Supplementary Materials: Evolution of Cooperation in Social Dilemmas with Assortative Interactions

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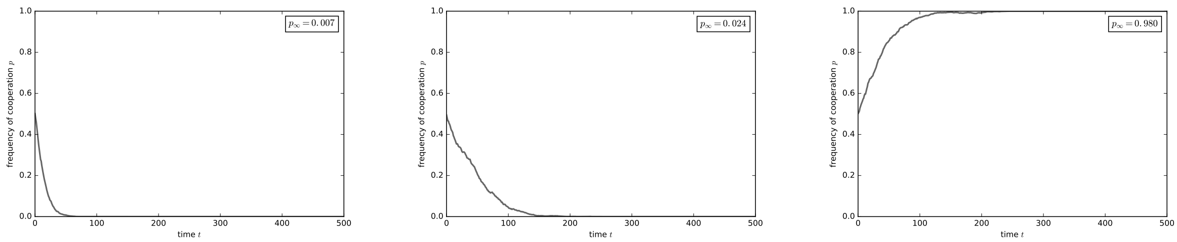
1. Additional Results from Individual-Based Simulations

1.1. Discrete Games

1.1.1. Donation Game

Figures S1(a)-(c) show the evolution of the fraction p of cooperators in the donation game with payoff matrix given by equation 9, for different values of the assortativity r. The inset in the plots indicates the long-term fraction p_∞ of cooperators, averaged over the last 10% of the generations.

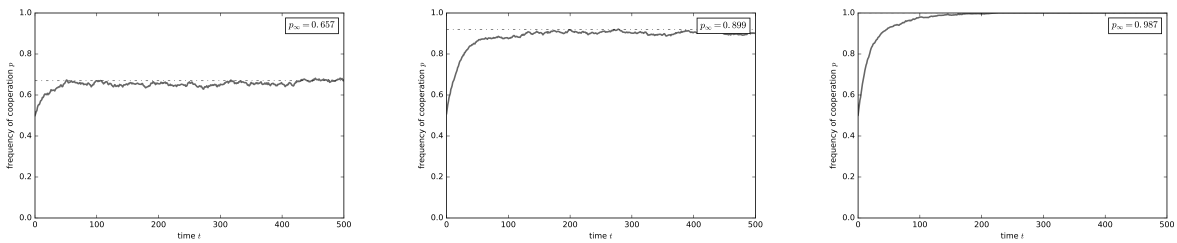


(a) **(b)** **(c)**

Figure S1. Evolution of the frequency p of cooperators in the donation game for different values of assortativity r. (a) $r = 0$. (b) $r = 0.2$. (c) $r = 0.4$. Parameters: $\rho = 0.3$, $n = 10000$, $p_0 = 0.5$, and $\beta = 1$.

1.1.2. Snowdrift Game

Figures S2(a)-(c) show the evolution of the fraction p of cooperators in the snowdrift game with payoff matrix given by equation 11, for different values of the assortativity r. The inset in the plots indicates the long-term fraction p_∞ of cooperators, averaged over the last 10% of the generations, and the dotted line indicates the analytically predicted value corresponding either to the stable internal equilibrium $p^\star$ or to the stable boundary equilibrium $\hat{p}$.

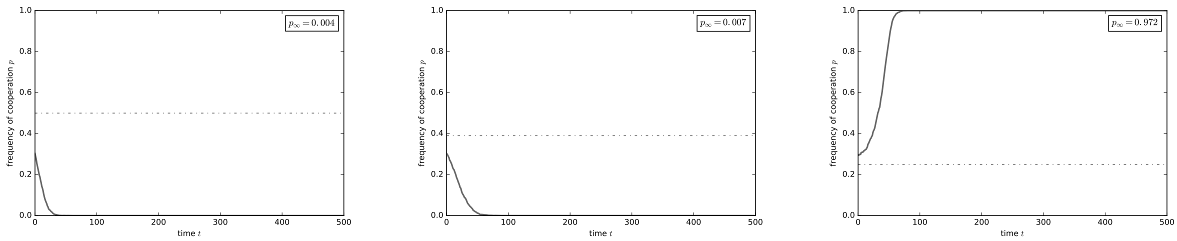


(a) **(b)** **(c)**

Figure S2. Evolution of the frequency p of cooperators in the snowdrift game for different values of assortativity r. (a) $r = 0$ ($p^\star \approx 0.67$). (b) $r = 0.2$ ($p^\star \approx 0.92$). (c) $r = 0.3$ ($p^\star \approx 1.0$). Parameters: $\rho = 0.5$, $n = 10000$, $p_0 = 0.5$, and $\beta = 1$.

1.1.3. Sculling Game

Figures S3(a)-(c) show the evolution of the fraction p of cooperators in the sculling game with payoff matrix given by equation 14, for different values of the assortativity r. The inset in the plots indicates the long-term fraction p_∞ of cooperators, averaged over the last 10% of the generations and the dotted line indicates the value of the unstable internal equilibrium $p^\star$.



(a) **(b)** **(c)**

Figure S3. Evolution of the frequency p of cooperators in the sculling game for different values of assortativity r. (a) $r = 0$. (b) $r = 0.1$. (c) $r = 0.2$. Parameters: $\rho = 1$, $n = 10000$, $p_0 = 0.3$, and $\beta = 1$.

1.2. Continuous Games

1.2.1. Continuous Donation Game

Figures S4(a)-(c) show the evolution of the distribution of strategies for different values of r in the CD game with linear cost and benefit functions $C(x) = cx$ and $B(x) = bx$, where $b > c$. We also show in this figure the corresponding pairwise invasibility plots (PIPs), in which the regions where a mutant strategy y can invade a resident strategy x (i.e., the set $\mathcal{I}_+ = \{(x, y) \in [0, 1] : f_x(y) > 0\}$) are shown in black (and marked "+") and the uninvadable regions (i.e., the set $\mathcal{I}_- = \{(x, y) \in [0, 1] : f_x(y) < 0\}$) are shown in white (and marked "−").

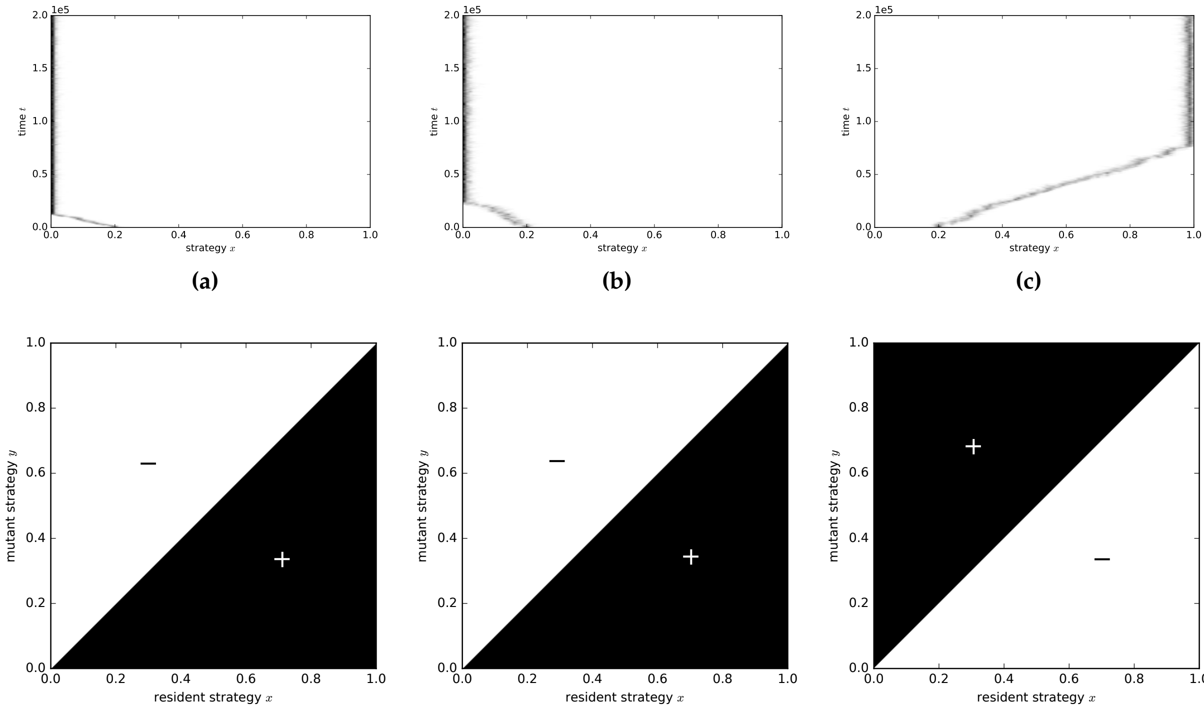
Figures S5(a)(b) show the evolution of the distribution of strategies x for different values of assortativity r, in the CD game with quadratic cost and benefit functions $C(x) = c_1x^2$ and $B(x) = -b_2x^2 + b_1x$, where $c_1, b_1, b_2 > 0$. We let $b_1 = 2b_2$; the dotted line in the plots indicates the singular strategy $x^\star$ given by equation 24. Figures S5(c)(d) show the corresponding PIPs.

1.2.2. Continuous Snowdrift Game

Figures S6(a)(b) and S7(a)(b) show the evolution of the distribution of strategies x for different values of assortativity r, in hte CSD game with quadratic cost and benefit functions $C(x) = -c_2x^2 + c_1x^2$ and $B(x) = -b_2x^2 + b_1x$, where $c_1, c_2, b_1, b_1 > 0$. The dotted line in the plots indicates the singular strategy $x^\star$ given by equation 28. Figures S6(c)(d) and S7(c)(d) show the corresponding PIPs.

1.2.3. Continuous Tragedy of the Commons Game

Figures S8(a)(b) show the evolution of the distribution of strategies x for different values of assortativity r, in the CTOC game with quadratic cost and cubic benefit functions $C(x) = c_1x^2$ and $B(x) = -b_3x^3 + b_2x^2 + b_1x$. If we let $b_2 = 2b_1$ and $c_1 = b_1$; the dotted line in the plots indicates the singular strategy $x^\star$ given by equation 34. Figures S8(c)(d) show the corresponding PIPs.



(d) (e) (f)

Figure S4. Evolution of the distribution of strategies x (a-c) and the corresponding pairwise invasibility plots (d-f) in the CD game with linear cost and benefit functions: $C(x) = 0.3x$ and $B(x) = x$. (a) $r = 0$. (b) $r = 0.2$. (c) $r = 0.4$. Parameters: $n = 10000$, $x_0 = 0.2$, $x_m = 1$, $\mu = 0.01$, $\sigma = 0.005$, and $\beta = 1$.

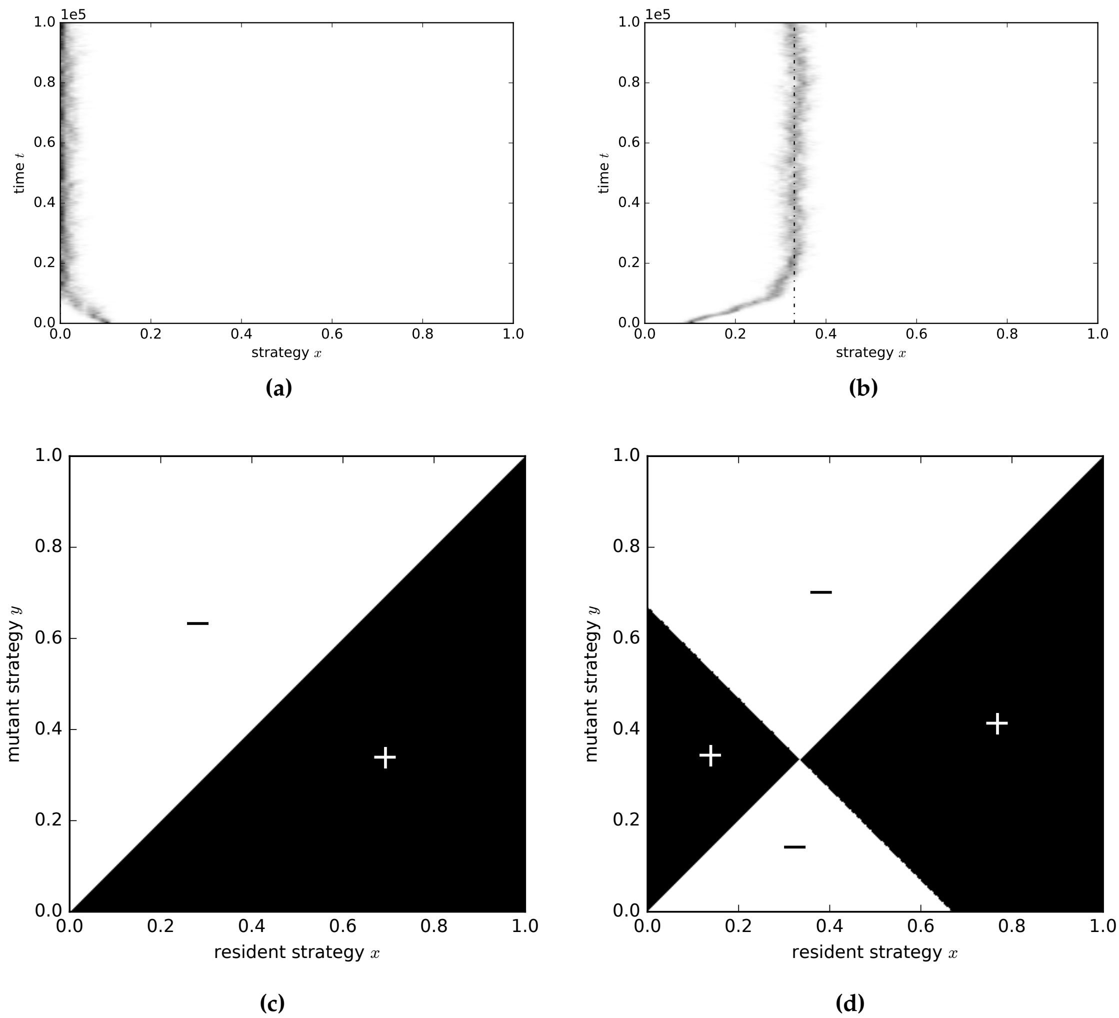


Figure S5. Evolution of the distribution of strategies x (a, b) and the corresponding pairwise invasibility plots (c, d) in the CD game with quadratic cost and benefit functions: $C(x) = x^2$ and $B(x) = -x^2 + 2x$. (a) $r = 0$ ($x^\star = 0$ is an ESS). (b) $r = 0.5$ ($x^\star = 0.33$ is an ESS). Parameters: $n = 10000$, $x_0 = 0.1$, $x_m = 1$, $\mu = 0.01$, $\sigma = 0.005$, and $\beta = 1$.

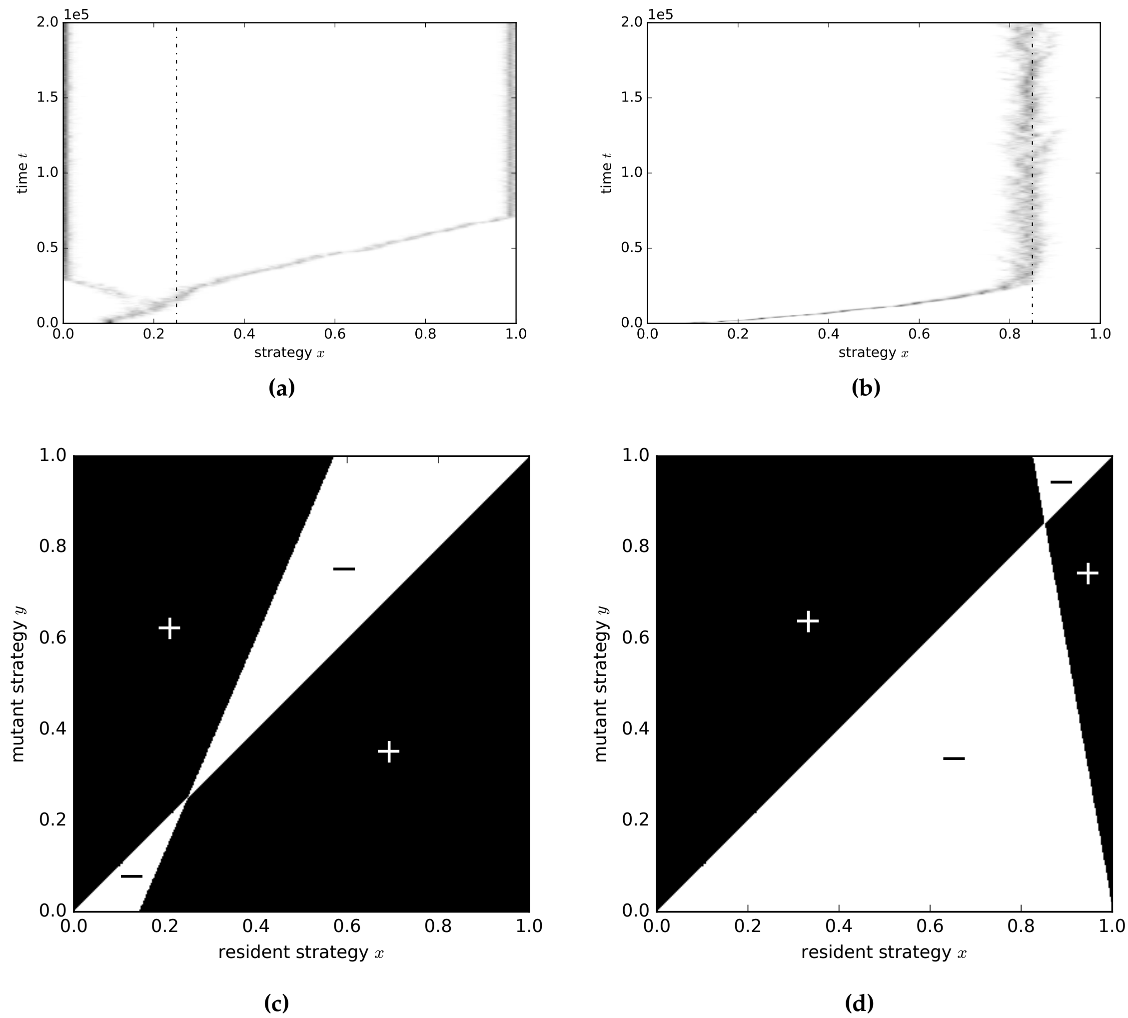


Figure S6. Evolution of the distribution of strategies x (a, b) and the corresponding pairwise invasibility plots (c, d) in the CSD game with quadratic cost and quadratic benefit functions: $C(x) = -c_2x^2 + c_1x$ and $B(x) = -b_2x^2 + b_1x$, with $c_1 = 4.8, c_2 = 1.6, b_1 = 5, b_2 = 1$. (a) $r = 0$ ($x^\star = 0.25$ is an EBP). (b) $r = 0.3$ ($x^\star = 0.85$ is an ESS). Parameters: $n = 10000$, $x_0 = 0.1$, $x_m = 1$, $\mu = 0.01$, $\sigma = 0.005$, and $\beta = 1$.

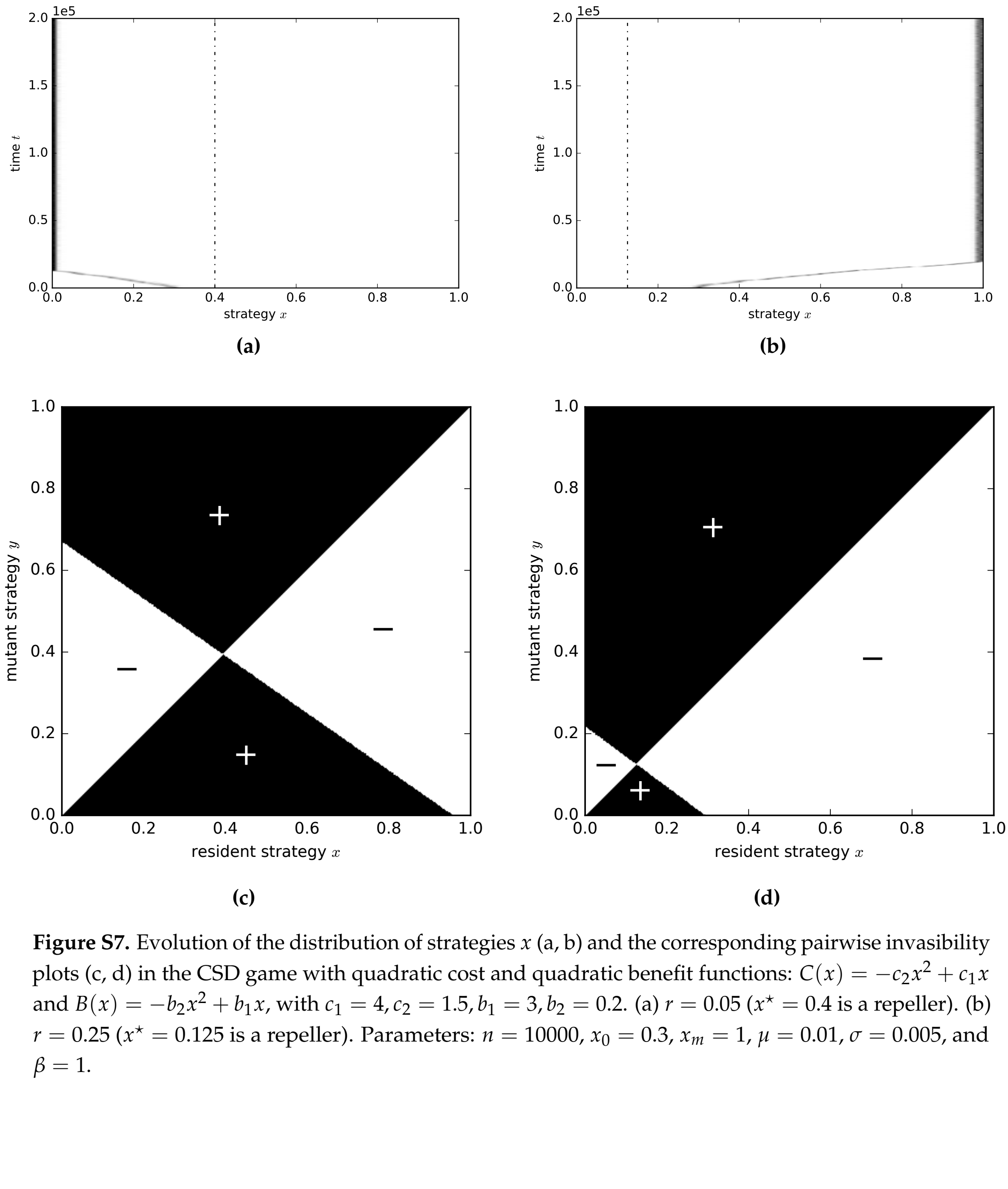


Figure S7. Evolution of the distribution of strategies x (a, b) and the corresponding pairwise invasibility plots (c, d) in the CSD game with quadratic cost and quadratic benefit functions: $C(x) = -c_2x^2 + c_1x$ and $B(x) = -b_2x^2 + b_1x$, with $c_1 = 4, c_2 = 1.5, b_1 = 3, b_2 = 0.2$. (a) $r = 0.05$ ($x^\star = 0.4$ is a repeller). (b) $r = 0.25$ ($x^\star = 0.125$ is a repeller). Parameters: $n = 10000$, $x_0 = 0.3$, $x_m = 1$, $\mu = 0.01$, $\sigma = 0.005$, and $\beta = 1$.

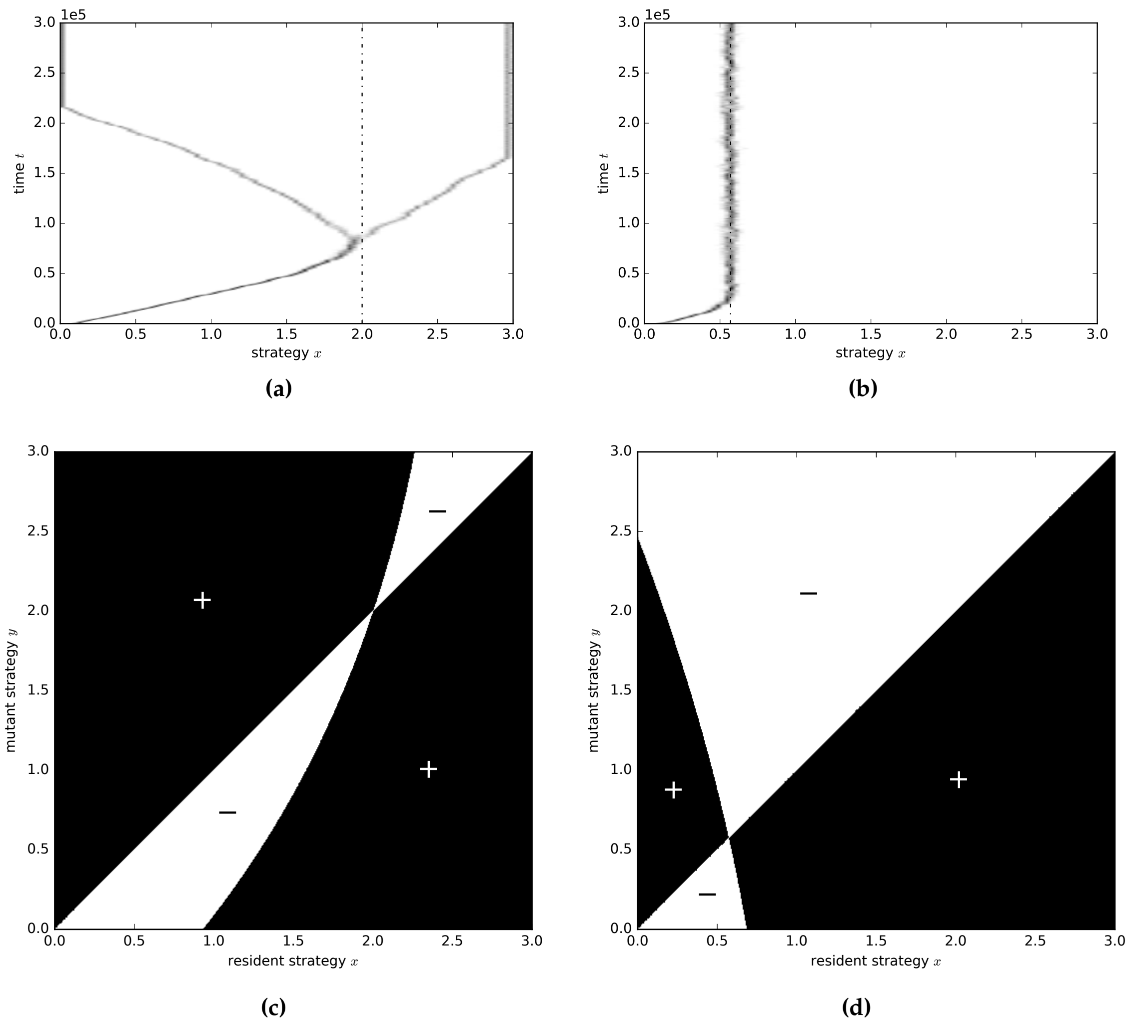


Figure S8. Evolution of the distribution of strategies x (a-b) and the corresponding pairwise invasibility plots (c-d) in a CTOC game with quadratic cost and cubic benefit functions: $C(x) = x^2$ and $B(x) = -0.0834x^3 + 2x^2 + x$. (a) $r = 0$ ($x^\star = 2$ is an EBP); (b) $r = 0.4$ ($x^\star = 0.57$ is an ESS). Parameters: $n = 10000$, $x_0 = 0.1$, $x_m = 3$, $\mu = 0.01$, $\sigma = 0.005$, and $\beta = 1$.