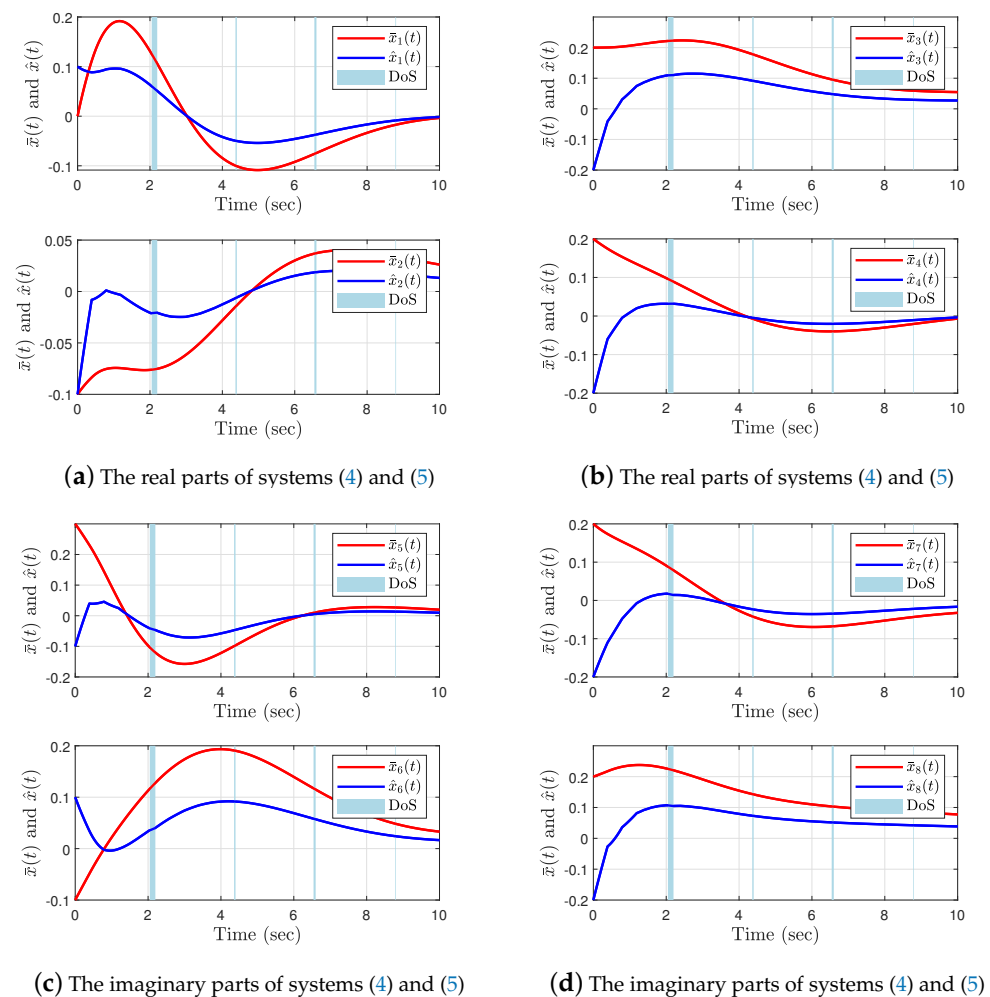
With reference to the weighting matrix and gain matrix discussed earlier, the improved AETM (7), along with release instants, release intervals, and the ATP  $\phi(t_{k,n}h)$ , is presented in Figure 9, which ensures the stabilization of the system. In other words, the designed AETM proves effective in defending against aperiodic DoS attacks. As shown in Figure 9, after the aperiodic DoS attacks cease, the triggering frequency of the AETM rises significantly, and the execution time is extended, thereby stabilizing the system. This illustrates that the AETM is capable of effectively counteracting aperiodic DoS attacks.

**Example 3.** Let the parameters be set as follows:  $\bar{\vartheta} = -1$ ,  $h = 0.2$ ,  $\varphi_M = 0.1$ ,  $\tau = 0.5$ ,  $b_1 = 2$ ,  $b_2 = 0.2$ . By incorporating these parameter values with the system parameters specified in Example 1 and applying Corollary 1, the following theoretical result is obtained:

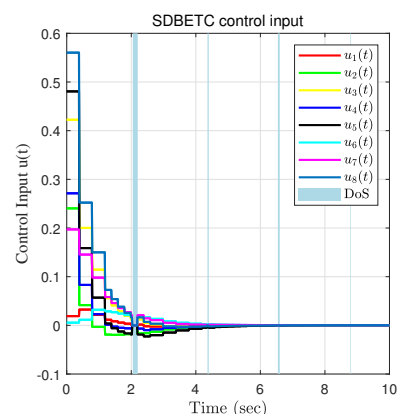
$$\mathcal{K}_1 = \begin{bmatrix} -4.6678 & 1.2746 & 0.0239 & 0.8922 \\ 1.0599 & -1.6919 & -0.6684 & -0.0316 \\ -0.0239 & -0.8922 & -4.6678 & 1.2746 \\ 0.6684 & 0.0316 & 1.0599 & -1.6919 \end{bmatrix},$$

$$\Phi = \begin{bmatrix} 48.7608 & -1.4685 & 0.0000 & -2.0089 \\ -1.4685 & 55.8123 & 2.0089 & 0.0000 \\ 0.0000 & 2.0089 & 48.7608 & -1.4685 \\ -2.0089 & 0.0000 & -1.4685 & 55.8123 \end{bmatrix}.$$

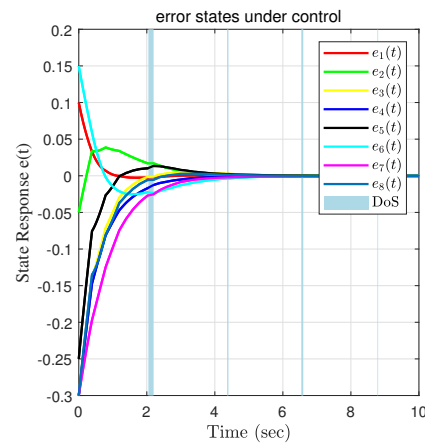
Taking the initial value as  $\bar{x}(t) = [0; -0.1; 0.2; 0.2]^T$  and  $\hat{x}(t) = [0.1; -0.1; -0.2; -0.2]^T$ , the anti-synchronization state trajectory of the real and imaginary parts of FOCVNNs are shown in Figure 10. Obviously, the master-slave system of FOCVNNs are able to achieve anti-synchronization under the proposed AETMs and aperiodic DoS attacks. Meanwhile, the real and imaginary parts of the control input  $u(t)$  are shown in Figure 11. In this case, once the aperiodic DoS attacks occur, the control channel is blocked and the signal cannot be transmitted, the curve of the control input becomes zero. Figure 12 then plots the real-imaginary part of the state response of the anti-synchronization error system. It is clear that all its state trajectories converge to zero, i.e., it indicates that systems (4) and (5) achieve anti-synchronization.



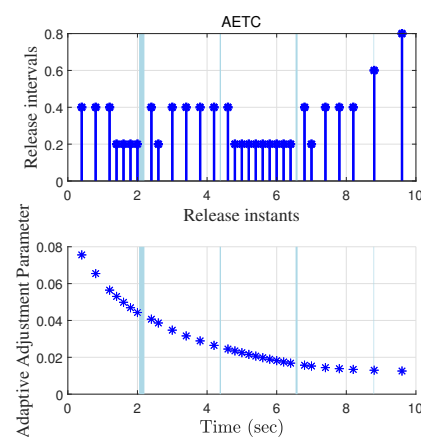
**Figure 6.** State responses of systems (4) and (5) for Example 2.



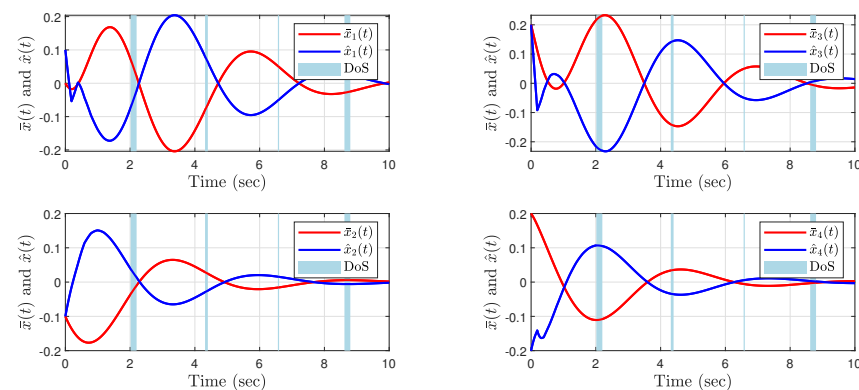
**Figure 7.** Control input under aperiodic DoS attacks for Example 2.



**Figure 8.** State responses of the error system for Example 2.



**Figure 9.** Release instants and intervals. The evolution of triggering threshold  $\varphi(t_{k,n}h)$ . The shaded area represents the DoS attacks, and the symbols \* denote the triggering threshold at each discrete time instant.

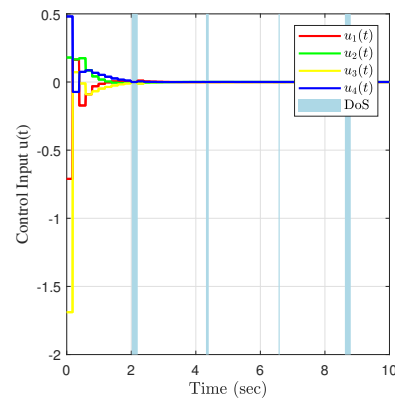


**(a)** The real parts of systems (4) and (5)

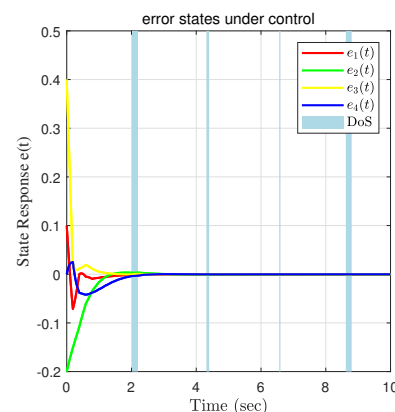
**(b)** The imaginary parts of systems (4) and (5)

**Figure 10.** State responses of systems (4) and (5) for Example 3.

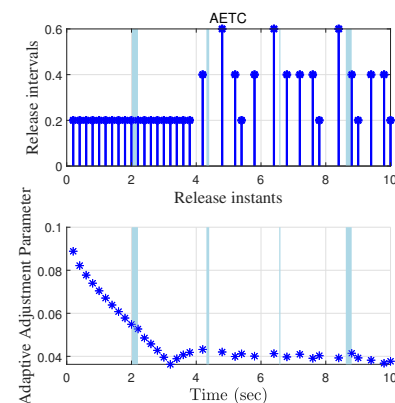
Based on the above weighting matrix and gain matrix, through the improved AETM (7), release instants and release intervals, the adaptive adjustment threshold parameter  $\varphi(t_{k,n}h)$  are drawn in Figure 13. This leads to the stabilization of the system. In other words, the designed AETM can effectively counteract aperiodic DoS attacks. It is obvious that, as shown in Figure 13, the triggering frequency of the AETM is significantly increased and the execution time is significantly prolonged after the end of the aperiodic DoS attacks, which leads to the stabilization of the system. In other words, the designed AETM can effectively counteract aperiodic DoS attacks.



**Figure 11.** Control input under aperiodic DoS attacks for Example 3.



**Figure 12.** State responses of error system for Example 3.



**Figure 13.** Release instants and intervals. The evolution of triggering threshold  $\varphi(t_{k,n}h)$ . The shaded area represents the DoS attacks, and the symbols \* denote the triggering threshold at each discrete time instant.

## 5. Conclusions

This paper has investigated the projective synchronization control problem for fractional-order complex-valued neural networks with additive time-varying delays using adaptive event-triggered mechanisms under aperiodic DoS attacks. First, a novel adaptive event-triggered mechanism has been designed by incorporating a switch adaptive threshold parameter to mitigate aperiodic DoS attacks and conserve network bandwidth. Second, the switching controller, integrating synchronization and event-triggered information, has been designed. Moreover, a delay-switching fractional-order complex-valued neural network model has been developed to mitigate the effects of aperiodic DoS attacks on data transmission. Third, a new common Lyapunov–Krasovskii functional, with fewer constraints

and less conservatism, has been constructed. Based on this, a novel projective synchronization criterion has been derived by utilizing linear matrix inequalities and fractional-order inequality methods. Finally, numerical simulations have validated the effectiveness and feasibility of the proposed control strategy, highlighting its ability to ensure robust synchronization performance under complex attack scenarios. Future research may address more challenging cybersecurity issues by considering broader classes of cyber attacks and developing advanced control strategies, such as memory-based adaptive event-triggered mechanisms. Moreover, the robust projective synchronization of FOCVNNs subject to both aperiodic DoS attacks and external disturbances will also be investigated in future work.

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**Data Availability Statement:** No new data were created or analyzed in this study.

**Conflicts of Interest:** The authors declare no conflicts of interest.

## Appendix A. Supplementary Notations of Theorem 1

To save space and present the theoretical result of Theorem 1 in a more concise and clear manner, the following notations are introduced.

$$\begin{aligned}\Xi_1 &= \Xi_{11} + \Xi_{12} + \Xi_{13} + \Xi_{14} + \Xi_{15}, \\ \Xi_{11} &= 2E_{11}^T P(-\bar{C}E_{11} + \bar{A}E_{12} + \bar{B}E_{14} + \mathcal{K}_1(E_{114} - E_{113})) \\ &\quad + \beta_1 E_{11}^T P E_{11} + E_{113}^T \Phi E_{113} - \varphi_M E_{114}^T \Phi E_{114}, \\ \Xi_{12} &= -\frac{(b-\mu)\Gamma(2-\alpha)}{b_1^{1-\alpha}}(E_{11} - E_{17})^T Q_1(E_{11} - E_{17}) - \frac{3(b-\mu)}{\Gamma(2-\alpha)b_1^{1-\alpha}}(\Gamma(3-2\alpha)E_{116} \\ &\quad - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{17} - \Gamma(2-\alpha)(E_{11} - E_{17}))^T Q_1\left(\Gamma(3-2\alpha)E_{116} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{17} \right. \\ &\quad \left. - \Gamma(2-\alpha)(E_{11} - E_{17}))\right) \\ &\quad - \frac{(b-\mu)\Gamma(2-\alpha)}{b_1^{1-\alpha}}(E_{11} - E_{18})^T Q_2(E_{11} - E_{18}) - \frac{3(b-\mu)}{\Gamma(2-\alpha)b_1^{1-\alpha}}(\Gamma(3-2\alpha)E_{117} \\ &\quad - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{18} - \Gamma(2-\alpha)(E_{11} - E_{18}))^T Q_2\left(\Gamma(3-2\alpha)E_{117} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{18} \right. \\ &\quad \left. - \Gamma(2-\alpha)(E_{11} - E_{18}))\right) \\ &\quad - \frac{(b-\mu)\Gamma(2-\alpha)}{b_2^{1-\alpha}}(E_{11} - E_{19})^T Q_3(E_{11} - E_{19}) - \frac{3(b-\mu)}{\Gamma(2-\alpha)b_2^{1-\alpha}}(\Gamma(3-2\alpha)E_{118} \\ &\quad - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{19} - \Gamma(2-\alpha)(E_{11} - E_{19}))^T Q_3\left(\Gamma(3-2\alpha)E_{118} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{19} \right. \\ &\quad \left. - \Gamma(2-\alpha)(E_{11} - E_{19}))\right) \\ &\quad - \frac{(b-\mu)\Gamma(2-\alpha)}{b_2^{1-\alpha}}(E_{11} - E_{110})^T Q_4(E_{11} - E_{110}) - \frac{3(b-\mu)}{\Gamma(2-\alpha)b_2^{1-\alpha}}(\Gamma(3-2\alpha)E_{119} \\ &\quad - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{110} - \Gamma(2-\alpha)(E_{11} - E_{110}))^T Q_4\left(\Gamma(3-2\alpha)E_{119} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{110} \right. \\ &\quad \left. - \Gamma(2-\alpha)(E_{11} - E_{110}))\right),\end{aligned}$$

$$\begin{aligned}\Xi_{13} = & -\frac{(b-\mu)\Gamma(2-\alpha)}{b^{1-\alpha}}(E_{11}-E_{111})^T R_1(E_{11}-E_{111}) - \frac{3(b-\mu)}{\Gamma(2-\alpha)b^{1-\alpha}}(\Gamma(3-2\alpha)E_{120} \\ & - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{111} - \Gamma(2-\alpha)(E_{11}-E_{111}))^T R_1\left(\Gamma(3-2\alpha)E_{120} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{111}\right. \\ & \left. - \Gamma(2-\alpha)(E_{11}-E_{111}))\right) \\ & - \frac{(b-\mu)\Gamma(2-\alpha)}{b^{1-\alpha}}(E_{11}-E_{112})^T R_2(E_{11}-E_{112}) - \frac{3(b-\mu)}{\Gamma(2-\alpha)b^{1-\alpha}}(\Gamma(3-2\alpha)E_{120} \\ & - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{112} - \Gamma(2-\alpha)(E_{11}-E_{112}))^T R_2\left(\Gamma(3-2\alpha)E_{120} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{112}\right. \\ & \left. - \Gamma(2-\alpha)(E_{11}-E_{112}))\right),\end{aligned}$$

$$\begin{aligned}\Xi_{14} = & -\frac{(b-\mu)\Gamma(2-\alpha)}{b^{1-\alpha}}\left(\begin{bmatrix} E_{11} \\ E_{12} \end{bmatrix} - \begin{bmatrix} E_{13} \\ E_{14} \end{bmatrix}\right)^T S_1\left(\begin{bmatrix} E_{11} \\ E_{12} \end{bmatrix} - \begin{bmatrix} E_{13} \\ E_{14} \end{bmatrix}\right) \\ & - \frac{3(b-\mu)}{\Gamma(2-\alpha)b^{1-\alpha}}\left(\Gamma(3-2\alpha)\begin{bmatrix} E_{122} \\ E_{123} \end{bmatrix} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}\begin{bmatrix} E_{13} \\ E_{14} \end{bmatrix} - \Gamma(2-\alpha)\left(\begin{bmatrix} E_{11} \\ E_{12} \end{bmatrix} - \begin{bmatrix} E_{13} \\ E_{14} \end{bmatrix}\right)\right)^T \\ & \times S_1\left(\Gamma(3-2\alpha)\begin{bmatrix} E_{122} \\ E_{123} \end{bmatrix} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}\begin{bmatrix} E_{13} \\ E_{14} \end{bmatrix} - \Gamma(2-\alpha)\left(\begin{bmatrix} E_{11} \\ E_{12} \end{bmatrix} - \begin{bmatrix} E_{13} \\ E_{14} \end{bmatrix}\right)\right) \\ & - \frac{(b-\mu)\Gamma(2-\alpha)}{b^{1-\alpha}}\left(\begin{bmatrix} E_{11} \\ E_{12} \end{bmatrix} - \begin{bmatrix} E_{15} \\ E_{16} \end{bmatrix}\right)^T S_2\left(\begin{bmatrix} E_{11} \\ E_{12} \end{bmatrix} - \begin{bmatrix} E_{15} \\ E_{16} \end{bmatrix}\right) \\ & - \frac{3(b-\mu)}{\Gamma(2-\alpha)b^{1-\alpha}}\left(\Gamma(3-2\alpha)\begin{bmatrix} E_{124} \\ E_{125} \end{bmatrix} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}\begin{bmatrix} E_{15} \\ E_{16} \end{bmatrix} - \Gamma(2-\alpha)\left(\begin{bmatrix} E_{11} \\ E_{12} \end{bmatrix} - \begin{bmatrix} E_{15} \\ E_{16} \end{bmatrix}\right)\right)^T \\ & \times S_2\left(\Gamma(3-2\alpha)\begin{bmatrix} E_{124} \\ E_{125} \end{bmatrix} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}\begin{bmatrix} E_{15} \\ E_{16} \end{bmatrix} - \Gamma(2-\alpha)\left(\begin{bmatrix} E_{11} \\ E_{12} \end{bmatrix} - \begin{bmatrix} E_{15} \\ E_{16} \end{bmatrix}\right)\right),\end{aligned}$$

$$\begin{aligned}\Xi_{15} = & -\frac{(b-\mu)\Gamma(2-\alpha)}{\mu^{1-\alpha}}(E_{11}-E_{114})^T T_1(E_{11}-E_{114}) - \frac{3(b-\mu)}{\Gamma(2-\alpha)\mu^{1-\alpha}}(\Gamma(3-2\alpha)E_{126} \\ & - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{114} - \Gamma(2-\alpha)(E_{11}-E_{114}))^T T_1\left(\Gamma(3-2\alpha)E_{126} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{114}\right. \\ & \left. - \Gamma(2-\alpha)(E_{11}-E_{114}))\right) \\ & - \frac{(b-\mu)\Gamma(2-\alpha)}{\mu^{1-\alpha}}(E_{11}-E_{115})^T T_2(E_{11}-E_{115}) - \frac{3(b-\mu)}{\Gamma(2-\alpha)\mu^{1-\alpha}}(\Gamma(3-2\alpha)E_{127} \\ & - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{115} - \Gamma(2-\alpha)(E_{11}-E_{115}))^T T_2\left(\Gamma(3-2\alpha)E_{127} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{115}\right. \\ & \left. - \Gamma(2-\alpha)(E_{11}-E_{115}))\right),\end{aligned}$$

$$\Xi_2 = \Xi_{21} + \Xi_{22} + \Xi_{23} + \Xi_{24} + \Xi_{25},$$

$$\Xi_{21} = 2E_{21}^T(P)(-\tilde{C}E_{21} + \tilde{A}E_{22} + \tilde{B}E_{24}) + \beta_1 E_{21}^T P E_{21},$$

$$\begin{aligned}\Xi_{22} = & -\frac{(b-\mu)\Gamma(2-\alpha)}{b_1^{1-\alpha}}(E_{21}-E_{27})^T Q_1(E_{21}-E_{27}) - \frac{3(b-\mu)}{\Gamma(2-\alpha)b_1^{1-\alpha}}(\Gamma(3-2\alpha)E_{215} \\ & - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{27} - \Gamma(2-\alpha)(E_{21}-E_{27}))^T Q_1\left(\Gamma(3-2\alpha)E_{215} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{27}\right. \\ & \left. - \Gamma(2-\alpha)(E_{21}-E_{27}))\right) \\ & - \frac{(b-\mu)\Gamma(2-\alpha)}{b_1^{1-\alpha}}(E_{21}-E_{28})^T Q_2(E_{21}-E_{28}) - \frac{3(b-\mu)}{\Gamma(2-\alpha)b_1^{1-\alpha}}(\Gamma(3-2\alpha)E_{216} \\ & - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{28} - \Gamma(2-\alpha)(E_{21}-E_{28}))^T Q_2\left(\Gamma(3-2\alpha)E_{216} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{28}\right. \\ & \left. - \Gamma(2-\alpha)(E_{21}-E_{28}))\right) \\ & - \frac{(b-\mu)\Gamma(2-\alpha)}{b_2^{1-\alpha}}(E_{21}-E_{29})^T Q_3(E_{21}-E_{29}) - \frac{3(b-\mu)}{\Gamma(2-\alpha)b_2^{1-\alpha}}(\Gamma(3-2\alpha)E_{217} \\ & - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{29} - \Gamma(2-\alpha)(E_{21}-E_{29}))^T Q_3\left(\Gamma(3-2\alpha)E_{217} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{29}\right. \\ & \left. - \Gamma(2-\alpha)(E_{21}-E_{29}))\right) \\ & - \frac{(b-\mu)\Gamma(2-\alpha)}{b_2^{1-\alpha}}(E_{21}-E_{210})^T Q_4(E_{21}-E_{210}) - \frac{3(b-\mu)}{\Gamma(2-\alpha)b_2^{1-\alpha}}(\Gamma(3-2\alpha)E_{218} \\ & - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{210} - \Gamma(2-\alpha)(E_{21}-E_{210}))^T Q_4\left(\Gamma(3-2\alpha)E_{218} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{210}\right. \\ & \left. - \Gamma(2-\alpha)(E_{21}-E_{210}))\right),\end{aligned}$$

$$\begin{aligned}\Xi_{23} = & -\frac{(b-\mu)\Gamma(2-\alpha)}{b^{1-\alpha}}(E_{21}-E_{211})^T R_1(E_{21}-E_{211}) - \frac{3(b-\mu)}{\Gamma(2-\alpha)b^{1-\alpha}}(\Gamma(3-2\alpha)E_{219} \\ & - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{211} - \Gamma(2-\alpha)(E_{21}-E_{211}))^T R_1\left(\Gamma(3-2\alpha)E_{219} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{211} \right. \\ & \left. - \Gamma(2-\alpha)(E_{21}-E_{211}))\right) \\ & - \frac{(b-\mu)\Gamma(2-\alpha)}{b^{1-\alpha}}(E_{21}-E_{212})^T R_2(E_{21}-E_{212}) - \frac{3(b-\mu)}{\Gamma(2-\alpha)b^{1-\alpha}}(\Gamma(3-2\alpha)E_{220} \\ & - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{212} - \Gamma(2-\alpha)(E_{21}-E_{212}))^T R_2\left(\Gamma(3-2\alpha)E_{220} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{212} \right. \\ & \left. - \Gamma(2-\alpha)(E_{21}-E_{212}))\right),\end{aligned}$$

$$\begin{aligned}\Xi_{24} = & -\frac{(b-\mu)\Gamma(2-\alpha)}{b^{1-\alpha}}\left(\begin{bmatrix} E_{21} \\ E_{22} \end{bmatrix} - \begin{bmatrix} E_{23} \\ E_{24} \end{bmatrix}\right)^T S_1\left(\begin{bmatrix} E_{21} \\ E_{22} \end{bmatrix} - \begin{bmatrix} E_{23} \\ E_{24} \end{bmatrix}\right) \\ & - \frac{3(b-\mu)}{\Gamma(2-\alpha)b^{1-\alpha}}\left(\Gamma(3-2\alpha)\begin{bmatrix} E_{221} \\ E_{222} \end{bmatrix} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}\begin{bmatrix} E_{23} \\ E_{24} \end{bmatrix} - \Gamma(2-\alpha)\left(\begin{bmatrix} E_{21} \\ E_{22} \end{bmatrix} - \begin{bmatrix} E_{23} \\ E_{24} \end{bmatrix}\right)\right)^T \\ & \times S_1\left(\Gamma(3-2\alpha)\begin{bmatrix} E_{221} \\ E_{222} \end{bmatrix} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}\begin{bmatrix} E_{23} \\ E_{24} \end{bmatrix} - \Gamma(2-\alpha)\left(\begin{bmatrix} E_{21} \\ E_{22} \end{bmatrix} - \begin{bmatrix} E_{23} \\ E_{24} \end{bmatrix}\right)\right) \\ & - \frac{(b-\mu)\Gamma(2-\alpha)}{b^{1-\alpha}}\left(\begin{bmatrix} E_{21} \\ E_{22} \end{bmatrix} - \begin{bmatrix} E_{25} \\ E_{26} \end{bmatrix}\right)^T S_2\left(\begin{bmatrix} E_{21} \\ E_{22} \end{bmatrix} - \begin{bmatrix} E_{25} \\ E_{26} \end{bmatrix}\right) \\ & - \frac{3(b-\mu)}{\Gamma(2-\alpha)b^{1-\alpha}}\left(\Gamma(3-2\alpha)\begin{bmatrix} E_{223} \\ E_{224} \end{bmatrix} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}\begin{bmatrix} E_{25} \\ E_{26} \end{bmatrix} - \Gamma(2-\alpha)\left(\begin{bmatrix} E_{21} \\ E_{22} \end{bmatrix} - \begin{bmatrix} E_{25} \\ E_{26} \end{bmatrix}\right)\right)^T \\ & \times S_2\left(\Gamma(3-2\alpha)\begin{bmatrix} E_{223} \\ E_{224} \end{bmatrix} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}\begin{bmatrix} E_{25} \\ E_{26} \end{bmatrix} - \Gamma(2-\alpha)\left(\begin{bmatrix} E_{21} \\ E_{22} \end{bmatrix} - \begin{bmatrix} E_{25} \\ E_{26} \end{bmatrix}\right)\right),\end{aligned}$$

$$\begin{aligned}\Xi_{25} = & -\frac{(b-\mu)\Gamma(2-\alpha)}{\mu^{1-\alpha}}(E_{21}-E_{213})^T T_1(E_{21}-E_{213}) - \frac{3(b-\mu)}{\Gamma(2-\alpha)\mu^{1-\alpha}}(\Gamma(3-2\alpha)E_{225} \\ & - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{213} - \Gamma(2-\alpha)(E_{21}-E_{213}))^T T_1\left(\Gamma(3-2\alpha)E_{225} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{213} \right. \\ & \left. - \Gamma(2-\alpha)(E_{21}-E_{213}))\right) \\ & - \frac{(b-\mu)\Gamma(2-\alpha)}{\mu^{1-\alpha}}(E_{21}-E_{214})^T T_2(E_{21}-E_{214}) - \frac{3(b-\mu)}{\Gamma(2-\alpha)\mu^{1-\alpha}}(\Gamma(3-2\alpha)E_{226} \\ & - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{214} - \Gamma(2-\alpha)(E_{21}-E_{214}))^T T_2\left(\Gamma(3-2\alpha)E_{226} - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}E_{214} \right. \\ & \left. - \Gamma(2-\alpha)(E_{21}-E_{214}))\right).\end{aligned}$$

## Appendix B. Proof of Theorem 1

The following expression defines the LKFs:

$$V(t) = \sum_{j=1}^5 V_j(t), \quad (\text{A1})$$

where

$$V_1(t) = e^T(t)Pe(t) \quad (\text{A2})$$

$$\begin{aligned}V_2(t) = & (b - (t - i_1h)) \int_t^{i_{l+1}h} e^{-\beta_1(t-s)} [{}_{r-b_1(r)}\mathfrak{D}_r^{1-\alpha}e(r)]^T Q_1 [{}_{r-b_1(r)}\mathfrak{D}_r^{1-\alpha}e(r)] dr \\ & + (b - (t - i_1h)) \int_t^{i_{l+1}h} e^{-\beta_1(t-s)} [{}_{r-b_1(r)}\mathfrak{D}_r^{1-\alpha}e(r)]^T Q_2 [{}_{r-b_1(r)}\mathfrak{D}_r^{1-\alpha}e(r)] dr \\ & + (b - (t - i_1h)) \int_t^{i_{l+1}h} e^{-\beta_1(t-s)} [{}_{r-b_2(r)}\mathfrak{D}_r^{1-\alpha}e(r)]^T Q_3 [{}_{r-b_2(r)}\mathfrak{D}_r^{1-\alpha}e(r)] dr \\ & + (b - (t - i_1h)) \int_t^{i_{l+1}h} e^{-\beta_1(t-s)} [{}_{r-b_2(r)}\mathfrak{D}_r^{1-\alpha}e(r)]^T Q_4 [{}_{r-b_2(r)}\mathfrak{D}_r^{1-\alpha}e(r)] dr\end{aligned} \quad (\text{A3})$$

$$V_3(t) = (b - (t - i_l h)) \int_t^{i_l+1h} e^{-\beta_1(t-s)} [{}_{r-b_1-b_2(r)}\mathfrak{D}_r^{1-\alpha} e(r)]^T R_1 [{}_{r-b_1-b_2(r)}\mathfrak{D}_r^{1-\alpha} e(r)] dr \\ + (b - (t - i_l h)) \int_t^{i_l+1h} e^{-\beta_1(t-s)} [{}_{r-b_2-b_1(r)}\mathfrak{D}_r^{1-\alpha} e(r)]^T R_2 [{}_{r-b_2-b_1(r)}\mathfrak{D}_r^{1-\alpha} e(r)] dr \quad (A4)$$

$$V_4(t) = (b - (t - i_l h)) \int_t^{i_l+1h} e^{-\beta_1(t-s)} [{}_{r-b(r)}\mathfrak{D}_r^{1-\alpha} \chi(r)]^T S_1 [{}_{r-b(r)}\mathfrak{D}_r^{1-\alpha} \chi(r)] dr \\ + (b - (t - i_l h)) \int_t^{i_l+1h} e^{-\beta_1(t-s)} [{}_{r-b}\mathfrak{D}_r^{1-\alpha} \chi(r)]^T S_2 [{}_{r-b}\mathfrak{D}_r^{1-\alpha} \chi(r)] dr \quad (A5)$$

$$V_5(t) = (b - (t - i_l h)) \int_t^{i_l+1h} e^{-\beta_1(t-s)} [{}_{r-\mu(r)}\mathfrak{D}_r^{1-\alpha} e(r)]^T T_1 [{}_{r-\mu(r)}\mathfrak{D}_r^{1-\alpha} e(r)] dr \\ + (b - (t - i_l h)) \int_t^{i_l+1h} e^{-\beta_1(t-s)} [{}_{r-\mu}\mathfrak{D}_r^{1-\alpha} e(r)]^T T_2 [{}_{r-\mu}\mathfrak{D}_r^{1-\alpha} e(r)] dr, \quad (A6)$$

with  $\chi(r) = \text{col}\{e(r), \mathfrak{F}(e(r))\}$ . Obviously, taking the fractional derivative of  $V(t)$  yields:

$$\mathfrak{D}^\alpha V(t) = \mathfrak{D}^\alpha \left( \sum_{j=1}^5 V_j(t) \right),$$

where

$$\mathfrak{D}^\alpha V_1(t) \leq -\beta_1 V_1(t) + \beta_1 e^T(t) P e(t) + \mathfrak{D}^\alpha (e^T(t) P e(t)) \\ \leq -\beta_1 V_1(t) + \beta_1 e^T(t) P e(t) + 2e^T(t) P (-\bar{C}e(t) + \bar{A}\mathfrak{F}(e(t)) \\ + \bar{B}\mathfrak{F}(e(t - b(t))) + K(e(t - \mu(t)) - \psi(i_l h))), \quad (A7)$$

$$\mathfrak{D}^\alpha V_2(t) = \mathfrak{D}^\alpha \left\{ (b - (t - i_l h)) \int_t^{i_l+1h} [{}_{r-b_1(t)}\mathfrak{D}_r^{1-\alpha} e(r)]^T Q_1 [{}_{r-b_1(t)}\mathfrak{D}_r^{1-\alpha} e(r)] dr \right\} \\ + \mathfrak{D}^\alpha \left\{ (b - (t - i_l h)) \int_t^{i_l+1h} [{}_{r-b_1}\mathfrak{D}_r^{1-\alpha} e(r)]^T Q_2 [{}_{r-b_1}\mathfrak{D}_r^{1-\alpha} e(r)] dr \right\} \\ + \mathfrak{D}^\alpha \left\{ (b - (t - i_l h)) \int_t^{i_l+1h} [{}_{r-b_2(r)}\mathfrak{D}_r^{1-\alpha} e(r)]^T Q_3 [{}_{r-b_2(r)}\mathfrak{D}_r^{1-\alpha} e(r)] dr \right\} \\ + \mathfrak{D}^\alpha \left\{ (b - (t - i_l h)) \int_t^{i_l+1h} [{}_{r-b_2}\mathfrak{D}_r^{1-\alpha} e(r)]^T Q_4 [{}_{r-b_2}\mathfrak{D}_r^{1-\alpha} e(r)] dr \right\} \quad (A8)$$

Building on Lemmas 5 and 6, it can be concluded that:

$$D^\alpha V_2(t) \leq -\frac{(b-\mu)\Gamma(2-\alpha)}{b_1^{1-\alpha}} (e(t) - e(t - b_1(t)))^T Q_1 (e(t) - e(t - b_1(t))) \\ - \frac{3(b-\mu)}{\Gamma(2-\alpha)b_1^{1-\alpha}} \left( \frac{\Gamma(3-2\alpha)}{b_1(t)} \left( {}_{t-b_1(t)}I_t^{1-\alpha} e(t) \right) - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)} e(t - b_1(t)) \right) \\ - \Gamma(2-\alpha) (e(t) - e(t - b_1(t)))^T Q_1 \left( \frac{\Gamma(3-2\alpha)}{b_1(t)} \left( {}_{t-b_1(t)}I_t^{1-\alpha} e(t) \right) \right. \\ \left. - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)} e(t - b_1(t)) - \Gamma(2-\alpha) (e(t) - e(t - b_1(t))) \right) \\ - \frac{(b-\mu)\Gamma(2-\alpha)}{b_1^{1-\alpha}} (e(t) - e(t - b_1(t)))^T Q_2 (e(t) - e(t - b_1(t))) \\ - \frac{3(b-\mu)}{\Gamma(2-\alpha)b_1^{1-\alpha}} \left( \frac{\Gamma(3-2\alpha)}{b_1} \left( {}_{t-b_1}I_t^{1-\alpha} e(t) \right) - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)} e(t - b_1) \right) \\ - \Gamma(2-\alpha) (e(t) - e(t - b_1(t)))^T \left( \frac{\Gamma(3-2\alpha)}{b_1} \left( {}_{t-b_1}I_t^{1-\alpha} e(t) \right) \right. \\ \left. - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)} e(t - b_1) - \Gamma(2-\alpha) (e(t) - e(t - b_1(t))) \right) \\ - \frac{(b-\mu)\Gamma(2-\alpha)}{b_2^{1-\alpha}} (e(t) - e(t - b_2(t)))^T Q_3 (e(t) - e(t - b_2(t))) \\ - \frac{3(b-\mu)}{\Gamma(2-\alpha)b_2^{1-\alpha}} \left( \frac{\Gamma(3-2\alpha)}{b_2(t)} \left( {}_{t-b_2(t)}I_t^{1-\alpha} e(t) \right) - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)} e(t - b_2(t)) \right) \\ - \Gamma(2-\alpha) (e(t) - e(t - b_2(t)))^T Q_3 \left( \frac{\Gamma(3-2\alpha)}{b_2(t)} \left( {}_{t-b_2(t)}I_t^{1-\alpha} e(t) \right) \right. \\ \left. - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)} e(t - b_2(t)) - \Gamma(2-\alpha) (e(t) - e(t - b_2(t))) \right) \\ - \frac{(b-\mu)\Gamma(2-\alpha)}{b_2^{1-\alpha}} (e(t) - e(t - b_2(t)))^T Q_4 (e(t) - e(t - b_2(t))) \\ - \frac{3(b-\mu)}{\Gamma(2-\alpha)b_2^{1-\alpha}} \left( \frac{\Gamma(3-2\alpha)}{b_2} \left( {}_{t-b_2}I_t^{1-\alpha} e(t) \right) - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)} e(t - b_2) \right) \\ - \Gamma(2-\alpha) (e(t) - e(t - b_2(t)))^T Q_4 \left( \frac{\Gamma(3-2\alpha)}{b_2} \left( {}_{t-b_2}I_t^{1-\alpha} e(t) \right) \right. \\ \left. - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)} e(t - b_2) - \Gamma(2-\alpha) (e(t) - e(t - b_2(t))) \right). \quad (A9)$$

Similarly, the fractional-order derivatives of  $V_3(t)$ ,  $V_4(t)$ ,  $V_5(t)$  are given as follows:

$$\begin{aligned} \mathfrak{D}^\alpha V_3(t) \leq & -\frac{(b-\mu)\Gamma(2-\alpha)}{b^{1-\alpha}}(e(t) - e(t-b_1-b_2(t)))^T R_1(e(t) - e(t-b_1-b_2(t))) \\ & -\frac{3(b-\mu)}{\Gamma(2-\alpha)b^{1-\alpha}} \left( \frac{\Gamma(3-2\alpha)}{(b_1+b_2(t))^{1-\alpha}} \left( {}_{t-b_1-b_2(t)}\mathcal{I}_t^{1-\alpha} e(t) \right) \right. \\ & \left. -\frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)} e(t-b_1-b_2(t)) - \Gamma(2-\alpha)(e(t) - e(t-b_1-b_2(t))) \right) \\ & \times R_1 \left( \frac{\Gamma(3-2\alpha)}{(b_1+b_2(t))^{1-\alpha}} \left( {}_{t-b_1-b_2(t)}\mathcal{I}_t^{1-\alpha} e(t) \right) \right. \\ & \left. -\frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)} e(t-b_1-b_2(t)) - \Gamma(2-\alpha)(e(t) - e(t-b_1-b_2(t))) \right) \\ & -\frac{(b-\mu)\Gamma(2-\alpha)}{b^{1-\alpha}}(e(t) - e(t-b_2-b_1(t)))^T R_2(e(t) - e(t-b_2-b_1(t))) \\ & -\frac{3(b-\mu)}{\Gamma(2-\alpha)b^{1-\alpha}} \left( \frac{\Gamma(3-2\alpha)}{(b_2+b_1(t))^{1-\alpha}} \left( {}_{t-b_2-b_1(t)}\mathcal{I}_t^{1-\alpha} e(t) \right) \right. \\ & \left. -\frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)} e(t-b_2-b_1(t)) - \Gamma(2-\alpha)(e(t) - e(t-b_2-b_1(t))) \right) \\ & \times R_2 \left( \frac{\Gamma(3-2\alpha)}{(b_2+b_1(t))^{1-\alpha}} \left( {}_{t-b_2-b_1(t)}\mathcal{I}_t^{1-\alpha} e(t) \right) - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)} e(t-b_2-b_1(t)) \right. \\ & \left. -\Gamma(2-\alpha)(e(t) - e(t-b_2-b_1(t))) \right), \end{aligned} \quad (\text{A10})$$

$$\begin{aligned} \mathfrak{D}^\alpha V_4(t) \leq & -\frac{(b-\mu)\Gamma(2-\alpha)}{b^{1-\alpha}}(\chi(t) - \chi(t-b(t)))^T S_1(\chi(t) - \chi(t-b(t))) \\ & -\frac{3(b-\mu)}{\Gamma(2-\alpha)b^{1-\alpha}} \left( \frac{\Gamma(3-2\alpha)}{b(t)^{1-\alpha}} \left( {}_{t-b(t)}\mathcal{I}_t^{1-\alpha} \chi(t) \right) - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)} \chi(t-b(t)) \right. \\ & \left. -\Gamma(2-\alpha)(\chi(t) - \chi(t-b(t))) S_1 \left( \frac{\Gamma(2-\alpha)}{b(t)^{1-\alpha}} \left( {}_{t-b(t)}\mathcal{I}_t^{1-\alpha} \chi(t) \right) \right. \right. \\ & \left. \left. -\frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)} \chi(t-b(t)) - \Gamma(2-\alpha)(\chi(t) - \chi(t-b(t))) \right) \right) \\ & -\frac{(b-\mu)\Gamma(2-\alpha)}{b^{1-\alpha}}(\chi(t) - \chi(t-b))^T S_2(\chi(t) - \chi(t-b)) \\ & -\frac{3(b-\mu)}{\Gamma(2-\alpha)b^{1-\alpha}} \left( \frac{\Gamma(3-2\alpha)}{b^{1-\alpha}} \left( {}_{t-b}\mathcal{I}_t^{1-\alpha} \chi(t) \right) - \frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)} \chi(t-b) \right. \\ & \left. -\Gamma(2-\alpha)(\chi(t) - \chi(t-b)) S_2 \left( \frac{\Gamma(2-\alpha)}{b^{1-\alpha}} \left( {}_{t-b}\mathcal{I}_t^{1-\alpha} \chi(t) \right) \right. \right. \\ & \left. \left. -\frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)} \chi(t-b) - \Gamma(2-\alpha)(\chi(t) - \chi(t-b)) \right) \right), \end{aligned} \quad (\text{A11})$$

$$\begin{aligned}
\mathfrak{D}^\alpha V_5(t) \leq & -\frac{(b-\mu)\Gamma(2-\alpha)}{\mu^{1-\alpha}}(e(t)-e(t-\mu(t)))^T T_1(e(t)-e(t-\mu(t))) \\
& -\frac{3(b-\mu)}{\Gamma(2-\alpha)\mu^{1-\alpha}}\left(\frac{\Gamma(3-2\alpha)}{\mu(t)^{1-\alpha}}\left({}_{t-\mu(t)}\mathcal{I}_t^{1-\alpha}e(t)\right)-\frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}e(t-\mu(t))\right. \\
& -\Gamma(2-\alpha)(e(t)-e(t-\mu(t))))T_1\left(\frac{\Gamma(2-\alpha)}{\mu(t)^{1-\alpha}}\left({}_{t-\mu(t)}\mathcal{I}_t^{1-\alpha}e(t)\right)\right. \\
& \left.-\frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}e(t-\mu(t))-\Gamma(2-\alpha)(e(t)-e(t-\mu(t)))\right) \\
& -\frac{(b-\mu)\Gamma(2-\alpha)}{\mu^{1-\alpha}}(e(t)-e(t-\mu))^T T_2(e(t)-e(t-\mu)) \\
& -\frac{3(b-\mu)}{\Gamma(2-\alpha)\mu^{1-\alpha}}\left(\frac{\Gamma(3-2\alpha)}{\mu^{1-\alpha}}\left({}_{t-\mu}\mathcal{I}_t^{1-\alpha}e(t)\right)-\frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}e(t-\mu)\right. \\
& -\Gamma(2-\alpha)(e(t)-e(t-\mu)))T_2\left(\frac{\Gamma(2-\alpha)}{\mu^{1-\alpha}}\left({}_{t-\mu}\mathcal{I}_t^{1-\alpha}e(t)\right)\right. \\
& \left.-\frac{\Gamma(3-2\alpha)}{\Gamma(2-\alpha)}e(t-\mu)-\Gamma(2-\alpha)(e(t)-e(t-\mu))\right),
\end{aligned} \tag{A12}$$

Additionally, considering condition (11) and diagonal matrices  $M_i \in \mathbb{R}_+^{n \times n}$  ( $i = 1, 2, 3$ ), we can conclude

$$\begin{aligned}
0 \leq & 2(\mathfrak{F}(e(t)) - L^-e(t))^T M_1(L^+e(t) - \mathfrak{F}(e(t))) \\
& + 2(\mathfrak{F}(e(t-b)) - L^-e(t-b))^T M_2(L^+e(t-b) - (e(t-b))) \\
& + 2(\mathfrak{F}(e(t-b(t))) - L^-e(t-b(t)))^T M_3(L^+e(t-b(t)) - \mathfrak{F}(e(t-b(t)))).
\end{aligned} \tag{A13}$$

In conclusion, combining (14), (A7) and (A9) through (A13) yields that

$$\mathfrak{D}^\alpha V(t) \leq -\beta_1 V(t) + \zeta_1^H(t) \Xi_1 \zeta_1(t), \tag{A14}$$

The next step is to analyze the case where  $t \in \mathcal{D}_n$ , following the same approach as before:

$$\mathfrak{D}^\alpha V(t) \leq -\beta_1 V(t) + \zeta_2^H(t) \Xi_2 \zeta_2(t). \tag{A15}$$

Assume  $\beta_1 + \beta_2 > 0$ , and inequalities (16) and (17), it is easy to obtain:

$$\mathfrak{D}^\alpha V(t) \leq -\beta_1 V(t), t \in \mathcal{U}_l \cap \mathcal{H}_n, \tag{A16}$$

$$\mathfrak{D}^\alpha V(t) \leq \beta_2 V(t), t \in \mathcal{D}_n. \tag{A17}$$

By combining inequalities (A16) and (A17) with Lemma 2, we can obtain:

$$\begin{cases} V(t) \leq V(h_n)E_\alpha(-\beta_1(t-h_n)^\alpha), t \in \mathcal{U}_l \cap \mathcal{H}_n, \\ V(t) \leq V(h_n+d_n)E_\alpha(\beta_2(t-h_n-d_n)^\alpha), t \in \mathcal{D}_n, \end{cases} \tag{A18}$$

then

$$\begin{cases} V(h_n+d_n) \leq V(h_n)E_\alpha(-\beta_1(d_n)^\alpha), t \in \mathcal{U}_l \cap \mathcal{H}_n, \\ V(h_{n+1}) \leq V(h_n+d_n)E_\alpha(\beta_2(h_{n+1}-h_n-d_n)^\alpha), t \in \mathcal{D}_n. \end{cases} \tag{A19}$$

Next, we focus on the stability analysis of the FOEs (15). According to Lemma 3, inequality (A18) and (A19), when  $t \in \mathcal{U}_l \cap \mathcal{H}_n$ , it has

$$\begin{aligned}
V(t) &\leq V(h_n)E_\alpha(-\beta_1(t-h_n)^\alpha) \\
&\leq V(h_{n-1}+d_{n-1})E_\alpha(\beta_2(h_n-h_{n-1}-d_{n-1})^\alpha)E_\alpha(-\beta_1(t-h_n)^\alpha) \\
&\dots \\
&\leq V(0)(E_\alpha(-\beta_1(d_{\min})^\alpha))^{n+1}(E_\alpha(\beta_2(b_{\max})^\alpha))^n
\end{aligned}$$

Under the conditions of Assumptions 1 and 2, and Lemma 4, it follows that

$$V(t) \leq \ell_1 e^{-\lambda t} V(0) \quad (\text{A20})$$

where  $\ell_1 = e^{-v\Gamma_D\lambda} E_\alpha(-\beta_1 d_{\min}^\alpha)$ ,  $\lambda = \frac{-1}{\Gamma_D} \left( \ln E_\alpha(-\beta_1 d_{\min}^\alpha) - \ln \alpha + (\beta_2 b_{\max}^\alpha)^{\frac{1}{\alpha}} \right)$ . When  $t \in \mathcal{D}_n$ , one has

$$\begin{aligned}
V(t) &\leq V(h_n+d_n)E_\alpha(\beta_2(t-h_n-d_n)^\alpha) \\
&\leq V(h_n)E_\alpha(-\beta_1(d_n)^\alpha)E_\alpha(\beta_2(t-h_n-d_n)^\alpha) \\
&\dots \\
&\leq V(0)(E_\alpha(-\beta_1(d_{\min})^\alpha)E_\alpha(\beta_2(b_{\max})^\alpha))^{n+1}
\end{aligned}$$

As per Assumptions 1 and 2, and Lemma 4, it has

$$V(t) \leq \ell_2 e^{-\lambda t} V(0) \quad (\text{A21})$$

with  $\ell_2 = e^{-(v+1)\Gamma_D\lambda}$ .

Finally, applying the inequalities (A20), (A21) and (18) with  $\ell = \max\{\ell_1, \ell_2\}$ , we get

$$V(t) \leq \ell e^{-\lambda t} V(0). \quad (\text{A22})$$

In summary, the error system (15) is GES. The proof is completed.

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