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Article

Salience and Strategy Choice in 2 × 2 Games

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Abstract: We present a model of boundedly rational play in single-shot 2 × 2 games. Players choose strategies based on the perceived salience of their own payoffs and, if own-payoff salience is uninformative, on the perceived salience of their opponent's payoffs. When own payoffs are salient, the model's predictions correspond to those of Level-1 players in a cognitive hierarchy model. When it is the other player's payoffs that are salient, the predictions of the model correspond to those of traditional game theory. The model provides unique predictions for the entire class of 2 × 2 games. It identifies games where a Nash equilibrium will always occur, ones where it will never occur, and ones where it will occur only for certain payoff values. It also predicts the outcome of games for which there are no pure Nash equilibria. Experimental results supporting these predictions are presented.

Keywords: behavioral game theory; bounded rationality; salience; heuristics

JEL Classification: D03; C72

1. Introduction

Game theorists often note that strategic interactions are ubiquitous in everyday life, from the occasional salary negotiation, to the more frequent negotiation regarding the division of labor on a project, to the daily dance of the carts in aisle 6 at the grocery store. In some respects, this ubiquity is a blessing—the domain over which the theory potentially applies is quite large. But the frequency of strategic interaction is also a curse—given any cost to considering all possible actions, computing exact payoffs or reasoning strategically, the sheer number of interactions we encounter on a daily basis would make it implausible to solve each by calculating and maximizing expected utility. The standard retort to this type of argument is that we do not really believe people do the sorts of computations we ascribe to them when we analyze games, but they somehow behave "as if" they did. This would close the discussion if the predictions of game theory were right. However, game theory fails in two systematic and distinctive ways: in certain circumstances, the theory is imprecise and in others, it is inaccurate.

The experimental results for a Stag-hunt game from Leland [1] shown in Figure 1 serve to illustrate both problems. Percentages shown along the right and lower borders of the matrix reflect the number of U *versus* D and L *versus* R choices subjects made, while the percentages in the interior of the matrix indicate the implied relative frequency with which outcomes in the game should occur.

		Player 2		
		L	R	
Player 1	U	8.00 8.00 41%	2.00 2.10 4%	17 (45%)
	D	7.9 2.00 51%	7.9 2.10 4%	21 (55%)
		35 (92%)	3 (8%)	

Figure 1. The Stag Hunt Game.

There are two pure strategy Nash equilibria in this game (UL and DR) but game theory gives no guidance as to which of these to expect. This is the imprecision problem. Neither of the outcomes predicted by game theory predominates. Instead, the majority outcome is the non-equilibrium outcome DL. This illustrates the inaccuracy problem.

These problems have spawned two primary responses from behavioral game theorists. One has been to posit that people have other-regarding preferences. Models by Fehr and Schmidt [2] and Bolton and Ockenfels [3] assume people are "inequality averse"—they prefer higher payoffs to lower ones but also prefer more equal payoffs across players. Models by Andreoni and Miller [4] among others assume agents have social-welfare maximizing preferences—they prefer that all players get more to less but also prefer to get more for themselves when they are behind than when they are ahead.

A second explanation for departures from the predictions of game theory is that people may not be as strategically sophisticated as the theory assumes. Camerer, Ho and Chong [5], for example, assume that players may differ in their strategic sophistication.[1] The most naive players, often referred to as Level-0 agents, randomly choose between available strategies. Level-1 players presume their opponents are Level-0 players and choose the strategy that maximizes their expected utility conditional on an opponent who chooses between strategies with equal probability. The analysis is extended to a "cognitive hierarchy" of still higher level players who are increasingly sophisticated in how they formulate their strategy choices based on their beliefs about the distribution of lower level players.

[1] Also see work by Nagel [6], and Stahl and Wilson [7].

Both modifications to the corpus of game theory can address some of game theory's descriptive inadequacies. However, both do so largely within the confines of a rational model. Agents with other-regarding preferences maximize just like the ones assumed in game theory, their preference function is simply different from (and more complex than!) the one assumed in the standard model. Level-1 agents also maximize expected utility, it is just that their beliefs differ from those assumed in the standard rational model. To the extent these solutions have such a strong rational bent they are subject to the ubiquity critique mentioned above—that the sheer frequency with which we encounter strategic situations rules out *computing* optimal strategies. They are also subject to an even more pedestrian criticism—that they all assume players realize they are involved in strategic interactions. While this might seem like an odd criticism, the reason people trained in game theory think about social situations as games is because they are trained to do so. Students being introduced to game theory for the first time generally do not do this. Instead, the instructor has to work sometimes very hard to get the student to see social situations as ones in which (1) the outcome depends jointly upon the decisions of the people involved; (2) the decision she will want to make depends on the decisions others make; and (3) that the process of revising decisions based on others' choices only ends, if it does, when no one can make him or herself better off by changing strategy.

Two related literatures in psychology suggest an approach to modelling how real people play games that avoids strong rationality assumptions and, indeed, even the idea that people necessarily understand that they are involved in a strategic interaction. The first is work by Gigerenzer and colleagues on fast and frugal heuristics. Consistent with the ubiquity critique of traditional game theory and its variants, Todd and Gigerenzer [8] argue that rationality is impossible "in a world where knowledge is limited, time is pressing and deep thought is often an unattainable luxury". Instead, they suggest that people rely on simple rules, often ones involving a single reason, for making choices. Research, beginning with Tversky [9], suggests that the process whereby we reach this single reason is one involving comparisons of attributes of one option *versus* another. In this process, we ignore comparisons of magnitudes that appear similar or, alternatively, overweight differences perceived as large and thus salient.[2] Here we extend this intuition to examine the implications of players choosing strategies based on payoff differences that are perceived as salient.[3]

Section 2 presents a model of strategy choice in which a player chooses his dominant strategy or the strategy associated with his largest salient payoff when such a strategy exists, otherwise best-responds to his opponent's dominant or salience-based strategy choice when that strategy exists, and chooses at random otherwise. The model provides predictions for all 2 × 2 games. For games where a player faces a configuration of payoffs for which one strategy dominates another, henceforth referred to as "Dom" configurations, that strategy is played. In games where one of the comparisons of payoffs involves the best and worst possible outcome (henceforth referred to as "H(igh)L(ow)" payoff configurations) the model predicts that the strategy a player chooses will correspond to the one offering the best payoff. This choice will be independent of the values of the intermediate payoffs facing the player. Finally, and in contrast to the predictions of game theory, there will be cases where the strategy chosen varies

[2] Examples of models of this type include [10–12] in psychology, and [13–20] in economics.

[3] For a related approach, see [1] and [21].

systematically with the relative values of the payoffs the player faces. Payoff arrangements producing this result are referred to as High Intermediate/Intermediate Low (HI/IL) configurations.

When we move from considering the play of a single player to the interaction between two players engaged in a game, these observations imply that in games involving payoff configurations for which both players have dominating strategies (Dom-Dom games), a single game outcome corresponding to the Nash equilibrium will result. A single outcome also obtains for Dom–HL and HL-HL games. In some of these games, the predicted outcome will always correspond to a Nash equilibrium, in others it will never correspond to an equilibrium, and in still others there are no pure strategy Nash equilibrium outcomes. For all other configurations of payoffs (Dom-HI/IL, HL-HI/IL, HI/IL-HI/IL), the model predicts that the outcome of the game will vary systematically from one cell to another as a function of the relative values of the payoffs to the player(s) facing the HI/IL payoff configuration(s). Interestingly, all these predictions also follow from a Cognitive Hierarchy model assuming Level-1 players. That is, the predictions of a model involving simple comparisons of payoff differences exactly mirror those of a model that assumes expected utility maximization, albeit with uniform priors on opponents' behavior.

For certain HI/IL payoff configurations, neither of a player's strategies will be perceived as salient because their associated payoff differences are equal. Here, what will stand out to the player is her opponent's dominant or salient strategy, to the extent one exists, and what will be clear is that the right strategy choice is the best response to the opponent's choice.[4] It is not that people cannot understand that a social situation is strategic but that this realization must come about endogenously in response to specific features of the situation.

Interestingly, in these cases the predictions that follow if players base their choices on salience diverge from those implied by the hypothesis that players are Level-1 boundedly rational in favor of the Nash equilibrium predictions that follow from perfectly rational behavior. Thus, as Todd and Gigerenzer [8] require, this simple heuristic for play in 2 × 2 games performs "comparably to more complex algorithms". Section 3 presents experimental results supporting these predictions. Section 4 applies the model to other two-person games. Section 5 concludes.

2. Salience and Play in 2 × 2 Games

We develop a model of salience-based play in 2 × 2 games and apply the model to identify payoff structures which always result in Nash equilibrium play for salience-based players, and games which never result in Nash equilibrium play for such players. Consider a generic 2 × 2 game of the form in Table 1.

Table 1. A generic 2 × 2 game.

	Left ($\boldsymbol{l}$)	**Right ($\boldsymbol{r}$)**
Up ($\boldsymbol{u}$)	$x_1(u,l), x_2(u,l)$	$x_1(u,r), x_2(u,r)$
Down ($\boldsymbol{d}$)	$x_1(d,l), x_2(d,l)$	$x_1(d,r), x_2(d,r)$

4 If neither player has dominant or salient strategy, players choose at random.

Index payoffs of the row player (P1), by $x_1(s_1, s_2)$ where $s_1 \in \{u, d\}$ denotes his strategy, and $s_2 \in \{r, l\}$ denotes the strategy of the column player (P2). Analogously, index the payoffs of P2 by $x_2(s_1, s_2)$.

For the purposes of examining the implications of salience in games, we begin by defining those payoffs in the game that are perceived as salient by a player.

Definition 1 (Salient Payoffs): Two payoffs, $x_1(s_1, s_2')$ and $x_1(s_1', s_2')$ are *salient* (for P1) if $|\, x_1(s_1, s_2) - x_1(s_1', s_2)| < |\, x_1(s_1, s_2') - x_1(s_1', s_2')|$.

If $s_1 = u, s_1' = d, s_2 = l$ and $s_2' = r$, then payoffs $x_1(u, r)$ and $x_1(d, r)$ are salient if

$$|\, x_1(u, l) - x_1(d, l)| < |\, x_1(u, r) - x_1(d, r)| \quad (1)$$

When the difference in payoffs to P1 from choosing U or D, conditional on P2 choosing L, are not equal to the difference in payoffs to P1, conditional on P2 choosing R, only those payoffs to P1 associated with the larger difference are perceived as salient. When the differences are equal and of the same or opposite sign, no payoffs are perceived as more salient than the others.

Definition 1 captures the intuition that larger differences in payoffs are more salient than smaller differences. This intuition is also a feature of salience applied to decisions under risk [19]. The behavior of a player guided by salient payoffs can be summarized in the following lexicographic decision rule:

Strategy $\mathbf{s_0(P1)}$: Choose the strategy with the larger salient payoff for P1. If no payoffs are salient for P1, but one strategy is dominant, choose the dominant strategy. If this is not possible and P2 has a strategy recommended by his largest salient payoff or dominant strategy, best respond to that strategy choice. Otherwise, choose randomly.

Strategy $\mathrm{s_0(P2)}$ is defined analogously for P2. When our statements apply to either player we will simply refer to this strategy as s_0. In essence, a player following $\mathrm{s_0}$ is naïve to the strategic nature of the interaction and bases his strategy choice solely on his own salient payoffs when they are salient, and only realizes the strategic nature of the interaction when his own payoffs are not salient and the only distinctive features of the game involve the opponent's payoffs. One might think of players who choose according to $\mathrm{s_0}$ as ones who jump to what appears to be the "right" answer based only on the evaluation of their own payoffs and only come to reflect on the strategic nature of the situation if they are not struck by an obvious or intuitive solution.[5] Bordalo *et al.* [19] refer to decision makers whose choices are guided by salient payoffs in risky decisions as *local thinkers*. Similarly, we refer to agents who follow strategy $\mathrm{s_0}$ as *local players*.

We contrast $\mathrm{s_0}$ with the strategy of a Level-1 player who best-responds as if his opponent is a Level-0 player who chooses either strategy with equal probability:

Strategy $\overline{\mathbf{s}}_{\mathbf{0}}$ (for P1): Suppose player 1 in a 2 × 2 game forms a uniform prior over the probability that Player 2 plays his two strategies, s_2 and s_2'. Let $x_1(s_1, s_2)$ be the payoff to Player 1 if she plays s_1 and Player 2 plays s_2. Choose strategy s_1 over strategy s_1' if condition Equation (2) holds:

$$0.5x_1(s_1, s_2) + 0.5x_1(s_1, s_2') = E[x_1(s_1)] > 0.5x_1(s_1', s_2) + 0.5x_1(s_1', s_2') = E[x_1(s_1')] \quad (2)$$

where $E[x_1(s_1)]$ denotes the subjective expected value to Player 1 from playing strategy s_1.

5 This type of sequential decision making process is discussed by Frederick [22] in the context of the Cognitive Reflection Test.

Under strategy $\bar{s}_0$, Player 1 acts as a subjective expected utility maximizer. Intriguingly, for 2 × 2 games in which each player has payoffs which are salient, strategy s_0 and $\bar{s}_0$ prescribe the same behavior.

Definition 2: For a given player, two strategies, s and s', are *observationally equivalent* in game, Γ, if an outside observer cannot infer whether s or s' was played by that agent, after observing her actions.

Proposition 1: For all 2 × 2 games in which each player has salient payoffs, strategies s_0 and $\bar{s}_0$ are observationally equivalent.

Proof: Suppose, without loss of generality that $x_1(s_1, s_2)$ and $x_1(s_1', s_2)$ are salient (for P1) and that $x_1(s_1, s_2) > x_1(s_1', s_2)$. Also suppose $x_1(s_1', s_2') > x_1(s_1, s_2')$ Then $x_1(s_1, s_2) - x_1(s_1', s_2) > x_1(s_1', s_2') - x_1(s_1, s_2')$. Rearranging terms and scaling each payoff by the same constant, we have:

$$0.5x_1(s_1, s_2) + 0.5(x_1(s_1, s_2')) = E[x_1(s_1)] > 0.5x_1(s_1', s_2') + 0.5x_1(s_1', s_2) = E[x_1(s_1')] \quad (3)$$

A player guided by s_o and a player guided by $\bar{s}_0$ would each play s_1. Note that if $x_1(s_1', s_2') < x_1(s_1, s_2')$, then s_1 is a dominant strategy for x_1, and it yields a higher expected payoff. Thus, a player guided by $\bar{s}_0$ again chooses s_1 which is also the strategy with the larger salient payoff. ■

Salience-based choice may thus explain *why* it appears that players often play *as if* they are Level-1 boundedly rational for the class of games we consider. Camerer [23], among others, has noted the strong performance of the Level-1 model in such games. While the results in Section 2.1 will be stated for local players, Proposition 1 implies that these results also extend to players who are Level-1 boundedly rational, as in $\bar{s}_0$.

2.1. Implications of Own-Payoff Salience

Depending on whether there are ties in the payoffs of one or both players and whether the identity of the players is viewed as important there are between 78 [24] and 1431 [25–26] unique 2 × 2 games. In a compromise between completeness and brevity, we will focus our discussion on Robinson and Goforth's [27] typology of 144 strictly ordinal games, depicted in blocks of 36 games in Figures 2–5. A bird's eye view of the entire set is provided in Figure A1 in the appendix.

In Robinson and Goforth's typology there are 12 possible payoff structures for each player. For six of these, there is a dominating strategy so 36 of the games are solved by dominance. The games where both players have dominating strategies are the ones clustered in purple. According to game theory, the outcome in these will be the dominating Nash equilibrium (where here and throughout the figures in Section 2, Nash equilibria are indicated by yellow cells containing a payoff in bold.) An additional 72 games are dominance solvable—one player has a dominating strategy and the other best responds. These are the clusters of games shaded in beige. The two clusters of games in blue involve payoff configurations that produce coordination and anti-coordination games, respectively. For these games, traditional game theory predicts that one of the two Nash outcomes will obtain but does not specify which will occur. Finally, there are two clusters of games in green. These are cyclic games for which game theory makes no pure strategy predictions.

Figure 2. The class of 144 strictly ordinal 2 × 2 games (games 1–1 through 6–6).

Figure 3. The class of 144 strictly ordinal 2 × 2 games (games 1–7 through 6–12).

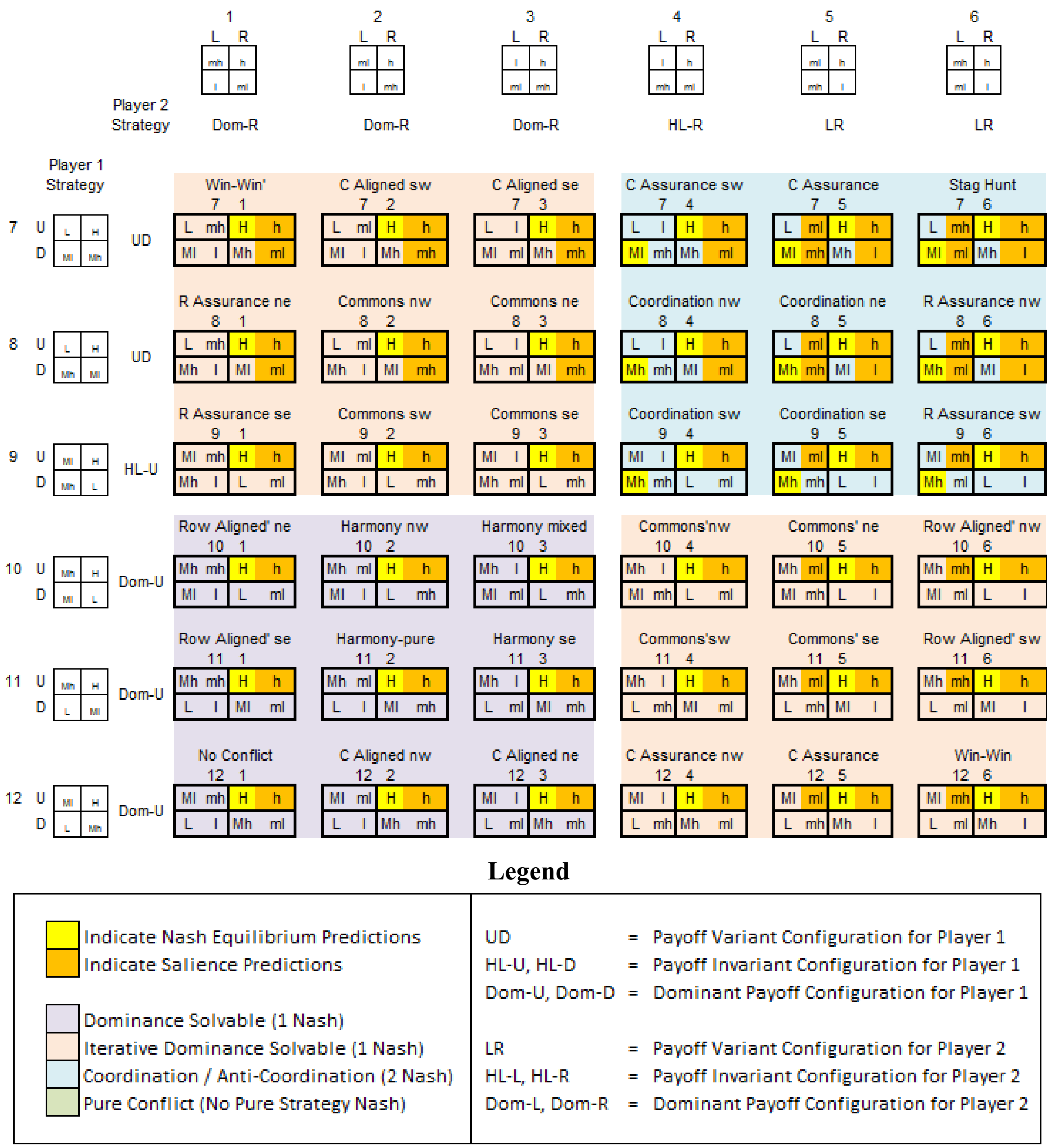

Figure 4. The class of 144 strictly ordinal 2 × 2 games (games 7–1 through 12–6).

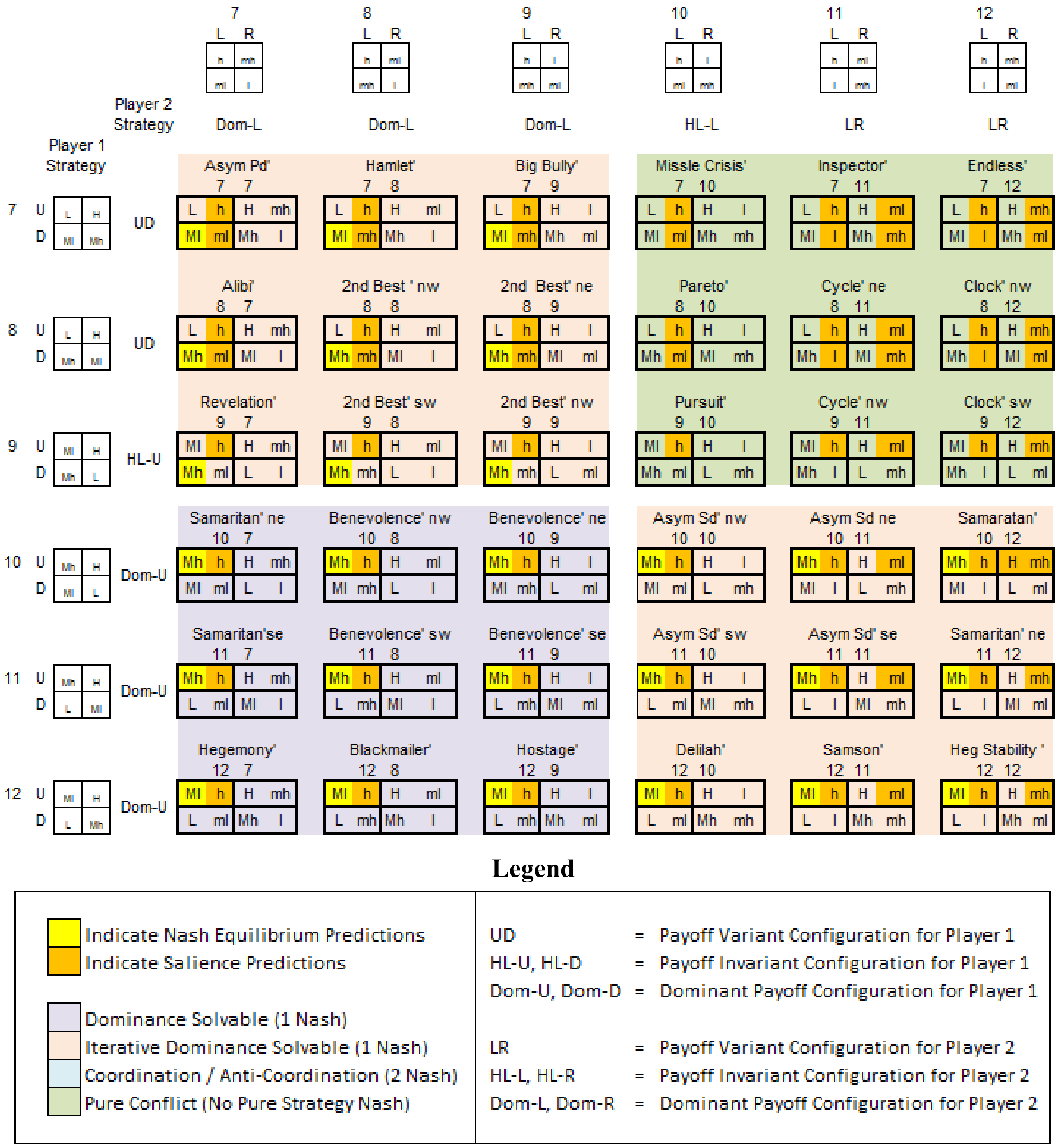

Figure 5. The class of 144 strictly ordinal 2 × 2 games (games 7–7 through 12–12).

With the overall structure of the figures summarized, we now turn to finer details of the topology. In the tables, payoffs for Pl are ordered H, M(edium)h(igh), M(edium)l(ow), L(ow) and are ordered h, mh, ml, l for P2. In each of Figures 2–5, payoff structures for Player 1 are shown in the un-shaded matrices in the leftmost column. Payoff structures for Player 2 are shown in the un-shaded matrices in the top row. Notice that in all four tables there are sets of games solved by dominance (in purple) and by iterated dominance (in beige.) Figure 5, like Figure 2, contains a subset not solvable by game theory (in green). In contrast, Figures 3 and 4 each contain a subset of games in blue. Those in Figure 3 are anti-coordination games while those in Figure 4 are coordination games.

Games that have been the focus of most inquiry in economics are those along the diagonal running from the lower-left to upper right in Figues 3 and 4. These include games involving varying degrees of preference incompatibility like the Prisoner's Dilemma in the lower left and the Chicken game in the upper right (for Figure 3) and games involving varying degrees of preference compatibility (from the "no conflict" game at one extreme to the stag-hunt at the other in Figure 4).

Now consider the implications of strategy s_0 for behavior in these games. For a local player facing a payoff configuration with a dominating strategy (configurations labeled Dom-U(p) or Dom-D(own) and Dom-L(eft) or Dom-R(ight) in the figures), both differences in payoffs have the same sign. The dominant strategy will be selected either because one of the differences is larger, the payoffs associated with that difference are salient, and the player selects the strategy with the largest salient payoff, or because the differences are equal and the player checks for a dominant strategy.

For two other payoff configurations facing either player, the largest and smallest payoffs are juxtaposed. For these configurations, labeled H(igh)L(ow), the strategy corresponding to the larger payoff will be selected and this will be true independent of the values of the other payoffs. Payoff configurations for which this is the case are denoted HL-U(p) and HL-D(own) in the column labeled "Player 1 Strategy" and HL-L(eft) and HL-R(ight) in the row labeled Player 2 Strategy.

The third possible configuration of payoffs involves comparisons of the best and worst outcomes with outcomes of intermediate values where the differences are of opposing signs and different or equal magnitudes. These are the H(igh)I(ntermediate)/I(ntermediate)L(ow) payoff configurations and are simply labeled UD configurations (for Player 1) or LR configurations (for Player 2) in the figures since either strategy may be played depending on the precise payoff differences. For HI/IL payoff configurations where the differences are not equal, local players will choose the strategy associated with the larger salient payoff. Note however, that the identity of this strategy will vary with the values of the intermediate payoffs to the extent that changes in these values may reverse the absolute magnitudes of the differences. For HI/IL configurations where the differences are of equal magnitude and opposite sign, a local player will move on to consider the salience of his opponent's payoffs. In Figures 2–5, the strategy choices and game outcomes with local players when their own payoffs are salient are denoted in orange.

To provide a clearer picture of how the play of local players compares to that of fully rational ones, Figures 6–9 reorder the payoff configurations from Figures 2–5, presenting those payoff structures where there is a dominating strategy first, then the HL configurations, and then the HI/IL configurations. As before, the combined Figure A2 is provided in the appendix.

As is clear in Figure 6, because local players obey dominance, the predictions following from the model of salience-based play and from the Nash solution concept coincide in games where both players have dominating strategies. For cases depicted in Figures 7 and 8 where one player has a dominant strategy but the other faces a HL or a HI/IL payoff structure, the former chooses the dominant strategy. The player facing a HL structure will choose the strategy offering the best possible outcome. As a consequence, in half of the Dom-HL cases, the outcomes following from iterated dominance and salience coincide (indicated by cells in games that are half yellow and half orange) whereas in the other half they diverge (where one cell in a game is yellow and another is orange). For a player facing a HI/IL configuration, the choice will vary as the relative magnitudes of the payoffs change the differences in payoffs across strategy choices. In Dom-HI/IL cases, the outcome predicted by salience coincides with that predicted by iterated dominance half the time but deviates the other half of the time.

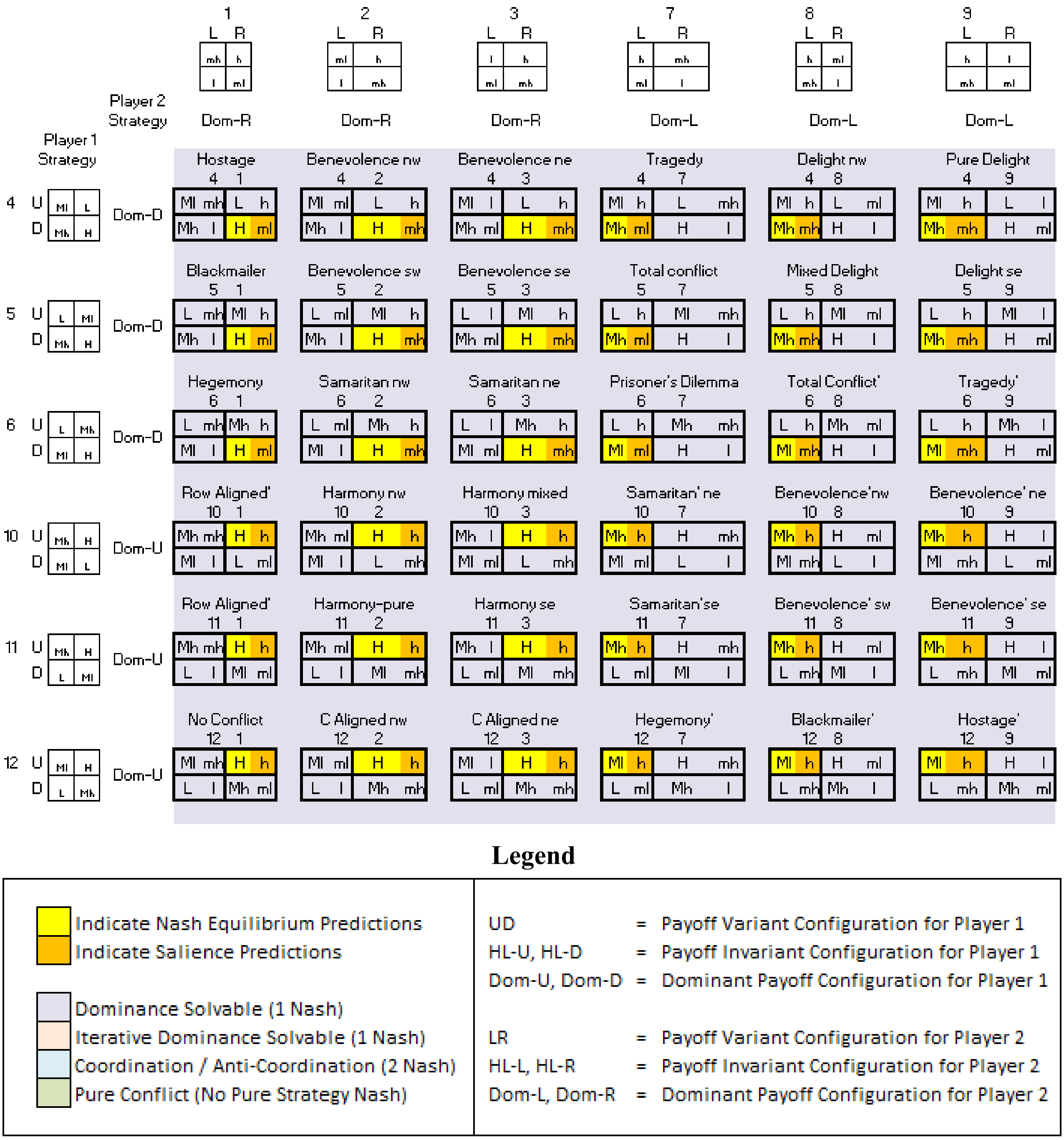

Figure 6. Strictly ordinal 2 × 2 games containing a dominant strategy equilibrium.

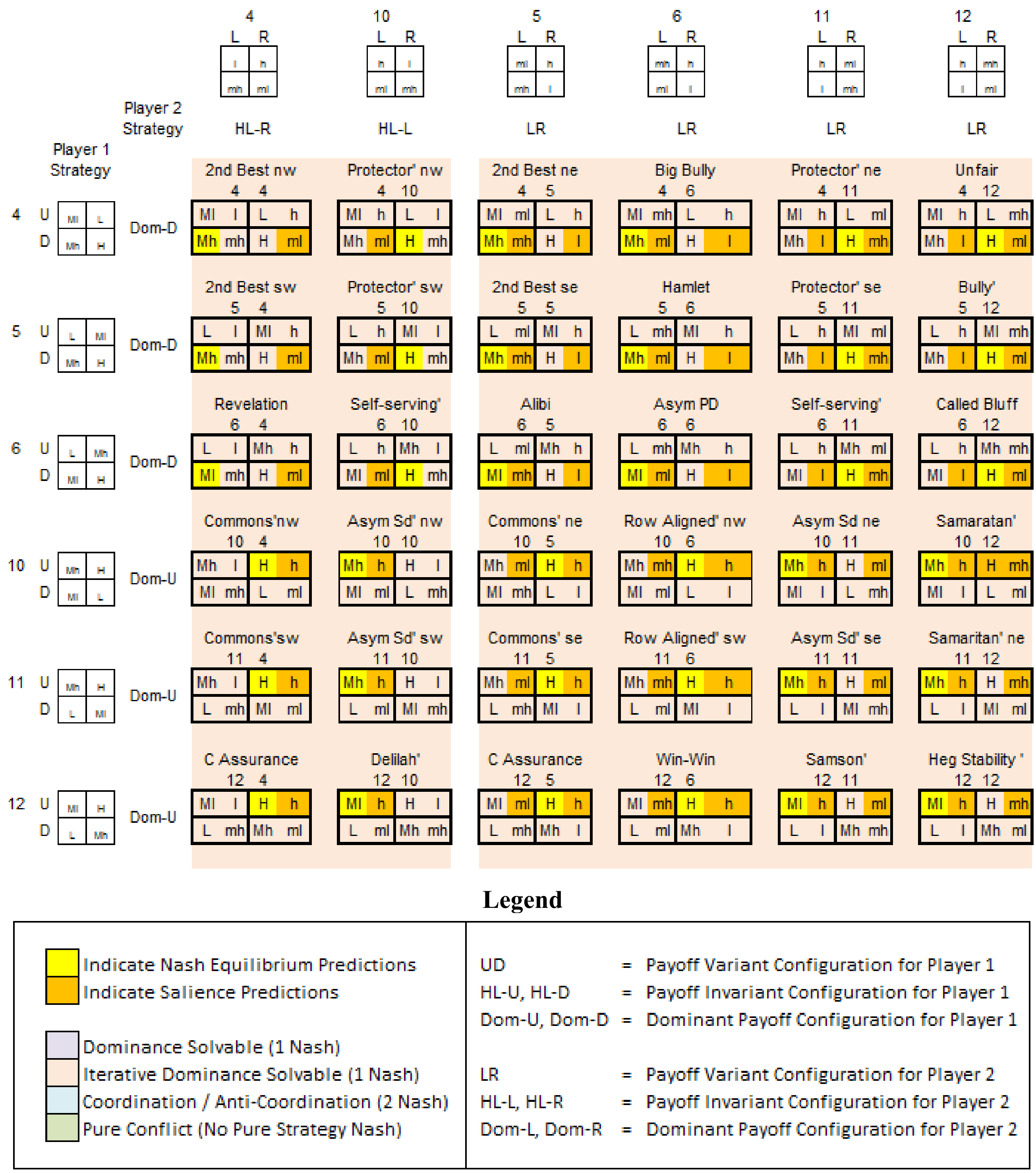

Figure 7. Strictly ordinal 2 × 2 dominance solvable games with dominant strategy for P1.

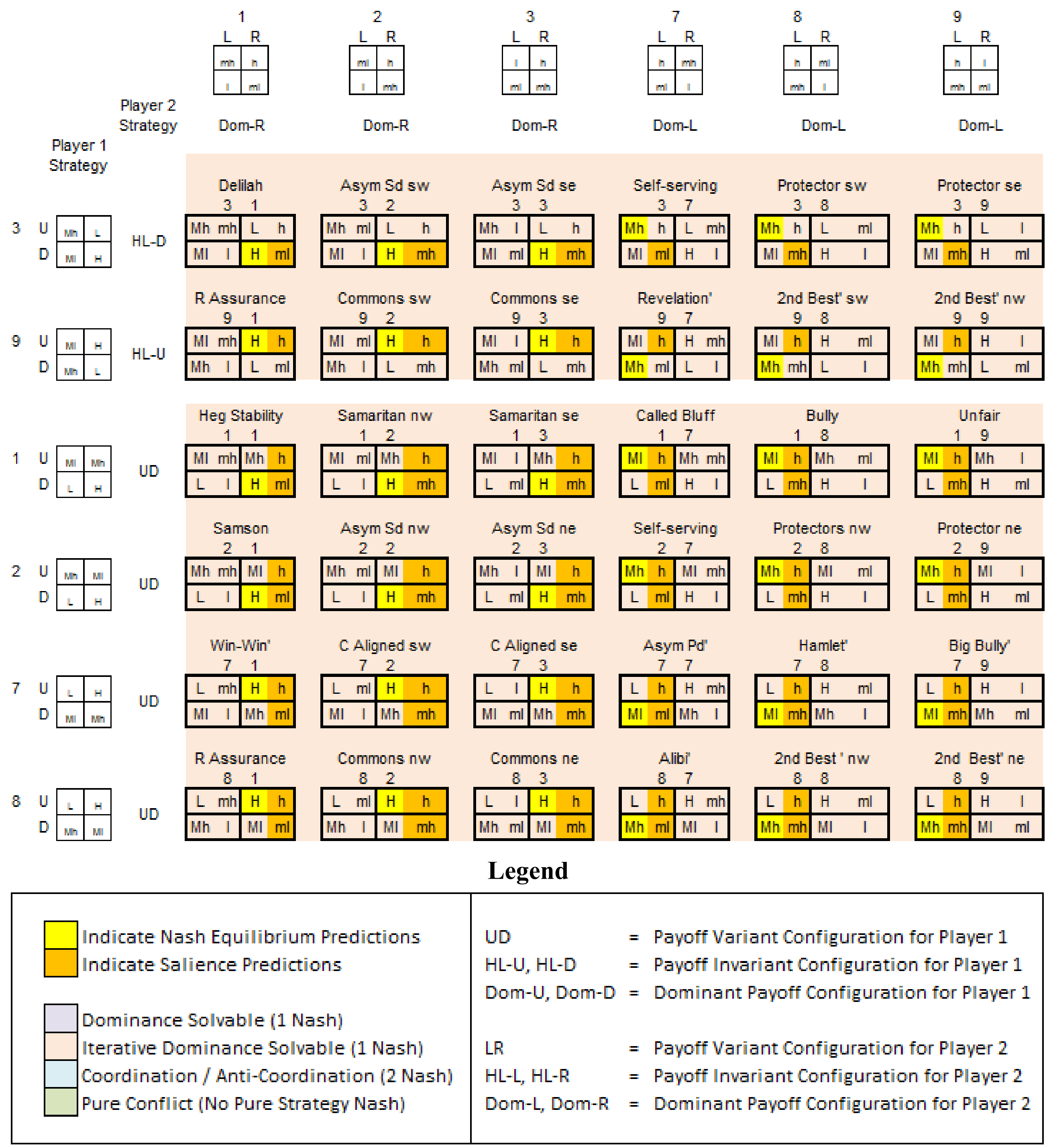

Figure 8. Strictly ordinal 2 × 2 dominance solvable games with dominant strategy for P2.

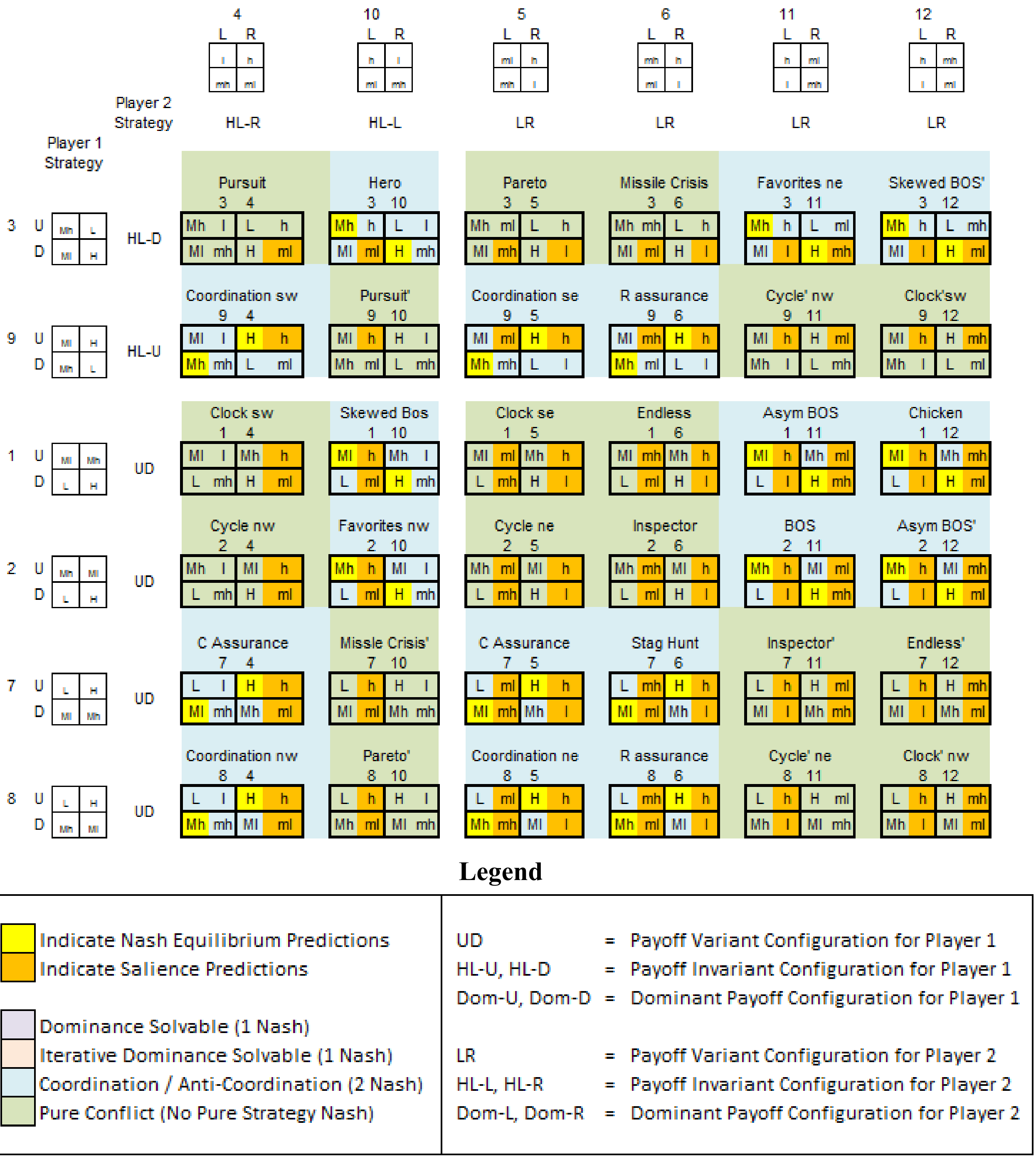

Figure 9. Strictly ordinal 2 × 2 games with zero or two pure-strategy Nash equilibria.

Figure 9 contains the games for which players face either HL or HI/IL payoff configurations. There are four games (those in the upper left corner of the matrix) where both players face HL payoff configurations. In these games, the model predicts that play will be invariant to the relative magnitudes of the payoffs (as is predicted for all games in traditional game theory). The predicted outcome in the Coordination sw game on the lower left corresponds to the payoff dominant Nash equilibrium. In the anti-coordination Hero game to the upper right (a variant of the Battle of the Sexes game), the predicted outcome is neither of the Nash equilibria. The other two HL/HL games are cyclic games in which there are no pure strategy Nash equilibria. In the remaining games in Figure 9 (all involving either coordination, anti-coordination or cycles), own-payoff salience predicts either two or four outcomes as a function of the relative values of the intermediate payoffs for one or both players. [6]

To summarize, in 2 × 2 games where the choice can be resolved by evaluation of a players' own payoffs, there will be dominance solvable, coordination, anti-coordination and cyclic games involving Dom-HL or HL-HL payoff configurations for which the strategy choices and game outcomes will be invariant to the relative values of the payoffs. Henceforth, these are referred to as *payoff-invariant* games.

In contrast to the predictions of traditional game theory, there will also be dominance solvable, coordination, anti-coordination and cyclic games where strategy choices and the outcome of the game will be sensitive to the relative values of payoffs. In Dom-HI/IL games and HL-HI/IL games, 2 outcomes are possible. For HI/IL-HI/IL games, four outcomes are possible. We call these *payoff-variant* games. More generally, we state:

Definition 3: For a fixed ordinal ranking of payoffs, a game is *payoff invariant* if the outcome that occurs does not depend on the magnitudes of payoffs.

Proposition 2: A strictly ordinal 2 × 2 game played by local players is payoff invariant, if and only if it has the form of either Dom-Dom, Dom-HL, or HL-HL.

Proof: As illustrated in Figures 2–5, there are six classes of payoff structures, depending on the configurations of the high, low, and intermediate payoffs for each player. These structures are Dom-Dom, Dom-HL, Dom-HI/IL, HL-HL, HL-HI/IL, and HI/IL-HI/IL. All 144 games in the figures can be classified into one of these structures. A configuration where both local players have dominant strategies (Dom-Dom) is payoff invariant (and will result in a Nash equilibrium) since the players always play their dominant strategies (each player either chooses the strategy with the largest salient payoff, which will be the dominant strategy if one exists, or chooses the dominant strategy when no payoffs are salient). When one player has a dominant strategy and the other player does not, but has his highest and lowest payoff in the same row (if he is a column player) or in the same column (if he is a row player) we have a Dom-HL configuration. This configuration is also payoff invariant (but will not necessarily induce a Nash equilibrium) since the local player with the dominant strategy continues to play that strategy, and the other local player always plays the strategy with his highest payoff (since the highest and lowest payoffs are salient, as the inequality $|\mathrm{H}-\mathrm{L}| > |\mathrm{Mh}-\mathrm{Ml}|$ always holds for strict-ordinal games). Analogous reasoning leads us to conclude that HL-HL configurations are payoff invariant (but do not necessarily result in a Nash equilibrium). For an HI/IL configuration a local player either compares $|\mathrm{H}-\mathrm{Mh}|$ and $|\mathrm{Ml}-\mathrm{L}|$ or $|\mathrm{H}-\mathrm{Ml}|$ and $|\mathrm{Mh}-\mathrm{L}|$. The particular direction of the inequality determines which payoffs are salient, and can thus influence the strategy choice since a local player plays the strategy with the larger salient payoff. Thus, games where either player has an HI/IL configuration are not payoff invariant. ■

6 These predictions, to the extent they are borne out empirically, have interesting implications for *engineering* outcomes in strategic settings. Within the confines of traditional game theory, the only ways to influence outcomes in games are through side-payments and penalties that change the game structure or by changing players' preferences over the outcomes. In the model presented here, side-payments and penalties can influence the outcome in payoff-variable games even if the fundamental structure remains the same. We may also be able to construct situations in which the preferred outcome will always obtain (e.g., the payoff dominant outcome in the Coordination sw game) or ones where it will never occur like the Hero game (in case we do not like the players.).

Note that the top row and the left-most column (not shaded) in Figures 2–9 provide a more detailed characterization of the possible payoff structures for each player. Using these payoff structures, we can characterize the set of games played by local players (or by Level-1 boundedly rational players) for which a Nash equilibrium will *always* obtain out of the entire set of 144 2 × 2 games.

As shown in Figures 2–9, Player 1 has two possible strategies, U and D, and Player 2 has two possible strategies, L and R. We refer to particular strategies and payoff structures in the figures by a string of letters first noting the payoff structure and then the strategy determined by that structure for local players. For instance, HL-U corresponds to a HL payoff structure in which Player 1 plays U.

Proposition 3: A pure-strategy Nash equilibrium will always obtain in a 2 × 2 strictly ordinal game with local players if and only if the game has a dominant strategy equilibrium *or* if it has one of the following payoff structures:

(i) HL-U/HL-R
(ii) HL-D/Dom-R
(iii) HL-U/Dom-R
(iv) Dom-U/HL-R
(v) Dom-U/HL-L

For general representations of these payoff structures, see the upper row and left-most column in Figures 2–9. The observation that each of these payoff structures always yields a Nash equilibrium can be seen directly in Figures 6–8 since in these games, the orange cells (played by local players) always coincide with a yellow (Nash equilibrium) cell. The proof is straightforward and is thus omitted. There are a total of 49 games classified under Proposition 3 (36 games with dominant strategy equilibria, and 13 games classified in (i) through (v)). Thus, 49 out of 144 strictly ordinal 2 × 2 games will *always* produce a Nash equilibrium for local players and players who are Level-1 boundedly rational, even though these players are not as strategically sophisticated as assumed in traditional game theory. There are additional games which may produce an equilibrium, depending on the payoff structure.

We can also characterize the set of 2 × 2 games played by local players (or by Level-1 players) for which a pure strategy Nash equilibrium will *never* obtain.

Proposition 4: A pure-strategy Nash equilibrium will never obtain in a 2 × 2 game with local players if and only if the game is a cyclic game *or* if it has one of the following payoff structures:

(i) HL-D/HL-L
(ii) HL-D/Dom-L
(iii) HL-U/Dom-L
(iv) Dom-D/HL-R
(v) Dom-D/HL-L

The result that each of these payoff structures never yields an equilibrium can be seen directly in Figures 7–9 since in these games, the orange cells never coincide with a yellow (Nash equilibrium) cell. The proof is straightforward and is thus omitted. There are a total of 31 games classified under Proposition 4 (18 cyclic games, and 13 games classified in (i) through (v)). Thus, 31 out of 144 strictly ordinal 2 × 2 games will *never* produce a Nash equilibrium for local players.

In total, Propositions 3 and 4 identify 80 payoff invariant games, indicating that in over half of all ordinal 2 × 2 games, the games are "rigged" for local players. They cannot escape the outcome of the strategic interaction they face. As noted in Proposition 1, these same predictions follow if subjects behave as Level-1 players in a Cognitive Hierarchy model. A Level-1 player chooses his strategy based on the assumption that the other player chooses between her strategies with equal probability—(*i.e.*, choosing the strategy that maximizes expected utility given a uniform prior on the other player's choices). For Dom payoff structures, the dominant strategy will always have a higher expected value than the dominated strategy. Similarly, for HL games, the expected value associated with the strategy containing H and Ml or H and Mh will always be greater than the expected value associated with the strategy containing Mh and L or Ml and L, respectively. For HI/IL structures, on the other hand, the expected value of the two strategies (e.g., H and L *versus* Mh and Ml) will vary with the values of the intermediate payoffs, but they do so in lockstep with differences H−Ml and Mh−L or H−Mh and Ml−L.

2.2. Implications of Other-Payoff Salience

We now consider the implications of the model when no payoffs are salient and neither strategy is dominant for Player 1. In this case, the cardinality of payoff differences must be the same for each pair of payoffs, obtained when holding Player 2's strategy fixed:

$$x_1(u,l) - x_1(d,l) = -(x_1(u,r) - x_1(d,r)) \tag{4}$$

These cases can only arise in situations where one or both players' payoff configurations are of the HI/IL variety. When this occurs, s_0 implies that an agent is spurred to think strategically and best responds to the strategy he anticipates his opponent will play given the perceived salience of the opponent's payoffs. Figures 10 and 11 summarize Player 1's strategy choices conditional on the salience of Player 2's payoffs for the relevant subset of the 144 games. The predicted outcomes of s_0 are again in orange and the Nash equilibrium outcomes are in bold and yellow.

As is clear from Figures 10 and 11, games that had two outcomes in Figures 2–9 now have a single outcome and those that had four outcomes now have two. Note also that the outcome or outcomes predicted by s_0 now coincide entirely with the Nash equilibrium outcome(s) when they exist. The reason is simple. The disequilibria outcomes in Figures 2–9 only arise for two sets of cases. One set involves one player having a dominating strategy and the other's salience perceptions recommending the non-equilibrium response. The second set involve coordination and anti-coordination games where one player's salience perceptions lead him to choose the strategy consistent with one of the Nash equilibrium outcomes while the other player's perceptions lead him to choose the strategy corresponding to the other equilibrium outcome.[7] When one player becomes strategic and best responds to the opponent's salience-based strategy, the opponent's play basically determines the outcome of the game. Note finally that s_0 still makes unique predictions for games where there are no pure Nash equilibria.

[7] Here again, the model suggests that we may be able to engineer the outcome that obtains in strategic settings by using side-payments or penalties to convert a HI/IL payoff structure to a Dom or HL structure for which one of the Nash equilibrium outcomes is assured.

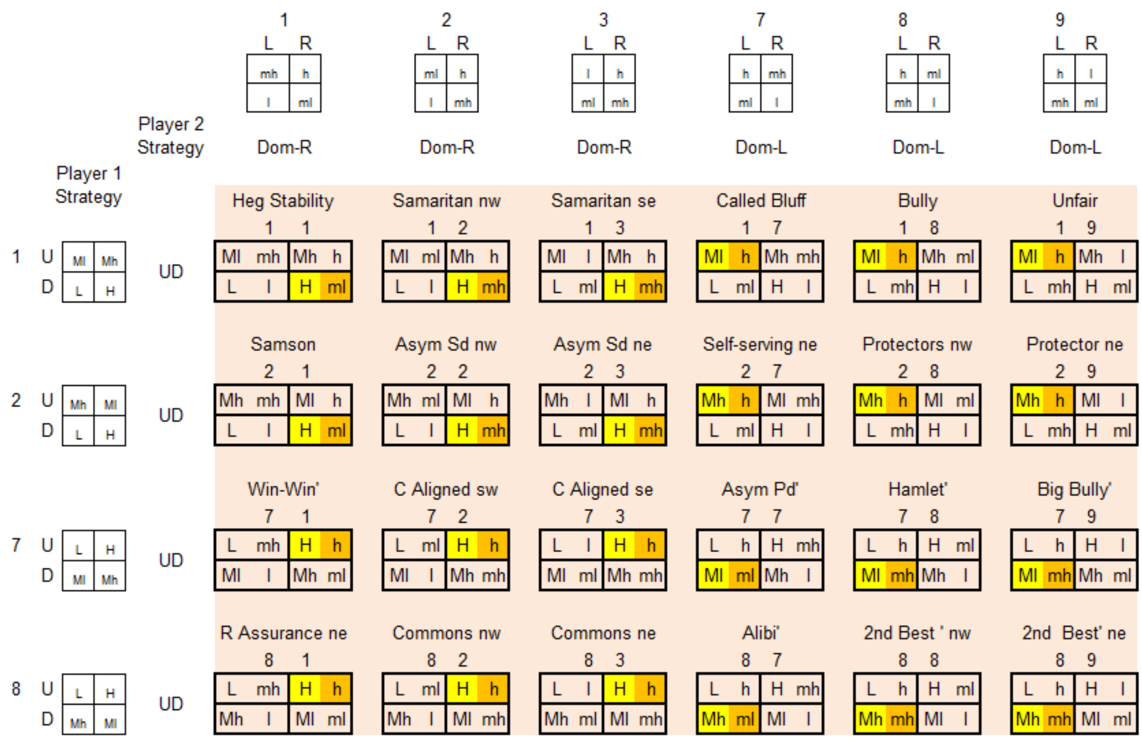

Figure 10. Implications of "other-payoff salience" in Dominance Solvable Games for Player 1. [8]

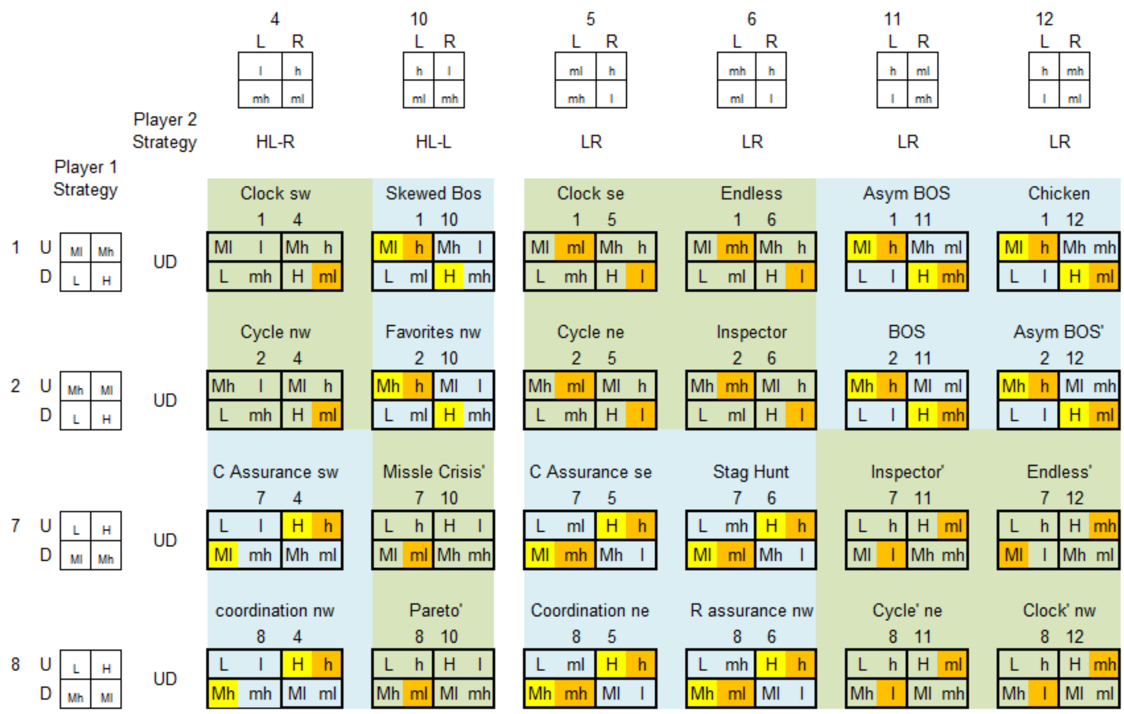

Figure 11. Implications of "other-payoff salience" in Cycle and Coordination Games for Player 1.

[8] Predicted outcomes are the same for Player 2 except for cyclic games with HI/IL structures for both players. In these games, Player 2s best responses would result in the upper left and lower right (rather than lower left and upper right) outcomes in each game.

Recall that in Proposition 1, we showed the observational equivalence of salience and Level-1 strategies for cases where each local player has salient payoffs. In this section, we considered the case where one local player has no salient payoffs. For games with a unique pure strategy Nash equilibrium, denote the Nash equilibrium strategy for P1 by $s_N(P1)$.

Proposition 5: Consider any 2 × 2 strictly ordinal game with a unique pure-strategy Nash equilibrium. If Player 1 has no salient payoffs or dominant strategies, and Player 2 does have salient payoffs, then strategies $s_0(P1)$ and $s_N(P1)$ are observationally equivalent.[9]

Proof: In the absence of salient payoffs or dominant strategies for P1, strategy $s_o(P1)$ dictates that P1 best-responds to the salient or dominant strategy of P2. Note that since P2 has salient payoffs, the differences in payoffs between P2's strategies are not equal and any dominant strategy for P2 will correspond to the strategy with the larger salient payoff. If P2 has a dominant strategy, then P1's best-response to P2's strategy clearly results in an equilibrium. If P2 does not have a dominant strategy, but has either an HL or HI/IL configuration, P2 will choose the strategy with the larger salient payoff. This strategy is always a best-response to an equilibrium strategy played by P1 (when a pure strategy equilibrium exists), and thus it is an equilibrium strategy for P2. Since P1 is playing his best-response to this equilibrium strategy for P2, the result will be the pure strategy Nash equilibrium. ■

Proposition 5 extends straightforwardly to 2 × 2 strictly ordinal games with multiple pure-strategy Nash equilibria, in which case strategy $s_0(P1)$ is observationally equivalent to one of the Nash equilibrium strategies.

Propositions 1 and 5 collectively predict instances when a local player will appear to be Level-1 boundedly rational, and when the same player will appear to be the rational economic agent assumed in game theory.

Note that the behavior predicted in Proposition 5 does not follow if players are Level-1. Level-1s base their strategy choices on the assumption that their opponent will choose randomly irrespective of the payoff values that the opponent faces.

For cases where both players' own payoff comparisons are uninformative, local and Level-1 players choose at random.

[9] This result suggests yet a third way to engineer outcomes in games and "solve" coordination problems. If we can convince one player in a game to consider the salience of his opponent's payoffs and best respond before considering the salience of his own payoffs, the resulting game outcomes will be the same as those that arise if he has no salient payoffs and will correspond to Nash equilibria in games where they exist. Interestingly, if both players consider the salience of their opponent's payoffs first, the non-Nash equilibrium outcomes predicted by the model are mirror images of the ones when players consider salience in their own payoffs first. In summarizing the impact of communication for solving coordination problems, Camerer [22] (p. 357) notes that one-way communication "work like a charm" but 2-way communication is almost as bad as no communication at all. This is exactly what we would expect if one-way communication prompts one player to think in terms of his opponent's salient payoffs but two-way communication prompts both players to do so.

3. Experimental Results

To test these predictions, we analyze results from an experiment conducted at the Computable and Experimental Economics Laboratory (CEEL) at the University of Trento. The experiment involved 22 games, 20 of which are relevant here.[10] In all but one of these games, both players confronted three outcome variants of games in Figures 2–5 for which payoffs to Player 1 are H(igh), M(edium) and L(ow) and payoffs to Player 2 are h(igh), m(edium), and l(ow). The games played included five payoff-variant Battle of the Sexes games, three payoff-variant Stag-hunt games, three payoff-variant Cycle games (a pure conflict situation), four payoff-invariant Coordination SW games, a payoff-variant Coordination NW game, a payoff-invariant version of the Battle of the Sexes game known as the Hero game, three instantiations of a dominance solvable Hegemony Stability' game and a coordination SW game in which one player faced only two payoffs.

3.1. Design and Implementation

The experiment was conducted in four sessions, three of which involved 20 participants and one of which involved 18. Participants were recruited from the university community. Of the 78 participants, 49 were male and 29 were female. Experiments were conducted in Italian. At the beginning of each experimental session, subjects were seated at individual computer terminals separated by partitions. Half of the participants were assigned the role of Player 1 for the duration of the experiment and the other half the role of Player 2. Each participant was asked to read the instructions shown on his or her terminal screen while the laboratory administrator read them aloud. The instructions for Player 1s, translated into English, are shown in Table 2.

After reading the instructions, subjects proceeded with the experiment at their own pace. The screen shot in Figure 12 shows how games were presented. Subjects registered their choices by clicking their mouse on buttons at the bottom of the screens labeled "U" and "D" (or "L" and "R" for Player 2s).

After each choice they were asked to confirm their choice. Subjects received no feedback regarding the outcome of any of their choices until the end of the experiment. Games were presented to subjects in different randomly chosen orders. The presentation of payoffs corresponding to U or D and to L or R was counterbalanced.

3.2. Results and Analysis

The strategy for testing predictions following from S_0 is particularly straightforward for games containing three outcomes. To illustrate, consider the nine-game matrix shown in Figure 13 depicting a payoff-variable Battle of the Sexes game.

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[10] Two additional games, always presented to subjects last, examined how players behave when their opponent has more than 2 strategies available.

Table 2. Experiment Instructions for Player 1. [11]

For each of the following screens you are randomly paired with another participant in the experiment referred to as "Other". For each screen, you are presented with a choice between two options U or D. Simultaneously, Other is presented with a choice between two options L and R. You and Other will receive payoffs depending on the decision you make and the decision Other makes. Information about what you and Other will receive depending on the choices you both make will be provided in a table like the one shown below. For this table, if Other chooses his or her option L and you choose your option U, you receive 5 and Other receives 2. If instead you choose your option D, you receive 7 Euros and Other received 7 Euros. If Other chooses his or her option R and you choose your option U, you receive 5 Euros and Other receives 4 Euros. If you instead choose your option D, you receive 1 Euro and Other receives 2 Euros.

If Other Choose L and you choose U	You receive 5.00 and Other receives 2.00
D	You receive 7.00 and Other receives 7.00
If Other Choose R and you choose U	You receive 5.00 and Other receives 4.00
D	You receive 1.00 and Other receives 2.00

For each question please indicate the option you would rather have. At the end of the experiment, one of these situations will be selected at random and you and the person with whom you are matched will be paid according to the decisions you both made. As such, in addition to the 5 Euro participation fee you can earn an additional amount between 0 and 13 Euros.

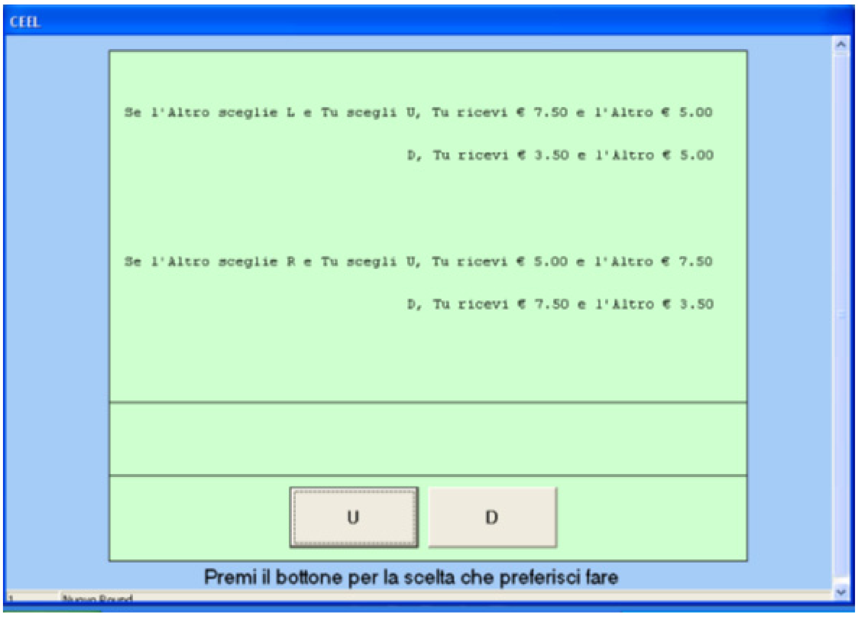

Figure 12. A Screen Shot of One of the Games Presented to Player 1s.

[11] Instructions for Player 2s were identical except they were to choose between strategies L and R (and their opponent's between U and D).

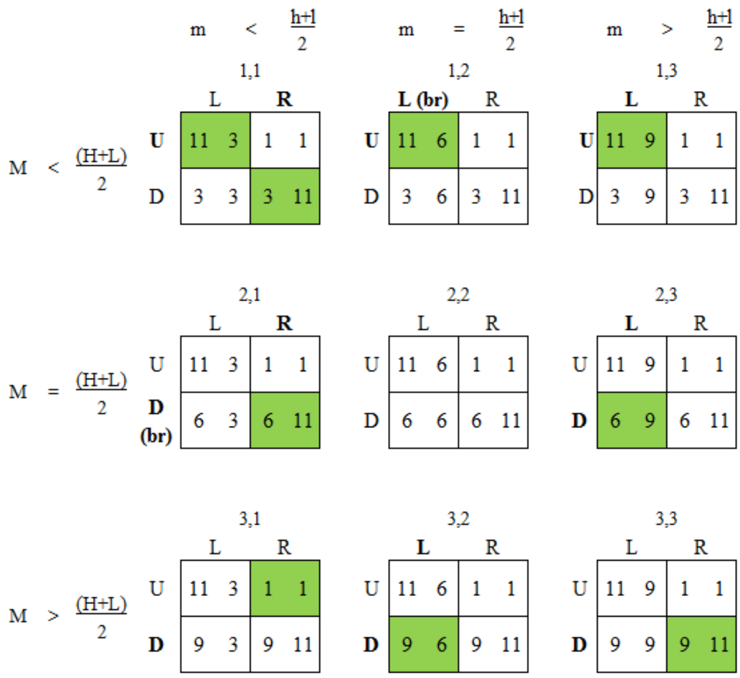

Figure 13. Testing Sensitivity of Play in a Payoff Variable Game.

For HI/IL−HI/IL payoff configurations such as those in Figure 13, game 1,1 corresponds to a situation where the value of the intermediate payoffs are less than half the sum of the best and worst outcomes. Thus, the difference H−M will be greater than M−L for Player 1 and the difference h−m will be greater than m−l for Player 2. In this case, Player 1s and Player 2s will choose the strategy containing the largest salient payoff, H and h, respectively. If we hold Player 1's intermediate payoff M constant and increase m to the point where h−m = m−l (e.g., Game 1,2), Player 1 will continue to choose the strategy containing the largest salient outcome H while Player 2 will best respond to that choice. If we continue to hold M constant and increase m such that it exceeds (h + l)/2 (as in Game 1,3), Player 1 continues choosing the strategy containing H but Player 2's salient payoffs are now m and l. Thus, Player 2 chooses the strategy containing m. The point is that for HI/IL-HI/IL games, strategy choices and game outcomes will vary systematically as M and m vary in value from below to above the sum of H and L (for Player 1), and the sum of h and l (for Player 2).

In contrast, for Dom-Dom, Dom-HL and HL-HL structures, players' strategy choices will be the same for all nine instantiations in the figure, although for Dom-HL and HL-HL games they may systematically deviate from the choices consistent with Nash equilibrium.

Dom-HI/IL and HL-HI/IL games are intermediate cases between the entirely payoff-invariant and entirely payoff-variant games with the strategy choice of Player 1, for instance, being insensitive to changes in his intermediate payoff, but the strategy choice of Player 2, and thus the outcome of the game, varying with the value of the intermediate payoff to Player 2.

For clarity, we will focus first on the results obtained for the subset of 14 games in which both players "own" salient payoffs predict their choices and then on those cases where one or both players must consider the salience of the other player's payoffs to determine which strategy to choose.

3.2.1. Own-Payoff Solvable Games with Payoff Invariant Structures

The Coordination SW game is one game in the upper left of Figure 9 for which the payoff configuration facing both players is of the HL variety. Figure 14 shows the three instantiations of this game played by subjects. In all three games, the payoffs 11 and 3 will be perceived as salient [12] and the strategies U and R are predicted to be chosen as a consequence. Shaded cells indicate the percentages of players choosing these strategies and the expected outcome in the game. In every case, the predicted strategies, U and R, are chosen more frequently than D and L and the most frequent game outcome, UR, is the one predicted by salience. Our hypothesis is that the proportion of U *versus* D and L *versus* R responses will not vary with the value of the intermediate payoff. A Cochrane's Q test cannot reject this null hypothesis for either Player 1 (Cochrane's Q = 0.67 $p = 0.72$, 2 df) nor Player 2 (Cochrane's Q = 4.333, $p = 0.1146$, 2 df).

Coord sw 1,1

	M=3.5	m=3.5	
	L	R	
U	3.5 3_s 26%	11^s 11^s 53%	79%
D	11 11 7%	3_s 3.5 14%	21%
	33%	67%	

Coord sw 3,1

	M=10.5	m=3.5	
	L	R	
U	10.5 3_s 24%	11^s 11^s 61%	85%
D	11 11 4%	3_s 3.5 11%	15%
	28%	72%	

Coord sw 3,3

	M=10.5	m=10.5	
	L	R	
U	10.5 3_s 13%	11^s 11^s 72%	85%
D	11 11 2%	3_s 10.5 13%	15%
	15%	85%	

Figure 14. Coordination sw: A Payoff Invariant Coordination Game.

To the extent that the observed outcome, UR, is also one of the Nash equilibria (along with DL) these results do not provide a strong test of s_0 relative to the hypothesis that players are rational.[13] However, now consider Figure 15, which compares the responses obtained in Coord sw 3,3 and those in the game Coord nw 3,3. The payoff values in the two games are identical but in Coord nw 3,3, Player 1's payoffs for UL and DR have been transposed.[14] In this case, Player 1's largest salient payoff of 11 now dictates choosing D and 95% of subjects choose that strategy. McNemar's test for correlated proportions confirms that the change in payoff structure has a significant effect on Player 1's strategy choices in the predicted direction ($p < 0.000001$, 1-tailed, 1 df). The payoff structure for Player 2s in Coord nw is unchanged from Coord sw and Player 2s continue to choose R. The result is that the predominant outcome in Coord nw is the disequilibrium DR, as predicted by s_0.

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[12] Superscript s and subscript s here and in the subsequent figures, indicate the larger and smaller salient payoffs in the games.

[13] Although it is worth noting that allowing for the additional Nash outcome DL adds very little (2% to 7%) in terms of explaining the outcomes observed in the games.

[14] Coord nw 3,3 is obtained by equating H and Mh and h and mh in the Coord NW game in Figure 9 (8,4). This converts the game from a payoff-variant HI/IL-HI/IL structure to a payoff invariant HL-HL structure. This illustrates a general consequence of moving from four distinct outcomes to three, namely, that the number of payoff configurations corresponding to a HI/IL structure declines and the number of configurations corresponding a Dom structure proliferate. For four distinct payoffs there are 12 possible payoff configurations, six of which are Dom, two of which are HL, and four of which are UD. For three distinct payoffs, there are six possible payoff configurations, four of which are Dom, and two of which are UD.

Coordination SW

	L	R
U	M l	H h
D	H h	L m

Coordination NW

	L	R
U	L l	H h
D	H h	M m

Coord sw 3,3

M=10.5 m=10.5

	L	R	
U	10.5 3_s 13%	11^s 11^s 72%	85%
D	11 11 2%	3_s 10.5 13%	15%
	15%	85%	

Coord nw 3,3

M=10.5 m=10.5

	L	R	
U	3_s 3_s 1%	11 11^s 5%	6%
D	11^s 11 2%	10.5 10.5 92%	94%
	3%	97%	

Figure 15. Equilibrium and Disequilibrium Play in Coordination Games.

3.2.2. Payoff Variant Games

Figure 16 shows responses obtained from players for the two instantiations of a dominance solvable game, a Stag-hunt game, three Battle of the Sexes games, and three instantiations of the Cycle game. In all cases s_0 predicts that the strategy choice for one or both players should vary with M and/or m. In the dominance solvable games, Player 1s face a Dom structure in which no payoffs are salient and payoff differences are of the same sign. As predicted by s_0, between 80% and 87% of Player 1s choose the strategy with the larger salient payoff, U. Player 2s face a HI/IL payoff structure. When m is low, the payoffs of 8 and 1 are salient and 51% of Player 2s choose L, the strategy with the larger salient payoff of 8. When m = 7.5, it is the largest salient payoff and here 100% of Player 2s chose the best response strategy R, as predicted by s_0. A McNemar's test for correlated proportions confirms that the proportions choosing L between the two games vary in the predicted direction with changes in the value of m ($p < 0.000001$, 1-tailed, 1 df).

In the Stag-hunt, Battle of Sexes, and Cycle games, both players face HI/IL payoff structures. In the Stag Hunt, the strategy choices predicted by salience (U and R) predominate, resulting in 88% of the predicted outcomes being the payoff dominant equilibrium. In the Battle of the Sexes, the strategy chosen by P1 varies as predicted from D to U as M varies from low to high (Cochrane's Q = 45.5152, $p < 0.0001$, 2 df) while P2's choice varies as predicted from L to R as m varies from low to high (Cochrane's Q = 51.1875, $p < 0.0001$, 2 df).

Behavior in the Cycle games exhibits the same type of pattern. Player 1s shift from playing U to D when the value of M goes from 1 to 9 between games 1,3 and 3,3, while Player 2s shift strategy from L to R between game 1,1, and 1,3. Cochrane's Q test confirm that the value of the intermediate payoffs M and m have significant effects on player' choices (for Player 1, Cochrane's Q = 27.5294, $p < 0.0001$, 2 df and for Player 2 Cochrane's Q = 44.24, $p < 0.0001$, 2 df).

Hegemony Stability' (a dominance solvable game)

HS 3,1

M=7.5 m=1

	L	R	
U	7.5^s 0 41%	8 1 39%	80%
D	0_s 8^s 10%	8 1_s 10%	20%
	51%	49%	

HS 3,3

M=7.5 m=7.5

	L	R	
U	7.5^s 0_s 0%	8 7.5^s 87%	87%
D	0_s 8 0%	8 7.5 13%	13%
	0%	100%	

The Stag-hunt (a coordination game)

SH1,1

M=0.1 m=0.1

	L	R	
U	0.1 0.1_s 5%	6^s 6^s 88%	92%
D	0.1 0.1 0%	0.1_s 0 7%	8%
	5%	95%	

The Battle of the Sexes (an anti-coordination game)

BOS 1,1

M=2 m=2

	L	R	
U	2 10^s 29%	2_s 2_s 7%	36%
D	1 1 51%	10^s 2 13%	64%
	79%	21%	

BOS 1,3

M=2 m=9

	L	R	
U	2 10 0%	2_s 9 10%	10%
D	1 1_s 3%	10^s 9^s 87%	90%
	3%	97%	

BOS 3,3

M=9 m=9

	L	R	
U	9^s 10 9%	9 9 81%	90%
D	1_s 1_s 1%	10 9^s 9%	10%
	10%	90%	

The Cycle Game (a conflict game)

1,1

M=1 m=1

	L	R	
U	0 10^s 43%	10^s 1_s 24%	67%
D	1 0 21%	1_s 1 12%	33%
	64%	36%	

1,3

M=1 m=9

	L	R	
U	0 10 4%	10^s 9 78%	82%
D	1 0_s 1%	1_s 9^s 17%	18%
	5%	95%	

3,3

M=9 m=9

	L	R	
U	0_s 10 1%	10 9 20%	21%
D	9^s 0_s 2%	9 9^s 77%	79%
	3%	97%	

Figure 16. Payoff Variant Games.

To summarize, in every case above, the value of the intermediate payoff systematically effects the strategy choice and in every case the modal (and typically the majority) game outcome is the one uniquely predicted by s_0. In comparison with the predictions of Nash equilibrium, note that in BOS 1,3 and Stag-hunt 1,1 there are two Nash equilibria but only the one consistent with s_0 occurs. In BOS 1,1 the frequency with which subjects played the Nash equilibria UL and DR combined is lower than the frequency with which subjects played the disequilibrium outcome DL, predicted by s_0.

Similarly, for BOS 3,3, the frequency with which subjects played the Nash equilibria UL and DR, combined, is lower than the frequency with which subjects played the disequilibrium outcome UR, predicted by s_0. Moreover, salience uniquely predicted the modal outcome in each of the three cycle games even though these games have no pure strategy Nash equilibria.[15]

In addition to playing the payoff-variant instantiations of the Battle of the Sexes game, players played an instantiation of its payoff-invariant cousin—the Hero game (Game (3,10) in Figures 3 and 9). The results of the Hero game and how they compare to the Battle of the Sexes 3,3 game with identical payoff values are shown in Figure 17.

Battle of the Sexes

	L	R
U	M h	M m
D	L l	H m

Hero

	L	R
U	M h	L l
D	L l	H m

BOS 3,3

	L (M=9)	R (m=9)	
U	9^s 10 9%	9 9 81%	90%
D	1_s 1_s 1%	10 9^s 9%	10%
	10%	90%	

H 3,3

	L (M=9)	R (m=9)	
U	9 10^s 21%	1_s 1_s 15%	36%
D	1 1 38%	10^s 9 26%	64%
	59%	41%	

Figure 17. Payoff Variant and Invariant Anti-coordination Games.

The largest salient payoff in BOS 3,3 is 9 for both players, yielding the disequilibrium outcome UR. In the Hero game, the largest salient payoff is now 10 resulting in a majority of Player 1s and 2s choosing their strategies D (64%) and L (59%). As a consequence, the opposite disequilibrium outcome DL predominates in the Hero game.

3.2.3. "Other-Payoff" Solvable Games

The results discussed above appear to provide strong support for the hypothesis that players base their choices on salience. But, as noted earlier, these same predictions follow if subjects behave as Level-1 players in a Cognitive Hierarchy model. However, predictions of the two models diverge for the subset of HI/IL payoff configurations in which the differences between the high and intermediate and intermediate and low outcomes are equal but have an opposite sign. In these instances, a Level-1 player believing his opponent is equally likely to choose either strategy would be indifferent between his own strategies as each offers the same expected value. A player choosing according to s_0 would, instead, attempt to infer the strategy choice of his opponent given the salience of the opponents' payoffs and best respond. Subjects played six games that had payoff structures where own-payoff comparisons were uninformative for one or both players (1 variant on the Coord sw game denoted

15 It is not quite clear what a mixed strategy means in the context of a single-shot game. However, if players did randomize optimally given the payoff matrices, we would expect 81% of the outcomes in 1,1 to be DR (rather than the 12% observed), 81% of the outcomes in 1,3 to be UL (rather than the 4% observed) and 81% of the outcomes in 3,3 to be UR (rather than the 20% observed.)

Coord b(est)r(response), 1 dominance solvable Hegemony Stability' game, two Battle of the Sexes games, and two Stag-hunt games). Results are shown in Figure 18. In game Coord br, Player 2 has the same payoff invariant structure as in the coordination games discussed earlier, the larger salient payoff recommends R and 87% of subjects choose that strategy. In contrast, Player 1 has no salient payoffs. Strategy U is the best response to R and 59% of Player 1s choose that strategy. Likewise, in Stag-Hunt 2,3, 74% of Player 2s choose Left consistent with salience or Level-1. Among Player 1s for whom there are no salient payoffs, 62% choose the best response, D, consistent only with s_0. However, given the number of each response pattern observed across the two games (11 UU, 12 UD, 4 DU and 12 DD) we cannot reject the hypothesis that responses are random ($\chi^2 = 4.59$, 3 df, $p = 0.2044$).

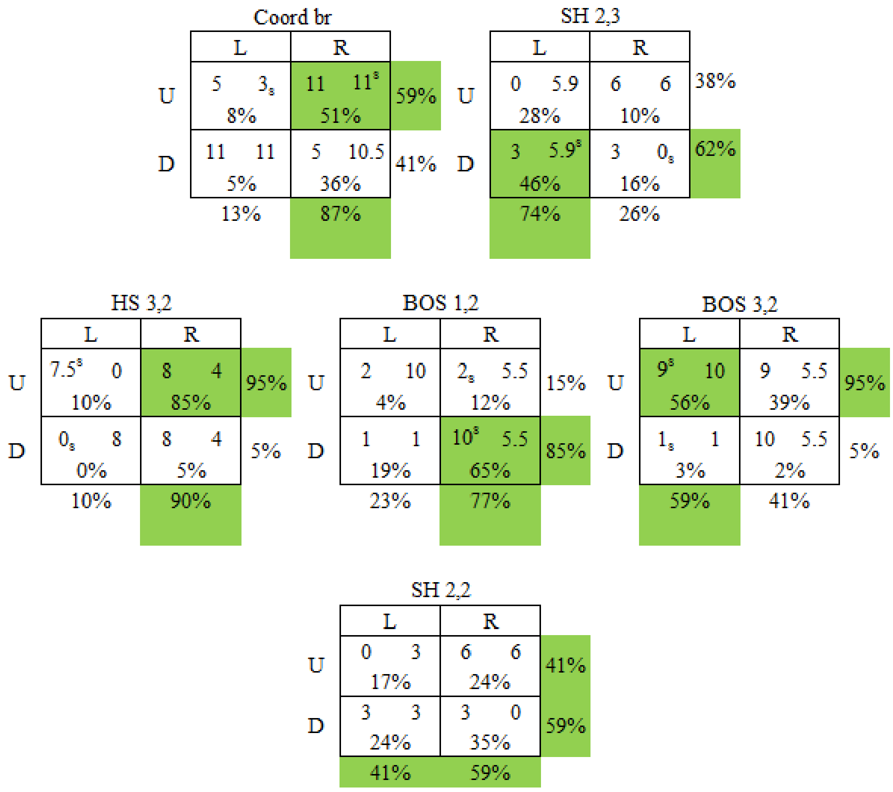

Figure 18. "Other Payoff" Solvable Games.

In game HS 3,2 Player 1s chose their dominating strategy U (95%) and a significant majority of Player 2s best responded with the choice of R (90%). In BOS 1,2 and 3,2, the play of Player 1 is consistent with choice based on own-payoff salience or Level-1 play. However, Player 2s played their best response to Player 1 choices by choosing R in BOS 1,2 (77%) and L in BOS 3,2 (59%) consistent only with s_0. Figure 19 shows the observed response patterns for Player 2s across all three of these games. If Player 2s were responding randomly, we would expect each of the four possible response patterns to occur 4.85 times on average. The observed results differ significantly from uniform ($\chi^2 = 57.21$, $p = 0.001$, 7 df) and a plurality, and nearly a majority, of the response patterns across all three games observed are consistent with best responding to the opponent's salience based choices.

HS 3,2	BOS1,2	BOS3,2	# off	#	%	Cum %
R	R	L	0	18	46.15%	46.15%
R	R	R	1	11	28.21%	74.36%
R	L	L	1	4	10.26%	84.62%
R	L	R	2	2	5.13%	89.74%
R	L	L	1	1	2.56%	92.31%
L	R	R	2	1	2.56%	94.87%
L	L	R	3	1	2.56%	97.44%
L	L	L	2	1	2.56%	100.00%

Figure 19. Player 2 Frequency of Best-Response Patterns.

Finally, in Stag-hunt 2,2 in Figure 18, where evaluation of own-payoff and other-payoff salience is uninformative for both players, responses appear unambiguously random.[16] This is consistent[17] with the predictions of both s_0 and Level-1.

3.3. Overall Response Patterns and Game Outcomes

The discussion to this point has focused on behavior within specific games for different values of the intermediate payoffs and behavior between games of a given type (e.g., coordination games) but with different payoff structures. We now consider how well the model of salience-based strategy choice accords with the majority response patterns observed across all the individual choices and how the predicted pattern of responses over all choices compares to the patterns subjects' actually exhibit. These results are summarized in Figure 20. The entries under the titles "Percentage Exhibiting the Predicted Choice" indicate the number and percentage of Player 1s and Player 2s choosing U and D and L and R, respectively, for each game. As indicated by the highlighted cells, the majority of subjects choose as predicted by s_0 in each of the 20 games.

Individual response patterns observed were, overall, close to the predicted pattern. Of the 1,048,576 response patterns possible, Player 1s and Player 2s exhibit a total of 32 each. Among Player 1s, the most common patterns observed were the predicted pattern (four subjects) and a pattern differing from the predicted one only in game Coord nw 2,3* (four subjects)[18]. Among Player 2s, the most common

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[16] Here 39 of 78 subjects choose strategies offering payoffs of either 0 or 6 (23 P1s chose U and 16 P2s chose R) and 39 choose strategies offering payoffs of 3 or 3 (16 P1s choose D and 23 P2s choose L).

[17] One additional plausible prediction one might construct for salience-based payers is that response times will be longer for situations where players are predicted to best-respond. After analyzing the response time data from the experiment, we find that the results testing this prediction are ambiguous and small–for Player 1s the average reaction time for the 3 strategy choices in which they best responded was 25.79 seconds versus 25.99 seconds for the 17 choices when they did not have to best respond (opposite from the predicted direction). For Player 2s, the difference went in the predicted direction (24.63 seconds for the four strategy choices where they best responded versus 23.44 seconds for the 16 choices where they do not).

[18] Although the modal pattern for Player 1s appears to be two off the predicted pattern in Figure 20, Player 1s were two off the predicted pattern in different ways such that the modal response pattern apparently exhibited by nine subjects conceals the fact there were several different response patterns by which Player 1s differed by two choices from the predicted pattern. A similar observation holds for Player 2s.

single pattern (exhibited by three subjects) deviated from the predicted pattern on only the Hero game and HS 3,1.

The right-hand columns in Figure 20 under "Deviations from Predicted" shows the number of subjects (#P1s or #P2s) exhibiting response patterns that were 0 off, 1 off, 2 off, *etc.* from the predicted pattern. As indicated in the columns titled "%", almost half of the response patterns observed for Player 1s (49%) and Player 2s (44%) are within two responses of the pattern predicted by s_0 and 61.5% of all individual response patterns are within three responses of the predicted pattern. Thus, for almost half of the subjects, strategy s_0 predicted at least 90% of subjects' responses over all 20 choices.

Percentage Exhibiting the Predicted Choice

Player 1

	Cd sw 1,1	Cd sw 3,1	Cd sw 3,3	Coord nw	HS 3,1	HS 3,3	BOS 1,1	BOS 1,3	BOS 3,3	Hero	SH 1,1	C 1,1	C 1,3	C 3,3	Coord br	SH 2,3	HS 3,2	BOS 1,2	BOS 3,2	SH 2,2
Pred.	U	U	U	D	U	U	D	D	U	U	U	U	U	D	U	D	U	D	U	either
U	31	33	33	2	31	34	14	4	35	14	36	26	32	8	23	15	37	6	37	16
D	8	6	6	37	8	5	25	35	4	25	3	13	7	31	16	24	2	33	2	23
%U	79%	85%	85%	5%	79%	87%	36%	10%	90%	36%	92%	67%	82%	21%	59%	38%	95%	15%	95%	41%
%D	21%	15%	15%	95%	21%	13%	64%	90%	10%	64%	8%	33%	18%	79%	41%	62%	5%	85%	5%	59%

Deviations From Predicted

Player 1 Patterns

# off	# P1s	%
0	4	10.26%
1	6	15.38%
2	9	23.08%
3	6	15.38%
4	5	12.82%
5	4	10.26%
6	3	7.69%
8	1	2.56%
11	1	2.56%

Player 2

	Cd sw 1,1	Cd sw 3,1	Cd sw 3,3	Coord nw	HS 3,1	HS 3,3	BOS 1,1	BOS 1,3	BOS 3,3	Hero	SH 1,1	C 1,1	C 1,3	C 3,3	Coord br	SH 2,3	HS 3,2	BOS 1,2	BOS 3,2	SH 2,2
Pred.	R	R	R	R	L	R	L	R	R	L	R	L	R	R	R	L	R	R	L	either
L	13	11	6	1	20	0	31	1	4	23	2	25	2	1	5	29	4	9	23	16
R	26	28	33	38	19	39	8	38	35	16	37	14	37	38	34	10	35	30	16	23
%L	33%	28%	15%	3%	51%	0%	79%	3%	10%	59%	5%	64%	5%	3%	13%	74%	10%	23%	59%	41%
%R	67%	72%	85%	97%	49%	100%	21%	97%	90%	41%	95%	36%	95%	97%	87%	26%	90%	77%	41%	59%

Player 2 Patterns

# off	# P2s	%
0	1	2.56%
1	5	12.82%
2	11	28.21%
3	6	15.38%
4	4	10.26%
5	3	7.69%
6	3	7.69%
7	3	7.69%
8	2	5.13%
9	1	2.56%

Figure 20. Overall Response Patterns for Individual Strategy Choices.

Figure 21 summarizes the results in terms of game outcomes predicted based on the strategy choices of Player 1s and Player 2s for each game. In addition to displaying the observed frequency of all 20 game outcomes, Figure 21 also provides the Nash equilibrium predictions and the salience predictions. As indicated by the highlighted cells, in 19 of the 20 games, the modal outcome is the one predicted by s_0. Notice that salience predictions are unique for these 19 games as opposed to the Nash solutions which have multiple equilibria in some of these games, and no pure strategy equilibria in others. Only in game Sh 2,2 does the salience model not make a unique prediction, predicting that play will appear random. This prediction is also not far from the observed behavior with each of the four game outcomes being played between 17% and 35% of the time. Note also that the modal (and predicted) outcome was played over 50% of the time for 15 of the 20 games, even though four game outcomes were possible. For those games where the unique s_0 prediction corresponds to one of the Nash equilibrium predictions, allowing the additional Nash equilibrium outcome provides very little in terms of additional predictive power. Where the s_0 prediction and the Nash equilibrium predictions differ, the latter do poorly.

Frequency of Implied Game Outcomes

	Cd sw 1,1	Cd sw 3,1	Cd sw 3,3	Coord nw	HS 3,1	HS 3,3	BOS 1,1	BOS 1,3	BOS 3,3	Hero	SH 1,1	C 1,1	C 1,3	C 3,3	Coord br	SH 2,3	HS 3,2	BOS 1,2	BOS 3,2	SH 2,2
UL	26%	24%	13%	0%	41%	0%	29%	0%	9%	21%	5%	43%	4%	1%	8%	29%	10%	4%	56%	17%
UR	53%	61%	72%	5%	39%	87%	7%	10%	81%	15%	88%	24%	78%	20%	51%	10%	85%	12%	39%	24%
DL	7%	4%	2%	2%	11%	0%	51%	2%	1%	38%	0%	21%	1%	2%	5%	46%	1%	20%	3%	24%
DR	14%	11%	13%	92%	10%	13%	13%	87%	9%	26%	7%	12%	17%	77%	36%	16%	5%	65%	2%	35%

Frequency of Nash Equilibrium Predictions in Implied Game Outcomes

UR DL	UR DL	UR DL	UR DL	UR	UR	UL DR	UL DR	UL DR	UL DR	UR DL				UR DL	UR DL	UR	UL DR	UL DR	UR DL
53%	61%	72%	5%	39%	87%	29%	0%	9%	21%	88%				51%	10%	85%	4%	56%	24%
7%	4%	2%	2%			13%	87%	9%	26%	0%				5%	46%		65%	2%	24%

Frequency of Salience Predictions in Implied Game Outcomes

UR	UR	UR	DR	UL	UR	DL	DR	UR	DL	UR	UL	UR	DR	UR	DL	UR	DR	UL	Random
53%	61%	72%	92%	41%	87%	51%	87%	81%	38%	88%	43%	78%	77%	51%	46%	85%	65%	56%	

Figure 21. Observed Frequency of Implied Game Outcomes across all 20 Games.

4. Extensions to Other Two-Person Games

The theory developed here applies most directly to 2 × 2 normal form games. An obvious challenge is to extend the model of the perceptual process to apply to more complex settings, or specify the rules whereby more complex problems are simplified so as to be amenable to evaluation by s_0. While these extensions are beyond the scope of this paper, we can extend the model to predict behavior in other 2-person games. We consider two examples from Goeree and Holt [28]—The Traveler's Dilemma, and a minimum effort coordination game.

4.1. The Traveler's Dilemma

In the Traveler's Dilemma [29], two players, P1 and P2, each select an integer between (and including) 180 and 300. Denote the chosen numbers by n_1 and n_2. The player who provided the smaller number receives $\min(n_1, n_2) + r$ for some $r > 1$. The player who provided the larger number receives $\min(n_1, n_2) - r$. If $n_1 = n_2 = n$, then both players receive n.

The unique rationalizable equilibrium [30–31] is for both players to state 180, since it is always to each player's advantage to undercut the other player. Goeree and Holt [28] conducted two variants of this game—one in which $r = 5$ and the other in which $r = 180$.

To consider the predictions of a theory based on salience, we note that the strategies 180 and 300 are salient, for at least two reasons: First, the biggest difference in possible strategies occurs by taking the difference between 180 and 300. In addition, 180 and 300 may be salient to the extent that they are the only two strategies that are explicitly mentioned.

If players eliminate (or ignore) non-salient strategies, we can represent the Salient Traveler's Dilemma as in Tables 3 and 4. By deleting (or ignoring) non-salient strategies, the Traveler's Dilemma can be approximated as a 2 × 2 game. It can then by solved by the model in this paper.

Table 3. Salient Traveler's dilemma with $r = 5$ (TD 1).

	Player 2		
	Strategy	$s_2 = 180$	$s_2' = 300$
Player 1	$s_1 = 300$	175, 185	300, 300
	$s_1' = 180$	180, 180	185, 175

Table 4. Salient Traveler's dilemma with $r = 180$ (TD 2).

	Player 2		
	Strategy	$s_2 = 180$	$s_2' = 300$
Player 1	$s_1 = 300$	0, 360	300, 300
	$s_1' = 180$	180, 180	360, 0

Consider Game TD1 with local players in Table 3, where $r = 5$. Analyzing this game according to the model in Section 2, we obtain that payoffs 300 and 185 are salient for each player. If local players play the game, both will choose $n_1 = n_2 = 300$ (since this is the strategy with the larger salient payoff). Goeree and Holt found that approximately 80% of respondents indeed chose 300, when $r = 5$.

Next, consider Game TD2 in Table 4, where $r = 180$. Proceeding as before, we note that payoffs 180 and 0 are salient for each player. If the game is played by local players, both players will choose $n_1 = n_2 = 180$. Interestingly, Goeree and Holt found that approximately 80% of respondents now chose 180.

4.2. A Minimum Effort Coordination Game

Goeree and Holt [28] also consider a coordination game as follows: Two players, Player 1 and Player 2, choose "effort" levels, e_1 and e_2 simultaneously. Player 1 receives $\min(e_1, e_2) - ce_1$ where $c < 1$ is a coefficient indicating the cost of effort. Likewise, Player 2 receives $\min(e_1, e_2) - ce_2$. Effort levels are integers between and including 110 and 170. Goeree and Holt considered two variants of the game, one in which c = 0.10 and the other in which c = 0.90. As in the Traveler's Dilemma, the endpoints of the interval (110 and 170) are the salient strategies. If players eliminate (or ignore) non-salient strategies, then the salient coordination game (with cost of effort c = 0.10) is represented as in Table 5.

Table 5. Minimum Effort Coordination Game with c = 0.10.

	Player 2		
	Strategy	$s_2 = 110$	$s_2' = 170$
Player 1	$s_1 = 110$	99, 99	99, 93
	$s_1' = 170$	93, 99	153, 153

If the game with c = 0.10 is played by local players, both Player 1 and Player 2 will each choose to play 170, since this is the strategy with the larger salient payoff.

If players eliminate (or ignore) non-salient strategies, then the salient coordination game with cost of effort c = 0.90 is shown in Table 6:

Table 6. Minimum Effort Coordination Game with c = 0.90.

		Player 2	
	Strategy	$s_2 = 110$	$s_2' = 170$
Player 1	$s_1 = 110$	11, 11	11, -43
	$s_1' = 170$	-43, 11	17, 17

If the game with c = 0.9 is played by local players, both Player 1 and Player 2 will each choose to play 110 out of all integer effort levels between and including 110 and 170. Consistent with these predictions, for c = 0.10, Goeree and Holt observe "*behavior is quite concentrated at the highest effort level of 170*; *subjects coordinate on the Pareto-dominant outcome. The high effort cost treatment (c = 0.9)*, *however*, *produced a preponderance of efforts at the lowest possible level.*" ([28], p. 1408).

5. Discussion

We have presented a model of play in 2 × 2 games in which the course of action is determined not by complex chains of strategic reasoning and calculation but rather by what aspects of the game and, specifically, the payoffs attract players' attention. We have shown that for one subset of 2 × 2 games, outcomes will be sensitive to theoretically inconsequential changes in the relative values of the payoffs to one or both players. For another subset, play will be insensitive to relative payoff values. We have presented propositions that completely characterize which game structures will be payoff variant and which will be payoff invariant for the entire class of strictly ordinal 2 × 2 games. In the process, we have identified games where a Nash equilibrium (or one of the Nash equilibria) will always obtain, games where the Nash equilibrium outcome will never obtain, and games where a Nash equilibrium will obtain for certain payoff values but not for others. We also obtain predictions regarding the outcome of games for which there are no pure Nash equilibrium solutions. For the games we examined experimentally, the predictions of the model appear correct. In particular, the model correctly (and uniquely) predicts the modal outcome in 19 of the 20 experimental 2 × 2 games, and in the remaining case, the model predicts random play which appears close to what is observed. At the individual level, the model predicts at least 90% of all 20 choices for each of 36 of the 78 subjects in the experiment.

Taken as a whole, the results reported may explain why the outcome in certain strategic interactions seems stable in some cases (in payoff invariant games) but unstable in others (payoff variant games), amicable in some cases (*i.e.*, Coord sw) but intractable in others (the Hero game). In this respect, the model presented here accounts for a different notion of stability than that captured by Nash equilibrium: Nash equilibria are robust to unilateral deviations from a best-response profile for rational players, but are not necessarily robust to changes in cardinal payoffs for salience-based players. It is possible that the notion of payoff invariant games provides a stability concept for single-shot games, whereas Nash equilibria may be more predictive for long-run or repeated interactions.

At the individual level, we have also provided a simple model of salience-based strategy selection that shows how as-if expected value maximizing behavior (in games with own-payoff salience) and Nash equilibrium behavior (in games with other payoff salience) each arise from a plausible feature of the perceptual system—attention to larger differences in payoffs. In this regard, we have presented propositions that characterize when salience-based players will behave as if they are Level-1 boundedly rational, and cases when the same players will behave as if they conform to the Nash equilibrium predictions of traditional game theory. We also found in our experiments that salience-based play did coincide with Level 1 behavior in games solvable by own-payoff salience, whereas behavior conformed to Nash equilibrium play in games with "other payoff" salience, although the frequency of best replies was not always significantly different from random.

From a broader perspective, the approach taken in this paper follows in the tradition of Schelling [32] in its emphasis on certain aspects of the payoff matrix as being salient or focal. It differs from Schelling in that the origins of focality emanate from shared properties of our perceptual system rather than shared knowledge of the social milieu.

The approach also follows on Simon's work [33] on bounded rationality and satisficing as well as Gigerenzer and Goldstein's work [34] on ecological rationality. Simon [33] discusses how an organism employs simple perceptual and choice mechanisms to interact with the environment, without requiring any "elaborate procedure for calculating marginal rates of substitution among different wants" ([33], p. 129). We hope our paper contributes to this research program, as it demonstrates, for instance, that the strength of assumptions needed to produce apparent expected value-maximizing behavior and Nash equilibrium behavior (focusing on salient differences in payoffs) is much weaker than is generally recognized, and provides a more psychologically plausible description of such behavior, at least for 2×2 games.

Salience-based play is not programmed as an optimization process. Nevertheless, the process may produce reasonably good outcomes a large portion of the time. To make this point, consider again the set of 144 games in Figures 2–5 and, for the sake of argument, imagine an experiment in which players play each game once for randomly generated payoffs. Considering the situation from Player 1's perspective, for half of these games, he has a dominating strategy. Feedback from these games will not indicate there is anything wrong with s_0. For another 25% of the games, the other player has a dominating strategy and Player 1 does not. In these cases, on average, Player 1 will receive positive feedback regarding his decisions half the time and wish he had chosen otherwise the other half of the time. The same will be true for all the cyclic games, eight of the nine coordination games and eight of nine of the anti-coordination games. A local player in a Coord sw game will never wish to change his play while he will always wish he had chosen differently in the Hero game.[19] Taken together, a Player 1 basing choices on s_0 will be happy with the outcome (or at least will not change his strategies) 75% of the time. While this is not optimal performance, it is not obviously dismal and certainly better than chance. This kind of performance seems to characterize decision making in general. People generally do not act in patently stupid ways nor do models that posit rationality fail egregiously. Instead, people seem to make the right choice most of the time and rational models predict those choices. Where things seem to go wrong is in a narrow corridor in the choice space where the available options are, at

[19] This suggests that the Hero game might be an ideal candidate for studies of learning in games to the extent it provides unambiguous negative feedback to either salience or Level-1 players. However, the general implication of this discussion is that opportunities for learning about the quality of decisions prescribed by s_0 are likely to be limited. Opportunities for learning in this type for setting will be even more circumscribed for Level-1 players to the extent that instances where they would want to have chosen differently can be attributed to their opponent having chosen randomly.

least in a relative sense, close in expected value or utility. Here, choices can be swayed by theoretically irrelevant aspects of the decision like the values of intermediate payoffs in the 2 × 2 games studied here.

The model and game characterizations introduced here may also have implications for mechanism design. In particular, cardinal differences in payoffs may be changed to engineer equilibrium play, whereas such changes to a game are irrelevant in the classical framework that depends only on ordinal payoff rankings. In addition, side payments may be used to change payoff variant games to payoff invariant ones, thereby increasing the robustness of the game outcome, at least for games played once.

As noted in the introduction, salience-based decision-making provides an explanation for many of the anomalies observed in individual choice behavior. The results presented here show it also explains behavior in strategic settings, albeit simple ones. These findings seem to suggest that further exploration of the role of salience in decision-making is in order.

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Author Contributions

This paper is the result of a now 4 year old collaboration between the authors. The motivations for the model build on prior work by Leland examining the implications of similarity judgments in games and the authors' work on the role of salience in risky and intertemporal choice. Schneider formalized the model of salience-based play in 2 × 2 games and derived many of the model's implications. The empirical results come from an experiment conducted and analyzed by Leland. The authors collaboratively developed the idea of payoff variant and invariant games and participated in the drafting of the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

Appendix: Taxonomy of 2 × 2 Games

Figure A1. Robinson and Goforth's Taxonomy of 144 Strictly Ordinal Games (Composite of Figures 2–5).

Figure A2. Taxonomy of 144 Strictly Ordinal Games (Composite of Figures 6–9).

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