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Cycles in Team Tennis and Other Paired-Element Contests

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Abstract: Team Tennis competitions produce aggregate scores for teams, and thus team rankings, based on head-to-head matchups of individual team members. Similar scoring rules can be used to rank any two groups that must be compared on the basis of paired elements. We explore such rules in terms of their strategic and social choice characteristics, with particular emphasis on the role of cycles. We first show that cycles play an important role in promoting competitive balance, and show that cycles allow for a maximum range of competitive balance within a league of competing teams. We also illustrate the impact that strategic behavior can have on the unpredictability of competition outcomes, and show for a general class of team tennis scoring rules that a rule is strategy-proof if and only if it is acyclic (dictatorial) and manipulable otherwise. Given the benefits of cycles and their relationship with manipulability, a league valuing competitive balance may invite such social choice violations when choosing a scoring rule.

Keywords: social choice theory; competitive balance; ranking cycles; strategic voting; sports

1. Introduction

Tennis is typically viewed as an individual sport. At school, club, and professional levels, however, individual matchups are aggregated into a team result. A National Collegiate Athletic Association (NCAA) team tennis competition, for example, features six individual matches and either one or three doubles matches.[1] Each singles player within a team is assigned an ability ranking 1 through 6 prior to a given match (an assignment process we will consider further below). A player then competes against the player of corresponding assigned ranking on the opposing team. In an NCAA team competition, or dual meet, featuring six singles matches and one doubles match, each individual match victory is worth one point. A team must score at least four points to win the overall competition. Team tennis competition of this form is, therefore, decided by majority rule aggregation of primitive matches.

More generally, what we refer to as *team tennis scoring rules* are rank-ordering mechanisms based on the aggregation of paired-element comparisons. Any conflict or contest situation that requires indivisible elements of groups to compete or to be compared head-to-head against one another to

1 The NCAA division a team is in dictates whether it plays one or three doubles matches. This is discussed a bit more in Section 3.

decide a ranking of groups could be determined by such a rule. One example could be a form of Blotto game situation, in which two generals face off with indivisible units pitted against one another on separate battle fields. The outcome of the game could be decided by what we refer to as *standard* team tennis scoring, with each battle field worth one point and the general who wins the larger number of battle fields winning—a form of majority rule. Or it could be that winning some battle fields is more important than others. Similarly, it could be that in team tennis competitions, some individual matches are more important than others.

In the present paper we examine the social choice characteristics of team tennis scoring rules as they relate to strategic manipulability and ambiguity. In particular we are interested in the role of *cycles*, in which team A defeats team B in a dual meet, and team B defeats team C in a dual meet, but team C defeats team A in a dual meet. When we say that a rule is *strategy-proof*, we mean a team never has an incentive to falsely list its players in terms of their ability levels; otherwise a rule is *manipulable*. These phenomena are well-known in the social choice literature, especially as applied to voting, but we show that they have special importance as applied to team tennis scoring rules and sporting competitions.

For a general class of team tennis scoring rules, we show that a rule is strategy-proof if and only if it is dictatorial, meaning only one of the element pairs matters for the entire group ranking. This in turn means that a rule is strategy-proof if and only if it is acyclic, and manipulable whenever it allows cycles. Though such results are often portrayed as negative in the social choice literature, we argue that this is not the case in the context of team tennis, since we show that both of the noted social choice violations promote competitive balance between opposing teams. In fact, we demonstrate that a full range of competitive balance, from cycles between the absolute best and worst teams in a league to perfect balance, can be achieved with a family of team tennis scoring rules that we call weighted-sum team tennis scoring rules.

Unlike traditional arenas of social choice analysis, third-party (spectator) welfare is paramount in sport. Following the seminal contributions of Neale [1] and Rottenberg [2], there is strong evidence that competitive balance is an important source of league-wide sport demand. Zimbalist [3] writes, "... sports leagues need to cooperate in maintaining a certain level of competitive balance among their teams to preserve and enhance fan interest." Rather than a symptom of poor design, then, the allowance of ranking cycles and strategic intra-squad ranking within a sports league may act to promote competitive balance and, in so doing, increase demand and spectator welfare.

A few previous studies have examined the social choice characteristics of team sport. Avgerinou [4], Hammond [5], and Sloane [6] note that sporting events (outcomes) carry increasing economic and social significance, as evidenced by China's $43 billion outlay for the 2008 Summer Olympic Games (Demick [7]). Therefore, it is important to understand ranking methodologies used to generate outcomes in sport. Hammond shows that sum of positions team scoring in cross country competition can generate ranking cycles. He also shows that said scoring methodology can generate team rankings that violate the weak axiom of revealed preference. Team A may defeat team B when team C is scored, for example, but lose to team B when team C is not scored. Hammond shows an alternative scoring arrangement ("sum of times" scoring) to have superior social choice properties. Saari [8] finds that team scoring in track and field can also violate the weak axiom of revealed preference. MacKay [9] demonstrates problems with preference aggregation (event scoring aggregation) in the Olympic decathlon.

After providing some basic results regarding cycles and competitive balance and the uncertainty of outcome in the second section, we provide our general team tennis model and establish some results on team tennis scoring and cycles in the third section. Then in the fourth section we follow Boudreau et al. [10] and Hammond [5] by using computational methods to ascertain the proportion of match "types" that produce a ranking cycle in team tennis. Employing Java programming, we extend this method to computations over very large (realistic) sets involving three teams and seven primitive elements (individual players or doubles pairs) per team. In the penultimate section we examine the role of strategic behavior and its impact on competitive balance, and provide our general result on manipulation and cycles. The final section concludes with discussion.

2. Ranking Cycles and Competitive Balance

We begin with just a few of the most basic aspects of the model and will build upon them further in the next section. Consider a league of N teams, $L = \{\mathbf{x}, \mathbf{y}, \mathbf{z}, \ldots\}$. When two teams compete in dual meets, their scores are determined by elemental (player) pairings, which we refer to as individual matches. Most of our analysis focuses on the scoring of those individual matches and how those scores are aggregated, but for now all that matters is that when two teams meet a strict ranking between the two teams is determined. That is, in a dual meet between **x** and **y**, either $\mathbf{y} \succ^* \mathbf{x}$, meaning **y** defeats **x**, or $\mathbf{x} \succ^* \mathbf{y}$, meaning **x** defeats **y**.

However the scoring rule aggregates individual match outcomes, we say that the rule is *acyclic* if it does not permit any cycles to occur in dual meet outcomes. Otherwise, the rule is said to permit cycles, though whether cycles actually occur depends on the specific elemental pairings involved. What is most important is that cycles are possible under some rules and impossible under others.

We can now establish a very simple proposition, adapted from Boudreau and Sanders [11], that relates to competitive balance and transitivity. In doing so we assume that each team in the league plays each other team once, and that competitive balance is measured by the standard deviation of wins among teams after a full schedule of play. More discrepancy between the win totals of teams therefore means less balance in a league. When we refer to expected competitive balance, we think of the perspective of a spectator or fan forming expectations over the league's competitive balance at the beginning of the season.

Proposition 1. *A scoring rule that permits ranking cycles increases expected competitive balance relative to one that does not.*

Proof. Given an acyclic scoring rule, after a full schedule it must be the case that one team has $N-1$ wins, one has $N-2$ wins, and so on, with the last-ranked team having zero wins. Competitive balance will therefore always be the same. Without loss of generality assume that team **x** beats team **y**, **y** beats **z**, and (since there are no cycles) **x** beats **z** under this acyclic rule. Now consider an alternative rule in which **x**, **y**, and **z** form a cycle. Since **x** beat the other two teams under the acyclic rule, it must have had more wins and **z** fewer. The presence of the cycle under the alternative rule means that **x** has one less win and **z** has one more, thereby improving competitive balance, and this holds for any additional cycles relative to the case of an acyclical rule. The possibility of cycles (i.e., a positive *ex ante* probability of a cycle occurring) therefore unambiguously improves expected competitive balance within a league. □

We also develop a proposition regarding match uncertainty of outcome, which will be explained more fully after the statement of the proposition itself.

Proposition 2. *A scoring rule that permits cycles increases ex ante uncertainty of a marginal match outcome relative to one that does not.*

In a league consisting of **x**, **y**, and **z** engaging in an exhaustive schedule of (three) pairwise matches, let matches occur sequentially in time. Then, suppose the outcome of *every* match is a priori uncertain (cannot be determined based upon properties of the scoring rule) if the aggregation rule is cyclic. This is not the case if the ranking methodology is acyclic. Thus, cyclicality increases uncertainty of outcome.

Proof. Whether the ranking methodology is cyclic or acyclic, the first two match outcomes are a priori uncertain. As the outcomes of the first two matches could never generate a cycle, there are no restrictions on the outcomes of the first two matches. Under the acyclic rule, the third match outcome is a priori known whenever the alternative third match outcome would generate a cycle. Under the cyclic rule, the third match outcome is never known a priori. □

3. Team Tennis Scoring

We now flesh out the model. Consider a match between teams **x** and **y**, where each team consists of $n \geq 3$ (odd)[2] elements that we refer to as players. The elements could also include doubles pairs, but for brevity we do not distinguish. Let player x_i, $i \in \{1, 2, \ldots, n\}$, be the i^{th} ranked player on team **x**, so **x** is a vector of players in strict rank order. That is, $\mathbf{x} = [x_1, x_2, \ldots, x_n]$.[3] For now, let us assume that each team knows the true quality rank-ordering among its own set of players and organizes accordingly, meaning there is no strategic intra-squad ranking (yet).

We assume that the rank ordering of players for each team is given, fixed, and transitive, perhaps decided by the team's coach or by prior intra-squad competition.[4] This sort of assumption, which means that chance does not play a role in forming an outcome, pervades the social choice literature. Gibbard [12] and Satterthwaite [13], for example, make such an assumption in the case of voting, as does Saari [14] in the case of track and field (athletic) performance. Herein, rankings are assumed to be invariable and transitive so as to theoretically isolate the social choice characteristics of team rank ordering. If team ranking cycles are found to occur in the assumed setting, then, it is certainly the cause of the team ranking methodology.

We define a team tennis match as a series of events in which each competitive element of **x** engages in a match against each equivalent element of **y**. Hence, there are n individual matches that comprise a team match. In a given individual match, we can have that $x_i \succ^* y_i$, meaning x_i defeats y_i, or $y_i \succ^* x_i$, meaning y_i defeats x_i. As in a typical one-on-one tennis match, we do not allow for draws. That is to say, the match will eventually define a unique winner for each pairing. [5]

In a team tennis dual meet between **x** and **y**, the outcome is determined by a scoring rule that aggregates the outcomes of the n individual matches. A *weighted-sum team tennis scoring rule* assigns a point value to the winner of each individual match, and winning team is the one with the highest summed point total. Formally, let γ_i be the value of winning individual matchup i, and assume that $\gamma_i \geq \gamma_j \ \forall i < j$. Then the aggregate score for team **x** is $\boldsymbol{\Gamma}_\mathbf{x} = \sum_i \gamma_i I(x_i \succ^* y_i)$, where the $I(\cdot)$ is the indicator function taking on a value of 1 if the statement in parentheses is true and 0 otherwise. If $\boldsymbol{\Gamma}_\mathbf{x} > \boldsymbol{\Gamma}_\mathbf{y}$ we have $\mathbf{x} \succ^* \mathbf{y}$; otherwise, we have that $\mathbf{y} \succ^* \mathbf{x}$ (for our purposes here we can assume the γ_is and n are such that ties are not an issue).

Standard team tennis scoring is a case of $\gamma_i = 1 \ \forall i$, but even in mainstream team tennis this scoring rule is not always used. In some NCAA divisions, for example, team competitions involve 6 singles matches worth 1 point each, but 3 doubles matches, with the doubles matches worth a collective 1 point for the team that wins 2 of the three. In this sense the 3 elements that represent the doubles matches are weighted significantly less than the singles matches.

We have already established that cyclical scoring rules improve competitive balance and uncertainty of outcome in leagues, so we now investigate the degree to which weighted-sum team tennis rules allow for cycles. To begin we use standard rule of $\gamma_i = 1 \ \forall i$ for convenience just to illustrate to concept of cycles, then provide more general results.

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2 The assumption that n must be odd is for convenience only, in order to avoid ties. It can be done away with, but n even adds additional cases for consideration, thereby impeding exposition.

3 For simplicity, we abstract from the reality that individual players and doubles pairs are ranked separately.

4 In reality, opposing NCAA team tennis coaches do submit intra-squad rankings simultaneously just before a match. The rankings cannot be edited upon submission.

5 John Isner and Nicolas Mahut met in a first round match of the 2010 Wimbledon Championships. Isner was victorious in the longest professional tennis match ever played (11 h and 5 min of play over parts of three days; 183 games). The two players essentially traded 136 games in the fifth set before Isner won the 137th and 138th games of the set.

By assumption, the following individual player quality rankings obtain for any three teams, **x**, **y** and **z**.

$$\mathrm{x_1} \succ^* \mathrm{x_2} \ldots \succ^* \mathrm{x_{n-1}} \succ^* \mathrm{x_n} \tag{1a}$$

$$\mathrm{y_1} \succ^* \mathrm{y_2} \ldots \succ^* \mathrm{y_{n-1}} \succ^* \mathrm{y_n} \tag{1b}$$

$$\mathrm{z_1} \succ^* \mathrm{z_2} \ldots \succ^* \mathrm{z_{n-1}} \succ^* \mathrm{z_n} \tag{1c}$$

Example 1. *(Basic Cycles). To illustrate the notion of a cycle, let $n = 3$ for ease of exposition and further let counterpart rankings across team be specified as follows.*

$$\mathrm{x_1} \succ^* \mathrm{y_1} \succ^* \mathrm{z_1} \tag{2a}$$

$$\mathrm{z_2} \succ^* \mathrm{x_2} \succ^* \mathrm{y_2} \tag{2b}$$

$$\mathrm{y_3} \succ^* \mathrm{z_3} \succ^* \mathrm{x_3} \tag{2c}$$

More succinctly, player quality ordering across the three teams can be represented as

$$\mathrm{x_1} \succ^* \mathrm{y_1} \succ^* \mathrm{z_1} \succ^* \mathrm{z_2} \succ^* \mathrm{x_2} \succ^* \mathrm{y_2} \succ^* \mathrm{y_3} \succ^* \mathrm{z_3} \succ^* \mathrm{x_3} \tag{3}$$

or $\{\mathrm{x_1, y_1, z_1, z_2, x_2, y_2, y_3, z_3, x_3}\}$. These individual orderings lead to the following *dual meet* outcomes:

$$\mathbf{x} \succ^* \mathbf{y},\ \mathbf{y} \succ^* \mathbf{z},\ \text{and } \mathbf{z} \succ^* \mathbf{x}.$$

In Example 1, one notes immediately the absence of a Condorcet winner in the set of team rankings in Equation 2a-c. When **x** and **y** compete, **x** is victorious. When **y** and **z** compete, **y** is victorious. When **z** and **x** compete, **z** is victorious. In this case, the absence of a Condorcet winner is due to the method by which team rank orderings are established. That is, an outcome of each dual meet is decided by a simple majority rule aggregation of (primitive) individual player rankings. Intransitive outcomes of plurality or majority rule are well documented (see, e.g., Arrow [15] or Johnson [16]). However, they have not been considered extensively with respect to (competitive balance within) sports leagues.

Our next example shows that cycles among teams with a maximum amount of disparity between their win differentials, or between any teams, within a league are always possible with weighted-sum team tennis scoring rules, so long as they are not dictatorial. We say that a team tennis scoring rule is *dictatorial* if the matchup between the top player on each team is the only one that matters for the outcome of a dual meet. Formally, a rule is dictatorial if $\gamma_1 > \sum_{j>1} \gamma_j$; otherwise it is non-dictatorial. To be clear, this is not the same as the standard definition of dictatorship in the social choice literature as applied to contexts such as voting rules, but we feel that it is appropriate for this setting. Essentially, if one team can defeat another simply by winning one match and losing all others, that one winning player is its "dictator", or that one all-important match dictates the entire outcome. Given that we are by definition considering a team scoring rule, it seems reasonable to rule out such scoring rules.

To work with a general, non-dictatorial weighted-sum team tennis scoring rule, we must also define the index ω as the minimum index such that, if two teams, **x** and **y**, have their players ranked as $\{x_1,\ x_2, \ldots,\ x_\omega,\ y_1,\ y_2, \ldots, y_n, x_{\omega+1},\ \ldots x_n\}$, then $\mathbf{x} \succ^* \mathbf{y}$. This critical index can be thought of as the minimum number of individual match victories necessary for a team victory. Note that since the rule is non-dictatorial $\omega > 1$, and since $\gamma_i \geq \gamma_j\ \forall \mathrm{i} < \mathrm{j}$ and n is odd, $\omega \leq \frac{n+1}{2}$. We can now establish the following result.

Proposition 3. *For any league of $N \geq 3$ (odd) teams each composed of $n \geq 3$ players each using a non-dictatorial weighted-sum team tennis scoring rule for dual meets, it is possible for a team with fewer* *wins to defeat a team with more wins and form a cycle with that team, no matter how large is the difference between their win totals.*

Proof. Consider a league of $\mathrm{N} \geq 3$ teams with $\mathrm{n} \geq 3$ players each, $\mathrm{L} = \{\ell^1, \ \ell^2, \ \ldots, \ \ell^{\mathrm{N}}\}$, and suppose the individual elements collectively rank as

$$\ell^1_{1,..,(\omega-1)} \succ^* \ell^2_{1,..,(\omega-1)} \succ^* \ldots \succ^* \ell^{\mathrm{N}-1}_{1,..,\omega} \succ^* \ell^{\mathrm{N}}_{1,\ldots,\mathrm{n}} \succ^* \ell^1_{\omega,\ldots,\mathrm{n}} \succ^* \ell^2_{\omega,\ldots,\mathrm{n}} \succ^* \ell^{\mathrm{N}-1}_{(\omega+1),\ldots,\mathrm{n}} \tag{4}$$

where ℓ^i_j indicates the jth player on team i, and $\ell^{\mathrm{i}}_{\mathrm{j},..,\mathrm{k}}$ indicates that the jth through kth elements of team i are uninterrupted in the overall ranking list.

Based on the definition of ω, team ℓ^N defeats every team except ℓ^{N-1}; team ℓ^{N-1}, meanwhile is defeated by every team except for team ℓ^N. The remaining teams rank in order as ℓ^1 defeats all teams except ℓ^N, ℓ^2 defeats all teams except ℓ^1 and ℓ^N, and so on, with team ℓ^{N-2} defeating only team ℓ^{N-1}. Teams ℓ^N and ℓ^{N-1} therefore tie as the best and worst teams, respectively, and form a cycle with every other team in the league—every team that ℓ^N defeats is one that defeats ℓ^{N-1}. This covers the range of win discrepancies. □

We also note that the example in the proof above can be augmented to illustrate cycles between teams with any number of wins by simply expanding or contracting the number of teams within the template ordering above, or by including "dummy" teams whose n players rank entirely ahead of or behind the teams in the template ordering.

Our next result shows that perfect competitive balance is possible for a league using a non-dictatorial weighted-sum team tennis scoring rule. A state of perfect balance is one in which all teams have the same win total. To show this, we must define another threshold index, $\hat{\omega} < \omega$, as the maximum index such that if two teams, $\mathbf{x}$ and $\mathbf{y}$, have their players ranked as $\{x_1, \ x_2, \ldots, \ x_{\hat{\omega}}, \ y_1, \ y_2 \ldots, y_n, \ x_{\hat{\omega}+1}, \ \ldots, \ x_n\}$, then $\mathbf{y} \succ^* \mathbf{x}$. If the ranking were changed so that any more players from team $\mathbf{x}$ were ranked ahead of their counter parts on the other team, $\mathbf{x}$ would win.

Proposition 4. *For any league of $N \geq 3$ (odd) teams each composed of $n \geq 3$ players each using a non-dictatorial weighted-sum team tennis scoring rule for dual meets, perfect competitive balance is possible.*

Proof. Suppose a league's elements rank as follows:

$$\begin{gathered}
\ell^1_{1,\ \ldots,\hat{\omega}} \succ^* \ell^2_{1,\ldots,\hat{\omega}} \succ^* \ldots \succ^* \ell^{\mathrm{N}}_{1,\ldots,\hat{\omega}} \succ^* \ell^{\frac{\mathrm{N}+3}{2}}_{\hat{\omega}+1,\ldots,\omega} \succ^* \ell^{\frac{\mathrm{N}+5}{2}}_{\hat{\omega}+1,\ldots,\omega} \succ^* \ldots \succ^* \ell^{\mathrm{N}}_{\hat{\omega}+1,\ldots,\omega} \\
\succ^* \ell^1_{\hat{\omega}+1,\ldots,\omega} \succ^* \ell^2_{\hat{\omega}+1,\ldots,\omega} \succ^* \ldots \succ^* \ell^{\frac{\mathrm{N}-1}{2}}_{\hat{\omega}+1,\ldots,\omega} \succ^* \ell^{\frac{\mathrm{N}+1}{2}}_{\hat{\omega}+1,\ldots,\mathrm{n}} \succ^* \ell^{\mathrm{N}}_{\omega+1,\ldots,\mathrm{n}} \\
\succ^* \ell^{\frac{\mathrm{N}-1}{2}}_{\omega+1,\ldots,\mathrm{n}} \succ^* \ell^{\mathrm{N}-1}_{\omega+1,\ \ldots,\mathrm{n}} \succ^* \ell^{\frac{\mathrm{N}-3}{2}}_{\omega+1,\ldots,\mathrm{n}} \ldots \succ^* \ell^{\mathrm{N}-2}_{\omega+1,\ldots,\mathrm{n}} \succ^* \ell^{\frac{\mathrm{N}-5}{2}}_{\omega+1,\ldots,\mathrm{n}} \succ^* \ldots \succ^* \ell^1_{\omega+1,\ldots,\mathrm{n}}
\end{gathered} \tag{5}$$

From the definition of ω, team ℓ^i defeats ℓ^j for all $1 \leq \mathrm{i} \ \leq \mathrm{j} \leq \ \frac{N+1}{2}$ since those teams rank as $\{\ell^i_{1,\ldots\omega}, \ell^j_{1,\ldots\omega}, \ \ldots\}$ in a dual meet. From the definition of $\hat{\omega}$, team ℓ_i defeats ℓ_j for all $2 \ \leq \mathrm{i} \ \leq \frac{N+1}{2}$ and $\frac{N+1}{2} + 1 \leq \ \mathrm{j} \leq \mathrm{N}$ since those teams rank as $\{\ell^i_{1,\ldots\hat{\omega}}, \ell^j_{1,\ldots\omega}, \ \ell^i_{\hat{\omega}+1,\ldots,n}, \ \ell^j_{\omega+1,\ldots,n}\}$, so ℓ^i has a sufficient number of wins to ensure a victory. The definition of ω again makes sure that team ℓ^i defeats ℓ^j for all $\frac{N+1}{2} \ +1 \leq \ \mathrm{i} \ < N$ and $\frac{N+1}{2} + 1 < \ \mathrm{j} \leq \ \mathrm{N}$ since those teams rank as $\{\ell^{1,\ldots\omega}_i, \ell^{1,\ldots\omega}_j, \ \ldots\}$. We can then rely again on $\hat{\omega}$ to know that ℓ^i defeats ℓ^j for all $\frac{N+1}{2} + 1 \leq \ \mathrm{i} \leq \mathrm{N}$ and $1 \leq \mathrm{j} \leq \frac{N+1}{2} - 1$ since those teams rank as $\{\ell^j_{1,\ldots\hat{\omega}}, \ell^i_{1,\ldots\omega}, \ \ell^j_{\hat{\omega}+1,\ldots,\omega}, \ \ell^i_{\omega+1,\ldots,n}, \ \ell^j_{\omega+1,\ldots,n}\}$. Since we assume the teams rank players truthfully and players meet pairwise in individual matches so all that matters is one player's ranking relative to their counterpart's, this ranking is the same as $\{\ell^j_{1,\ldots\hat{\omega}}, \ell^i_{1,\ldots n}, \ \ell^j_{\omega+1,\ldots,n}\}$, and the definition of $\hat{\omega}$ applies.

Each team therefore has exactly $\frac{(N+1)}{2}$ victories each. Specifically, every team ℓ^i defeats teams ℓ^{i+1} through $\ell^{i+\frac{N-1}{2}}$ (where the subscripts are taken modulo N), and is defeated by teams $\ell^{i-\frac{N-1}{2}}$ through ℓ^{i-1}. □

The last result major of this section sets up the rest of the paper by relating the presence of cycles and dictatorship to the prospect of strategic behavior in team tennis. Rather than truthfully listing the ranking of their players, some tennis coaches may instead choose to list their stronger players lower in the order in order to give them easier victories, sacrificing one match in order to win others. A similar situation, dubbed the "tennis coach problem", was considered by Arad [17] [6], and can also be thought of as a type of Colonel Blotto game in which two generals have battle units of different strengths and must simultaneously deploy them on discrete battle fields. Each coach's strategy set is the set of all permutations of their players, and we examine an example of such a game in detail in Section 5.

As mentioned in the introduction, we consider a scoring rule to be strategy-proof if it is a (weakly) dominant strategy for a team to rank its players truthfully. We assume that it is each team's priority to win a dual meet, and that they will misrepresent their player ranking if doing so will help them to that end.

Proposition 5. *A weighted-sum team tennis scoring rule is strategy-proof if, and only if, it is dictatorial.*

Proof. For the "if" part of the statement, note that when $\gamma_1 > \sum_{\mathrm{j}>1} \gamma_{\mathrm{j}}$ only the result of the first, most important match determines the outcome of the competition. In that case, clearly a team cannot change whether it wins or loses by placing a weaker player in Match 1, and all other matches are irrelevant to the competition's final outcome, so truthful ranking is weakly dominant.

For the "only if" part of the statement, suppose $\gamma_1 < \sum_{j>1} \gamma_j$. Specifically, suppose that by winning all matches j through n, $2 \leq \mathrm{j} \leq \left(\left\lceil \frac{\mathrm{n}}{2} \right\rceil - 1\right)$, a team wins the competition. In that case there is always a possibility that two teams' players collectively rank as $\{x_1,\ y_1,\ x_2, y_2, x_3, y_3, \ldots, x_{n-1,}\ ,\ y_{n-1},\ x_n,\ y_n\}$. In that case, $\Gamma(\mathbf{x}) > \Gamma(\mathbf{y})$ if both teams truthfully represent themselves in all matchups. But team $\mathbf{y}$ could win the match simply by setting $y_i' = y_{i-1}$ for all $2 \leq \mathrm{i} \leq n$ and $y_1' = y_n$. □

Based on that result, in particular, the example used in the proof, and the fact that we assume all players must be strictly ranked overall, so a dictatorial rule must be acyclic, we have the following.

Corollary 1. *In a team tennis competition with $n \geq 3$ matches with weighted-sum scoring, strategic manipulation is possible for some true team rankings unless the scoring rule is acyclic.*

4. Cycle Computations

We know that cycles are possible for team tennis scoring, and that they are linked to interesting properties of leagues. But just because they are possible does not mean they are likely. To determine the proportion of ranking sequences that generate a cycle under standard scoring, we constructed a Java program to search the substantial space of possible rankings across three teams, each with three components, in the case that each team engages in a dual meet against each of the other two teams. In the simplified case of three teams and three individuals per team, there are $\left(\mathrm{n} = \frac{9!}{3!3!3!} = 1,680\right)$ possible player orderings.[7] We find a ranking cycle to occur in 30 of these orderings (1.79 percent of orderings). In the case of three teams and five individuals per team, there

6 Arad [17] considered the case of only four players per team, with each team having players of equal strengths, standard scoring, ties allowed, and teams playing in tournaments, using the games to test level-k thinking experimentally.

7 This calculation assumes that player orderings within team are known and invariant.

are $\left(n = \frac{15!}{5!5!5!} = 756,756\right)$ possible player orderings. We find a ranking cycle to occur in 13,140 of these orderings (1.74 percent of orderings). In the realistic case, where each team features seven individuals, there are $\left(n = \frac{21!}{7!7!7!} = 399,072,960\right)$ possible player orderings. Of these orderings, 6,259,992 (1.57 percent) generate a ranking cycle between the three teams. Empirically, the likelihood of obtaining one player quality ordering may be different from the likelihood of obtaining another (i.e., individual quality levels within a given team may be dependent). Therefore, the previous combinatorial results are not empirical in nature. However, the magnitudes of these results suggest that ranking cycles may occur with only modest frequency under the conditions of the model. The analysis of the subsequent section suggests that strategic representation of intra-squad ranking influences team match outcomes for a much larger set of orderings, but is also importantly linked to the possible presence of cycles.

5. Strategic Ranking and Competitive Balance

Strategic voting occurs when individuals have an incentive to falsely represent their preferences. Gibbard [12] and Satterthwaite [13] show conditions under which a voting rule leads to such incentives. Similarly, it has already been shown that a tennis team may have an incentive to falsely represent the quality rankings of its players. As in the case of strategic voting, strategic intra-squad ranking may alter the outcome of a competition. In the example that will soon be considered, a team that is strictly dominated in the case of true intra-squad ranking can win a majority of individual matches with positive probability (i.e., win the team contest with positive probability) given strategic intra-squad ranking. When strategic intra-squad ranking occurs, we show that it increases competitive balance and ex ante uncertainty of outcome between two opponents.

Consider two teams, **x** and **y**, each composed of three individual players, and assume standard team tennis scoring. As in the earlier case, let $x_1 \succ^* x_2 \succ^* x_3$ and $y_1 \succ^* y_2 \succ^* y_3$. In this arrangement, there are three individual matches in a dual meet between the two teams. Without loss of generality, assume a case in which **x** strictly dominates **y** in the absence of strategy. That is to say, $x_1 \succ^* y_1$, $x_2 \succ^* y_2$, and $x_3 \succ^* y_3$. And let us further assume that $y_1 \succ^* x_2$ and $y_2 \succ^* x_3$. As there are assumed to be no intransitive rankings at the individual level, we can infer from the above information the following player hierarchy matrix. In Table 1 below, a "1" indicates victory for player x_i over player y_j, and a "0" indicates the opposite outcome.

Table 1. All potential individual matches involving **x** and **y**.

	y_1	y_2	y_3
x_1	1	1	1
x_2	0	1	1
x_3	0	0	1

From Table 1, we construct the following player ranking sequence:

$$x_1 \succ^* y_1 \succ^* x_2 \succ^* y_2 \succ^* x_3 \succ^* y_3 \quad (6)$$

Note that **x** defeats **y** in 6 of 9 possible individual pairings. If each team represents its true intra-squad preference ranking, **y** will lose the team competition 0 (individual) matches to 3. One can verify this by examining the left-to-right diagonal of the above matrix. However, **y** could rank its players strategically to gain a positive probability of team victory. For example, **y** could (mis)represent its player rankings as follows: $y_3 \succ^* y_1 \succ^* y_2$. If **x** does not misrepresent its player rankings in kind, **y** will win the team competition (2 individual matches to 1) under this arrangement. This outcome obtains because $y_1 \succ^* x_2$ and $y_2 \succ^* x_3$. Of course, **x** can also misrepresent its player ability rankings. Therefore, we consider a strategic ranking *game* between teams **x** and **y** that occurs before the match proper. In the present analysis of the game, **x** and **y** are defined by the player quality ordering in condition 6. We will subsequently generalize the analysis to all possible match types between **x** and **y**. In the representation game, it is assumed that each team knows the true quality rank-ordering of the six participating players prior to the match. This is taken as a form of inside information that is not shared by the typical fan (i.e., that is derived from recruiting and from potentially obscure third-party comparisons).[8] Either team may misrepresent its true quality rank-ordering of players. However, it is difficult for the relevant sport regulatory body to prove misrepresentation of rankings, as such organizations do not typically know the results of intra-squad matches. Thus, we treat the practice of misrepresentation as feasible and as carrying no punishment cost. Let teams choose player rankings simultaneously. Further assume that, once stated, a team's rank-ordering must hold for the match proper. In NCAA tennis, for example, coaches simultaneously (and irretrievably) commit player rankings before a given team match. Below, we characterize the six possible representations within a three-player team (**x**).

In Table 2, note that rank-ordering A represents the team's true intra-squad rank-ordering. Rank-orderings B–F represent all possible misrepresentations. Each team in a dual team competition can represent its individual rank-ordering in one of six manners. That is, $S_i = \{A, B, C, D, E, F\}$, $i \in \{x, y\}$. As such, there are 36 strategy profiles associated with this representation game. We list these profiles and the outcomes associated with them in Table 3.

Table 2. Possible misrepresentations of player rank-ordering within team x.

Position	True Rank	Chosen Rank A	Chosen Rank B	Chosen Rank C	Chosen Rank D	Chosen Rank E	Chosen Rank F
1	x_1	x_1	x_1	x_2	x_2	x_3	x_3
2	x_2	x_2	x_3	x_1	x_3	x_1	x_2
3	x_3	x_3	x_2	x_3	x_1	x_2	x_1

Table 3. Normal Form of a representation game between x and y.

S_x \ S_y	A	B	C	D	E	F
A	(1,0)	(1,0)	(1,0)	(1,0)	(0,1)	(1,0)
B	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)	(0,1)
C	(1,0)	(0,1)	(1,0)	(1,0)	(1,0)	(1,0)
D	(0,1)	(1,0)	(1,0)	(1,0)	(1,0)	(1,0)
E	(1,0)	(1,0)	(1,0)	(0,1)	(1,0)	(1,0)
F	(1,0)	(1,0)	(0,1)	(1,0)	(1,0)	(1,0)

For each representation strategy employed by **x**, there exists a different strategy available to **y** that defeats the given strategy of **x**. Therefore, there exist no pure strategy Nash equilibria in the present representation game. Specifically, if **x** (**y**) were to unilaterally employ a given pure strategy, then **y** (**x**) could win the team competition (i.e., win the majority of individual matches) with certainty. There is a mixed strategy Nash equilibrium in the game that can be characterized as follows.

$$p(A) = p(B) = p(C) = p(D) = p(E) = p(F) = q(A) = q(B) = q(C) = q(D) = q(E) = q(F) = \tfrac{1}{6} \quad (7)$$

where $p(A)$ represents the likelihood that **x** chooses rank representation A and $q(A)$ is the likelihood that **y** chooses rank representation A.

[8] Numerous betting scandals involve betting fans seeking information from athletes and coaches prior to a match.

This mixed strategy Nash equilibrium can be described as a pair of evenly mixed strategies. Across the two teams, each representation strategy is chosen (independently) with likelihood $\frac{1}{6}$. In the equilibrium outcome, **x** wins the team competition with likelihood $\frac{5}{6}$, and **y** wins the team competition with likelihood $\frac{1}{6}$. Let us demonstrate that this is indeed the equilibrium of the representation game. Let **x** unilaterally deviate from this equilibrium candidate. If **x** were to choose one of the six strategies (strategy j) with likelihood less than $\frac{1}{6}$, this implies that **x** will choose at least one other strategy (strategy k) with likelihood greater than $\frac{1}{6}$. As **y** mixes evenly between the six strategies, however, **x** cannot do better than an expected win likelihood of $\frac{5}{6}$. By similar reasoning, y cannot do better than a win likelihood of $\frac{1}{6}$ via unilateral deviation.

As $x_1 \succ^* x_2 \succ^* x_3$ and $y_1 \succ^* y_2 \succ^* y_3$ by definition, there are $\left(\frac{6!}{3!3!} = 20\right)$ possible true orderings of the six players (i.e., 20 match types). To generalize the analysis of the representation game, we list these possible true orderings as follows:

$$x_1 \succ^* x_2 \succ^* x_3 \succ^* y_1 \succ^* y_2 \succ^* y_3 \tag{8a}$$
$$x_1 \succ^* x_2 \succ^* y_1 \succ^* x_3 \succ^* y_2 \succ^* y_3 \tag{8b}$$
$$x_1 \succ^* x_2 \succ^* y_1 \succ^* y_2 \succ^* x_3 \succ^* y_3 \tag{8c}$$
$$x_1 \succ^* x_2 \succ^* y_1 \succ^* y_2 \succ^* y_3 \succ^* x_3 \tag{8d}$$
$$x_1 \succ^* y_1 \succ^* x_2 \succ^* x_3 \succ^* y_2 \succ^* y_3 \tag{8e}$$
$$x_1 \succ^* y_1 \succ^* x_2 \succ^* y_2 \succ^* x_3 \succ^* y_3 \text{ (case previously considered)} \tag{8f}$$
$$x_1 \succ^* y_1 \succ^* x_2 \succ^* y_2 \succ^* y_3 \succ^* x_3 \tag{8g}$$
$$y_1 \succ^* x_1 \succ^* x_2 \succ^* x_3 \succ^* y_2 \succ^* y_3 \tag{8h}$$
$$y_1 \succ^* x_1 \succ^* x_2 \succ^* y_2 \succ^* x_3 \succ^* y_3 \tag{8i}$$
$$y_1 \succ^* x_1 \succ^* x_2 \succ^* y_2 \succ^* y_3 \succ^* x_3 \tag{8j}$$
$$y_1 \succ^* y_2 \succ^* y_3 \succ^* x_1 \succ^* x_2 \succ^* x_3 \tag{8k}$$
$$y_1 \succ^* y_2 \succ^* x_1 \succ^* y_3 \succ^* x_2 \succ^* x_3 \tag{8l}$$
$$y_1 \succ^* y_2 \succ^* x_1 \succ^* x_2 \succ^* y_3 \succ^* x_3 \tag{8m}$$
$$y_1 \succ^* y_2 \succ^* x_1 \succ^* x_2 \succ^* x_3 \succ^* y_3 \tag{8n}$$
$$y_1 \succ^* x_1 \succ^* y_2 \succ^* y_3 \succ^* x_2 \succ^* x_3 \tag{8o}$$
$$y_1 \succ^* x_1 \succ^* y_2 \succ^* x_2 \succ^* y_3 \succ^* x_3 \tag{8p}$$
$$y_1 \succ^* x_1 \succ^* y_2 \succ^* x_2 \succ^* x_3 \succ^* y_3 \tag{8q}$$
$$x_1 \succ^* y_1 \succ^* y_2 \succ^* y_3 \succ^* x_2 \succ^* x_3 \tag{8r}$$
$$x_1 \succ^* y_1 \succ^* y_2 \succ^* x_2 \succ^* y_3 \succ^* x_3 \tag{8s}$$
$$x_1 \succ^* y_1 \succ^* y_2 \succ^* x_2 \succ^* x_3 \succ^* y_3 \tag{8t}$$

In 12 of the 20 true orderings, there is no incentive to misrepresent one's intra-squad player rankings and thus no representation game. These include orderings Equation (8a–e,h,k–o,r). In each such case, there is a dominant team that wins regardless of the strategy profile that would obtain in a representation game. If one of true orderings Equation (8f,g,i,j,p,q,s), or (8t) obtains, a representation game ensues. By the same reasoning as was applied in the previous analysis of ordering Equation (8f), the Nash equilibrium for each of these eight true orderings is found to be an evenly distributed mixed strategy (i.e., each representation chosen with probability $\frac{1}{6}$). However, payoffs (in the form of win likelihood) vary according to the true ordering that obtains. Payoffs are depicted in Table 4 as follows for each of the eight true orderings that elicit a representation game.

Table 4. Representation Game Nash equilibrium outcomes.

Ordering	Outcome without Strategy	Outcome with Strategy
(8f)	**x** wins with probability **1**	**x** wins with probability $\mathbf{\frac{5}{6}}$
(8g)	**x** wins with probability **1**	**x** wins with probability $\mathbf{\frac{2}{3}}$
(8i)	**x** wins with probability **1**	**x** wins with probability $\mathbf{\frac{2}{3}}$
(8j)	**x** wins with probability **0**	**x** wins with probability $\mathbf{\frac{1}{3}}$
(8p)	**x** wins with probability **0**	**x** wins with probability $\mathbf{\frac{1}{6}}$
(8q)	**x** wins with probability **0**	**x** wins with probability $\mathbf{\frac{1}{3}}$
(8s)	**x** wins with probability **0**	**x** wins with probability $\mathbf{\frac{1}{3}}$
(8t)	**x** wins with probability **1**	**x** wins with probability $\mathbf{\frac{2}{3}}$

In 8 of 20 possible player quality orderings, the representation game occurs and allows a team that would never win in the absence of strategy to win with positive probability. We therefore conclude that representation strategy, like ranking cycles, is expected to increase competitive balance and *ex ante* uncertainty of marginal match outcome between two opponents. It is clear from the previous table that competitive balance increases under strategic misrepresentation. It also follows that team match outcome uncertainty increases under strategic misrepresentation. This is because strategic misrepresentation creates *ex ante* uncertainty as to which individual matchups will occur. Whereas the victorious team follows from individual player rankings in the absence of strategic misrepresentation, such a mapping does not occur under strategic misrepresentation.

Under the uncertainty of outcome hypothesis, we expect representation strategy to increase fan welfare in team tennis. Moreover, strategic ranking obtains over a much larger set of orderings than do ranking cycles. As in the section concerning ranking cycles, consider a set of three teams, each of which features three individual players. As was found previously, there are 1680 possible player orderings across three such teams, and 30 orderings (1.79 percent) result in a ranking cycle. We now examine the likelihood that strategic ranking behavior occurs in all three team matches between **x**, **y**, and **z** (thus improving competitive balance between **x** and **y**, **y** and **z**, and **z** and **x**, respectively).

In the rank-ordering $x_1 \succ^* y_1 \succ^* z_1 \succ^* z_2 \succ^* x_2 \succ^* y_2 \succ^* y_3 \succ^* z_3 \succ^* x_3$, which was found to produce a ranking cycle in Section 2, three sub-orderings can be derived (each defining the outcome of a dual team match among the three teams). When **x** and **y** play, the sub-ordering $x_1 \succ^* y_1 \succ^* x_2 \succ^* y_2 \succ^* y_3 \succ^* x_3$ obtains. When **y** and **z** play, the sub-ordering $y_1 \succ^* z_1 \succ^* z_2 \succ^* y_2 \succ^* y_3 \succ^* z_3$ obtains. When **z** and **x** play, the sub-ordering $x_1 \succ^* z_1 \succ^* z_2 \succ^* x_2 \succ^* z_3 \succ^* x_3$ obtains. It can be verified from the three sub-orderings, that strategic intra-squad ranking occurs in each of these three dual matches. Therefore, the sequence $x_1 \succ^* y_1 \succ^* z_1 \succ^* z_2 \succ^* x_2 \succ^* y_2 \succ^* y_3 \succ^* z_3 \succ^* x_3$ results in strategic ranking for each of the three dual team matches between, **y,** and **z**. Across the set of 1680 rank-orderings, 216 orderings (12.9 percent) result in strategic ranking for each of the three dual team matches between **x**, **y**, and **z**. While this is not an empirical result, it indicates that strategic ranking influences competitive balance over a much larger set of orderings than do ranking cycles. Moreover, strategic ranking occurs in at least one of the three dual team matches in 1272 orderings (75.7 percent of orderings) and in at least two of the three dual team matches in 528 orderings (31.4 percent of orderings).

6. Conclusions

In what many believe to be the earliest journal article in sports economics (see, e.g., Sanderson and Siegfried [18]), Rottenberg [19] developed the uncertainty of outcome hypothesis.[9] This hypothesis holds that spectator demand (value) for a match rises in the uncertainty of its outcome. Fort [20] writes, "Rottenberg [19] . . . noted that there must be equal 'size' among competitors if any are to be successful (p. 242) and that 'uncertainty of outcome' shifts demand functions to the right (p. 246)." As discussed in the introduction, the uncertainty of outcome hypothesis has received considerable empirical support. Knowles, Sherony, and Haupert [21] use a measure of outcome uncertainty derived from betting lines and find competitive balance to significantly improve attendance in major league baseball. Pinnuck and Potter [14] find that uncertainty of outcome positively influences attendance and team financial viability in the Australian Football League. Similarly, Forrest and Simmons [22] find that "admissions at English soccer matches relate positively to the quality of teams involved and negatively to a measure of the relative win probabilities of the competing teams" (p. 229). Forrest, Simmons, and Buraimo [23] find that the same result holds when examining the television demand for English soccer matches. In describing the hypothesis, Sanderson and Siegfried [24] state:

> Although the absolute quality of play influences demand and absolute investments in training are socially efficient (Lazear and Rosen [25]), the relative aspects of demand and quality of competition also loom large in sports. In cases when consumer demand depends, to a large extent, on inter-team competition and rivalry, the necessary interactions across 'firms' (i.e., teams) define the special nature of sports. Contests between poorly matched competitors would eventually cause fan interest to wane and industry revenues to fall.
>
> (p. 256)

Schmidt and Berri [26] find a positive relationship between competitive balance and attendance in major league baseball "whether one explores the relationship strictly across time or with the use of a panel data set" (p. 145). Schmidt and Berri (p. 146) further explain the effect of uncertainty of outcome upon demand and team talent allocation:

> The seminal work of El-Hodiri and Quirk [27] laid forth the essential dilemma facing a professional sports league. Each team strives to put together a level of talent that increases the probability that it will defeat its opponents. However, if the team achieves too much success with respect to the objective of win maximization, the objective of profit maximization may be compromised. In the words of Walter Neale [25], whose seminal work preceded El-Hodiri and Quirk [27] by 7 years, the prayer of a premier team such as the New York Yankees must be, 'Oh Lord, make us good, but not that good'.
>
> (Neale, [25] (p. 2))

To understand the uncertainty of outcome hypothesis, one might consider three types of fans in a dual match: team A supporters, team B supporters, and neutral fans. Neutral fans may consist of bettors and of those who simply like to observe a new or uncertain event. Under the uncertainty of outcome hypothesis, neutral fans are expected to lose interest as the match becomes more certain in favor of one team. It becomes more difficult to establish gambles (i.e., simplistic betting markets cease and a handicap betting market must be designed), and neutral fans who do not gamble may feel that the match outcome is a foregone conclusion.[10] As competitive balance (uncertainty of outcome)

[9] Several economists have credited Rottenberg [19] with beating Ronald Coase to the Coase Theorem in the same paper through his Invariance Principle (see, e.g., Fort [20]).

[10] Handicap betting markets, in which a "handicapper" or "market-maker" attempts to establish a fair bet through artificial scoring compensation or other means, are commonly thought to be more manipulable than traditional betting markets. For

increases, there may be a transfer of "fan welfare" from team A (B) supporters to team B (A) supporters. If these two groups are equal in numbers and fervor, this transfer will be welfare neutral. Following the uncertainty of outcome hypothesis, however, one expects the welfare of neutral fans to increase in competitive balance. In terms of its incentive to generate fan interest, team tennis is not dissimilar from many higher revenue sports. In the first two years of the recent economic recession, for example, it is estimated that 227 NCAA sports teams would be discontinued, with a disproportionate number occurring in low (but variable) revenue sports such as tennis (Watson [28]). Despite the non-profit nature of university athletic departments in the United States, athletic departments apparently consider financial viability when deciding whether to maintain a given sports team.

In Section 4 of the present paper, it was found that ranking cycles improve competitive balance among sets of three opposing tennis teams. In Section 5, it was found that strategic ranking improves competitive balance between two teams (when employed). It does so by causing a team that is dominant in the absence of strategy to lose with positive probability in the presence of strategy. Under the uncertainty of outcome hypothesis, ranking cycles and strategic ranking in tennis promote demand and enhance third-party (spectator) welfare. Rather than a symptom of poor design, the noted social choice violations in sport may promote demand and enhance welfare (e.g., under the uncertainty of outcome hypothesis). That is to say, certain social choice violations *in sport* may be purposeful.

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example, the "market-maker" may have an incentive to under-compensate or over-compensate a team. Moreover, a player or coach can sometimes alter the outcome of a handicapped bet without altering the overall match outcome. In college basketball, several point-shaving scandals have arisen from handicapped betting markets. Such problems are explored in, e.g., Wolfers [29], Borghesi [30], and Bernhardt and Heston [31].

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