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# The Impact of Organizer Market Structure on Participant Entry Behavior in a Multi-Tournament Environment

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**Abstract:** A multi-tournament environment is analyzed, focusing on the impact of organizer market structure on agent entry behavior. Two high ability agents first decide which tournament to enter (with fields then filled by low ability agents). If the marginal benefit of high ability agents in an event is weakly increasing, a monopsonist organizer sets prizes so that the high ability agents enter the same event. If this marginal benefit is diminishing, a monopsonist organizer will either: always set prizes for which the high ability agents enter different events; or set prizes for which the high ability agents enter different events if and only if the difference in ability between the high ability and low ability agents is sufficiently small. Sequentially competing organizers set prizes for which both high ability agents enter the same event if and only if the marginal benefit of having two high ability agents in one event is relatively large. For competing organizers there may be either a first or second mover advantage. Finally, Social Welfare may be higher or lower with competing organizers, implying greater organizer competition does not necessarily increase Social Welfare. Parallels are noted throughout to the labor market for professional golfers both over years when the PGA TOUR was essentially a monopsonist and more recently when LIV Golf emerged as a competitor.

**Keywords:** tournament; entry decision; competing organizers; employee compensation

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## 1. Introduction

Labor market tournaments (in which payments to agents depend upon relative performance) have been examined extensively in the economics literature (pioneering works include [1–4]). The primary focus has been an environment in which agents compete in a single tournament. However, in practice agents often have a choice over the competitive environment in which they will compete. The present study examines the entry decision by agents in a multi-tournament setting. Our primary objective is to develop and analyze a game-theoretic model to gain insights on how organizer market structure impacts the choice of tournament prizes and resulting tournament fields in such an environment.

The participation decision of agents and the resulting field of entrants in a single tournament has been examined previously. Ref. [1] identified an adverse selection problem arising when the organizer cannot observe the ability of agents and determines the field of a single tournament by randomly choosing a pair of agents from a common applicant pool. Agents will not sort themselves into applicant pools of different abilities, since: if there were a pool of high ability agents and a separate low ability pool, low ability agents would prefer to enter the high ability pool.

More recently, the endogenous entry decision of agents across multiple competitive environments of this type has been examined. Ref. [5] examines a multi-tournament market in which heterogeneous firms (differing in bankruptcy probability) compete for workers by varying the magnitude of a pre-announced prize. Workers decide which firm to enter based upon the bankruptcy rates and prizes of each firm. The two primary insights are that: workers exert less effort in firms with higher bankruptcy rates; however, firms with higher bankruptcy rates may offer larger prizes. Ref. [6] considers a multi-tournament setting in which agents self select the competitive environment in which they will compete. The unique equilibrium outcome may be such that the tournament offering larger prizes attracts a field of lower quality entrants than the tournament offering smaller prizes. Ref. [7] obtains a qualitatively similar result when examining the participation decision by agents of differing abilities over two all-pay auctions. They show that equilibria exist in which the highest ability agent chooses to compete for the less valued item. The present study distinguishes itself from these by treating the choice of prizes as endogenous, and focusing on how organizer market structure impacts the choice of prizes and entry behavior of tournament participants.

Considering the choice of prizes as a tool for organizers in the model, first note that in general an agent would exert more effort when competing against rivals of relatively similar ability and would exert less effort when competing against rivals of relatively different ability, all else constant. This intuition is explored in depth by the literature on *bias* in tournament environments, where the optimal design for an organizer seeking to maximize total effort is often to "level the playing field," restoring symmetry between players by augmenting the success function technology [8,9]. By that logic, in order to maximize effort, an organizer would want to group agents of similar abilities in the same tournaments.

There are multiple differences between the current model and traditional bias models such as [8] or [9], however. Traditional bias models allow organizers to directly adjust (or "handicap") the marginal benefit of agent effort toward the probability of victory based on their heterogeneity within a single competition; this is their primary policy tool to influence effort, by making the field of participants more or less balanced. We instead allow organizers to offer prizes of differing value to agents of heterogeneous ability in a multi-tournament setting, then allow the agents themselves to self-select which competition to enter and therefore how balanced (or unbalanced) the resulting fields in each individual competition are. This indirect method of influencing the balance of ability leads to a similar impact on effort maximization within each individual competition, but this leads to another major difference between the current paper and others.

Most models of labor tournament competition, including those involving bias or "handicapping," explicitly assumes that the benefits to the organizer depend only upon effort and not upon the identity of tournament entrants; the model developed here allows the organizer's benefits to depend upon levels of effort, the identity of tournament participants, or both. Furthermore, most neglect the possibility of a multi-tournament setting, a major feature of the current model, which may be an important consideration even if effort is the designer's only objective. Refs. [10,11], for example, show that biasing a dynamic, multi-stage tournament so that participants are *less* symmetric can increase total effort due to increased participation when incentives are linked across stages. Whether a designer will prefer agents of similar or differing abilities in the same tournament therefore remains a question.

Finally, as previously mentioned, another feature making the current model distinct is its treatment of participation as a choice. While past work such as [12] has considered the impact of the number of competitors on effort provision in tournaments, and [13] models the choice of prizes versus punishments (prizes of negative value) when agents may choose not to participate in a single competition, our model differs in that agents choose which tournament to enter from a set of options–and therefore whom to compete against. The entry decision therefore depends not only on the prizes offered by different events, but also the field of competitors making their own entry choices.

As a motivating example for the model's focus on multi-tournament participation, consider the labor market for elite professional tournament golfers. Since establishing itself as a separate entity from the PGA of America in 1968, the PGA TOUR has been the premier circuit on which the best professional golfers have chosen to compete for decades.[1] In 2022, the LIV Golf Invitational Series was launched in an attempt to compete directly with the PGA TOUR, trying to attract the most talented and highest profile names in golf as entrants in its events.[2] The fact that LIV Golf cares about the identity of the entrants in its events (as opposed to just the amount of effort they exert in a tournament) is evidenced by the large amounts of up front guaranteed money they paid to several prominent players, including over $100 million each to Phil Mickelson, Dustin Johnson, and Bryson DeChambeau.[3] Moreover, as LIV Golf was assembling its group of 48 participants, most of the individuals that were invited to join the circuit had membership status on the PGA TOUR and were therefore presented with a choice regarding in which circuit of events to compete.

Consider a multi-tournament setting in which agents self select the competitive environment in which they will compete. The primary focus is on how prizes and resulting fields depend upon tournament organizer market structure. Suppose there are two tournaments, each with a field limited to two entrants. Agents are of two different ability levels (high ability and low ability), and the identity of tournament participants is potentially valued in and of itself by an organizer. First, two high ability agents individually choose which event to enter, after which two low ability agents fill any remaining vacancies in the fields. In this setting, the resulting composition of entrants across events can be broadly thought of as either a "pooling" composition (with both high ability agents in the same event) or a "separating" composition (with the high ability agents in different events). The focus of the analysis is on the choice of prizes by the organizer(s) and the subsequent agent entry behavior. When setting prizes, an organizer must account for how the prizes impact both the resulting tournament fields as well as the within tournament effort choice by agents. Two alternative organizer market structures are considered: monopsony (with a single organizer setting prizes in both events) and sequential competition (with two independent, competing organizers sequentially setting prizes). In a loose sense, the monopsony model would apply over the decades when the PGA TOUR was the only circuit for the premier players in the world, whereas the model of sequential competition would apply as LIV Golf enters and attempts to compete directly with the PGA TOUR.[4]

With a monopsonist organizer (whose benefits potentially depend on the identities of tournament participants) either a pooling or a separating composition could be best, depending upon how the "marginal benefit of having high ability agents in a particular tournament" behaves. If the marginal value of having high ability agents in a tournament is constant or increasing, then a monopsonist organizer will set prizes for which both high ability agents enter the same event. If instead there is a diminishing marginal benefit from having high ability entrants in a particular event, a monopsonist organizer will either: always set prizes for which the high ability agents enter different events; or set prizes for which the high ability agents enter different events if and only if the difference in ability between the high ability agents and low ability agents is sufficiently small.

The relevance of this scenario for a monopsonist organizer can be illustrated by considering the PGA TOUR during the first decade of the twenty-first century. During this time, Tiger Woods was unquestionably the best golfer in the world and Phil Mickelson was the second best.[5] The appeal to fans and ability of Tiger and Phil over these years was such that we could think of them as the two high ability agents on the PGA TOUR with the rest of the members as "filler competitors" of low ability. As a monopsonist organizer, the PGA TOUR needed to figure out if they wanted to set prizes for which Tiger and Phil would compete directly against each other as often as possible or if it was better to set prizes for which these two top players would spread themselves out over as many events as possible (i.e., was it ideal to try to maximize the number of events that had at least one of these top tier players in the filed, even at the expense of rarely having them compete head-to-head against each other).

In contrast, when organizers are competing against each other they only care about the effort exerted and identity of competitors in their own event. Of the 48 competitors that LIV Golf eventually attracted to its circuit in its first year of play, four had been ranked as the best player in the world at some point in time by the Official World Golf Ranking

(Dustin Johnson, Martin Kaymer, Brooks Koepka, and Lee Westwood) plus ten others are past major championship winners, including Phil Mickelson, Bryson DeChambeau, and Cameron Smith (who was ranked as the second best player in the world when he left the PGA TOUR for LIV Golf).[6] Clearly LIV Golf has been able to effectively recruit and attract many top tier players, although many big names (such as Tiger Woods, Rory McIlroy, Scottie Scheffler, Jon Rahm, and Xander Schauffele) have remained on the PGA TOUR.

For the model developed and analyzed, with competing organizers we also have that either a pooling or a separating composition could result: a pooling composition will typically result when the marginal benefit of having high ability agents in a single event is relatively large; while a separating composition will typically result when the marginal benefit of having high ability agents in a single event is relatively small. Further, when competition between organizers will give rise to a separating composition, it is shown that there is a second mover advantage in that the organizer choosing their prize first earns a smaller profit than the organizer choosing their prize second.[7]

Comparing the outcome across the alternate organizer market structures, it is shown that (depending upon the values of the parameters of the model) the high ability agents are: in some instances pooled in the same event regardless of organizer market structure, and in other instances separated across the two events regardless of organizer market structure. Further, there are parameter values for which the high ability agents are pooled by a monopsonist organizer but separated by competing organizers, and also there are parameter values for which the high ability agents are separated by a monopsonist organizer but pooled by competing organizers. Finally, Total Social Welfare may be either higher or lower with competing organizers versus a monopsonist organizer. That is, greater competition within the tournament organizer market does not necessarily lead to greater Social Welfare.

A model is fully developed and described in Section 2. Tournament participant behavior (both the within tournament choice of effort and the entry decision) is examined in Section 3. The choice of prizes by a monopsonist organizer is analyzed in Section 4, while the interaction between competing organizers is analyzed in Section 5. Comparisons between the outcome under a monopsonist organizer to the outcome with competing organizers are made in Section 6. Section 7 concludes.

## 2. Overview of Model

Consider a series of two rank order tournaments ("Event 1" and "Event 2"), each with a field limited to two entrants. Suppose there are two "high ability" agents ($H$), each wishing to participate in one and only one event. After the type $H$ agents decide which events to enter, the field of any tournament which has not been filled will be filled by "low ability" agents ($L$).[8] As a result of this entry process, one of two compositions of entrants across the tournaments will result: a "pooling composition," with both high ability agents in the same event; or a "separating composition," with the high ability agents in different events.

First focusing on a monopsonist tournament organizer, conditions are determined specifying which type of composition will be implemented in order to maximize profit. Subsequently, an environment in which two independent tournament organizers compete by sequentially choosing prizes is analyzed. A comparison is made between these alternative environments.

The players, strategies, and timing of the game are broadly as follows:

- Stage 1. prize levels for each event are set,[9]
- Stage 2. tournament participants/entrants decide which events to enter,
- Stage 3. competition takes place in each tournament (by way of participants/entrants exerting effort) and prizes are awarded.

In total, the players of the game consist of either one or two tournament organizers (depending upon market structure) and four tournament participants/entrants (or agents). The organizer(s) strategy is a choice of prize levels in Stage 1. The high ability agents have a strategic choice of which event to event in Stage 2 (low ability agents also have an entry decision at this stage, but it is trivial—they will enter events to fill the field). All agents, both high ability and low ability, then have a strategic choice of effort level in Stage 3.

This situation is analyzed via backward induction to identify a subgame perfect Nash Equilibrium by first focusing on the tournament level competition, then analyzing the entry decision of tournament participants, and finally examining the choice of prizes.[10] Subgame perfect Nash Equilibrium is chosen as the solution concept since it is standard for games of this nature with sequential moves.

Let Event $j$ denote a tournament with two entrants in which the first place finisher receives a prize of $p_j$, while the second place finisher receives nothing. Suppose the costs to a tournament organizer are simply equal to prizes paid, while the benefits to a tournament organizer depend upon (potentially) both the field of entrants as well as the level of effort exerted by tournament participants as follows. Let $V_j \geq 0$ denote the value to an organizer of conducting a tournament with the field of entrants realized by Event $j$ (this value depends only upon the identity of the entrants in Event $j$ and not upon their levels of effort). Additionally, let $E_j$ denote the total effort exerted by participants in Event $j$. Assume that total effort in Event $j$ is valued according to $r(E_j) = r\sqrt{E_j}$, with $r \geq 0$.

The payoff of the organizer of a single Event $j$ is thus

$$\gamma_j(p_j) = V_j + r\sqrt{E_j} - p_j,$$

while the payoff of a monopsonist organizer of both events is:

$$\gamma_M(p_1, p_2) = V_1 + V_2 + r\sqrt{E_1} + r\sqrt{E_2} - (p_1 + p_2).$$

In Stage 1 of the game, a tournament organizer chooses prizes to maximize their payoff function. An organizer's payoff depends critically upon the prizes across the events. This is true not only because the cost of organizing an event depends directly upon prizes, but also because the entry decisions of participants (and thus the realized $(V_1, V_2)$ for a monopsonist organizer or the realized $V_j$ for a competing organizer) and choices of effort levels within each tournament (and therefore the resulting $(E_1, E_2)$ or the resulting $E_j$) depend upon prizes.

In order for an organizer to optimally set prizes, it is first necessary to determine how the tournament level choice of effort by each agent in Stage 3 depends upon the tournament prize and the identity of the within tournament rival. Once this is done, the entry decision of the participants in Stage 2 can be examined. Finally, the initial choice of organizer prizes in Stage 1 will be analyzed (under the aforementioned alternative organizer market structures), taking the subsequent behavior of tournament participants as given.

## 3. Decisions of Tournament Participants

As noted, prizes influence two decisions made by tournament entrants: the decision of in which event to compete, and the decision of how much effort to exert. An analysis of these decisions is presented in this section.

### 3.1. Tournament Level Competition

When competing in a tournament in Stage 3, the strategy available to an agent $i$ is a choice of effort level $e_i \geq 0$. Consider a tournament in which entrants $A$ and $B$ compete by simultaneously choosing $e_A \geq 0$ and $e_B \geq 0$. Let $\delta(e_A, e_B) = \frac{\delta e_A}{\delta e_A + (1-\delta)e_B}$ be the contest success function for agent $A$, specifying the probability with which $A$ is the winner of the tournament when at least one agent chooses positive effort.[11] If both agents choose zero effort, define $\delta(0, 0) = \delta$. Supposing $A$ is of (weakly) higher ability than $B$, consider

$\frac{1}{2} \leq \delta < 1$. Assume the cost of exerting effort is simply equal to level of effort: $c(e) = e$. The payoff of an agent is equal to the value of the prize ($p$) multiplied by the probability of winning the prize (which depends upon efforts levels of both agents and is equal to $\delta(e_A, e_B)$ for agent $A$ and $1 - \delta(e_A, e_B)$ for agent $B$), minus the cost of exerting effort ($c(e_i)$ for agent $i$). Thus, $A$ and $B$ competing for a prize of $p$ have respective payoffs of: $\Pi_A = p\delta(e_A, e_B) - e_A$ and $\Pi_B = p[1 - \delta(e_A, e_B)] - e_B$.

The simultaneous choice of effort by $A$ and $B$ in the subgame consisting of a single tournament is analyzed in Appendix A. A unique pure strategy equilibrium is shown to exist in which $e_A^* = e_B^* = e^* = p\delta(1-\delta)$. These effort levels result in $\delta(e_A^*, e_B^*) = \delta$ and payoffs for $A$ and $B$ of: $\Pi_{A,\{A,B\}} = p\delta^2$ and $\Pi_{B,\{A,B\}} = p(1-\delta)^2$.

With agents of two different skill levels, a particular tournament realizes one of three fields of entrants: $\{H, H\}$, $\{H, L\}$ , or $\{L, L\}$. It is straightforward to apply the results above to each of these situations. For example, in an event with a field of either $\{H, H\}$ or $\{L, L\}$, the tournament participants are of equal ability, so that $\delta = \frac{1}{2}$. Therefore, a participant in such a tournament exerts effort of $\frac{1}{4}p$ and realizes a payoff of $\frac{1}{4}p$. In an event with a field of $\{H, L\}$, $\delta > \frac{1}{2}$ since the participants are of different abilities.[12] While agents $H$ and $L$ will exert a common level of effort, $p\delta(1-\delta)$, they realize different payoffs of $\Pi_{H,\{H,L\}} = p\delta^2$ and $\Pi_{L,\{H,L\}} = p(1-\delta)^2$, respectively.

### 3.2. *Entry Decision of Tournament Participants*

In Stage 2 the two high ability agents have a strategy choice of which event to enter. Suppose they make this decision sequentially. Let $H_i$ and $H_{ii}$ denote the two high ability agents, and suppose $H_i$ is the agent who makes the initial entry decision. That is, $H_i$ first decides to enter Event 1 or Event 2. After $H_i$ makes this observable choice, $H_{ii}$ then decides to enter Event 1 or Event 2. Finally, the field of any tournament that does not have two entrants is filled by low ability agents. Focusing on the entry decisions of the two high ability agents, a subgame perfect equilibrium will be determined for all possible values of $\delta$ and values of prizes.[13]

Let $p_1$ denote the larger and $p_2$ denote the smaller of the prizes across the two events (i.e., $p_1 \geq p_2$). First note that if $H_i$ enters Event 2, then $H_{ii}$ has: an expected payoff of $p_1\delta^2$ from instead entering Event 1; versus an expected payoff of $\frac{1}{4}p_2$ from also entering Event 2. Since $\delta^2 > \frac{1}{4}$ for any $\delta > \frac{1}{2}$ and $p_1 \geq p_2$, it follows that if $H_i$ enters Event 2, then $H_{ii}$ will enter Event 1.

Next note that if $H_i$ enters Event 1, then $H_{ii}$ has: an expected payoff of $\frac{1}{4}p_1$ from also entering Event 1; versus an expected payoff of $p_2\delta^2$ from instead entering Event 2. From here two cases arise. First suppose $\frac{1}{4}p_1 \geq \delta^2 p_2$. In this case, following a choice by $H_i$ to enter Event 1, $H_{ii}$ will enter Event 1.[14] The initial decision by $H_i$ is now between competing against $H_{ii}$ in Event 1 (leading to an expected payoff of $\frac{1}{4}p_1$) or competing against a low ability agent in Event 2 (leading to an expected payoff of $\delta^2 p_2$). If $\frac{1}{4}p_1 \geq \delta^2 p_2$, the former clearly gives a higher expected payoff than the latter, so that a "pooling composition" is realized in which both high ability agents enter Event 1. As a result, Event 1 realizes a field of $(H, H)$ while Event 2 realizes a field of $(L, L)$.

Instead suppose $\frac{1}{4}p_1 < \delta^2 p_2$. Now, following a choice by $H_i$ to enter Event 1, $H_{ii}$ will enter Event 2. The decision of $H_i$ is now one of competing against a low ability rival in either Event 1 or Event 2. Since $p_1 \geq p_2$, $H_i$ realizes a greater expected payoff from entering Event 1. Thus, for $\frac{1}{4}p_1 < \delta^2 p_2$ a "separating composition" results in which one high ability agent enters each event. Thus, both Event 1 and Event 2 realize fields of $(H, L)$.

It is worth noting that if instead the entry process had been modelled as a simultaneous choice by the two high ability agents, essentially the same conditions would arise. Specifically, if $\frac{1}{4}p_1 \geq \delta^2 p_2$, then each agent has a dominant strategy of entering Event 1. Thus, the pooling composition arises. If instead $\frac{1}{4}p_1 < \delta^2 p_2$, then the best reply for each high ability agent (to any choice by their rival) is to enter the event that their rival does not enter. As such, there are two pure strategy equilibria, each characterized by a separating composition of entrants. However, in this case there is also a mixed strategy equilibrium, in which each high ability agent randomizes between the two events.[15] To proceed with a unique prediction of the resulting composition of entrants across the events, one of two equivalent (in terms of predicted outcome) approaches could be taken: assume the entry decision of the high ability agents is sequential, or assume it is simultaneous and focus on pure strategy equilibria.

Define $\Omega(\delta) = \frac{1}{4\delta^2}$. A separating composition (with each event realizing a field of $(H, L)$) results if $\frac{p_2}{p_1} > \Omega(\delta)$; a pooling composition (in which the event with larger prize attracts a field of $(H, H)$ while the event with the smaller prize attracts a field of $(L, L)$) results if $\frac{p_2}{p_1} \leq \Omega(\delta)$. Note that: $\Omega\left(\frac{1}{2}\right) = 1$; $\Omega(1) = \frac{1}{4}$; and $\Omega'(\delta) = -\frac{1}{2\delta^3} < 0$. Thus, $\Omega(\delta) \in \left[\frac{1}{4}, 1\right)$ for all $\delta \in \left(\frac{1}{2}, 1\right]$. Since $0 < \Omega(\delta) < 1$, it follows that for any arbitrary $p_1 > 0$, there exists a unique $\bar{p}_2 \in (0, p_1)$ such that: the separating composition results for $p_2 \in (\bar{p}_2, p_1]$, while the pooling composition results for $p_2 \in [0, \bar{p}_2]$.

From here the analysis will shift to the endogenous choice of tournament prizes in Stage 1 of the game. When choosing prize levels, it is assumed that the tournament entrants subsequently choose which event to enter in Stage 2 and how much effort to exert in Stage 3 as derived thus far.

## 4. Monopsonist Tournament Organizer

First suppose there is a single, monopsonist organizer of the two events. The analysis of the choice by such an organizer proceeds as follows. First, the optimal prizes for realizing the separating composition are determined. Second, the optimal prizes for realizing the pooling composition are determined. In each of these cases, the resulting payoff of the organizer is determined. Finally, these payoffs are compared to each other, to determine if the organizer prefers to realize a pooling or separating composition. Recall that the payoff of a monopsonist organizer is:

$$\gamma_M(p_1, p_2) = V_1 + V_2 + r\sqrt{E_1} + r\sqrt{E_2} - (p_1 + p_2).$$

The monopsonist maximizes this expression by choosing $p_1 \geq p_2 \geq 0$, carefully accounting for the subsequent behavior of tournament entrants.

### *4.1. Optimal Prizes to Realize Separating Composition*

To realize a field of $(H, L)$ in each event, the chosen prizes must satisfy $\frac{p_2}{p_1} > \Omega(\delta)$. Letting $V_{\{H,L\}}$ denote the value of having a field of $(H, L)$, we have $V_1 = V_2 = V_{\{H,L\}}$ in this case. Further, each agent in Event 1 exerts effort of $p_1\delta(1-\delta)$, so that $E_1 = 2p_1\delta(1-\delta)$. Likewise, each agent in Event 2 exerts effort of $p_2\delta(1-\delta)$, so that $E_2 = 2p_2\delta(1-\delta)$. It follows that choosing prizes such that $\frac{p_2}{p_1} > \Omega(\delta)$ gives the monopsonist organizer a payoff of

$$\gamma_{MS}(p_1, p_2) = 2V_{\{H,L\}} + r\sqrt{2p_1\delta(1-\delta)} + r\sqrt{2p_2\delta(1-\delta)} - (p_1 + p_2).$$

Lemma 1 characterizes the optimal prizes and resulting payoff for the monopsonist organizer in this case.

**Lemma 1.** *For values of $(p_1, p_2)$ satisfying $p_1 \geq p_2$ and $\frac{p_2}{p_1} > \Omega(\delta)$, $\gamma_{MS}(p_1, p_2)$ is maximized by $p_1^{MS*} = p_2^{MS*} = \frac{r^2\delta(1-\delta)}{2}$. This choice results in $\gamma_{MS}^* = 2V_{\{H,L\}} + r^2\delta(1-\delta)$.*

**Proof of Lemma 1.** Begin by noting that: $\frac{\partial\gamma_{MS}}{\partial p_1} = r\sqrt{\frac{\delta(1-\delta)}{2p_1}} - 1$ and $\frac{\partial\gamma_{MS}}{\partial p_2} = r\sqrt{\frac{\delta(1-\delta)}{2p_2}} - 1$. From here, $\frac{\partial^2\gamma_{MS}}{\partial p_1^2} < 0$ and $\frac{\partial^2\gamma_{MS}}{\partial p_2^2} < 0$.

$\frac{\partial \gamma_{MS}}{\partial p_1} = 0$ for $p_1 = \frac{1}{2}\delta(1-\delta)r^2$, and $\frac{\partial \gamma_{MS}}{\partial p_2} = 0$ for $p_2 = \frac{1}{2}\delta(1-\delta)r^2$. The constraint of $\Omega(\delta) < \frac{p_2}{p_1}$ is clearly satisfied at this pair of $(p_1, p_2)$, since $\frac{p_2}{p_1} = 1$ while $\Omega(\delta) \in \left[\frac{1}{4}, 1\right)$ in general. Therefore, in order to realize the separating composition, the optimal prizes are: $p_1^{MS*} = p_2^{MS*} = \frac{r^2\delta(1-\delta)}{2}$, resulting in $\gamma^*_{MS} = 2V_{\{H,L\}} + r^2\delta(1-\delta)$. *Q.E.D.* □

It is clear that $\gamma^*_{MS} = 2V_{\{H,L\}} + r^2\delta(1-\delta)$ is: increasing in $V_{\{H,L\}}$; increasing in $r$; and decreasing in $\delta$ (since $\delta \in \left(\frac{1}{2}, 1\right)$).

### *4.2. Optimal Prizes to Realize Pooling Composition*

To realize the pooling composition (for which Event 1 attracts a field of $(H, H)$, while Event 2 attracts a field of $(L, L)$), the prizes must be such that $\frac{p_2}{p_1} \leq \Omega(\delta)$. Let $V_{\{H,H\}}$ denote the value to the organizer from a field of $(H, H)$; let $V_{\{L,L\}}$ denote the value to the organizer from a field of $(L, L)$. The relevant pooling composition results in $V_1 = V_{\{H,H\}}$ and $V_2 = V_{\{L,L\}}$. Since each participant in each event is competing against a rival of identical ability, each entrant in Event 1 exerts effort of $\frac{1}{4}p_1$ while each entrant in Event 2 exerts effort of $\frac{1}{4}p_2$. Thus, $E_1 = \frac{1}{2}p_1$ and $E_2 = \frac{1}{2}p_2$. The resulting payoff of the organizer from choosing $\frac{p_2}{p_1} \leq \Omega(\delta)$ is

$$\gamma_{MP}(p_1, p_2) = V_{\{H,H\}} + V_{\{L,L\}} + r\sqrt{\frac{p_1}{2}} + r\sqrt{\frac{p_2}{2}} - (p_1 + p_2).$$

Lemma 2 characterizes the optimal prizes and subsequent payoff of the monopsonist organizer in this case.

**Lemma 2.** *For values of* $(p_1, p_2)$ *satisfying* $\frac{p_2}{p_1} \leq \Omega(\delta)$, $\gamma_{MP}(p_1, p_2)$ *is maximized by* $p_1^{MP*} = \frac{r^2\delta^2}{2}\left(\frac{1+2\delta}{1+4\delta^2}\right)^2$ *and* $p_2^{MP*} = \frac{r^2}{8}\left(\frac{1+2\delta}{1+4\delta^2}\right)^2$. *This choice results in* $\gamma^*_{MP} = V_{\{H,H\}} + V_{\{L,L\}} + \frac{r^2(1+2\delta)^2}{8(1+4\delta^2)}$.

**Proof of Lemma 2.** First note that:

$$\frac{\partial \gamma_{MP}}{\partial p_1} = \frac{r}{2}\sqrt{\frac{1}{2p_1}} - 1 \tag{1}$$

and

$$\frac{\partial \gamma_{MP}}{\partial p_2} = \frac{r}{2}\sqrt{\frac{1}{2p_2}} - 1. \tag{2}$$

It immediatelyfollows that $\frac{\partial^2 \gamma_{MP}}{\partial p_1^2} < 0$ and $\frac{\partial^2 \gamma_{MP}}{\partial p_2^2} < 0$.

To have $\frac{\partial \gamma_{MP}}{\partial p_1} = 0$ and $\frac{\partial \gamma_{MP}}{\partial p_2} = 0$ simultaneously would require $p_1 = p_2$, implying that the constraint of $\frac{p_2}{p_1} \leq \Omega(\delta)$ is binding. Thus, the optimal $(p_1, p_2)$ must satisfy $p_2 = \Omega(\delta)p_1$, which can be expressed as

$$p_2 = \frac{1}{4\delta^2}p_1. \tag{3}$$

Consider the choice of prizes in $(p_1, p_2)$-space. The constraint of $\frac{p_2}{p_1} \leq \Omega(\delta)$ holds with equality along a straight line through the origin with slope of $\Omega(\delta)$. Let $\gamma_{MP}(p_1, p_2) = C$ denote the locus of points $(p_1, p_2)$ for which the value of the objective function is equal to a constant arbitrary value $C$. By the Implicit Function Theorem, the slope of any such locus in $(p_1, p_2)$-space is given by $-\frac{\frac{\partial \gamma_{MP}}{\partial p_1}}{\frac{\partial \gamma_{MP}}{\partial p_2}}$. From here it follows that the optimal $(p_1, p_2)$ must also satisfy $-\frac{\frac{\partial \gamma_{MP}}{\partial p_1}}{\frac{\partial \gamma_{MP}}{\partial p_2}} = \Omega(\delta)$, which can be expressed as

$$-\frac{\partial \gamma_{MP}}{\partial p_1} = \frac{\partial \gamma_{MP}}{\partial p_2}\frac{1}{4\delta^2}. \tag{4}$$

Equations (3) and (4) provide a system of two equations with two unknowns that the optimal $(p_1, p_2)$ must satisfy. Substituting (1), (2), and (3) into (4) yields: $-\left[\frac{r}{2}\sqrt{\frac{1}{2p_1}} - 1\right] = \left[\frac{r}{2}\sqrt{\frac{2\delta^2}{p_1}} - 1\right]\frac{1}{4\delta^2}$. Solving for $p_1$ gives rise to

$$p_1^{MP*} = \frac{r^2\delta^2}{2}\left(\frac{1+2\delta}{1+4\delta^2}\right)^2. \tag{5}$$

From (3) and (5) it follows that

$$p_2^{MP*} = \frac{r^2}{8}\left(\frac{1+2\delta}{1+4\delta^2}\right)^2. \tag{6}$$

The prizes from (5) and (6) give a payoff of $\gamma_{MP}^* = V_{\{H,H\}} + V_{\{L,L\}} + \frac{r^2(1+2\delta)^2}{8(1+4\delta^2)}$. *Q.E.D.* □

It is clear that $\gamma_{MP}^* = V_{\{H,H\}} + V_{\{L,L\}} + \frac{r^2(1+2\delta)^2}{8(1+4\delta^2)}$ is: increasing in $V_{\{H,H\}}$; increasing in $V_{\{L,L\}}$; increasing in $r$; and decreasing in $\delta$ (since $\delta \in \left(\frac{1}{2}, 1\right)$).

### *4.3. Separating Composition or Pooling Composition?*

Define

$$g(\delta) = \frac{(1+2\delta)^2}{8(1+4\delta^2)} - \delta(1-\delta)$$

and

$$\beta\left(V_{\{H,H\}}, V_{\{H,L\}}, V_{\{L,L\}}, r\right) = \frac{\left(V_{\{H,L\}} - V_{\{L,L\}}\right) - \left(V_{\{H,H\}} - V_{\{H,L\}}\right)}{r^2}.$$

Theorem 1 describes the optimal choice by a monopsonist organizer.[16]

**Theorem 1.** *A monopsonist tournament organizer will choose prizes of: (i) $p_1^{MS*}$ and $p_2^{MS*}$ (and realize the separating composition) if $g(\delta) < \beta$, and (ii) $p_1^{MP*}$ and $p_2^{MP*}$ (and realize the pooling composition) if $g(\delta) \geq \beta$.*

**Proof of Theorem 1.** The optimal prizes and resulting payoff for a monopsonist organizer from realizing a separating composition are specified in Lemma 1 as $p_1^{MS*}$, $p_2^{MS*}$, and $\gamma_{MS}^*$. Likewise, Lemma 2 states similar expressions for such an organizer implementing a pooling composition by $p_1^{MP*}$, $p_2^{MP*}$, and $\gamma_{MP}^*$. From here it follows that the monopsonist organizer will: choose $p_1^{MS*}$ and $p_2^{MS*}$ (and realize the separating composition) if $\gamma_{MS}^* > \gamma_{MP}^*$; and choose $p_1^{MP*}$ and $p_2^{MP*}$ (and realize the pooling composition) if $\gamma_{MP}^* \geq \gamma_{MS}^*$.[17]

That is, the monopsonist organizer will choose prizes leading to the pooling composition if and only if $\gamma^*_{MP} \geq \gamma^*_{MS}$, or equivalently

$$\begin{aligned}
& V_{\{H,H\}} + V_{\{L,L\}} + \frac{r^2(1+2\delta)^2}{8(1+4\delta^2)} \geq 2V_{\{H,L\}} + r^2\delta(1-\delta) \\
\Leftrightarrow \quad & r^2\left\{\frac{(1+2\delta)^2}{8(1+4\delta^2)} - \delta(1-\delta)\right\} \geq 2V_{\{H,L\}} - \left(V_{\{H,H\}} + V_{\{L,L\}}\right) \\
\Leftrightarrow \quad & g(\delta) \geq \frac{\left(V_{\{H,L\}} - V_{\{L,L\}}\right) - \left(V_{\{H,H\}} - V_{\{H,L\}}\right)}{r^2} \\
\Leftrightarrow \quad & g(\delta) \geq \beta.
\end{aligned}$$

*Q.E.D.* □

More insight into the result of Theorem 1 can be obtained by examining $g(\delta)$ and $\beta$. First observe that $g\left(\frac{1}{2}\right) = 0$ and $g(1) = \frac{9}{40}$. Further,

$$g'(\delta) = (2\delta - 1)\left[1 - \frac{1+2\delta}{2(1+4\delta^2)^2}\right] = \frac{2\delta - 1}{2(1+4\delta^2)^2}\left[(1-\delta)^2 + 15\delta^2 + 32\delta^4\right].$$

Thus, $g'(\delta) > 0$ for all $\delta \in \left(\frac{1}{2}, 1\right)$, implying $g(\delta) > 0$ for all $\delta \in \left(\frac{1}{2}, 1\right)$.

Next, note that $\beta$ is: increasing in $V_{\{H,L\}}$, but decreasing in $V_{\{H,H\}}$ and $V_{\{L,L\}}$. Further, $\beta$ is increasing in $V_{\{H,L\}} - V_{\{L,L\}}$ and decreasing in $V_{\{H,H\}} - V_{\{H,L\}}$.[18] To recognize the impact of $r$ on $\beta$, first observe that both $\beta > 0$ and $\beta < 0$ are possible. If $\beta > 0$, then $\beta$ is decreasing in $r$. If instead $\beta < 0$, then $\beta$ is increasing in $r$ (i.e., for a larger value of $r$, $\beta$ is closer to zero in absolute terms but remains negative).[19]

Thus, with a monopsonist organizer market conditions would be more conducive to realizing a pooling composition if (all other factors fixed):

1. $\delta$ were larger (so the difference in abilities between $H$ and $L$ would be greater, and agents would exert less effort in an event with a field of $(H, L)$);
2. $V_{\{H,H\}}$ were larger, which would imply that $V_{\{H,H\}} - V_{\{H,L\}}$ would be larger (so that the marginal benefit to the organizer of having a second high ability agent in a particular tournament would be greater);
3. $V_{\{L,L\}}$ were larger, which would imply that $V_{\{H,L\}} - V_{\{L,L\}}$ would be smaller (so that the marginal benefit to the organizer of having a first high ability agent in a particular tournament would be smaller);
4. $V_{\{H,L\}}$ were smaller, which would imply that $V_{\{H,H\}} - V_{\{H,L\}}$ would be larger (so that the marginal benefit to the organizer of having a second high ability agent in a particular tournament would be greater) and $V_{\{H,L\}} - V_{\{L,L\}}$ would be smaller (so that the marginal benefit to the organizer of having a first high ability agent in a particular tournament would be smaller);
5. $r$ were larger (so that the organizer valued effort to a greater degree).

Focusing on the marginal value of having a high quality entrant in a particular field, two cases are possible. Either $V_{\{H,H\}} - V_{\{H,L\}} < V_{\{H,L\}} - V_{\{L,L\}}$ (i.e., the marginal value of a high ability agent is diminishing), or $V_{\{H,H\}} - V_{\{H,L\}} \geq V_{\{H,L\}} - V_{\{L,L\}}$ (i.e., the marginal value of a high ability agent is constant or increasing).

Standard economic intuition would suggest that this marginal benefit could reasonably be diminishing. This marginal benefit would be constant if the value of having a particular agent in an event does not at all depend on which other entrants are in the event. This is the case in standard tournament models, which assume the benefits to the organizer depend only on effort (so that $V_{\{H,H\}} = V_{\{H,L\}} = V_{\{L,L\}} = 0$). Finally, this marginal benefit could be increasing if there exists a synergy between the high ability agents, so that the value of having both high ability agents in the same event is greater than the sum of the values of having them in separate events.

The following corollaries relate the choice of the monopsonist organizer to the behavior of the marginal benefit of having high ability agents in an event.

**Corollary 1.** *If $V_{\{H,H\}} - V_{\{H,L\}} \geq V_{\{H,L\}} - V_{\{L,L\}}$ (so that the marginal benefit of having high ability agents in an event is constant or increasing), then for all $\delta \in \left(\frac{1}{2}, 1\right)$ a monopsonist organizer will choose prizes so that the pooling composition results.*

**Proof of Corollary 1.** If $V_{\{H,H\}} - V_{\{H,L\}} \geq V_{\{H,L\}} - V_{\{L,L\}}$, then $\beta \leq 0$. Since $g(\delta) > 0$, Theorem 1 implies that a monopsonist organizer will set prizes leading to the pooling composition. *Q.E.D.* □

The result of Corollary 1 is intuitive. The organizer potentially values both effort and the identity of the participants in each event. Since agents exert greater effort when competing against rivals of relatively equal ability, effort is maximized by pairing the high ability agents with each other and pairing the low ability agents with each other. When the marginal benefit of having high ability agents in an event is constant or increasing, the portion of the organizer's payoff which depends upon the fields in the two events is also maximized by pairing the high ability agents in the same event. It seems reasonable that this was likely the case during the peak of the rivalry between Tiger Woods and Phil Mickelson when golf fans wanted to see the best compete head-to-head against each other as often as possible.

**Corollary 2.** *If $V_{\{H,H\}} - V_{\{H,L\}} < V_{\{H,L\}} - V_{\{L,L\}}$ (so that the marginal benefit of having high ability agents in an event is diminishing), then either: (i) for all $\delta \in \left(\frac{1}{2}, 1\right)$ a monopsonist organizer will choose prizes so that the separating composition results, or (ii) there exists a unique $\hat{\delta} \in \left(\frac{1}{2}, 1\right)$ such that a monopsonist organizer will choose prizes so that the separating composition results if and only if $\delta < \hat{\delta}$.*

**Proof of Corollary 2.** If $V_{\{H,H\}} - V_{\{H,L\}} < V_{\{H,L\}} - V_{\{L,L\}}$, then $\beta > 0$. Recall that $g(\delta)$ is such that: $g\left(\frac{1}{2}\right) = 0$, $g(1) = \frac{9}{40}$, and $g'(\delta) > 0$ for $\delta \in \left(\frac{1}{2}, 1\right)$. From here, two cases arise: $\beta \geq \frac{9}{40}$ and $\beta < \frac{9}{40}$.

First consider $\beta \geq \frac{9}{40}$. In this case, $g(\delta) < \beta$ for all $\delta \in \left(\frac{1}{2}, 1\right)$. Thus, Theorem 1 implies that the monopsonist organizer will choose prizes leading to the separating composition.

Next consider $\beta < \frac{9}{40}$. In this case, $g\left(\frac{1}{2}\right) < \beta$ while $g(1) > \beta$. Since $g'(\delta) > 0$, it follows that there exists a unique $\hat{\delta} \in \left(\frac{1}{2}, 1\right)$ such that $g(\hat{\delta}) = \beta$. From here: $g(\delta) \geq \beta$, for $\delta \geq \hat{\delta}$; and $g(\delta) < \beta$, for $\delta < \hat{\delta}$. By Theorem 1, a monopsonist organizer will: choose prizes leading to the separating composition for $\delta < \hat{\delta}$; and choose prizes leading to the pooling composition for $\delta \geq \hat{\delta}$.[20] *Q.E.D.* □

An implication of Corollary 2 is that if $V_{\{H,H\}} - V_{\{H,L\}} < V_{\{H,L\}} - V_{\{L,L\}}$, then the monopsonist organizer sets prizes so that the high ability agents enter different events for $\delta$ sufficiently close to $\frac{1}{2}$ (i.e., when $H$ and $L$ are of relatively equal ability). Intuitively this makes sense. When the marginal benefit of having high ability agents in an event is decreasing, then the part of the organizer's payoff which depends upon tournament fields is maximized by realizing a separating composition, with one high ability agent in each tournament. However, in comparison to a pooling composition, this composition makes agents exert less effort at the stage of tournament competition. The degree to which less effort is exerted under a separating composition (in comparison to a pooling composition) is smaller when $\delta$ is smaller (and approaches zero as $\delta \rightarrow \frac{1}{2}$). For $\delta$ sufficiently close to

$\frac{1}{2}$, the difference in effort becomes sufficiently small so that the decrease in payoff for the organizer from less effort (when realizing a separating composition) is less than the gain of $2V_{HL} - (V_{HH} + V_{LL})$ (which is positive in this case) that arises from realizing a separating instead of a pooling composition.

Further, if $V_{\{H,H\}} - V_{\{H,L\}} < V_{\{H,L\}} - V_{\{L,L\}}$, then a monopsonist organizer will realize the separating composition for any $\delta \in \left(\frac{1}{2}, 1\right)$ if the benefits to the organizer depend primarily upon the identities of the tournament participants as opposed to effort levels (that is, when $r$ is sufficiently close to zero). To see this, note that when $V_{\{H,H\}} - V_{\{H,L\}} < V_{\{H,L\}} - V_{\{L,L\}}$, not only is $\beta > 0$, but $\beta$ can be made arbitrarily large by sufficiently decreasing $r$. As a result, for sufficiently small $r$ we have $\beta \geq \frac{9}{40}$, in which case $g(\delta) < \beta$ for all $\delta \in \left(\frac{1}{2}, 1\right)$ (so the organizer would choose to realize the separating composition for all possible $\delta$). That is, if the value of high ability agents is diminishing, then a organizer who values effort to a sufficiently small degree will choose prizes so that the high ability agents enter different events.

## 5. Competing Tournament Organizers

Now suppose there are two tournament organizers competing with one another by sequentially choosing prizes. First, the organizer who is the "leader" (denoted by $l$) sets their prize, denoted $p_l$. After observing this choice, the organizer who is the "follower" (denoted by $f$) sets their prize, denoted $p_f$. Once both prizes are known, the tournament entrants choose which events to enter and how much effort to exert as described and analyzed in Section 3. In the context of professional golf, during 2022 we could think of the PGA TOUR as the leader and the entrant of LIV Golf as the follower.

Generally, the payoff of $l$ is $\gamma_l(p_l) = V_l + r\sqrt{E_l} - p_l$, while the payoff of $f$ is $\gamma_f(p_f) = V_f + r\sqrt{E_f} - p_f$. The choice of prizes by these competing organizers is analyzed by backward induction.

First consider the choice of $p_f$ by $f$, after observing the value of $p_l$ chosen by $l$. For every $p_l > 0$, there is a range of $p_f$ which will give rise to each of the three different fields that $f$ could realize. More precisely, $f$ could opt to attract both low ability entrants by choosing $p_f$ "sufficiently low" so that not only is $p_f < p_l$ but further $\frac{p_f}{p_l} \leq \Omega(\delta)$. This would give $f$ a payoff of

$$\gamma_f^{low}(p_f) = V_{\{L,L\}} + r\sqrt{\frac{p_f}{2}} - p_f.$$

If instead $f$ chose a "mid-range prize" of $p_f$ such that $\Omega(\delta) < \frac{p_l}{p_f} < \frac{1}{\Omega(\delta)}$ (or equivalently $p_l\Omega(\delta) < p_f < p_l\frac{1}{\Omega(\delta)}$), he would attract a field of $(H, L)$ and realize a payoff of

$$\gamma_f^{mid}(p_f) = V_{\{H,L\}} + r\sqrt{2p_f\delta(1-\delta)} - p_f.$$

Finally, $f$ could attract both high ability entrants by choosing $p_f$ "sufficiently high" so that not only is $p_f > p_l$ but further $\frac{p_l}{p_f} \leq \Omega(\delta)$. Such a prize of $p_f \geq p_l\frac{1}{\Omega(\delta)}$ would give $f$ a payoff of

$$\gamma_f^{high}(p_f) = V_{\{H,H\}} + r\sqrt{\frac{p_f}{2}} - p_f.$$

The payoff of $l$ in each case can be defined in a similar manner. For instance, if $l$ chose a value of $p_l$ after which $f$ would choose $p_f$ such that $p_f \geq p_l\frac{1}{\Omega(\delta)}$, then $l$ would realize a field of $(L, L)$ and a payoff of $\gamma_l\left(p_l, p_f\right) = V_{\{L,L\}} + r\sqrt{\frac{p_l}{2}} - p_l$.

Returning attention to the payoff of $f$, note that $\gamma_f^{low}(p_f)$, $\gamma_f^{mid}(p_f)$, and $\gamma_f^{high}(p_f)$ are each concave functions of $p_f$. If there were no restrictions on the value of $p_f$, both $\gamma_f^{low}(p_f)$ and $\gamma_f^{high}(p_f)$ would be maximized by $p_f = \frac{1}{8}r^2$. Likewise, without any restriction on $p_f$, $\gamma_f^{mid}(p_f)$ would be maximized by $p_f = \frac{\delta(1-\delta)}{2}r^2$. Since $\delta \in \left(\frac{1}{2}, 1\right]$ it follows that $\frac{\delta(1-\delta)}{2}r^2 < \frac{1}{8}r^2$. That is, without accounting for how the value of $p_f$ impacts the resulting tournament fields, we obtain the standard insight that an organizer would choose larger prizes in a tournament in which agents are of equal ability.

Even under the assumptions thus far, a general analysis of the sequential choice of prizes by competing organizers is not tractable. Therefore, the primary focus is on the results of a numerical analysis conducted as follows. First, accounting for the values of $p_l\Omega(\delta)$ and $p_l\frac{1}{\Omega(\delta)}$ which result for each possible $p_l \geq 0$, the payoff of the follower was determined for every possible $p_f \geq 0$ according to the functions defined above. Second, for each possible $p_l \geq 0$, the optimal choice of $p_f$ by $f$ (that is, the prize that $f$ would choose to maximize their own payoff) was determined. Viewing these values of $p_f$ collectively, they represent $p_f^{BR}(p_l)$: the "best response function" for $f$, which specifies the optimal choice of $p_f$ by $f$ for every possible $p_l$ that $l$ could initially choose. From here, the payoff that $l$ would ultimately realize for each possible $p_l \geq 0$ (accounting for the subsequent choice of $p_f$ by $f$) was determined. This essentially gives us the payoff of $l$ as a function of their own choice of prize, which can be denoted as: $\gamma_l\left(p_l, p_f^{BR}(p_l)\right)$. Third, $\gamma_l\left(p_l, p_f^{BR}(p_l)\right)$ was maximized with respect to $p_l$, to determine the initial best choice of $p_l$ by $l$.

After identifying the equilibrium $p_l$ and $p_f$, it is straightforward to determine which entrants will enter which tournament, as well as the profit of both $l$ and $f$. Further, the outcome with competing organizers can be compared to the outcome under a monopsonist organizer, in terms of the realized fields in the tournaments as well as Total Social Welfare (such comparisons are the focus of the discussion in Section 6).

Table A1a–c present the results of the numerical analysis of the equilibrium when competing organizers sequentially choose prizes.[21] Each table individually focuses on fixed values of $r$, $\delta$, and $V_{LL}$, while varying $V_{HH}$ and $V_{HL}$. Within each cell in the main body of each table, the resulting tournament fields and ratio of organizer profits is reported. An entry of $LP$ (for "leader, pooling") indicates that in equilibrium the pooling composition in which $l$ attracts both high quality entrants results. $CS$ (for "competing, separating") indicates that the equilibrium with competing organizers is such that the separating composition in which each event realizes a field of $(H, L)$ arises. The reported numerical value in each cell is $\frac{\gamma_l^*}{\gamma_f^*}$, the ratio of the profit of $l$ to the profit of $f$ in equilibrium. For example, from Table A1a we see that for $r = 1$, $\delta = 0.60$, $V_{LL} = 1$, $V_{HL} = 2.2$, and $V_{HH} = 4.8$, competing organizers will set prizes for which: $l$ attracts a field of $(H, H)$, $f$ attracts a field of $(L, L)$, and $l$ earns a profit 2.1992 times greater than the profit of $f$.

Examining Table A1a–c collectively we see that (all other factors fixed), a pooling composition tends to arise for smaller values of $V_{HL}$ while a separating composition tends to arise for larger $V_{HL}$. To understand why this outcome results, note that for fixed $V_{LL}$ and $V_{HH}$, a larger value of $V_{HL}$ makes $(V_{HH} - V_{HL})$ (i.e., the marginal value of a second high ability entrant) smaller and makes $(V_{HL} - V_{LL})$ (i.e., the marginal value of a first high ability entrant) larger. Thus, for $V_{HL}$ "sufficiently small" the relative benefit from having both high ability entrants in the same field is sufficiently large so that $l$ finds it worthwhile to choose $p_l$ large enough to attract both high ability entrants. The last row in each table indicates what "sufficiently small" means, by specifying the range of $V_{HL}$ (for each $V_{HH}$ considered) for which $l$ will attract both high ability entrants.[22]

Focusing on the values of equilibrium profit ratio: $\frac{\gamma_l^*}{\gamma_f^*} < 1$ for parameter values for which $CS$ results, and $\frac{\gamma_l^*}{\gamma_f^*} > 1$ for parameter values for which $LP$ results. That is, there appears to be a "second mover advantage" ($\gamma_f^* > \gamma_l^*$) for parameters such that $CS$ will result, while there appears to be a "first mover advantage" ($\gamma_l^* > \gamma_f^*$) for parameters such that $LP$ will result. Proposition 1 characterizes the relation between $\gamma_l^*$ and $\gamma_f^*$ when competition between the organizers leads to the separating composition.

**Proposition 1.** *If the sequential choice of prizes by competing tournament organizers results in a separating composition, then there is a "second mover advantage" in that* $\gamma_f^* \geq \gamma_l^*$.

**Proof of Proposition 1.** Let $p_l^*$ and $p_f^*$ denote the chosen prizes and let $\gamma_l^*$ and $\gamma_f^*$ denote the resulting payoffs under the conjectured equilibrium. Focus on the choice of $p_f$ by $f$ following a choice of $p_l = p_l^*$ by $l$. Recall that a separating composition will arise for any $p_f \in \left(\Omega(\delta)p_l^*, \frac{1}{\Omega(\delta)}p_l^*\right)$. Since $\Omega(\delta) < 1$, a choice of $\bar{p}_f = p_l^*$ is clearly in this range. Further, a choice of $\bar{p}_f = p_l^*$ would give $f$ a payoff of $\bar{\gamma}_f = \gamma_l^*$. Thus, the optimal $p_f^*$ must give $f$ a profit that is at least as large: $\gamma_f^* \geq \bar{\gamma}_f = \gamma_l^*$. *Q.E.D.* □

Continuing to focus on the profit ratio reported in Table A1a–c, observe that over the range of $V_{HH}$ and $V_{HL}$ leading to $LP$, this ratio appears to be: strictly increasing in $V_{HH}$; and weakly decreasing in $V_{HL}$. To understand the first of these observations, recognize that we are considering an increase in $V_{HH}$ over a range for which $l$ attracts both high quality entrants (both before and after the increase in $V_{HH}$). Such an increase in $V_{HH}$ will not alter $\gamma_f^*$, but will strictly increase $\gamma_l^*$. Thus, $\frac{\gamma_l^*}{\gamma_f^*}$ will strictly increase. Switching focus to the second of these observations, again recognize that we are considering an increase in $V_{HL}$ when neither tournament attracts a field of $(H, L)$. Thus, this change can only impact the equilibrium profits indirectly, by possibly altering the prize which $l$ must set to attract both high ability entrants. For $V_{HL}$ sufficiently close to $V_{LL}$, the optimal prizes are not affected by an increase in $V_{HL}$, so that neither $\gamma_l^*$ nor $\gamma_f^*$ change. However, for $V_{HL}$ sufficiently large, an increase in $V_{HL}$ will make it that $l$ must set a larger prize in order to make it not worthwhile for $f$ to attract a field other than $(L, L)$ to their own tournament. This makes $\gamma_l^*$ smaller (while not changing $\gamma_f^*$), leading to a decrease in $\frac{\gamma_l^*}{\gamma_f^*}$.

Over the range of $V_{HH}$ and $V_{HL}$ leading to $CS$, the profit ratio appears to be: strictly decreasing in $V_{HH}$; and strictly increasing in $V_{HL}$. Note that when $CS$ is relevant, $l$ chooses the smallest possible $p_l$ for which it is subsequently better for $f$ to choose a prize for which $f$ attracts a field of $(H, L)$ as opposed to a field of $(H, H)$. Consider an increase in $V_{HH}$ over a range for which both $l$ and $f$ attract fields of $(H, L)$ (both before and after the increase in $V_{HH}$). As $V_{HH}$ increases, $l$ must now choose a larger $p_l$ in order to make it that $f$ is still willing to ultimately accept a field of $(H, L)$ at their event (as opposed to setting $p_f$ to attract a field of $(H, H)$). As a result of this choice of a larger $p_l$ by $l$, the value of $p_f$ ultimately chosen by $f$ becomes larger as well. Together, these changes result in both $\gamma_l^*$ and $\gamma_f^*$ becoming smaller as $V_{HH}$ increases. Further, based upon the numerical results, the decreases in profits appear to be such that $\frac{\gamma_l^*}{\gamma_f^*}$ decreases. Now consider an increase in $V_{HL}$ over a range for which both $l$ and $f$ attract fields of $(H, L)$ (both before and after the increase in $V_{HL}$). As $V_{HL}$ increases, $l$ can choose a smaller $p_l$ in order to have it that $f$ is willing to accept a field of $(H, L)$ at their event (as opposed to setting $p_f$ to attract a field of $(H, H)$). As a result of this choice of a smaller $p_l$ by $l$, the value of $p_f$ ultimately chosen by $f$ also decreases. Together, these changes result in increased values of both $\gamma_l^*$ and $\gamma_f^*$ as $V_{HL}$ increases. Further, based upon the numerical results, the increases in profits appear to be such that $\frac{\gamma_l^*}{\gamma_f^*}$ increases. Finally, observe that as the relevant outcome changes from $CS$ to $LP$ as a result of either an increase in $V_{HH}$ or a decrease in $V_{HL}$, the profit ratio clearly increases (from less than 1, to greater than 1).

Comparing the results of Table A1a–c, insight can be gained into how the equilibrium tournament fields change as $\delta$ changes. From the bottom row of these tables we see that for most of the reported values of $V_{HH}$, the largest value of $V_{HL}$ for which $LP$ arises decreases as $\delta$ increases. However, this is not always true, as this cutoff value of $V_{HL}$ appears to be "u-shaped" for $V_{HH} = 1.2$ (decreasing from 1.0937 to 1.0791 as $\delta$ increases from 0.60 to 0.75, but then increasing from 1.0791 to 1.1309 as $\delta$ increases from 0.75 to 0.90).[23]

## 6. Impact of Organizer Market Structure

To see how the equilibrium outcome depends upon organizer market structure, a comparison is made between the outcome with sequentially competing organizers to the outcome with a monopsonist organizer. The results reported in Table A2a–c make this comparison. Again, each table individually focuses on fixed $r$, $\delta$, and $V_{LL}$, while varying $V_{HH}$ and $V_{HL}$. The following is reported within each cell in the main body of each table: the resulting fields with a monopsonist organizer, the resulting fields with competing organizers, and the ratio of Social Welfare with a monopsonist organizer to Social Welfare with competing organizers. An entry of $MP$ (for "monopsonist, pooling") indicates that a monopsonist organizer sets prizes for which both high ability agents are pooled into the event with larger prizes, while an entry of $MS$ (for "monopsonist, separating") indicates that a monopsonist organizer sets prizes for which the high ability agents are separated across the two events ($LP$ and $CS$ identify the resulting composition under competing organizers, as previously described).

Recall that a monopsonist organizer will set prizes and implicitly choose between pooling both high ability agents in one event or separating the high ability agents in different events based upon a comparison of $g(\delta)$ and $\beta$ as described by Theorem 1. Thus, for any chosen set of parameter values, the resulting values of $g(\delta)$ and $\beta$ are computed and Theorem 1 is applied to determine whether a monopsonist would set prizes resulting in a separating composition or pooling composition. If the monopsonist desires to separate the high ability agents, he would do so by setting prizes of $p_1^{MS*}$ and $p_2^{MS*}$ as specified in Lemma 1. Similarly, if the monopsonist desires to pool the high ability agents, he would do so by setting prizes of $p_1^{MP*}$ and $p_2^{MP*}$ as specified in Lemma 2. Again, it is straightforward to determine these prize levels numerically for any chosen parameter values.

Social Welfare is defined as the sum of the payoff(s) of the organizer(s) of the tournaments and the four tournament entrants. If a separating composition is realized, Social Welfare is:

$$\begin{aligned} W_s &= \left(V_{HL} + r\sqrt{2p_1\delta(1-\delta)} - p_1\right) + \left(V_{HL} + r\sqrt{2p_2\delta(1-\delta)} - p_2\right) \\ &\quad + p_1\delta^2 + p_1(1-\delta)^2 + p_2\delta^2 + p_2(1-\delta)^2 \\ &= 2V_{HL} + r\sqrt{2\delta(1-\delta)}(\sqrt{p_1} + \sqrt{p_2}) - 2\delta(1-\delta)(p_1 + p_2). \end{aligned}$$

If a pooling composition is realized, Social Welfare is:

$$\begin{aligned} W_p &= \left(V_{HH} + r\sqrt{\frac{1}{2}p_1} - p_1\right) + \left(V_{LL} + r\sqrt{\frac{1}{2}p_2} - p_2\right) + \frac{1}{2}p_1 + \frac{1}{2}p_2 \\ &= V_{HH} + V_{LL} + r\sqrt{\frac{1}{2}}(\sqrt{p_1} + \sqrt{p_2}) - \frac{1}{2}(p_1 + p_2). \end{aligned}$$

Let $W_m^*$ denote the equilibrium value of Social Welfare under a monopsonist organizer; let $W_c^*$ denote the equilibrium value of Social Welfare under competing organizers. The value reported in each cell is $\frac{W_m^*}{W_c^*}$.

First note that comparing the realized fields across the alternative market structures, each of the four possibilities of $(MP, LP)$, $(MS, LP)$, $(MP, CS)$, and $(MS, CS)$ can arise. In fact, each of these four outcomes can occur for common values of $r$, $\delta$, and $V_{LL}$. For instance, from Table A2a we see that for $r = 1$, $\delta = .60$, and $V_{LL} = 1$: $(MP, CS)$ arises for $(V_{HL}, V_{HH}) = (1.1, 1.2)$; $(MP, LP)$ arises for $(V_{HL}, V_{HH}) = (1.1, 2.4)$; $(MS, CS)$ arises for $(V_{HL}, V_{HH}) = (2.2, 2.4)$; and $(MS, LP)$ arises for $(V_{HL}, V_{HH}) = (4.4, 7.2)$.

Generally, $(MP, LP)$ arises (i.e., a pooling composition results for either organizer market structure) when $V_{HL}$ is "relatively small," while $(MS, CS)$ arises (i.e., a separating composition results for either organizer market structure) when $V_{HL}$ is "relatively large." Further, for an intermediate range of $V_{HL}$ the resulting fields differ for the two market structures considered. For instance, with $r = 1$, $\delta = 0.75$, and $V_{LL} = 1$ (results reported in Table A2b) for each reported $V_{HH}$ there is a range of $V_{HL} \in (V_{LL}, V_{HH})$ such that a monopsonist sets prizes for which the high ability agents are pooled, while competing organizers set prizes for which the high ability agents are separated (this range of $V_{HL}$ is reported in the last row of Table A2b).

Shifting focus to the value of the Welfare Ratio, we can have either $\frac{W_m^*}{W_c^*} > 1$ (i.e., $W_m^* > W_c^*$) or $\frac{W_m^*}{W_c^*} < 1$ (i.e., $W_m^* < W_c^*$). That is, Social Welfare can be either larger or smaller with competing organizers as opposed to a monopsonist organizer: increased competition within the tournament organizer market does not necessarily increase Social Welfare.

Further, the relation between $W_m^*$ and $W_c^*$ does not depend upon whether $(MP, LP)$, $(MS, LP)$, $(MP, CS)$, or $(MS, CS)$ is relevant. For instance, from the first row of results in Table (2b), it is clear that we can have either $\frac{W_m^*}{W_c^*} < 1$ or $\frac{W_m^*}{W_c^*} > 1$ when $(MP, LP)$ is relevant. Similarly, the results reported in the row corresponding to $V_{HL} = 4.4$ in Table A2a show that we can have either $\frac{W_m^*}{W_c^*} < 1$ or $\frac{W_m^*}{W_c^*} > 1$ for $(MS, CS)$. Likewise, from the results in Table A2b we see that (for the corresponding values of $r$, $\delta$, and $V_{LL}$) $(MP, CS)$ arises for both $(V_{HL}, V_{HH}) = (1.1, 1.2)$ and $(V_{HL}, V_{HH}) = (2.2, 3.6)$. $\frac{W_m^*}{W_c^*} < 1$ for the former pair, while $\frac{W_m^*}{W_c^*} > 1$ for the latter pair. Finally, as reported in Table (2a), (for the corresponding values of $r$, $\delta$, and $V_{LL}$) for $(V_{HL}, V_{HH}) = (4.4, 7.2)$, $(MS, LP)$ is relevant and $\frac{W_m^*}{W_c^*} > 1$. However, (although not reported in Appendix B) $(MS, LP)$ is relevant but $\frac{W_m^*}{W_c^*} < 1$ for $r = 1$, $\delta = 0.55$, $V_{LL} = 1$, $V_{HL} = 1.12$, and $V_{HH} = 1.2$.

## 7. Conclusions

Labor market tournaments have been examined extensively in the economics literature, with a primary focus on a single tournament environment. A common result in such models is that organizers are best off "leveling the playing field" by whatever means are at their disposal to maximize effort. However, in practice agents often have a choice over the environment in which they compete, a feature not included in existing models. The present study examined a multi-tournament environment in which agents self select their competitive environment, focusing on how organizer market structure impacts the outcome. In particular, the results identify conditions under which organizers will or will not choose to structure prizes so that high (and low) ability agents pool together, "leveling the playing field" as standard models would advise.

Appropriate parallels were observed throughout the discussion to the labor market for elite professional tournament golfers in recent years. At the start of the twenty-first century the PGA TOUR was essentially a monopsonist organizer setting prizes across events in which two high ability agents (Tiger Woods and Phil Mickelson) would ultimately compete. The PGA TOUR needed to decide if they wanted to set prizes for which these top agents would compete directly against each other as often as possible or if it was better to set prizes for which they would spread themselves out over as many events as possible. More recently, LIV Golf emerged as a competing organizer, trying to siphon off many high profile players to its series of events.

A monopsonist organizer may want to either pool or separate high ability agents, depending upon the behavior of the "marginal benefit of having high ability agents in a particular tournament." If this marginal benefit is constant or increasing, then a monopsonist organizer sets prizes for which high ability agents enter the same event. If instead this marginal benefit is diminishing, then a monopsonist organizer either: always sets prizes for which the high ability agents enter different events; or sets prizes for which the high ability agents enter different events if and only if the difference in ability between the high ability and low ability agents is sufficiently small.

With competing organizers, either a pooling or a separating composition could again result. A pooling composition would typically result when the marginal benefit of having high ability agents in a single event is relatively large, while a separating composition would typically result when the marginal benefit of having high ability agents in a single event is relatively small. Further, when competition between organizers leads to a separating composition, there is a second mover advantage in that the organizer setting their prize first earns a smaller profit than the organizer setting their prize second.

Comparing the outcome across the alternate organizer market structures, it was shown that the high ability agents were: in some instances pooled in the same event regardless of organizer market structure, and in other instances separated across the two events regardless of organizer market structure. Further, for some parameter values the high ability agents were pooled by a monopsonist organizer but separated by competing organizers, while for other parameter values the high ability agents were separated by a monopsonist organizer but pooled by competing organizers. Finally, it was shown that Total Social Welfare could be either larger or smaller with competing organizers versus a monopsonist organizer, implying that greater competition within the organizer market does not necessarily increase Social Welfare.

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## Appendix A

*Appendix A.1*

When $A$ and $B$ compete for a prize of $p$ as outlined in Section 3, their respective payoffs are: $\Pi_A = p\delta(e_A, e_B) - e_A$ and $\Pi_B = p[1 - \delta(e_A, e_B)] - e_B$. With $\delta(e_A, e_B) = \frac{\delta e_A}{\delta e_A + (1-\delta)e_B}$: $\frac{\partial\delta(e_A,e_B)}{\partial e_A} > 0$; $\frac{\partial\delta(e_A,e_B)}{\partial e_B} < 0$; $\frac{\partial^2\delta(e_A,e_B)}{\partial e_A^2} < 0$; and $\frac{\partial^2\delta(e_A,e_B)}{\partial e_B^2} > 0$ (for $e_A > 0$ and $e_B > 0$). Recall, by definition, $\delta(0,0) = \delta$.

There cannot be a pure strategy equilibrium with either agent choosing zero effort. To see this, consider the effort choice by $i$ if their rival, $-i$, chose $e_{-i} = 0$. Choosing $e_i = 0$ results in $i$ winning with probability $\delta < 1$, while any $e_i = \epsilon > 0$ results in $i$ winning with probability one. Thus, for any $\delta < 1$ and $p > 0$, the payoff for $i$ from $e_i = \epsilon > 0$ (when $e_{-i} = 0$) is greater than that from $e_i = 0$ for sufficiently small $\epsilon$. Therefore, a choice of zero effort by both agents is never an equilibrium. However, if $e_{-i} = 0$ and $e_i = \epsilon > 0$, $i$ can increase their payoff by instead choosing $e_i = \frac{1}{2}\epsilon$ (since $i$ still always wins the prize of $p$, but now incurs lower effort costs). Therefore, there are no pure strategy equilibria with either agent exerting zero effort.

To find an equilibrium in which both agents exert positive effort, note: $\frac{\partial \Pi_A}{\partial e_A} = p\frac{\partial \delta(e_A, e_B)}{\partial e_A} - 1$, $\frac{\partial \Pi_B}{\partial e_B} = -p\frac{\partial \delta(e_A, e_B)}{\partial e_B} - 1$, $\frac{\partial^2 \Pi_A}{\partial e_A^2} = p\frac{\partial^2 \delta(e_A, e_B)}{\partial e_A^2} < 0$, and $\frac{\partial^2 \Pi_B}{\partial e_B^2} = -p\frac{\partial^2 \delta(e_A, e_B)}{\partial e_B^2} < 0$. At an equilibrium with $e_A > 0$ and $e_B > 0$, both $\frac{\partial \Pi_A}{\partial e_A} = 0$ and $\frac{\partial \Pi_B}{\partial e_B} = 0$ must hold simultaneously. This requires $\frac{\partial \delta(e_A, e_B)}{\partial e_A} = -\frac{\partial \delta(e_A, e_B)}{\partial e_B}$, which for $\delta(e_A, e_B) = \frac{\delta e_A}{\delta e_A + (1-\delta) e_B}$ requires $e_A = e_B$ (i.e., in equilibrium the two agents exert equal effort). From here it readily follows (from either $\frac{\partial \Pi_A}{\partial e_A} = 0$ or $\frac{\partial \Pi_B}{\partial e_B} = 0$) that a unique pure strategy equilibrium exists in this subgame, with $e_A^* = e_B^* = e^* = p\delta(1-\delta)$. These effort levels lead to payoffs for $A$ and $B$ of: $\Pi_{A,\{A,B\}} = p\delta^2$ and $\Pi_{B,\{A,B\}} = p(1-\delta)^2$.

## Appendix B. Numerical Results

**Table A1.** Equilibrium When Competing Organizers Sequentially Choose Prizes.

| **(a). $r = 1$, $\delta = 0.60$, $V_{LL} = 1$.** | | | | | | |
|---|---|---|---|---|---|---|
| | | | $V_{HH}$ | | | |
| $V_{HL}$ | **1.2** | **2.4** | **3.6** | **4.8** | **6.0** | **7.2** |
| 1.1 | (CS,0.9679) | (LP,1.4805) | (LP,1.8438) | (LP,2.1992) | (LP,2.5501) | (LP,2.8982) |
| 2.2 | | (CS,0.9650) | (LP,1.6332) | (LP,2.1992) | (LP,2.5501) | (LP,2.8982) |
| 3.3 | | | (CS,0.9638) | (CS,0.7495) | (LP,2.3026) | (LP,2.8982) |
| 4.4 | | | | (CS,0.9631) | (CS,0.8119) | (LP,1.9178) |
| 5.5 | | | | | (CS,0.9626) | (CS,0.8458) |
| 6.6 | | | | | | (CS,0.9623) |
| (LP) | (1,1.0937] | (1,1.7688] | (1,2.4719] | (1,3.1836] | (1,3.8999] | (1,4.6193] |
| **(b). $r = 1$, $\delta = 0.75$, $V_{LL} = 1$.** | | | | | | |
| | | | $V_{HH}$ | | | |
| $V_{HL}$ | **1.2** | **2.4** | **3.6** | **4.8** | **6.0** | **7.2** |
| 1.1 | (CS,0.9756) | (LP,1.8443) | (LP,2.4968) | (LP,3.1365) | (LP,3.7691) | (LP,4.3972) |
| 2.2 | | (CS,0.9739) | (CS,0.7988) | (LP,1.7255) | (LP,2.7922) | (LP,3.8589) |
| 3.3 | | | (CS,0.9733) | (CS,0.8572) | (CS,0.7234) | (LP,1.5390) |
| 4.4 | | | | (CS,0.9730) | (CS,0.8861) | (CS,0.7894) |
| 5.5 | | | | | (CS,0.9728) | (CS,0.9033) |
| 6.6 | | | | | | (CS,0.9726) |
| (LP) | (1,1.0791] | (1,1.5030] | (1,1.9543] | (1,2.4139] | (1,2.8780] | (1,3.3451] |
| **(c). $r = 1$, $\delta = 0.90$, $V_{LL} = 1$.** | | | | | | |
| | | | $V_{HH}$ | | | |
| $V_{HL}$ | **1.2** | **2.4** | **3.6** | **4.8** | **6.0** | **7.2** |
| 1.1 | (LP,1.1067) | (LP,2.0220) | (LP,2.8261) | (LP,3.6158) | (LP,4.3978) | (LP,5.1746) |
| 2.2 | | (CS,0.9666) | (CS,0.8245) | (CS,0.6730) | (LP,2.1223) | (LP,3.1890) |
| 3.3 | | | (CS,0.9697) | (CS,0.8757) | (CS,0.7782) | (CS,0.6747) |
| 4.4 | | | | (CS,0.9713) | (CS,0.9012) | (CS,0.8293) |
| 5.5 | | | | | (CS,0.9723) | (CS,0.9165) |
| 6.6 | | | | | | (CS,0.9730) |
| (LP) | (1,1.1309] | (1,1.4678] | (1,1.8108] | (1,2.1555] | (1,2.5012] | (1,2.8475] |

**Table A2.** The Impact of Market Structure: Sequentially Competing Organizers vs a Monopsonist.

**(a).** $r = 1$, $\delta = 0.60$, $V_{LL} = 1$.

| | $V_{HH}$ | | | | | |
|---|---|---|---|---|---|---|
| $V_{HL}$ | **1.2** | **2.4** | **3.6** | **4.8** | **6.0** | **7.2** |
| 1.1 | (MP,CS,0.9645) | (MP,LP,1.0269) | (MP,LP,1.0799) | (MP,LP,1.1215) | (MP,LP,1.1546) | (MP,LP,1.1814) |
| 2.2 | | (MS,CS,0.9736) | (MP,LP,1.1007) | (MP,LP,1.1215) | (MP,LP,1.1546) | (MP,LP,1.1814) |
| 3.3 | | | (MS,CS,0.9819) | (MS,CS,1.0659) | (MP,LP,1.1753) | (MP,LP,1.1814) |
| 4.4 | | | | (MS,CS,0.9880) | (MS,CS,1.0571) | (MS,LP,1.3488) |
| 5.5 | | | | | (MS,CS,0.9926) | (MS,CS,1.0519) |
| 6.6 | | | | | | (MS,CS,0.9963) |
| (MP,CS) | [1.0938, 1.1039] | {∅} | {∅} | {∅} | {∅} | {∅} |
| (MS,LP) | {∅} | [1.7040,1.7688] | [2.3040,2.4719] | [2.9040,3.1836] | [3.5040,3.8999] | [4.1040,4.6193] |

**(b).** $r = 1$, $\delta = 0.75$, $V_{LL} = 1$.

| | $V_{HH}$ | | | | | |
|---|---|---|---|---|---|---|
| $V_{HL}$ | **1.2** | **2.4** | **3.6** | **4.8** | **6.0** | **7.2** |
| 1.1 | (MP,CS,0.9988) | (MP,LP,0.9930) | (MP,LP,1.0217) | (MP,LP,1.0451) | (MP,LP,1.0640) | (MP,LP,1.0794) |
| 2.2 | | (MS,CS,0.9799) | (MP,CS,1.0185) | (MP,LP,1.1637) | (MP,LP,1.1335) | (MP,LP,1.1127) |
| 3.3 | | | (MS,CS,0.9826) | (MS,CS,0.9772) | (MP,CS,1.0577) | (MP,LP,1.2942) |
| 4.4 | | | | (MS,CS,0.9848) | (MS,CS,0.9833) | (MS,CS,0.9954) |
| 5.5 | | | | | (MS,CS,0.9866) | (MS,CS,0.9870) |
| 6.6 | | | | | | (MS,CS,0.9881) |
| (MP,CS) | [1.0792,1.1264] | [1.5031,1.7264] | [1.9544,2.3264] | [2.4140,2.9264] | [2.8781,3.5264] | [3.3452,4.1264] |
| (MS,LP) | {∅} | {∅} | {∅} | {∅} | {∅} | {∅} |

**(c).** $r = 1$, $\delta = 0.90$, $V_{LL} = 1$.

| | $V_{HH}$ | | | | | |
|---|---|---|---|---|---|---|
| $V_{HL}$ | **1.2** | **2.4** | **3.6** | **4.8** | **6.0** | **7.2** |
| 1.1 | (MP,LP,0.9665) | (MP,LP,0.9791) | (MP,LP,0.9966) | (MP,LP,1.0120) | (MP,LP,1.0247) | (MP,LP,1.0351) |
| 2.2 | | (MS,CS,0.9777) | (MP,CS,1.0304) | (MP,CS,1.2685) | (MP,LP,1.1869) | (MP,LP,1.1566) |
| 3.3 | | | (MS,CS,0.9819) | (MS,CS,0.9654) | (MP,CS,1.0424) | (MP,CS,1.2103) |
| 4.4 | | | | (MS,CS,0.9844) | (MS,CS,0.9731) | (MS,CS,0.9691) |
| 5.5 | | | | | (MS,CS,0.9861) | (MS,CS,0.9779) |
| 6.6 | | | | | | (MS,CS,0.9874) |
| (MP,CS) | [1.1310,1.1705] | [1.4679,1.7705] | [1.8109,2.3705] | [2.1556,2.9705] | [2.5013,3.5705] | [2.8476,4.1705] |
| (MS,LP) | {∅} | {∅} | {∅} | {∅} | {∅} | {∅} |

## Notes

1 The abbreviation PGA stands for Professional Golfers Association. See https://www.pgatour.com/media-guide/brief-tour-history.html (last accessed on 30 December 2022) for a brief history.

2 LIV is not an abbreviation, but rather Roman numerals for 54 (which is both the number of holes played in each LIV Golf tournament and the total score that a golfer would record for a round on an 18 hole, par 72 course by getting a birdie (one stroke better than par) on each hole). See https://www.livgolf.com/ (last accessed on 30 December 2022) for more information about LIV Golf.

3 See https://www.golfmonthly.com/news/how-much-are-liv-players-being-paid (last accessed on 30 December 2022). For some perspective on these amounts, the leading money winner on the PGA TOUR for the 2021-22 season, Scottie Scheffler, earned just over $14 million for the year: https://www.pgatour.com/stats/stat.109.y2022.html (last accessed on 30 December 2022).

4 In reality, the PGA TOUR has always faced some competition for players over its entire existence, most notably from what had been know as the European Tour (now the DP World Tour; https://www.europeantour.com/dpworld-tour/, last accessed on 30 December 2022). However, in practice, the PGA TOUR has always treated the European Tour as a "partner" as opposed to an "adversary," in stark contrast to the way it has treated LIV Golf. For example, members of the PGA TOUR have always had to apply for a "conflicting event release" if they wanted to compete in an event on a non-PGA TOUR circuit. Such requests have always been granted, almost without exception, when PGA TOUR members have wanted to play in European Tour events. In contrast, before the first LIV Golf event was played, the PGA TOUR made it clear that they would take a hard stance and revoke the membership of any PGA TOUR player who chose to compete on the LIV Golf circuit.

5 The "Official World Golf Ranking" (https://www.owgr.com/, last accessed on 30 December 2022) is updated every week to provide an ordinal ranking of golfers based upon performance in sanctioned events over the most recent two years. In the 521 rankings released between 7 January 2001 and 26 December 2010: Tiger was ranked first 480 times (i.e., over 92% of the time); Phil was ranked second 265 times (i.e., over 50% of the time). In each of the 265 weeks that Phil was ranked second, Tiger was ranked first (thus, over this decade Tiger was ranked first and Phil was ranked second over half the time).

6 The modern day "major champsionships" consist of four tournaments per year: The Masters, PGA Championship, U.S. Open, and British Open. Dustin Johnson, Martin Kaymer, and Brooks Koepka have each won multiple majors.

7 Numerical results suggest that there is instead a first mover advantage when competition leads to a pooling composition.

8 The ability of agents is assumed to be common knowledge.

9 With a monopsonist organizer, a pair of prizes will be announced by the single organizer at this stage. With competing organizers, first "Organizer $l$" (i.e., the "leader") announces a prize, after which "Organizer $f$" (i.e., the "follower") announces a prize.

10 Many features of this model are similar to that analyzed by [6]. Their focus is on the entry decision of tournament participants of three different skill levels over two tournaments, with exogenously set prizes. The focus here is on the strategic, endogenous choice of tournament prizes. To conduct this analysis, we presently assume a less general contest success function, tournament participants of only two ability levels (not three ability levels), and a second place prize of zero (assumptions which allow the choice of the organizer to be examined with greater ease).

11 Tournaments with logit-form contest success functions have been considered by [14–16]. This form was axiomatized by [17].

12 Henceforth $\delta > \frac{1}{2}$ will specifically refer to the probability with which an agent of type $H$ will be the winner in a tournament with a field of $(H, L)$.

13 As previously noted, [6] provides a detailed analysis of the specific factors influencing such a tournament entry decisions in a framework more general than the one considered here. The restrictions imposed here allow the tournament prizes to be endogenously determined while keeping the model tractable.

14 In order to identify a unique choice by each agent at each decision node, it is assumed that if an agent has the same expected payoff in each tournament he will enter Event 1. Thus, for the present decision, if $\frac{1}{4}p_1 = \delta^2 p_2$ we assume $H_{ii}$ enters Event 1.

15 For this mixed strategy equilibrium, the two separating compositions, the pooling composition in which both enter Event 1, and the pooling composition in which both enter Event 2 would each arise with positive probability.

16 In Theorem 1, and hereafter, $\beta\left(V_{\{H,H\}}, V_{\{H,L\}}, V_{\{L,L\}}, r\right)$ is written as just $\beta$.

17 We assume that the organizer will implement the pooling composition if $\gamma^*_{MP} = \gamma^*_{MS}$.

18 We assume that the marginal value of having an additional high ability agent in any field is always non-negative (i.e., $\left(V_{\{H,L\}} - V_{\{L,L\}}\right) \geq 0$ and $\left(V_{\{H,H\}} - V_{\{H,L\}}\right) \geq 0$).

19 Recall, $g(\delta) > 0$ for all $\delta \in \left(\frac{1}{2}, 1\right)$. Thus, if $\beta < 0$, then $g(\delta) \geq \beta$ for all values of $r$. Therefore, a change in the value of $r$ can possibly alter the relation between $g(\delta)$ and $\beta$ only when $\beta > 0$ (in which case $\beta$ is decreasing in $r$). As a result, the only possible impact of a change in $r$ on the relation between $g(\delta)$ and $\beta$ is the following: if initially $g(\delta) < \beta$, then a larger value of $r$ may lead to $g(\delta) \geq \beta$ instead of $g(\delta) < \beta$.

20 A similar technique has been used in the study of opinion dynamics (c.f. [18]).

21 Numerical results were obtained for many more parameter values than those reported in Appendix B. All of the insights discussed in this subsection and the following section hold true for these non-reported results as well.

22 The larger value states the largest $V_{HL}$ rounded to four decimal places for which $LP$ arises. For example, from the last column of Table (1a) we have that for $r = 1$, $\delta = 0.60$, $V_{LL} = 1$, and $V_{HH} = 7.2$: $LP$ arises for $V_{HL} \leq 4.6193$, while $CS$ arises for $V_{HL} \geq 4.6194$.

23 Though not reported, results were obtained for different values of $r$, to see how the equilibrium depends upon this parameter. Generally, as $r$ increases, the maximum $V_{HL}$ leading to $LP$ appears to be "u-shaped." For example, with $\delta = 0.75$, $V_{LL} = 1$, and $V_{HH} = 4.8$, the largest $V_{HL}$ leading to $LP$: decreases from 2.5183 to 2.4139 as $r$ increases from 0.2 to 1, but then increases from 2.4139 to 2.6141 as $r$ increases from 1 to 5.

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