games

MDPI

Article

Efficient Decentralized Leadership under Hybrid Work and Attachment to Regions

Naoto Aoyama [1] and Emilson Caputo Delfino Silva [2,*]

1 Faculty of Management and Economics, Aomori Public University, 153-4 Yamazaki, Goshizawa, Aomori 030-0196, Japan
2 Department of Economics, University of Auckland, Auckland 1142, New Zealand
* Correspondence: emilson.silva@auckland.ac.nz

Abstract: Under the 'new normal' in the labor market, individuals can work remotely or in person, a hybrid work mode that became ubiquitous during the pandemic. This paper studies the efficiency of decentralized leadership in federations in which hybrid work is the modus operandi. Self-interested regional governments and a benevolent central government interact strategically in dynamic games in which there are provisions of federal and regional public goods and interregional income and fiscal transfers, the population is attached to regions and hybrid work creates a common labor market in the federation. In this setting, we first show that decentralized leadership is inefficient if the center controls income transfers only. This result provides an efficiency enhancing motivation for the center to additionally control earmarked transfers: we demonstrate that decentralized leadership is efficient whenever the center controls both income and earmarked transfers. However, this is not the only federal regime in which decentralized leadership is efficient. It is efficient in the absence of earmarked transfers if it is appropriately selective: when the regional governments commit to the provision of the federal public only and the center redistributes income across regions.

Keywords: decentralized leadership; earmarked transfers; common labor market; hybrid work

JEL Classification: C72; D62; D72; D78; H41; H77; H87; Q28; R3; R5

check for updates

Citation: Aoyama, N.; Silva, E.C.D. Efficient Decentralized Leadership under Hybrid Work and Attachment to Regions. *Games* **2023**, *14*, 26. https://doi.org/10.3390/g14020026

Academic Editors: Elkhan Richard Sadik-Zada, Andrea Gatto, Luigi Aldieri, Bimonte Giovanna, Luigi Senatore, Concetto Paolo Vinci, Isa Hafalir and Ulrich Berger

Received: 31 December 2022
Revised: 20 February 2023
Accepted: 14 March 2023
Published: 16 March 2023

1. Introduction

Brazil, Canada, Germany, Sweden, and USA are examples of nations where sub-national (e.g., regional) governments have considerable autonomy and fiscal capacity to provide public goods and services. Some of such public goods and services yield consumption benefits to both residents and non-residents of the jurisdictions where they are provided. In Canada, for example, the province of British Columbia has been a leader in climate change policy—it unilaterally moved forward with the levying of a carbon tax, even though a nationwide policy on carbon emissions was still lacking. In these federal regimes, the central government is an active participant and promotes significant interregional income and fiscal transfers.[1]

Caplan et al. [6] investigated strategic interactions between regional and central governments in a federation characterized by decentralized leadership. They showed that self-interested regional governments found it desirable to efficiently contribute to a federal pure public good in anticipation of interregional income transfers promoted by a utilitarian central government. Their model built on the models advanced by Mansoorian and Myers [7] and Wellisch [8], to examine the efficiency of decentralized provision of public goods in a federation where individuals/workers are imperfectly mobile because they derive idiosyncratic regional attachment benefits (e.g., language, culture, customs, family ties).

In Caplan et al. [6], as in Mansoorian and Myers [7] and Wellisch [8], workers were assumed to work in their regions of residence. This was in accordance with the modus operandi prior to the pandemic—most work activities required workers to work at the location of their residence. During the pandemic, remote work became necessary for the sustainability of many types of productive activities. Hybrid work is the new normal nowadays (see, e.g., https://www.gallup.com/workplace/398135/advantages-challenges-hybrid-work.aspx (accessed on 15 October 2022)). Hybrid work provides a worker with the choice of working remotely or at the workplace, or a combination of part-time remote working and part-time at the workplace. Since hybrid work does not require a person to work in the region in which the person resides, it has efficiency-enhancing properties relative to the previous modus operandi. Hybrid work creates a common labor market in the federation; hence, in equilibrium, marginal products for identical types of labor are the same across regions, satisfying an efficiency requirement in a competitive labor market.

Our main model builds on Caplan et al. [6]. We added regional public goods and fiscal transfers to the policy arsenals of regional and central governments, respectively, and consider situations in which hybrid work produces a common labor market.[2] To clearly demonstrate the efficiency-enhancing aspects promoted by hybrid work, we initially examine federal regimes in which there is decentralized leadership, but residents/workers are immobile. The distribution of the population across regions is asymmetric; namely, one region is more populous than the other. This realistic scenario enables us to show that the subgame perfect equilibria for the dynamic games played by regional and central governments in familiar decentralized leadership settings are typically inefficient. In contrast, in our main model, with imperfect resident mobility, hybrid work leads to equalization of regional populations in the subgame perfect equilibrium in which the central government implements interregional income and fiscal transfers, and in the subgame perfect equilibrium in which there is selective decentralized leadership and the center promotes interregional income transfers only. The endogenous equalization of regional populations and the efficiency-enhancing effects of interregional transfers implemented in such decentralized leadership settings yield perfect incentive equivalence in the federation—regional and central governments wish to maximize social welfare.

This paper is closely related to the study of Silva and Lucas [10], which investigated the decentralized provision of public goods that generate economywide, symmetric, interjurisdictional spillovers in a large metropolitan area; namely, a common labor market, which encompasses multiple autonomous jurisdictions. As in our main model, residents are imperfectly mobile due to attachment to localities and there is a policy authority, which cares about all local governments and promotes interjurisdictional income transfers. They showed that such transfers equalize marginal utilities of income whenever the competitive wage earned by a worker in any locality is the same, a fact that holds in a common labor market. Given the assumed symmetric spillover effects, equalization of marginal utilities of income implies equalization of consumption of private goods and, thus, equalization of utilities and population across regions. Therefore, the equilibria they examined yielded perfect incentive equivalence and, thus, an efficient allocation of resources.

Unlike Silva and Lucas [10], we consider decentralized provision of public goods that produce consumption benefits to regional residents only. Like Silva and Lucas [10], we also consider decentralized provision of a public good which generates economywide, symmetric, consumption benefits to all individuals in the economy. Our model is, therefore, more complex in that it incorporates asymmetric benefits associated with consumption of regional public goods. Even though interregional income transfers implemented by a central and benevolent government equalizes marginal utilities of income in the presence of a common wage across regions, we show that they are insufficient for perfect incentive equivalence in a federation in which the regional governments are Stackelberg leaders (i.e., in a federation with decentralized leadership). We demonstrate that the subgame perfect equilibrium for this decentralized leadership setting is inefficient—the regional governments overprovide the regional public goods. Keeping the regional actions and timing intact in the dynamic game, we find that it becomes necessary for the arsenal of policy instruments controlled by the central government to be expanded—the center must also have the ability to implement fiscal transfers to reduce (or eliminate) disparities in regional fiscal capacities. These transfers are earmarked. We show that the subgame perfect equilibrium for the decentralized leadership setting in which the center controls both interregional income and interregional earmarked fiscal transfers yields perfect incentive equivalence and, thus, was efficient. We also show that it is not necessary to add earmarked fiscal transfers to the arsenal of policy instruments controlled by the central government if the actions and timing for the dynamic game can be adjusted. If one of the regional governments is responsible for the provision of a federal public good, in addition to the provision of a regional public good, and this region provides this federal public good in anticipation of all other policy choices, the subgame perfect equilibrium for this selective decentralized leadership game also yields perfect incentive equivalence and, hence, efficient provision of federal and regional public goods.

To our knowledge, this is the first paper that examines the efficiency of decentralized leadership in a setting in which there is hybrid work, regional governments commit to the provision of regional and federal public goods and the center is endowed with instruments to implement income and earmarked interregional transfers. It is also the first to consider the efficiency of decentralized leadership under the above-mentioned policy actions in an asymmetric federation with immobile residents/workers. Therefore, we make multiple contributions to the literature.[3] Silva [3] considered a decentralized leadership setting in which regional governments provide regional and federal public goods and the center promotes interregional transfers. However, the population was immobile, and the center did not have an instrument to implement earmarked transfers. Motivated by the study of Smart and Bird [11], which offered an insightful and broad analysis of earmarking policy in federations, Silva [4] examined the efficiency of interregional earmarked fiscal transfers when used together with interregional income transfers in a federation where regional governments provide multiple regional public goods only. Akai and Watanabe [12] extended Silva [3] and Silva [4] in order to include general types of spillovers and also consider policy commitments in terms of expenditure or taxation authority. They showed that the efficiency results obtained by Silva [3] and Silva [4] also hold in settings with spillovers if local governments commit to expenditure policies. However, the subgame perfect equilibria for settings in which local governments commit to taxation policies are inefficient. Commitment to taxation policies is clearly an important aspect of federal regimes and it is, thus, an interesting avenue for future work in the context of hybrid work and imperfect labor mobility.

A key feature of the model with imperfect mobility and hybrid work considered here is that it implies a socially optimal allocation of resources whenever the combination of the prescribed policies, a common labor market and imperfect mobility yield a symmetric population distribution across regions. The endogenous symmetric nature of the equilibria produces perfect incentive equivalence and, thus, leads self-interested regional governments to find it desirable to internalize externalities. Silva [3] and Silva [4] also derived socially efficient equilibria in symmetric models. In order to highlight that symmetry is essential for efficiency, we first consider a setting with an immobile population in which the population distribution is asymmetric. We show that neither selective decentralized leadership nor decentralized leadership with interregional and earmarked transfers yields a socially efficient equilibrium.

In what follows, Section 2 introduces the basic model with an immobile and asymmetrically distributed population and derives the socially optimal allocation. Section 3 examines decentralized leadership games in this asymmetric federation with immobile residents. In Section 4, we build our main model, in which residents are imperfectly mobile. We examine the social optimum first. This is a useful benchmark. The dynamic games mirror those of Section 3 in terms of the strategies and timing. Section 5 concludes the paper.

2. Basic Model

Consider a federation with two regions in which one region is more abundant in labor and land resources. We initially assume that labor is immobile. Each resident in the federation is endowed with one unit of labor, which is offered inelastically in the region in which the individual resides. Let n_j and Z_j denote the amounts of labor and land inputs in region j, $j = 1, 2$. We assume that $n_1 > n_2 > 0$ and $Z_1 > Z_2 > 0$; that is, region 1 is the region that is more abundant in labor (population) and land. Let $N = n_1 + n_2$ denote the total population in the federation.

Each region produces a private good (numeraire) by employing labor and land. Labor is a variable input, while land is fixed. Assume that there are M_j perfectly competitive firms in region j. Let $F(l_j, z_j)$ be the quantity of numeraire good that the representative firm in region j produces. All firms utilize the same constant-returns-to-scale (CRS) technology. We assume that $F(.)$ denotes the concave production function that represents the CRS technology and l_j and z_j are the amounts of labor and land that the representative firm in region j employs in the production process, where $z_j \equiv Z_j / M_j$. The production function $F(.)$ is twice continuously differentiable, increases at decreasing rates in all arguments, and $F^j_{lz} \equiv \partial^2 F / \partial l_j \partial z_j \geq 0$, $j = 1, 2$. Let φ_j denote the wage in region j.

The representative firm in region j chooses non-negative $\{l_j\}$ to maximize $\pi(l_j, \varphi_j, z_j) = F(l_j, z_j) - \varphi_j l_j$, taking the choices of all other firms as given. Assuming an interior solution, the first order conditions yield $F^j_l = \varphi_j$,$j = 1, 2$, where $F^j_l \equiv \partial F / \partial l_j$. The amounts of labor that the representative firm hires satisfy the equalization of marginal products of labor to the prices of labor. Let $l(\varphi_j, z_j)$ denote labor demand functions for the representative firm in region j. The labor market in region j clears if, and only if, $M_j l(\varphi_j, z_j) = n_j$, conditions that we assume that hold in equilibrium. These market-clearing conditions define the regional wages as implicit functions of the relevant exogenous variables in the regional market: $\varphi_j = \varphi(M_j, Z_j)$. We simply write the market wage in region j as φ_j. We assume that each resident of region j earns an equal share of the profit produced by each firm in his/her region. The profit of the representative firm in region j is $\pi^j(\varphi_j, z_j) \equiv F(l(\varphi_j, z_j), z_j) - \varphi_j l(\varphi_j, z_j)$. Since each resident receives compensation from supplying labor in the input markets, the income of the representative resident in region j is $y_j = \varphi_j + M_j \pi^j(\varphi_j, z_j) / n_j$.

Let $u(x_j, g_j, Q)$ denote the utility for the representative resident of region j; this consumer derives utility from consumption of x_j units of a numeraire good, g_j units of a regional public good, and Q units of a federal pure public good. We assume that the utility function is strictly concave, increasing at a decreasing rate in each argument and satisfies the Inada conditions (i.e., all goods are essential in consumption). For simplicity, we assume that the utility function is strongly separable in private and public consumption: $u(x_j, g_j, Q) \equiv b(x_j) + r(g_j) + v(Q)$. We assume that it costs one unit of the numeraire good to provide one unit of each type of public good. Since region 1 is larger than region 2, we assume that region 1 is the sole provider of the federal public good. This asymmetry with respect to provision of the federal public good occurs naturally in a decentralized federation in which regional governments are utilitarian and, thus, make choices to maximize the sum of its residents' utility functions. In the Nash equilibrium, region 1 is the sole provider of the federal public good; region 2 free rides. The assumption is harmless for our analysis. Thus, region 1 contributes g_1 and Q units to the provision of the regional and federal public goods, respectively, and region 2 contributes g_2 units to the provision of the regional public good.

The budget constraints for the representative residents of region 1 and region 2 are

$$x_1 = y_1 + \frac{t_1 - g_1 - Q}{n_1} \text{ and } x_2 = y_2 + \frac{t_2 - g_2}{n_2} \quad (1)$$

where t_j, $j = 1, 2$, is the amount of income transfer that the individual receives (if positive) from the central government or pays (if negative) to the central government. Since the income transfers are redistributive, the center faces the following constraint:

$$\sum_{j=1}^{2} t_j = 0 \tag{2}$$

Social Optimum with Immobile Residents

The regional governments are benevolent and utilitarian. Their payoffs are $w_j = n_j\left[b\left(x_j\right) + r\left(g_j\right) + v(Q)\right]$, $j = 1, 2$. The central government is also benevolent and utilitarian. Its payoff is $W = \sum_{j=1}^{2} w_j$. The social optimum in this economy corresponds to the center's most desirable allocation. In the fully centralized allocation, the center chooses $\{x_1, x_2, g_1, g_2, Q\}$ to maximize $\sum_{j=1}^{2} n_j\left[b\left(x_j\right) + r\left(g_j\right) + v(Q)\right]$ subject to federation's resource constraint:

$$\sum_{j=1}^{2} n_j x_j + G + Q = X \tag{3}$$

where $G = \sum_{j=1}^{2} g_j$ and $X = \sum_{j=1}^{2} M_j F\left(l\left(\varphi_j, z_j\right), z_j\right)$. Letting $\rho \geq 0$ denote the Lagrangian multiplier associated with the federation's resource constraint. Assuming interior solutions, the first order conditions can be written as follows:

$$x_j : b'\left(x_j\right) = \rho, \quad j = 1, 2, \tag{4}$$

$$g_j : n_j r'\left(g_j\right) = \rho, \quad j = 1, 2, \tag{5}$$

$$Q : n_1 v'(Q) + n_2 v'(Q) = \rho. \tag{6}$$

Equation (4) imply

$$b'(x_1) = b'(x_2) \quad \Rightarrow \quad x_1 = x_2. \tag{7}$$

Combining Equation (4) with (5) yields

$$n_j \frac{r'\left(g_j\right)}{b'\left(x_j\right)} = 1, \quad j = 1, 2. \tag{8}$$

Combining Equation (4) with (6) yields

$$\sum_{j=1}^{2} n_j \frac{v'(Q)}{b'\left(x_j\right)} = 1. \tag{9}$$

Equations (8) and (9) are the Samuelson conditions for optimal provision of regional and federal public goods, respectively. The socially efficient allocation is characterized by Equations (7)–(9).

3. Decentralized Leadership with Immobile Residents

In this section, we consider the efficiency and equity properties of decentralized leadership. In the decentralized leadership regime, the regional governments are policy leaders and the federal government is a common policy follower. We examine three distinct federal arrangements: (i) decentralized leadership with centralized inter-regional income transfers; (ii) decentralized leadership with centralized inter-regional income and fiscal transfers; and (iii) selective decentralized leadership.

3.1. Centralized Interregional Income Transfers with Immobile Residents

We start by examining a decentralized leadership arrangement in which the federal government's role is simply to redistribute income across regions. The regional and central governments play a two-stage game as follows. In the first stage, regional government 1 chooses contributions to both regional good and federal public good and regional government 2 chooses contribution to its regional public good. The center observes these choices and in the second stage, chooses the interregional income transfers.

At the second stage of the game, the center chooses $\{t_1, t_2\}$ to maximize $n_1\left[b\left(y_1 + \frac{t_1 - g_1 - Q}{n_1}\right) + r(g_1) + v(Q)\right] + n_2\left[b\left(y_2 + \frac{t_2 - g_2}{n_2}\right) + r(g_2) + v(Q)\right]$ subject to constraint (2). Letting $\lambda \geq 0$ denote the Lagrangian multiplier associated with constraint (2). Assuming an interior solution, the first order conditions imply:

$$\lambda = b'(x_1) = b'(x_2) \quad \Rightarrow \quad x_1 = x_2. \tag{10}$$

Let $t^j(g_1, g_2, Q)$, $j = 1, 2$, denote the center's best response functions. These functions satisfy the following system of equations:

$$y_1 + \frac{t^1(g_1, g_2, Q) - g_1 - Q}{n_1} = y_2 + \frac{t^2(g_1, g_2, Q) - g_2}{n_2}, \tag{11}$$

$$t^1(g_1, g_2, Q) + t^2(g_1, g_2, Q) = 0. \tag{12}$$

Differentiating Equations (11) and (12) with respect to $\{g_1, g_2, Q\}$ and then solving those equations yields

$$\frac{\partial t^j}{\partial g_j} = -\frac{\partial t^k}{\partial g_j} = \frac{n_k}{N},\ j, k = 1, 2,\ k \neq j,\ \text{and}\ \frac{\partial t^1}{\partial Q} = -\frac{\partial t^2}{\partial Q} = \frac{n_2}{N} \tag{13}$$

Consider the first stage. Regional government 1 chooses non-negative $\{g_1, Q\}$ to maximize $n_1\left[b\left(y_1 + \frac{t^1(.) - g_1 - Q}{n_1}\right) + r(g_1) + v(Q)\right]$, taking the choices of regional government 2 as given, while regional government 2 chooses non-negative $\{g_2\}$ to maximize $n_2\left[b\left(y_2 + \frac{t^2(.) - g_2}{n_2}\right) + r(g_2) + v(Q)\right]$, taking the choices of regional government 1 as given. Assuming interior solutions, the first order conditions yield:

$$n_j \frac{r'(g_j)}{b'(x_j)} = 1 - \frac{\partial t^j}{\partial g_j}, \quad j = 1, 2, \tag{14}$$

$$n_1 \frac{v'(Q)}{b'(x_1)} = 1 - \frac{\partial t^1}{\partial Q}. \tag{15}$$

Combining Equations (13) and (14) yields

$$N \frac{r'(g_j)}{b'(x_j)} = 1,\ j = 1, 2 \tag{16}$$

Combining Equations (13) and (15) yields

$$N \frac{v'(Q)}{b'(x_1)} = 1. \tag{17}$$

Since $b'(x_1) = b'(x_2)$, Equation (17) implies

$$n_1 \frac{v'(Q)}{b'(x_1)} + n_2 \frac{v'(Q)}{b'(x_2)} = 1 \tag{18}$$

The subgame perfect equilibrium for decentralized leadership with centralized interregional income transfers and immobile residents is characterized by Equations (10), (16), and (18). By comparing Equations (16) and (18) to Equations (8) and (9), one can state that provision of a federal public good is socially optimal, but the regional public goods are overprovided.

Proposition 1. *Suppose that utility function is strongly separable in private and public consumption and individuals are immobile. The subgame perfect equilibrium for decentralized leadership with centralized inter-regional income transfers is inefficient.*

The interregional income transfers promoted induce the regional governments to view their regional public goods as if they are federal public goods—they overspend resources in their provision.

3.2. Centralized Interregional Income and Fiscal Transfers with Immobile Residents

In this section, we examine a setting in which the center can implement two types of interregional transfers, one type to reduce income disparities and another type to reduce disparities in fiscal capacity. As before, g_j denotes the expenditure in the provision of regional public good in region j. Let $e_j + s_j$ represent region j's total tax revenue available to finance the cost of providing its regional public good, where e_j is the amount of tax revenue that is collected in the region and s_j is the amount of a (negative or positive) fiscal transfer (paid or received). The fiscal transfer is earmarked. Each region must balance its budget with respect to provision of the regional public good: $g_j = e_j + s_j$, $j = 1, 2$. The budget constraints for the representative resident of regions 1 and 2 are

$$x_1 = y_1 + \frac{t_1 - s_1 - e_1 - Q}{n_1} \quad \text{and} \quad x_2 = y_2 + \frac{t_2 - s_2 - e_2}{n_2}. \tag{19}$$

Region 1's budget constraint is given by $n_1 x_1 + e_1 + s_1 + Q = n_1 y_1 + t_1$ and region 2's budget constraint is given by $n_2 x_2 + e_2 + s_2 = n_2 y_2 + t_2$. As in the studies of Boadway [13] and Silva [4], we assume that the fiscal transfers are redistributive, so that

$$s_1 + s_2 = 0 \tag{20}$$

Regional government 1 chooses $\{e_1, Q\}$ and regional government 2 chooses $\{e_2\}$ in the first stage. Having observed $\{e_1, e_2, Q\}$, the center chooses $\{s_1, s_2, t_1, t_2\}$ to maximize $n_1\left[b\left(y_1 + \frac{t_1 - s_1 - e_1 - Q}{n_1}\right) + r(g_1) + v(Q)\right] + n_2\left[b\left(y_2 + \frac{t_2 - s_2 - e_2}{n_2}\right) + r(g_2) + v(Q)\right]$ subject to constraints (2) and (20) in the second stage.

At the second stage of the game, let $\lambda \geq 0$ and $\mu \geq 0$ denote the Lagrangian multipliers associated with constraints (2) and (20), respectively. The conditions that maximize social welfare are Equations (2), (10), (20), and the following:

$$n_1 r'(g_1) = n_2 r'(g_2) \quad \Rightarrow \quad g_1 > g_2. \tag{21}$$

Let $t^j(e_1, e_2, Q)$ and $s^j(e_1, e_2, Q)$, $j = 1, 2$, denote the center's best response functions. These functions satisfy the following system of equations:

$$y_1 + \frac{t^1(e_1, e_2, Q) - e_1 - s^1(e_1, e_2, Q) - Q}{n_1} = y_2 + \frac{t^2(e_1, e_2, Q) - e_2 - s^2(e_1, e_2, Q)}{n_2} \tag{22}$$

$$t^1(e_1, e_2, Q) + t^2(e_1, e_2, Q) = 0, \tag{23}$$

$$n_1 r'\left(e_1 + s^1(e_1, e_2, Q)\right) = n_2 r'\left(e_2 + s^2(e_1, e_2, Q)\right) \tag{24}$$

$$s^1(e_1, e_2, Q) + s^2(e_1, e_2, Q) = 0 \tag{25}$$

Let $g^j(e_1, e_2, Q) = e_j + s^j(e_1, e_2, Q)$, $j = 1, 2$. Equations (22)–(25) imply that $(j, k = 1, 2, k \neq j)$

$$\frac{\partial t^j}{\partial e_j} = -\frac{\partial t^k}{\partial e_j} = \frac{n_k}{N} - \frac{n_j r''(g_j)}{n_1 r''(g_1) + n_2 r''(g_2)} \quad \text{and} \quad \frac{\partial s^j}{\partial e_j} = -\frac{\partial s^k}{\partial e_j} = -\frac{n_j r''(g_j)}{n_1 r''(g_1) + n_2 r''(g_2)}, \tag{26}$$

$$\frac{\partial t^1}{\partial Q} = -\frac{\partial t^2}{\partial Q} = \frac{n_2}{N} \text{ and } \frac{\partial s_j}{\partial Q} = 0. \quad (27)$$

Consider the first stage. Regional government 1 chooses non-negative $\{e_1, Q\}$ to maximize $n_1\left[b\left(y_1 + \frac{t^1(.)-e_1-s^1(.)-Q}{n_1}\right) + r\left(g^1(.)\right) + v(Q)\right]$, taking the choices of regional government 2 as given. Regional government 2 chooses non-negative $\{e_2\}$ to maximize $n_2\left[b\left(y_2 + \frac{t^2(.)-e_2-s^2(.)}{n_2}\right) + r\left(g^2(.)\right) + v(Q)\right]$, taking the choices of regional government 1 as given. Assuming interior solutions, the first order conditions yield:

$$b'(x_j)\left(\frac{\partial t^j}{\partial e_j} - \frac{\partial s^j}{\partial e_j} - 1\right) + n_j r'(g_j)\left(1 + \frac{\partial s^j}{\partial e_j}\right) = 0, j = 1, 2, \quad (28)$$

$$b'(x_1)\left(\frac{\partial t^1}{\partial Q} - \frac{\partial s^1}{\partial Q} - 1\right) + n_1 r'(g_1)\frac{\partial s^1}{\partial Q} + n_1 v'(Q) = 0. \quad (29)$$

For regional good provisions, combining Equations (26) and (28) yields

$$n_j \frac{r'(g_j)}{b'(x_j)} = \left(\frac{n_j}{N}\right) / \left(\frac{n_k r''(g_k)}{n_1 r''(g_1) + n_2 r''(g_2)}\right), \qquad j, k = 1, 2, \ k \neq j. \quad (30)$$

For a federal public good provision, combining Equations (27) and (29) yields

$$N\frac{v'(Q)}{b'(x_1)} = 1 \quad \Rightarrow \quad n_1\frac{v'(Q)}{b'(x_1)} + n_2\frac{v'(Q)}{b'(x_2)} = 1. \quad (31)$$

The subgame perfect equilibrium for decentralized leadership with centralized interregional income and fiscal transfers is characterized by Equations (10), (30), and (31). It involves $x_1 = x_2$

Proposition 2. *Suppose that utility function is strongly separable in private and public consumption and individuals are immobile. The subgame perfect equilibrium for decentralized leadership with centralized inter-regional income and fiscal transfers is inefficient.*

Equations (30) and (31) inform us that provision of a federal public good is socially optimal but provision of regional public goods is sensitive to the population distribution. It is efficient if population sizes are equal, but inefficient if population sizes differ. Since $n_1 > n_2$, provision of regional public goods is inefficient. To see this very clearly, suppose that $n_1 = n_2$. Then, Equation (21) implies that $g_1 = g_2$. Given $g_1 = g_2$, we have $r''(g_1) = r''(g_2)$. The right hand side of Equation (30) is equal to one, i.e., $\left(\frac{n_j}{N}\right) / \left(\frac{n_k r''(g_k)}{n_1 r''(g_1)+n_2 r''(g_2)}\right) = 1$. In such case, Equations (30) satisfy the Samuelson conditions. Therefore, a symmetric population distribution is essential for efficiency of decentralized leadership in a setting in which the center promotes interregional and fiscal transfers.

3.3. Selective Decentralized Leadership with Immobile Residents

Suppose that region 1 is the policy leader in the federation and respects provision of the federal public good. It selects the level of the federal public good in anticipation of the central government's policy regarding interregional redistribution of income and both regional governments' policies regarding provision of the regional public goods. We assume that the center's choice of interregional income transfers and the regional governments' choices of regional public good levels are simultaneous. The budget constraints for the representative resident of regions 1 and 2 are

$$x_1 = y_1 + \frac{t_1 - g_1 - Q}{n_1} \text{ and } x_2 = y_2 + \frac{t_2 - g_2}{n_2}. \quad (32)$$

In the second stage of the game, regional government 1 chooses non-negative $\{g_1\}$ to maximize $n_1\left[b\left(y_1+\frac{t_1-g_1-Q}{n_1}\right)+r(g_1)+v(Q)\right]$, regional government 2 chooses non-negative $\{g_2\}$ to maximize $n_2\left[b\left(y_2+\frac{t_2-g_2}{n_2}\right)+r(g_2)+v(Q)\right]$, and the center chooses $\{t_1,t_2\}$ to maximize $n_1\left[b\left(y_1+\frac{t_1-g_1-Q}{n_1}\right)+r(g_1)+v(Q)\right]+n_2[b\left(y_2+\frac{t_2-g_2}{n_2}\right)+r(g_2)$ $+v(Q)]$ subject to constraint (2). The equilibrium in the second stage is characterized by Equations (2), (10) and the following:

$$g_j:-b'(x_j)+n_jr'(g_j)=0 \quad\Rightarrow\quad n_j\frac{r'(g_j)}{b'(x_j)}=1,\ j=1,2. \tag{33}$$

Let $g^j(Q)$ and $t^j(Q)$, $j=1,2$, denote the best response functions. Inserting the response functions into (2), (10), and (33) and then differentiating the equations with respect to $\{Q\}$ yields

$$\begin{aligned}\frac{\partial t^1}{\partial Q}=-\frac{\partial t^2}{\partial Q}= \quad & \frac{1}{N}\left\{n_2+\frac{n_1b''(x_1)\left(n_1{}^2r''(g_1)-n_2{}^2r''(g_2)\right)}{b''(x_1)[n_1{}^2r''(g_1)-n_2{}^2r''(g_2)]+Nn_2r''(g_2)[n_1{}^2r''(g_1)+b''(x_1)]}\right\},\\ & \frac{\partial g^1}{\partial Q}=-\frac{n_2b''(x_1)r''(g_2)}{b''(x_1)[n_1r''(g_1)+n_2r''(g_2)]+Nn_1n_2r''(g_2)r''(g_1)}<0\\ & \frac{\partial g^2}{\partial Q}=-\frac{n_1b''(x_1)r''(g_1)}{b''(x_1)[n_1r''(g_1)+n_2r''(g_2)]+Nn_1n_2r''(g_2)r''(g_1)}<0\end{aligned} \tag{34}$$

Let us now examine the first stage. Regional government 1 chooses non-negative $\{Q\}$ to maximize $n_1\left[b\left(y_1+\frac{t^1(Q)-g^1(Q)-Q}{n_1}\right)+r\left(g^1(Q)\right)+v(Q)\right]$. Assuming an interior solution, the first order condition yields:

$$b'(x_1)\left(\frac{\partial t^1}{\partial Q}-\frac{\partial g^1}{\partial Q}-1\right)+n_1r'(g_1)\frac{\partial g^1}{\partial Q}+n_1v'(Q)=0 \quad\Rightarrow\quad n_1\frac{v'(Q)}{b'(x_1)}=1-\frac{\partial t^1}{\partial Q}. \tag{35}$$

Substituting $\partial t^1/\partial Q$ into Equation (35) yields

$$N\frac{v'(Q)}{b'(x_1)}=1-\frac{b''(x_1)\left(n_1{}^2r''(g_1)-n_2{}^2r''(g_2)\right)}{b''(x_1)[n_1{}^2r''(g_1)-n_2{}^2r''(g_2)]+Nn_2r''(g_2)[n_1{}^2r''(g_1)+b''(x_1)]} \tag{36}$$

The subgame perfect equilibrium for selective decentralized leadership is characterized by Equations (10), (33), and (36). It involves $x_1=x_2$. This novel result represents a major departure from the message that selective decentralized leadership incentivizes regional governments to behave efficiently. As Equation (36) reveals, the Samuelson condition for efficient provision of the federal public good is satisfied if, and only if, $n_1=n_2$. In such a case, $g_1=g_2$ and the fraction on the right-hand side of (36) vanishes.

Proposition 3. *Suppose that utility function is strongly separable in private and public consumption and individuals are immobile. The subgame perfect equilibrium for selective decentralized leadership is inefficient.*

4. Decentralized Leadership with Imperfectly Mobile Residents

We demonstrated that the equilibrium for decentralized leadership in a setting in which the center controls interregional and fiscal transfers and the equilibrium in a setting in which provision of the federal public good occurs prior to the provision of the regional public goods. We also showed that the center promotes interregional income transfers after it observes the choice with respect to the federal public good (i.e., selective decentralized leadership) are inefficient if region 1 is more abundant in labor and land and the population is immobile. We now consider the impacts of hybrid work and imperfect labor mobility.

In the presence of hybrid work, all firms operate in a common labor market. Under this 'new normal' in the labor market, which became ubiquitous during and after the COVID-19 pandemic, workers can work remotely. There is no obligation to work and reside in the same region. Let φ denote wage. The representative firm in region j chooses non-negative $\{l_j\}$ to maximize $\pi(l_j, \varphi, z_j) = F(l_j, z_j) - \varphi l_j$, taking the choices of all other firms as given. Assuming an interior solution, the first order conditions yield $F_l^j = \varphi, j = 1, 2$, where $F_l^j \equiv \partial F/\partial l_j$. The amounts of labor that the representative firm hires satisfy the equalization of marginal products of labor to the prices of labor. Let $l(\varphi, z_j)$ denote labor demand functions for the representative firm in region j. The labor markets clear if, and only if,

$$M_1 l(\varphi, z_1) + M_2 l(\varphi, z_2) = N, \tag{37}$$

where N is the total supply of labor in the economy since each individual supplies one unit of labor in the labor market. We use the market-clearing condition (37) to define wage as an implicit function of the relevant exogenous variables in the market: $\varphi = \varphi(M_1, M_2, N, Z_1, Z_2)$. We simply write the market wage as φ.

We consider situations in which individuals are free to choose their region of residence. Individuals are attached to regions because they derive idiosyncratic regional benefits associated with their language, culture, family relationships, etc. Let $n \in [0, N]$ denote an individual in the economy. This individual's attachment benefit is $a(N-n)$ if this individual resides in region 1 or an if this individual resides in region 2, where $a > 0$ denotes the intensity of attachment. Given this, this individual's utility is $b(x_1) + r(g_1) + v(Q) + a(N-n)$ for residing in region 1 or $b(x_2) + r(g_2) + v(Q) + an$ for residing in region 2. The migration equilibrium requires that an individual n_1 to be indifferent between residing in both regions:

$$b(x_1) + r(g_1) + a(N - n_1) = b(x_2) + r(g_2) + an_1 \tag{38}$$

Utilizing Equations (1), Equation (38) enables us to define the implicit function $n_1 = n^1(t_1, t_2, g_1, g_2, Q)$. This is the migration response function. The marginal responses are as follows:

$$n^1_{t_1} = \frac{-b'(x_1)/n_1}{b'(x_1)x_n^1 + b'(x_2)x_n^2 - 2a} > 0, \quad n^1_{t_2} = \frac{b'(x_2)/n_2}{b'(x_1)x_n^1 + b'(x_2)x_n^2 - 2a} < 0 \tag{39}$$

$$n^1_{g_1} = \frac{b'(x_1)/n_1 - r'(g_1)}{b'(x_1)x_n^1 + b'(x_2)x_n^2 - 2a}, \quad n^1_{g_2} = \frac{-b'(x_2)/n_2 + r'(g_2)}{b'(x_1)x_n^1 + b'(x_2)x_n^2 - 2a} \tag{40}$$

$$n^1_Q = \frac{b'(x_1)/n_1}{b'(x_1)x_n^1 + b'(x_2)x_n^2 - 2a} < 0, \tag{41}$$

where $n^1_{t_j} \equiv \partial n^1/\partial t_j$, $x_n^j \equiv \partial x^j/\partial n_j = -(x_j - \varphi)/n_j$, $n^1_{g_j} \equiv \partial n^1/\partial g_j$, and $n^1_Q \equiv \partial n^1/\partial Q$, $j = 1, 2$. Thus, the marginal responses account for the individual incomes are

$$y^j(n_j) = \varphi + M_j \pi^j(\varphi, z_j)/n_j, \qquad j = 1, 2, \tag{42}$$

where $\pi^j(\varphi, z_j) \equiv F(l(\varphi, z_j), z_j) - \varphi l(\varphi, z_j)$. Because we examine situations in which the migration equilibrium is stable, we assume throughout that $b'(x_j)x_n^j - a < 0$, $j = 1, 2$. This condition assures us that $b'(x_1)x_n^1 + b'(x_2)x_n^2 - 2a < 0$, the stable migration condition, is always satisfied. The marginal migration responses in (39) inform us that the population in region 1 increases (or decreases) as consumption of the center's interregional income transfer in region 1 (2) increases. The marginal migration responses in (40) demonstrate that one cannot, a priori, tell if the population in region 1 increases or decreases as the amount of regional public good in region 1 (2) increases. The marginal migration responses in (41) demonstrate that the population in region 1 decreases as the amount of federal pure public good increases.

As in Section 3, the regional governments are benevolent and utilitarian. Their payoffs are $w_1 = \int_0^{n_1} [b(x_1) + r(g_1) + v(Q) + a(N-n)]dn$ and $w_2 = \int_{n_1}^{N} [b(x_2) + r(g_2) + v(Q) + an]dn$. The central government is also benevolent and utilitarian. Its payoff is

$W = \sum_{j=1}^{2} w_j$. The social optimum in this economy corresponds to the center's most desirable allocation. We assume that the central government takes the migration response function into account when it chooses the policy variables. In the fully centralized allocation, the center chooses $\{t_1, t_2, g_1, g_2, Q\}$ to maximize

$$W = n^1(.)\left\{b(x_1) + r(g_1) + v(Q) + \frac{a\left[2N - n^1(.)\right]}{2}\right\} + n^2(.)\left\{b(x_2) + r(g_2) + v(Q) + \frac{a\left[2N - n^2(.)\right]}{2}\right\} \tag{43}$$

subject to constraints (1), (2), and $\sum_{j=1}^{2} n^j(.) = N$. Note that aggregating individual budget constraints (1) and then using Equation (2) and (42), we can obtain the economy wide resource constraint

$$\sum_{j=1}^{2} n^j(.)x_j + G + Q = X \tag{44}$$

where $G = \sum_{j=1}^{2} g_j$ and $X = \sum_{j=1}^{2} M_j F\left(l\left(\varphi, z_j\right), z_j\right)$. Therefore, Equations (1), (2) and (42) satisfy the federation's resource constraint. Let $\lambda \geq 0$ denote the Lagrangian multiplier associated with constraint (2). Assume an interior solution. Since $\sum_{j=1}^{2} n^j(.) = N$, $n^2_{g_j} = -n^1_{g_j}$, $n^2_{t_j} = -n^1_{t_j}$, and $n^2_Q = -n^1_Q$, $j = 1,2$, the first order conditions can be written as follows ($j,k = 1,2$ and $k \neq j$):

$$t_j : n_j b'\left(x_j\right)\left(x_n^j n_{t_j}^j + \frac{1}{n_j}\right) + n_k b'(x_k)\left(x_n^k n_{t_j}^k\right) = \lambda \tag{45}$$

$$g_j : n_j b'\left(x_j\right)\left(x_n^j n_{g_j}^j - \frac{1}{n_j}\right) + n_k b'(x_k)\left(x_n^k n_{g_j}^k\right) + n_j r'\left(g_j\right) = 0 \tag{46}$$

$$Q : n_1 b'(x_1)\left(x_n^1 n_Q^1 - \frac{1}{n_1}\right) + n_2 b'(x_2)\left(x_n^2 n_Q^2\right) + N v'(Q) = 0 \tag{47}$$

Combining Equations (39) and (45) yields

$$\begin{gathered} \tfrac{b'(x_1)}{n_1}\left(Nb'(x_2)x_n^2 - 2an_1\right) = \tfrac{b'(x_2)}{n_2}\left(Nb'(x_1)x_n^1 - 2an_2\right), \\ \Rightarrow b'(x_1) - b'(x_2) = \tfrac{Nb'(x_1)b'(x_2)(x_1 - x_2)}{2an_1 n_2}. \end{gathered} \tag{48}$$

Since $b'\left(x_j\right) > 0$, $j = 1,2$, Equation (48) implies that

$$x_1 = x_2 = x \Leftrightarrow b'(x_1) = b'(x_2) \tag{49}$$

One can prove that the equalities in (49) hold by contradiction. Suppose that $x_1 > x_2$. Then, the sign of the right-hand side of Equation (48) must be positive and it implies that $b'(x_1) > b'(x_2)$. However, this is impossible because $x_1 > x_2$ implies $b'(x_1) < b'(x_2)$ because $b'' < 0$, which contradicts Equation (48). Similarly, $x_1 < x_2$ is impossible.

Combining Equations (40) and (46) yields

$$\left(Nb'(x_k)x_n^k - 2an_j\right)\left(-\frac{b'\left(x_j\right)}{n_j} + r'\left(g_j\right)\right) = 0, \qquad j,k = 1,2, \quad k \neq j \tag{50}$$

Suppose that $-b'\left(x_j\right)/n_j + r'\left(g_j\right) \neq 0$. Then, $Nb'(x_1)x_n^1 - 2an_2 = Nb'(x_2)x_n^2 - 2an_1 = 0$ from Equations (48) and (50). Adding these equations yields $b'(x_1)x_n^1 + b'(x_2)x_n^2 - 2a = 0$. This equation violates our migration stable assumption: $b'(x_1)x_n^1 + b'(x_2)x_n^2 - 2a < 0$. Therefore,

$$n_j \frac{r'\left(g_j\right)}{b'\left(x_j\right)} = 1, \qquad j = 1,2 \tag{51}$$

Combining Equations (41) and (47) yields

$$\frac{b'(x_1)}{n_1}\left[\frac{n_1 b'(x_1)x_n^1 - n_2 b'(x_2)x_n^2}{b'(x_1)x_n^1 + b'(x_2)x_n^2 - 2a}\right] - b'(x_1) + Nv'(Q) = 0. \tag{52}$$

Given Equation (49), $n_1 b'(x_1)x_n^1 - n_2 b'(x_2)x_n^2 = 0$, Equation (52) implies that

$$\sum\nolimits_{j=1}^{2} n_j \frac{v'(Q)}{b'(x_j)} = 1 \tag{53}$$

Equations (51) and (53) are the Samuelson conditions for optimal provision of regional and federal public goods, respectively. The socially efficient allocation is characterized by Equations (48), (51) and (53).

In what follows, we consider the efficiency and equity properties of decentralized leadership. As in Section 3, we examine three distinct federal arrangements: (i) decentralized leadership with centralized inter-regional income transfers; (ii) decentralized leadership with centralized inter-regional income and fiscal transfers; and (iii) selective decentralized leadership. We show that a federation featuring decentralized leadership yields an efficient and equitable allocation of resources under alternatives (ii) and (iii) if individuals are mobile and work in a common labor market.

4.1. Centralized Interregional Income Transfers with Imperfectly Mobile Residents

Again, we start by examining a decentralized leadership arrangement in which the federal government's role is simply to redistribute income across regions. Since individuals are mobile across regions, all governments account for the migration responses in their maximization problems.

Consider the second stage. Having observed $\{g_1, g_2, Q\}$, the center chooses $\{t_1, t_2\}$ to maximize social welfare (43) subject to constraints (1), (2), and $\sum_{j=1}^{2} n^j(.) = N$. Let $\lambda \geq 0$ denote the Lagrangian multiplier associated with constraint (2). Hence, its choices yield Equations (45). Combining Equations (39) and (45) yields Equations (48) and (49). Let $t^j(g_1, g_2, Q)$ denote the center's best response functions, $j = 1, 2$. These functions satisfy the following system of equations:

$$y^1\left(\hat{n}^1(.)\right) + \frac{t^1(g_1, g_2, Q) - g_1 - Q}{\hat{n}^1(.)} = y^2\left(\hat{n}^2(.)\right) + \frac{t^2(g_1, g_2, Q) - g_2}{\hat{n}^2(.)}, \tag{54}$$

$$t^1(g_1, g_2, Q) + t^2(g_1, g_2, Q) = 0 \tag{55}$$

where $\hat{n}^1(.) \equiv n^1\left(t^1(g_1, g_2, Q), t^2(g_1, g_2, Q), g_1, g_2, Q\right)$ and $\hat{n}^2(.) \equiv N - \hat{n}^1(.)$. Differentiating Equations (54) and (55) with respect to $\{g_1, g_2, Q\}$ and then solving those equations yields

$$\frac{\partial t^j}{\partial g_j} = -\frac{\partial t^k}{\partial g_j} = \frac{1}{D}\left(\frac{1}{n_j} + \left(\frac{x_1 - \varphi}{n_1} + \frac{x_2 - \varphi}{n_2}\right) n^j_{g_j}\right), \quad j, k = 1, 2,\ k \neq j, \tag{56}$$

$$\frac{\partial t^1}{\partial Q} = -\frac{\partial t^2}{\partial Q} = \frac{1}{D}\left(\frac{1}{n_1} + \left(\frac{x_1 - \varphi}{n_1} + \frac{x_2 - \varphi}{n_2}\right) n^1_Q\right), \tag{57}$$

where $D \equiv \sum_{j=1}^{2}\left(\frac{1}{n_j} - \left(\frac{x_1 - \varphi}{n_1} + \frac{x_2 - \varphi}{n_2}\right) n^j_{t_j}\right)$.

Consider the first stage. Regional government 1 chooses non-negative $\{g_1, Q\}$ to maximize $\hat{n}^1(.)\left\{b\left(y^1(\hat{n}^1(.)) + \frac{t^1(g_1, g_2, Q) - g_1 - Q}{\hat{n}^1(.)}\right) + r(g_1) + v(Q) + \frac{a\left[2N - \hat{n}^1(.)\right]}{2}\right\}$, taking the choices of regional government 2 as given. Regional government 2 chooses non-negative $\{g_2\}$ to maximize $\hat{n}^2(.)\left\{b\left(y^2(\hat{n}^2(.)) + \frac{t^2(g_1, g_2, Q) - g_2}{\hat{n}^2(.)}\right) + r(g_2) + v(Q) + \frac{a\left[2N - \hat{n}^2(.)\right]}{2}\right\}$, taking the choices of regional government 1 as given. Assuming an interior solution, the first order conditions yield ($j = 1, 2$):

$$\begin{aligned}&\left(n^j_{t_1}\tfrac{\partial t^1}{\partial g_j}+n^j_{t_2}\tfrac{\partial t^2}{\partial g_j}+n^j_{g_j}\right)\left[b(x_j)+r(g_j)+v(Q)+a\left(N-n^j(.)\right)\right]\\&+n_j\left\{b'(x_j)\left[x^j_n\left(n^j_{t_1}\tfrac{\partial t^1}{\partial g_j}+n^j_{t_2}\tfrac{\partial t^2}{\partial g_j}+n^j_{g_j}\right)+\tfrac{1}{n_j}\left(\tfrac{\partial t^j}{\partial g_j}-1\right)\right]+r'(g_j)\right\}=0,\end{aligned} \tag{58}$$

$$\begin{aligned}&\left(n^1_{t_1}\tfrac{\partial t^1}{\partial Q}+n^1_{t_2}\tfrac{\partial t^2}{\partial Q}+n^1_{Q}\right)\left[b(x_1)+r(g_1)+v(Q)+a\left(N-n^1(.)\right)\right]\\&+n_1\left(b'(x_1)\left[x^1_n\left(n^1_{t_1}\tfrac{\partial t^1}{\partial Q}+n^1_{t_2}\tfrac{\partial t^2}{\partial Q}+n^1_{Q}\right)+\tfrac{1}{n_1}\left(\tfrac{\partial t^1}{\partial Q}-1\right)\right]+v'(Q)\right)=0.\end{aligned} \tag{59}$$

Since $n^2_{t_j} = -n^1_{t_j}$ and $n^2_{g_j} = -n^1_{g_j}$, $j = 1, 2$, combining Equations (39), (40), (56) and (58), for $j, k = 1, 2$, $k \neq j$, yields

$$\begin{aligned}&n_j\frac{r'(g_j)}{b'(x_j)}\\&=\left\{\frac{1}{n_k}-\frac{\dfrac{b'(x_k)(x_k-\varphi)}{n_k}\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}{\dfrac{b'(x_1)(x_1-\varphi)}{n_1}+\dfrac{b'(x_2)(x_2-\varphi)}{n_2}+2a}\right\}\Bigg/\left\{D+\frac{\dfrac{b(x_j)+r(g_j)+v(Q)+an_k}{n_j}\left(\dfrac{1}{n_1}+\dfrac{1}{n_2}\right)}{\dfrac{b'(x_1)(x_1-\varphi)}{n_1}+\dfrac{b'(x_2)(x_2-\varphi)}{n_2}+2a}\right\}.\end{aligned} \tag{60}$$

Each regional government provides their own regional public goods to satisfy Equation (60). For a federal public good, combining Equations (39), (41), (57), and (59) yields

$$\begin{aligned}&\left[\frac{b(x_1)+r(g_1)+v(Q)+a(N-n_1)}{b'(x_1)}-(x_1-\varphi)\right]\left(n^1_{t_1}\frac{\partial t^1}{\partial Q}+n^1_{t_2}\frac{\partial t^2}{\partial Q}+n^1_Q\right)\\&+\left(\frac{\partial t^1}{\partial Q}-1\right)+n_1\frac{v'(Q)}{b'(x_1)}=0.\end{aligned} \tag{61}$$

Since $b'(x_1) = b'(x_2)$ and $n^1_Q = -n^1_{t_1}$, we have $\left(n^1_{t_1}\frac{\partial t^1}{\partial Q}+n^1_{t_2}\frac{\partial t^2}{\partial Q}+n^1_Q\right)=0$. Additionally, then, we also have $\partial t^1/\partial Q = n_2/N$. Equation (61) is

$$n_1\frac{v'(Q)}{b'(x_1)}+n_2\frac{v'(Q)}{b'(x_2)}=1 \tag{62}$$

Equations (60) inform us that the subgame perfect Nash equilibrium for the decentralized leadership game features inefficient provision of the regional public goods because they violate Equations (51). Although the setting with imperfect resident mobility is more complex than the setting with immobile residents, the incentives faced by the regional governments are qualitatively identical in both settings. Intuitively, each regional government anticipates that the center will redistribute consumption of the private good, equating individual marginal utilities of income, and thus, has an incentive to deviate from efficient provision of its regional public good. The equilibrium, however, features efficient provision of the federal public good.

In sum, the equilibrium allocation for decentralized leadership with centralized interregional income transfers is characterized by Equations (48), (60) and (62). It involves $x_1 = x_2$.

Proposition 4. *Suppose that utility function is strongly separable in private and public consumption and individuals are imperfectly mobile in the presence of a common labor market. The subgame perfect equilibrium for decentralized leadership with centralized inter-regional income transfers is inefficient.*

4.2. Centralized Interregional Income and Fiscal Transfers with Imperfectly Mobile Residents

We now show that the center can induce efficient behavior at the regional level if it is endowed with earmarking transfers. As in Section 3.2, the center earmarks the provision of regional public goods. As before, the regional budgets must be balanced: $g_j = e_j + s_j$, $j = 1, 2$. The budget constraint for the representative resident of region j is (19). The fiscal transfers are redistributive to satisfy (20).

Regional government 1 chooses $\{e_1, Q\}$ and regional government 2 chooses $\{e_2\}$ in the first stage. Having observed $\{e_1, e_2, Q\}$, the center chooses $\{s_1, s_2, t_1, t_2\}$ to maximize social welfare (43) subject to constraints (2), (19), (20), and $\sum_{j=1}^{2} n^j(.) = N$.

Let $\lambda \geq 0$ and $\mu \geq 0$ denote the Lagrangian multipliers associated with constraint (2) and (20), respectively. The conditions that maximize social welfare are the constraints, Equations (45) and the following:

$$s_j : n_j \left[b'(x_j) \left(x_n^j n_{g_j}^j - \frac{1}{n_j} \right) + r'(g_j) \right] + n_k \left[b'(x_k) \left(x_n^k n_{g_j}^k \right) \right] = \mu, \qquad j, k = 1, 2,\ k \neq j. \tag{63}$$

Combining Equations (39) and (45) yields Equations (48) and (49). Given Equations (48) and (49), combining Equation (63) so as to eliminate μ and then substituting Equation (40) into the implied equation yields

$$r'(g_1) \left[N b'(x_2) x_n^2 - 2a n_1 \right] = r'(g_2) \left[N b'(x_1) x_n^1 - 2a n_2 \right]. \tag{64}$$

Given Equations (48) and (49), we obtain

$$n_1 r'(g_1) = n_2 r'(g_2). \tag{65}$$

Note that, given $x_1 = x_2$, Equation (38) simplifies to $r(g_1) + a n_2 = r(g_2) + a n_1$. For arbitrary a and N values, $n_1 r'(g_1) = n_2 r'(g_2)$, $r(g_1) + a n_2 = r(g_2) + a n_1$, and $N = n_1 + n_2$ hold simultaneously if, and only if, $n_1 = n_2 = N/2$ and $g_1 = g_2$. Hence, the central government makes the interregional income and fiscal transfers to satisfy the equity conditions $x_1 = x_2$ and $g_1 = g_2$. Let $t^j(e_1, e_2, Q)$ and $s^j(e_1, e_2, Q)$, $j = 1, 2$, denote the center's best response functions. Let $g^j(e_1, e_2, Q) = e_j + s^j(e_1, e_2, Q)$, $j = 1, 2$. These functions satisfy the following system of equations:

$$y^1\left(\frac{N}{2}\right) + \frac{t^1(e_1, e_2, Q) - e_1 - s^1(e_1, e_2, Q) - Q}{N/2} = y^2\left(\frac{N}{2}\right) + \frac{t^2(e_1, e_2, Q) - e_2 - s^2(e_1, e_2, Q)}{N/2} \tag{66}$$

$$t^1(e_1, e_2, Q) + t^2(e_1, e_2, Q) = 0, \tag{67}$$

$$e_1 + s^1(e_1, e_2, Q) = e_2 + s^2(e_1, e_2, Q) \tag{68}$$

$$s^1(e_1, e_2, Q) + s^2(e_1, e_2, Q) = 0 \tag{69}$$

Equations (66)–(69) imply that

$$\frac{\partial t^j}{\partial e_j} = -\frac{\partial t^k}{\partial e_j} = 0 \text{ and } \frac{\partial s^j}{\partial e_j} = -\frac{\partial s^k}{\partial e_j} = -\frac{1}{2}, \qquad j, k = 1, 2,\ k \neq j, \tag{70}$$

$$\frac{\partial t^1}{\partial Q} = -\frac{\partial t^2}{\partial Q} = \frac{1}{2} > 0 \text{ and } \frac{\partial s^j}{\partial Q} = 0, \qquad j = 1, 2. \tag{71}$$

Consider the first stage. Regional government 1 chooses non-negative $\{e_1, Q\}$ to maximize $\frac{N}{2}\left\{ b\left(y^1\left(\frac{N}{2}\right) + \frac{t^1(e_1,e_2,Q) - e_1 - s^1(e_1,e_2,Q) - Q}{N/2} \right) + r\left(g^1(e_1, e_2, Q)\right) + v(Q) + \frac{3aN}{4} \right\}$, taking the choices of region 2 as given. Regional government 2 chooses non-negative $\{e_2\}$ to maximize $\frac{N}{2}\left\{b\left(y^2\left(\frac{N}{2}\right)+\frac{t^2(e_1,e_2,Q)-e_2-s^2(e_1,e_2,Q)}{N/2}\right)+r\left(g^2(e_1,e_2,Q)\right)+v(Q)+\frac{3aN}{4}\right\}$, taking the choices of region 1 as given. Assuming interior solutions, the first order conditions yield:

$$\frac{b'(x_j)}{N/2}\left(\frac{\partial t^j}{\partial e_j}-\frac{\partial s^j}{\partial e_j}-1\right)+r'(g_j)\left(\frac{\partial s^j}{\partial e_j}+1\right)=0, \qquad j=1,2, \tag{72}$$

$$\frac{b'(x_1)}{N/2}\left(\frac{\partial t^1}{\partial Q}-\frac{\partial s^1}{\partial Q}-1\right)+r'(g_1)\frac{\partial s^1}{\partial Q}+v'(Q)=0. \tag{73}$$

Combining Equations (70) and (72) yields

$$\frac{N}{2}\frac{r'(g_j)}{b'(x_j)}=1, \qquad j=1,2 \tag{74}$$

Each regional public good is provided so as to satisfy Equation (74). For a federal public good, combining Equations (71) and (73) yields

$$N\frac{v'(Q)}{b'(x_j)}=1 \quad\Rightarrow\quad n_1\frac{v'(Q)}{b'(x_1)}+n_2\frac{v'(Q)}{b'(x_2)}=1. \tag{75}$$

Equation (74) inform us that the regional governments provide the regional public goods efficiently. Equation (75) shows that region 1 provides the federal good efficiently. Therefore, the combination of interregional and fiscal transfers induces efficient behavior at the regional level.

In sum, the equilibrium allocation for decentralized leadership with centralized inter-regional income and fiscal transfers is characterized by Equations (48), (74), and (75). It involves $x_1=x_2$ and $g_1=g_2$.

Proposition 5. *Suppose that utility function is strongly separable in private and public consumption and individuals are imperfectly mobile in the presence of a common labor market. The subgame perfect equilibrium for decentralized leadership with centralized inter-regional income and fiscal transfers is efficient.*

4.3. Selective Decentralized Leadership with Imperfectly Mobile Residents

We now show that both regions behave efficiently if region 1 can commit to provision of the federal public good. The budget constraints for the representative resident of region 1 and region 2 are shown in Equation (32), respectively. In the first stage of the game, regional government 1 chooses the amount of the federal public good. In the second stage of the game, the regional governments choose the amounts of regional public goods and the center chooses the interregional income transfers.

In the second stage, regional government 1 chooses non-negative $\{g_1\}$ to maximize $n^1(.)\left\{b\left(y^1(n^1(.))+\frac{t_1-g_1-Q}{n^1(.)}\right)+r(g_1)+v(Q)+\frac{a[2N-n^1(.)]}{2}\right\}$, taking the choices of the other governments as given, regional government 2 chooses non-negative $\{g_2\}$ to maximize $n^2(.)\left\{b\left(y^2(n^2(.))+\frac{t_2-g_2}{n^2(.)}\right)+r(g_2)+v(Q)+\frac{a[2N-n^2(.)]}{2}\right\}$, taking the choices of the other governments as given, and the central government chooses $\{t_1,t_2\}$ to maximize social welfare (43) subject to constraint (2), (32), and $\sum_{j=1}^{2} n^j(.)=N$, taking the choices of the regional governments as given.

Assuming interior solutions, the regional government's first order conditions yield

$$n^j_{g_j}\left[b(x_j)+r(g_j)+v(Q)+a(N-n_j)\right]+n_j\left[b'(x_j)\left(x^j_n n^j_{g_j}-\frac{1}{n_j}\right)+r'(g_j)\right]=0, \qquad j=1,2 \tag{76}$$

Combining Equation (76) with Equation (40) yields

$$\left(r'(g_j) - \frac{b'(x_j)}{n_j}\right)\left[\left(b'(x_k)x_n^k - a\right) - \frac{b(x_j) + r(g_j) + v(Q) + aN}{n_j}\right] = 0, \qquad j,k = 1,2,\, k \neq j. \tag{77}$$

Since the latter bracket is strictly negative, Equation (77) is

$$n_j \frac{r'(g_j)}{b'(x_j)} = 1, \qquad j = 1,2 \tag{78}$$

Hence, both regional governments provide their own regional public goods in an efficient manner. It satisfies Equation (51).

Let us consider the first order conditions for the center's maximization problem. Let $\lambda \geq 0$ denote the Lagrangian multiplier associated with constraint (2). The center's first order condition yields Equations (48) and (49). Let $g^j(Q)$, $j = 1,2$, denote regional government $j's$ best response function and $t^j(Q)$, $j = 1,2$, denote the center's best response functions. These functions satisfy the following center's redistribution rules:

$$y^1\left(\widetilde{n}^1(.)\right) + \frac{t^1(Q) - g^1(Q) - Q}{\widetilde{n}^1(.)} = y^2\left(\widetilde{n}^2(.)\right) + \frac{t^2(Q) - g^2(Q)}{\widetilde{n}^2(.)}, \tag{79}$$

$$t^1(Q) + t^2(Q) = 0, \tag{80}$$

where $\widetilde{n}^1(.) \equiv n^1\left(t^1(Q), t^2(Q), g^1(Q), g^2(Q), Q\right)$ and $\widetilde{n}^2(.) \equiv N - \widetilde{n}^1(.)$. As in Section 4.2., given Equation (49), Equations (38) and (78) simplify to $r(g_1) + an_2 = r(g_2) + an_1$ and $n_1 r'(g_1) = n_2 r'(g_2)$. We obtain $n_1 = n_2 = N/2$ and $g_1 = g_2$. Utilizing this, we have $\partial g^1/\partial Q = \partial g^2/\partial Q$. Differentiating Equations (79) and (80) with respect to $\{Q\}$ yields

$$\left(\frac{(x_1 - \varphi)}{n_1} + \frac{(x_2 - \varphi)}{n_2}\right)\left(\sum_{j=1}^{2}\left(n^1_{t_j}\frac{\partial t^j}{\partial Q} + n^1_{g_j}\frac{\partial g^j}{\partial Q}\right) + n^1_Q\right) - \frac{1}{n_1}\frac{\partial t^1}{\partial Q} + \frac{1}{n_2}\frac{\partial t^2}{\partial Q} = -\frac{1}{n_1} \tag{81}$$

$$\frac{\partial t^1}{\partial Q} + \frac{\partial t^2}{\partial Q} = 0. \tag{82}$$

Given Equation (78), $n^1_{g_j} = 0$, $j = 1,2$. Equation (81) is

$$\begin{aligned}&\left[\frac{1}{n_1} - \left(\frac{(x_1 - \varphi)}{n_1} + \frac{(x_2 - \varphi)}{n_2}\right)n^1_{t_1}\right]\frac{\partial t^1}{\partial Q} - \left[\frac{1}{n_2} + \left(\frac{(x_1 - \varphi)}{n_1} + \frac{(x_2 - \varphi)}{n_2}\right)n^1_{t_2}\right]\frac{\partial t^2}{\partial Q}\\ &= \frac{1}{n_1} + n^1_Q\left(\frac{(x_1 - \varphi)}{n_1} + \frac{(x_2 - \varphi)}{n_2}\right).\end{aligned} \tag{83}$$

Solving Equations (82) and (83) yields

$$\frac{\partial t^1}{\partial Q} = -\frac{\partial t^2}{\partial Q} = \frac{1}{D}\left(\frac{1}{n_1} + \left(\frac{x_1 - \varphi}{n_1} + \frac{x_2 - \varphi}{n_2}\right)n^1_Q\right) \tag{84}$$

where $D \equiv \sum_{j=1}^{2}\left(\frac{1}{n_j} - \left(\frac{x_1 - \varphi}{n_1} + \frac{x_2 - \varphi}{n_2}\right)n^j_{t_j}\right)$.

Consider the first stage. Regional government 1 chooses non-negative $\{Q\}$ to maximize $\widetilde{n}^1(.)\left\{b\left(y^1\left(\widetilde{n}^1(.)\right) + \frac{t^1(Q) - g^1(Q) - Q}{\widetilde{n}^1(.)}\right) + r\left(g^1(Q)\right) + v(Q) + \frac{a\left[2N - \widetilde{n}^1(.)\right]}{2}\right\}$. Assuming an interior solution, the first order condition is:

$$\begin{aligned}&\left(\frac{b(x_1) + r(g_1) + v(Q) + a\left(N - n^1(.)\right)}{b'(x_1)} - (x_1 - \varphi)\right)\left(\sum_{j=1}^{2}\left(n^1_{t_j}\frac{\partial t^j}{\partial Q} + n^1_{g_j}\frac{\partial g^j}{\partial Q}\right) + n^1_Q\right)\\ &+\left(\frac{\partial t^1}{\partial Q} - 1\right) + \left(n_1\frac{r'(g_1)}{b'(x_1)} - 1\right)\frac{\partial g^1}{\partial Q} + n_1\frac{v'(Q)}{b'(x_1)} = 0\end{aligned}. \tag{85}$$

Since $n^1_{g_j} = 0$, $j = 1, 2$, and $\left(n^1_{t_1}\frac{\partial t^1}{\partial Q} + n^1_{t_2}\frac{\partial t^2}{\partial Q} + n^1_Q\right) = 0$, Equation (85) is

$$n_1\frac{v'(Q)}{b'(x_1)} = 1 - \frac{\partial t^1}{\partial Q}. \tag{86}$$

Because $\partial t^1/\partial Q = n_2/N$, Equation (86) is

$$N\frac{v'(Q)}{b'(x_1)} = 1 \quad \Rightarrow \quad n_1\frac{v'(Q)}{b'(x_1)} + n_2\frac{v'(Q)}{b'(x_2)} = 1. \tag{87}$$

As revealed by Equation (78), the regional governments and the central government succeed in providing regional public goods efficiently. Equation (87) shows that regional government 1 provides the federal public good efficiently. In sum, the subgame perfect equilibrium for the selective decentralized leadership game under hybrid work and imperfect labor mobility is socially optimal. It is characterized by Equations (48), (78), and (87). It involves $x_1 = x_2$ and $g_1 = g_2$.

Proposition 6. *Suppose that utility function is strongly separable in private and public consumption and individuals are imperfectly mobile in the presence of a common labor market. The subgame perfect equilibrium for selective decentralized leadership game is efficient.*

5. Conclusions

In most developed nations, the new normal in the labor market is hybrid work. One of its direct implications is the creation of a national (or even international) common labor market. In such nations, we also tend to observe regional governments providing multiple types of public goods and services. The consumption benefits of some of these regionally provided public goods do not respect jurisdictional boundaries. It is, therefore, crucial to know if there is an arrangement of policy instruments and timing of policy actions within a federation that leads to a socially optimal allocation in the presence of hybrid work and imperfect residential mobility.

This paper demonstrated that there are circumstances under which the strategic interaction of regional and central governments yields a socially efficient allocation of resources in a federation characterized by a common labor market and attachment to regions. Provided that marginal products of labor are equalized across regions, which naturally occur in equilibrium for a common labor market, the policy actions of regional and central governments lead to a socially optimal allocation, if regional governments make policy choices in anticipation of the center's choices of interregional income and fiscal transfers. We also showed that the strategic interaction between regional and central governments yields a socially optimal allocation if the region that provides a federal public good is a leader in the federation: it commits to the provision of the federal public good in anticipation of simultaneous regional and central policy choices, where the regions choose the levels of regional public goods to be provided and the center chooses the amounts of interregional income transfers. A key common feature of the equilibria for the two policy settings was that hybrid work produces a symmetric population distribution. This symmetric nature implies that the regional governments' incentives are perfectly aligned.

Author Contributions: Conceptualization, N.A. and E.C.D.S.; methodology, N.A. and E.C.D.S.; formal analysis, N.A. and E.C.D.S.; investigation, N.A. and E.C.D.S.; writing—original draft preparation, N.A. and E.C.D.S.; writing—review and editing, N.A. and E.C.D.S. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: No new data created.

Conflicts of Interest: The authors declare no conflict of interest.

Notes

[1] See, e.g., Boadway and Cuff [1], Martinez-Velazquez et al. [2], Silva [3], Silva [4], and Silva [5]. These papers produce evidence that interregional income and earmarked transfers are ubiquitous in federations.

[2] See, e.g., Cornes and Itaya [9] for an alternative model with multiple public goods.

[3] See, e.g., Boadway and Cuff [1], Martinez-Velazquez et al. [2], Silva [3], Silva [4], and Silva and Lucas [10] for relevant contributions to fiscal federalism in related areas.

References

1. Boadway, R.; Cuff, C. The impressive contribution of Canadian economists to fiscal federalism theory and policy. *Can. J. Econ.* **2017**, *50*, 1348–1380. [CrossRef]
2. Martinez-Vazquez, J.; Lago-Peñas, S.; Sacchi, A. The impact of fiscal decentralization: A survey. *J. Econ. Surv.* **2017**, *31*, 1095–1129. [CrossRef]
3. Silva, E.C.D. Selective decentralized leadership. *J. Urban Econ.* **2014**, *83*, 1–5. [CrossRef]
4. Silva, E.C.D. Efficient earmarking under decentralized fiscal commitments. *Int. Tax Public Financ.* **2015**, *22*, 683–701. [CrossRef]
5. Silva, E.C.D. Tax competition and federal equalization schemes with decentralized leadership. *Int. Tax Public Financ.* **2017**, *24*, 164–178. [CrossRef]
6. Caplan, A.; Cornes, R.C.; Silva, E.C.D. Pure public goods and income redistribution in a federation with decentralized leadership and imperfect labor mobility. *J. Public Econ.* **2000**, *77*, 265–284. [CrossRef]
7. Mansoorian, A.; Myers, G.M. Attachment to home and efficient purchases of population in a fiscal externality economy. *J. Public Econ.* **1993**, *52*, 117–132. [CrossRef]
8. Wellisch, D. Interregional spillovers in the presence of perfect and imperfect household mobility. *J. Public Econ.* **1994**, *55*, 167–184. [CrossRef]
9. Cornes, R.C.; Itaya, J.-I. On the private provision of two or more public goods. *J. Public Econ. Theory* **2010**, *12*, 363–385. [CrossRef]
10. Silva, E.C.D.; Lucas, V.M. Common labor market, attachment and spillovers in a large metropolis. *Int. Tax Public Financ.* **2016**, *23*, 693–715. [CrossRef]
11. Smart, M.; Bird, R.M. Earmarked grants and accountability in government. In *General Grants Versus Earmarked Grants*; Kim, J., Lotz, J., Mau, N.J., Eds.; Korea Institute of Public Finance and Danish Ministry of Interior and Health: Copenhagen, Denmark, 2009.
12. Akai, N.; Watanabe, T. Delegation of taxation authority and multipolicy commitment in a decentralized leadership model. *Public Financ. Rev.* **2020**, *48*, 505–537. [CrossRef]
13. Boadway, R. The theory and practice of equalization. *CESifo Econ. Stud.* **2004**, *50*, 211–254. [CrossRef]