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Duopoly and Endogenous Single Product Quality Strategies

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Abstract: This research paper investigates a duopolistic market characterized by vertical product differentiation. The study considers both product qualities and consumer preferences represented as one-dimensional intervals. The focus is on analyzing the equilibrium in a duopoly game with convex production costs. In this setting, each firm has the option to present a multi-product strategy consisting of quality–price pairs, and their profits are determined by the decisions made by consumers. The findings of the study reveal that, under typical consumer preference conditions, both firms tend to offer a single quality–price pair. Additionally, the market is shown to be fully served, and firm profits decrease as the index of product quality increases. A comparative analysis is also conducted with the case of a monopoly.

Keywords: duopoly; single product quality strategy; fully served market

JEL Classification: C72; D43; L15

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1. Introduction

There is growing theoretical and practical interest in determining the optimal number of products and its implications for markets with few suppliers and differentiated products. The impact of this optimal number has received considerable attention in the fields of marketing (e.g., Shah and Wolford 2007, Scheibehenne et al., 2010 [1,2]) and theoretical economics (e.g., Mussa and Rosen, 1978 [3], Shaked and Sutton, 1982 [4], 1983 [5]).

From a marketing perspective, it is likely that a larger number of options within a category makes choice more difficult. Specifically, when the differences between attractive alternatives increase, there is a lesser amount of available information about each one (Fasolo et al., 2009 [6]; Timmermans 1993 [7]). On the other hand, a large variety of choices increases the likelihood that different consumers will be satisfied (Anderson 2006 [8]). However, from an economic theory perspective, another preliminary question needs to be entertained: how does the increase in the number of producers (in particular from the case of a monopoly[1] to the case of a duopoly) affect the total number of different products?

2. Literature Review

In the realm of marketing, Scheibehenne et al. (2010) [1] presented two contrasting lists of arguments that highlight the impact of the number of options on consumer satisfaction. The first list outlines reasons why an excessive number of choices can result in reduced satisfaction among consumers, while the second list puts forth arguments explaining how having more options can lead to increased satisfaction among consumers. In particular, Shah and Wolford (2007) [2] found that purchase behavior was a curvilinear function of the number of choices, but the function varied by product.

In the theoretical economics approach that we focus on, Mussa and Rosen (1978) [3] in their classic paper established a framework for multiple qualities under a monopoly with vertical product differentiation and a strictly convex production cost function. Building upon their foundation, the primary aim of this paper is to extend this theoretical framework to the context of a duopoly. We analyze a market with vertical product differentiation, where both qualities and consumers are represented by one-dimensional intervals and the two firms have different production costs.

The existing literature analyzing a continuum of consumers focuses on two types of models: models with a monopoly and models with an oligopoly.

In the case of a monopoly, Mussa and Rosen (1978) [3] showed that when the production cost function is strictly convex, the equilibrium offer spectrum is either a continuous curve or a single quality–price pair. This model was followed by Gabszewicz et al. (1986) [9], who analyzed the conditions under which the monopoly completely segments the market by offering the maximum range of qualities in the case of no production costs. Noh and Moschini (2006) [10] analyzed the potential entry of an innovative firm into a monopoly market, where the entry deterrence strategies of the incumbent firm rely on "limit qualities". Recently, Altan (2020) [11] analyzed a vertically differentiated market for imperfectly durable goods in a discrete time game. Their results suggest that the seller should offer different quality versions of the product when the innate durability of the product is high. Here, we extend the connected offer spectrum approach to the case of a duopoly.

For the case of an oligopoly with a single product quality strategy for each firm, Shaked and Sutton (1982) [4] showed that an endogenous duopoly with strictly positive profits emerges when there are no production costs. Shaked and Sutton (1983) [5] also showed that there are a limited number of firms in equilibrium with strictly positive profits in the case of different production costs. Motta (1993) [12] made two different assumptions about the nature of production costs: they are either fixed or variable with regard to quality improvement. Cremer and Thisse (1994) [13] introduced commodity taxation and showed that a uniform ad valorem tax lowers both qualities, distorts the allocation of consumers between firms and lowers the consumer prices of both variants. Recently, Bos and Marini (2019) [14] analyzed cartel stability with vertically differentiated cartelized products. They found that if market shares are maintained at pre-collusive levels, then the firm with the lowest competitive price–cost margin has the strongest incentive to deviate from the collusive agreement. This model was followed by Bos et al. (2020) [15], where their findings suggest that firms have a strong incentive to coordinate prices when the products involved are vertically differentiated, and by Bos and Marini (2022) [16] , who analyzed price collusion in a repeated game. They presented a characterization of the collusive pricing equilibrium and examined the corresponding effect on market shares and welfare. Finally, Morita and Nguyen (2021) [17] explored the quality-enhancing technology spillover in an asymmetric model where one of the firms can only choose a quality level for its product up to a certain upper bound value. Here, we extend the analysis of a single-product quality strategy to a multi-product quality strategy of individual firms. In the case of a duopoly with a multi-product quality strategy for each firm and identical production costs, Economides (1985) [18] analyzed zero profit equilibria. This model was followed by work by Champsaur and Rochet (1989) [19], who analyzed consumers who bought different quality brands. Finally, Cheng et al. (2011) [20] investigated a sequential game where firms simultaneously choose both the number of products and qualities in the first stage and then compete in terms of price in the second stage. In the case of an oligopoly with a multi-product quality strategy for each firm, Gayer (2017) [21] concluded that when there are different production costs, an endogenous duopoly with positive profits emerges in equilibrium. Building upon the aforementioned literature, this study delves into a specific scenario of a duopoly with varying production costs. The objective is to determine the conditions that justify the assumption of each firm offering a single product quality strategy.

Another approach in the literature is a monopoly model with a finite number of consumers. This analysis dates back to Spence (1980) [22] and Phlips (1983) [23] and was followed up by Shitovitz et al. (1989) [24], who derived the conditions under which the monopolistic firm will only offer the highest quality. Gayer (2007) [25] extended the monopoly model to the case of an oligopoly and showed the conditions under which an endogenous monopolist occurs in equilibrium.

In the model below, the duopoly firms compete in terms of quality and prices. The main result is that in a Nash equilibrium, the strategy of each firm is to offer a single quality–price pair.

3. The Model

Consider a product quality model where the range of qualities is given by the interval $D = (0, \bar{Q}]$, where $\bar{Q}$ is the highest quality. There is a continuum of buyers uniformly distributed on the interval $T = [a, b]$, which includes an ordering of their preferences for qualities, where $a > 0$. Let p denote the price of quality Q that generates a quality–price pair $(Q, p) \in D \times R_{++}$. We define $U_t(Q, p) : T \times D \times R_+ \longrightarrow R_+$, $U_t(Q, p) = t \cdot Q - p$, as the utility function for consumer t who chooses the pair (Q, p), for each $t \in T$. We denote by $U_t^* = U_t(0, 0) = 0$ the utility level that arises for consumer t when they do not purchase at all. Finally, the consumer $t \in T$ is characterized by their income e_t, where $e(\cdot) : T \longrightarrow R_{++}$. Denote by $y_t = e_t - p$ the net income of a consumer t who chooses the pair (Q, p), for each $t \in T$.

There are two firms with different production costs. Let $C_j(Q)$ be the cost function of firm j producing quality Q, where $j = 1, 2$.

Assumption 1. *Non-negative net income for all consumers.*

For any consumer $t \in T$ who chooses the pair $h = (Q, p)$, it holds that $e_t \geq t \cdot \bar{Q}$, for all $t \in T$[2].

Assumption 2. *Convex, monotonically increasing and continuously differentiable cost functions*

The production cost $C_j(Q)$ is twice continuously differentiable and convex, that is, $C_j(0) = 0$, $C_j'(.) > 0$ and $C_j''(.) \geq 0$, for $j = 1, 2$, where $a - C_j'(0) > 0$, for $j = 1, 2$.

Denote by $\bar{q}_j$ ($\underline{q}_j$) the maximum (minimum) quality that firm j can offer, where $j = 1, 2$. In particular, the following equations $a \geq C_1'(\underline{q}_1)$ and $b \geq C_2'(\underline{q}_2)$ hold for $\underline{q}_1$ and $\underline{q}_2$, respectively[3].

Assumption 3. *The unit costs cross at a single point—$\dot{Q}$.*

Denote by $\dot{Q}$ the quality where the two production costs intersect; that is, $C_1(\dot{Q}) = C_2(\dot{Q})$. Without loss of generality, we assume that firm 1 has superior technology relative to firm 2 in the low quality region, but firm 2 has superior technology in the high quality region. In particular, for all $0 < Q < \dot{Q}$, it is true that $C_1(Q) < C_2(Q)$, and for all $\dot{Q} < Q \leq \bar{Q}$, we have that $C_1(Q) > C_2(Q)$.

Assumption 4. *Marginal costs cross at a single point—$\underline{q}$.*

Denote by $\underline{q}$, where $\underline{q} < \dot{Q}$, the quality at which the slopes of production costs are equal, i.e., $C_1'(\underline{q}) = C_2'(\underline{q})$.

Assumption 5. *The slope of production costs.*

The slope of the production cost of firm 1 is greater than the slope of the production cost of firm 2, i.e., $C_1'(Q) > C_2'(Q)$ for all $Q > \underline{q}$, where $\underline{q}_j \geq \underline{q}$ for $j = 1, 2$[4].

The strategy of firm j, denoted by s_j, is a nonempty and closed set of $D \times R_{++}$, for $j = 1, 2$, where s_j is bounded. That is, for all $h = (Q, p) \in s_j$ with $j = 1, 2$, it holds that $Q \leq \bar{Q}$ and $p \leq b\bar{Q}$[5]. Moreover, for all $\hat{h} = (\hat{Q}, \hat{p})$, $\tilde{h} = (\tilde{Q}, \tilde{p}) \in s_j$ where $j = 1, 2$, $\tilde{Q} \geq \hat{Q} \iff \tilde{p} \geq \hat{p}$[6]. These two strategies are then used to build the offer spectrum s.

Assumption 6. *A constraint on the structure of strategies.*

We only consider strategies s_1 and s_2 consisting of a finite number of quality–price pairs and connected curves.

Each consumer $t \in T$ chooses one of the pairs from the offer spectrum s or decides not to buy at all.

Assumption 7. *Priority for the higher quality.*
If a consumer is indifferent between two pairs, they will buy the one with the higher quality.

The offer spectrum s generates the partition $(T_h)_{h \in s}$, where T_h denotes consumers $t \in T$ for whom $e_t \geq p$ holds and who prefer $h \in s$ over all other pairs in s[7].

Formally, $t \in T_h$ is a consumer t belonging to T_h, $h \equiv (Q, p) \in s$, if $e_t \geq p$, and

$$t \cdot Q \geq p \text{ and } t \cdot (Q - \tilde{Q}) \geq (p - \tilde{p})$$

for all $\tilde{h} \in s$, where the last inequality holds strictly for all $\tilde{h} \in s$, where $\tilde{h} > h$.

Similarly, T_0 denotes the set of consumers who choose not to buy a brand:

$$T_0 = \{t \in T : 0 > \underset{\{(Q,p)=h \in s\}}{Max} \{t \cdot Q - p\}\}.$$

We have that for all $h \in s$, T_h is an interval. Let $\underline{t}_h \in T_h$ be its left endpoint.

Denote by $\underline{h}_j = (\underline{Q}_j, \underline{p}_j)$ ($\bar{h}_j = (\bar{Q}_j, \bar{p}_j)$) the pair with the lowest (highest) quality offered by firm j for $j = 1,2$. Thus, for all $(Q_1, p_1) \in s_1$, $\underline{q}_1 \leq \underline{Q}_1 \leq Q_1 \leq \bar{Q}_1 < \dot{Q}$, and for all $(Q_2, p_2) \in s_2$, $\dot{Q} < \underline{Q}_2 \leq Q_2 \leq \bar{Q}_2 \leq Min\{\bar{q}_2, \bar{Q}\}$.

Since $\bar{h}_1, \underline{h}_2 \in s$, where $\bar{Q}_1 < \dot{Q} < \underline{Q}_2$, we denote $\Delta Q = \underline{Q}_2 - \bar{Q}_1 > 0$.

Thus, the indifferent consumer $\bar{t}$ is between the $\bar{h}_1$ and $\underline{h}_2$ purchase pairs:

$$\bar{t} = \frac{\underline{p}_2 - \bar{p}_1}{\Delta Q}. \quad (1)$$

Denote by $\Delta\underline{t} = \bar{t} - \underline{t}$ ($\Delta\bar{t} = b - \bar{t}$) the market share of firm 1 (2), respectively. Finally, we define $CS(t) = t \cdot Q - p$ as the consumer surplus of consumer t from the choice of the pair (Q, p), for each $t \in T$.

The model in a nutshell is as follows. We have a product quality model with a continuum of buyers uniformly distributed on an interval that includes an ordering of their taste preferences for qualities. We have two firms with different production costs, where one firm has superior technology relative to the other firm in the low-quality domain, and the other firm has superior technology in the high-quality domain. We only consider strategies that consist of a finite number of quality–price pairs and connected curves. The duopoly firms compete on qualities and prices under conditions of a simultaneous game with a Nash equilibrium.

This yields the following propositions:

Proposition 1.

I. *In equilibrium, the market is fully served. That is,* $\underline{t} = a$.

II. *In equilibrium, the consumer surplus of consumer a from the choice of pairs* $(Q_1, p_1) \in s_1^*$ *offered by firm* 1 *is positive. In particular,* $CS(a) > 0$[8].

Note 1

Economically, consumers who choose the quality–price pair from firm 1 can replace their choice by another pair offered by firm 2 in the case of a duopoly. This competition causes the price to fall compared a monopoly, thus increasing the consumer surplus.

Proposition 2. *In equilibrium with multi-product strategies, the profit per unit for each firm is strictly quality dependent. That is, for all* $\hat{h}_j = (\hat{Q}_j, \hat{p}_j)$, $\tilde{h}_j = (\tilde{Q}_j, \tilde{p}_j) \in s_j$, *where* $j = 1,2$, *we have:*

$$\tilde{Q}_j > \hat{Q}_j \iff \tilde{p}_j - C_j(\tilde{Q}_j) > \hat{p}_j - C_j(\hat{Q}_j).$$

Note 2

Economically, this proposition is similar to the monopoly case, where the profit per unit increases strictly with quality.

The intuition is that if this property does not hold for a particular quality–price pair, firms can either increase the price for that quality, accompanied by an identical price increase for all qualities above it, or can remove that pair from the offer spectrum, and as a result increase the firm's profit.

Proposition 3. *In equilibrium with multi-product strategies, the strategy of each firm is a connected curve.*

Note 3

Economically, and similar to Proposition 2, this case is like the one found for a monopoly in which the offer spectrum is connected (Mussa and Rosen 1978) [3]. The intuition is that if there are two consecutive quality–price pairs, we can insert a third pair between them and thereby increase the profit.

Applying these propositions leads to the main results.

The main result—part I

In equilibrium, firm 1 will offer an endogenous single quality price pair: $s_1^* = (Q_1, p_1)$, where

$$a = C_1^{'}(Q_1).$$

Note 4

i. Economically, firm 1 aims to distinguish itself from firm 2 by reducing its quality in order to relax competition.
ii. The intuition for the latter equation is that to maximize profit per unit, the slope of the indifferent consumer (a) and the slope of the cost function ($C_1^{'}(Q_1)$) must be equal in an interior solution[9].

Market Competition in Qualities and Prices

Proposition 1 part I and the main result part I yield the following profit functions:

$$\pi_1 = [(p_1 - C_1(Q_1)] \cdot \Delta \underline{t} = [(p_1 - C_1(Q_1)] \cdot \left(\frac{\underline{p}_2 - p_1}{\Delta Q} - a\right), \tag{2}$$

$$\pi_2 = [(\underline{p}_2 - C_2(\underline{Q}_2)] \cdot \Delta \bar{t} \cdot K = [(\underline{p}_2 - C_2(\underline{Q}_2)] \cdot \left(b - \frac{\underline{p}_2 - p_1}{\Delta Q}\right) \cdot K, \tag{3}$$

where $K \geq 1$ and $\frac{\partial K}{\partial \underline{Q}_2} = \frac{\partial K}{\partial \underline{p}_2} = 0$.

The first-order conditions yield the price best response functions $p_1 = \frac{\underline{p}_2 - a\Delta Q + C_1(Q_1)}{2}$ and $\underline{p}_2 = \frac{p_1 + b\Delta Q + C_2(\underline{Q}_2)}{2}$ of firms 1 and 2, respectively. Solving the two best response functions yields equilibrium prices given by

$$p_1 = \frac{(b - 2a)\Delta Q + 2C_1(Q_1) + C_2(\underline{Q}_2)}{3}. \tag{4}$$

$$\underline{p}_2 = \frac{(2b - a)\Delta Q + C_1(Q_1) + 2C_2(\underline{Q}_2)}{3}. \tag{5}$$

Substituting (4) and (5) into both profit functions (2) and (3) yields

$$\pi_1 = \frac{\left[(b-2a)\Delta Q - C_1(Q_1) + C_2(\underline{Q}_2)\right]^2}{9\Delta Q}. \tag{6}$$

$$\pi_2 = \frac{\left[(2b-a)\Delta Q + C_1(Q_1) - C_2(\underline{Q}_2)\right]^2}{9\Delta Q} \cdot K, \tag{7}$$

where $K \geq 1$ and $\frac{\partial K}{\partial \underline{Q}_2} = 0$.

The main result—part II

In equilibrium, firm 2 will offer an endogenous single quality–price pair $s_2^* = (\bar{Q}_2, p_2)$, where $\bar{Q}_2 = Min\{\bar{q}_2, \bar{Q}\}$. Specifically,

i. When $b \geq C_2'(\bar{Q})$, then $\bar{q}_2 = \bar{Q}$; in other words, firm 2 will offer top of the line quality[10].
ii. When $b < C_2'(\bar{Q})$, then firm 2 will offer $\bar{q}_2 < \bar{Q}$, where $b = C_2'(\bar{q}_2)$.

Note 5

Economically, firm 2 (as well as the other firm) wants to differentiate itself from firm 1 by raising its quality in order to relax competition.

The main result yields the following profit functions:

$$\pi_1 = \frac{\left[(b-2a)\Delta Q - C_1(Q_1) + C_2(\bar{Q}_2)\right]^2}{9\Delta Q}. \tag{8}$$

$$\pi_2 = \frac{\left[(2b-a)\Delta Q + C_1(Q_1) - C_2(\bar{Q}_2)\right]^2}{9\Delta Q}. \tag{9}$$

Proposition 4. *In equilibrium,*

i. *For both firms, there is a constant relationship between profit per unit and market share. That is,* $\Delta Q = \frac{p_1 - C_1(Q_1)}{\Delta \underline{t}} = \frac{p_2 - C_2(\bar{Q}_2)}{\Delta \bar{t}}$.
ii. *The profit per unit, market share and total profit of firm 1 are larger than firm 2*[11].
iii. *The following condition must be satisfied:* $b \leq 2a - \frac{3C_2(Q_1) - 2C_1(Q_1) - C_2(\bar{Q}_2)}{\bar{Q}_2 - Q_1}$, *where* $\bar{Q}_2 = Min\{\bar{q}_2, \bar{Q}\}$[12].

4. A Numerical Example

Example 1. *Assume an interval* $T = \left[\frac{9}{8}, \frac{10}{8}\right]$ *with a continuum of consumers, and their utilities are defined by* $U_t = tQ - p$, *where* $t \in T$. *The range of qualities is given by the interval* $[0, 3.125]$. *The income of consumer* t *is* 4, *for all* $t \in T$, *where* $4 > b \cdot \bar{Q} = \frac{10}{8} \cdot 3.125 = 3.90625$. *The cost function of firm* 1 *is* $C_1(Q) = 0.5Q^2$, *and the cost function of firm* 2 *is* $C_2(Q) = Q$. *In equilibrium, firms* 1 *and* 2 *will offer the following endogenous single quality–price pairs:* $s_1^* = (1.123, 0.795)$ *and* $s_2^* = (3.125, 3.21)$, *respectively*[13].

The offer spectrum is $s^* = ((1.123, 0.795), (3.125, 3.21))$. *The indifferent consumer is* $\bar{t} = 1.208$; *in other words, consumers* $t \in [1.125, 1.208)$ *will choose the quality–price pair* $(1.123, 0.795)$, *and consumers* $t \in [1.208, 1.25]$ *will choose the quality–price pair* $(3.125, 3.21)$.

The profit of firm 1 *is* $\pi_1(s^*) = 0.014$, *and the profit of firm* 2 *is* $\pi_2(s^*) = 0.0036$.

The market is fully served. Moreover, the consumer surplus of a is positive: $CS(a) = a \cdot Q_1 - p_1 = \frac{9}{8} \cdot 1.123 - 0.795 = 1.263 - 0.795 = 0.468 > 0$.

There is a constant ratio between the profit per unit and the market share for both firms: $\frac{p_1 - C_1(q_1)}{\Delta \underline{t}} = \frac{0.795 - 0.5 \cdot (1.123)^2}{1.208 - 1.125} = \frac{0.795 - 0.631}{0.083} = \frac{0.164}{0.083} \approx 2 \approx 3.25 - 1.123 = Q_2 - Q_1 = \Delta Q$, *and* $\frac{p_2 - C_2(\bar{Q}_2)}{\Delta \bar{t}} = \frac{3.21 - 3.125}{1.25 - 1.208} = \frac{0.085}{0.042} \approx 2 \approx \Delta Q$.

The marginal utility of consumer a is similar with respect to the marginal cost of firm 1*:* $a = \frac{9}{8} = 1.125 \approx 2 \cdot 0.5 \cdot 1.123 = 1.123 = C_1^{'}(\underline{q}_1)$.

The marginal utility of consumer b is larger than the marginal cost of firm 2*:* $b = \frac{10}{8} = 1.25 > 1 = C_2^{'}(\bar{Q})$*, yields that* $\bar{q}_2 = \bar{Q} = 3.125$[14].

5. Conclusions

This paper analyzed how increasing the number of producers can affect the total number of different products. Based on the literature, we reach these preliminary conclusions: (I) For the case of a finite number of consumers, Gayer (2007) [25] showed that under standard conditions, an endogenous monopoly will emerge in equilibrium. Therefore, we cannot increase the number of firms in this case in equilibrium. (II) For the case of a continuum of consumers, Gayer (2017) [21] showed that under standard conditions, an endogenous duopoly will arise in equilibrium. Therefore, we cannot increase the number of firms in this case to more than two in equilibrium. (III) From (I) and (II), by elimination, the case that still needs to be analyzed is the ways in which increasing the number of producers from one (monopoly) to two (duopoly) in the case of a continuum of consumers can affect the total number of different products, and we have done so.

The main result is that in a model with a continuum of consumers and vertical product differentiation, the strategy of each duopoly firm is to offer a single endogenous quality–price pair in a Nash equilibrium. Specifically, each firm chooses to maximize its quality differential with the competing firm to maximize differentiation between qualities and thus relax competition and increase profits. In other words, each firm's cost advantage decreases as it moves from the extremes to the middle, since this lowers differentiation. While this intuition can explain why firms in a duopoly offer fewer brands than a monopoly firm, such as in the parallel case described by Mussa and Rosen (1978) [3] under similar assumptions, where there were many more brands and a complete market segmentation, we still need to explain why each firm offers only a single brand. One explanation for this narrowing of the market stems from the standard assumption of vertical product differentiation. In conclusion, in markets with a less tight hierarchy between brands, each duopoly firm can offer more than one brand. Nevertheless, the expected total number of brands in the duopoly may decrease compared to a monopoly even without the standard assumption of vertical product differentiation. This suggests that firms in non-monopolistic markets should consider a smaller number of choices for consumers.

This paper took a theoretical economic standpoint. Future research may also give more weight to the marketing factors based on work by Shah and Wolford (2007) [2], who found that buying behavior depends on the number of choices, which varies by product. Future studies may empirically analyze whether the average number of brands produced in a monopoly is larger than the average total number of brands produced in an oligopoly (and especially in a duopoly).

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Appendix A

Proof of Proposition 1 part I. Assume by negation that the market is partially served. In this case, $\underline{t} > a$, $\underline{p}_1 = \underline{t}\underline{Q}_1$ and $\underline{t} = C_1^{'}(\underline{Q}_1)$ (the intuition for the latter equation is that to maximize the profit per unit, the slope of the price ($\underline{t}$) and the slope of the cost function ($C_1^{'}(\underline{Q}_1)$) must be equal in an interior solution).

Since $C_1^{'}(Q) > C_2^{'}(Q)$ and $C_2^{'}(Q) \cdot Q \geq C_2(Q)$ from weak convexity, for all $Q > \underline{q}$, we have:

$$\underline{p}_1 = \underline{t} \cdot \underline{Q}_1 = C_1^{'}(\underline{Q}_1) \cdot \underline{Q}_1 > C_2^{'}(\underline{Q}_1) \cdot \underline{Q}_1 \geq C_2(\underline{Q}_1)$$

by contradiction. □

Proof of Proposition 1 part II. Suppose a pair $(Q_1, p_1) \in s_1^*$ for which the consumer surplus of consumer a is not positive. Then, we have that $CS(a) = aQ_1 - p_1 \leq 0$ and $a = C_1^{'}(\underline{Q}_1)$. We have:

$$p_1 \geq a \cdot Q_1 = C_1^{'}(Q_1) \cdot Q_1 > C_2^{'}(Q_1) \cdot Q_1 \geq C_2(Q_1)$$

by contradiction. □

Before proving Proposition 2, note the following lemma:

Lemma A1. *In an equilibrium with a monopoly with multi-product strategies, the profit per unit of each firm increases strictly with quality.*

Proof of Lemma A1. Suppose by negation that in equilibrium under monopoly with a multi-product offer spectrum, there is a pair $\hat{h} < \bar{h}$, with $\hat{h} = (\hat{Q}, \hat{p})$ and $\bar{p} = (\bar{q}, \bar{p})$, where $\{\hat{p} - C(\hat{Q}) \geq \underset{\{(Q,p)=h \in [s]\}}{Max} \{p - C(Q)\}\}$. The strategy that the monopoly can consider is to omit all pairs above $\hat{h}$, and add the pair $\bar{h}' = (\bar{q}, \bar{p}')$, where $\bar{p}' = \bar{p} + \varepsilon$ and $\varepsilon > 0$ is sufficiently small. Since consumers who chose pair $\bar{h}$ and may switch to pair $\hat{h}$ suffer no loss because $\hat{p} - C(\hat{Q}) \geq \bar{p} - C(\bar{q})$, and since consumers who switch to quality $\bar{q}$ make an additional gain because $\bar{p}' > \bar{p}$, it follows that the gain will increase by contradiction. □

Proof of Proposition 2. Denote our duopoly market by A. We also examine the monopoly market B (C), which is similar to market A, except that we eliminate firm 2 (1) and all of the consumers in the interval $[\bar{t}, b]$ $([a, \bar{t}))$. Based on Lemma 1, since there is a correlation between the existence of equilibria in markets B (C) and A, we can derive that under a duopoly, the profit per unit of firm 1 (2) increases strictly with quality. □

Proof of Proposition 3. Denote our duopoly market by A. We also examine the monopoly market B (C), which is similar to market A, except that we eliminate firm 2 (1) and all of the consumers in the interval $[\bar{t}, b]$ $([a, \bar{t}))$. Based on work by Mussa and Rosen (1978) [3], the strategy of firm 1 (2) is a continuous curve. As a result of the correlation between duopoly and monopoly markets, the equilibrium in monopoly market B (C) is equivalent to the equilibrium in the duopoly market A. Therefore, firm 1 (2)'s strategy is a continuous curve under a duopoly. □

Proof of the main result—part I. Assume by negation that firm 1 in a duopoly offers a multi-product strategy. The strategy that firm 1 may consider is to only provide its highest pair $(\bar{Q}_1, \bar{p}_1)$. From Propositions 1 and 2, it can be automatically derived that the profit of firm 1 will increase for the following reasons: its profit per unit from this pair is maximal; the market is fully served; and all the consumers who chose one of the pairs from firm 1 will have a positive consumer surplus from the remaining pair and will choose it. By contradiction. □

Proof of the main result—part II. Assume by negation that in equilibrium $\underline{Q}_2 < Min\{\bar{q}_2, \bar{Q}\}$.

First, we will deal with the profit function of firm 1. From (6):

$$\frac{\partial \pi_1}{\partial Q_1} = \frac{1}{[9\Delta Q]^2} \cdot \left\{ 9\Delta Q \cdot 2 \cdot [(b-2a)\Delta Q - C_1(Q_1) + C_2(\underline{Q}_2)] \cdot [-b + 2a - C_1^{'}(Q_1)] + 9[(b-2a)\Delta Q - C_1(Q_1) + C_2(\underline{Q}_2)]^2 \right\}. \quad \text{(A1)}$$

From (A1):

$$\frac{\partial \pi_1}{\partial Q_1} = \frac{[(b-2a)\Delta Q - C_1(Q_1) + C_2(\underline{Q}_2)]}{9[\Delta Q]^2} \cdot \left\{\Delta Q \cdot 2 \cdot [-b + 2a - C_1'(Q_1)] + [(b-2a)\Delta Q - C_1(Q_1) + C_2(\underline{Q}_2)]\right\}. \quad \text{(A2)}$$

From (A2):

$$\frac{\partial \pi_1}{\partial Q_1} = \frac{[(b-2a)\Delta Q - C_1(Q_1) + C_2(\underline{Q}_2)]}{9[\Delta Q]^2} \cdot \left\{\Delta Q[-b + 2a - 2C_1'(Q_1)] + C_2(\underline{Q}_2) - C_1(Q_1)]\right\}. \quad \text{(A3)}$$

Since there is an interior solution, where $\frac{\partial \pi_1}{\partial Q_1} = 0$, from (A3), we have:

$$\Delta Q[-b + 2a - 2C_1'(Q_1)] = -C_2(\underline{Q}_2) + C_1(Q_1). \quad \text{(A4)}$$

Inserting (A4) into the profit function of firm 2 (7) yields:

$$\pi_2 = \frac{\{(2b-a)\Delta Q + \Delta Q[-b + 2a - 2C_1'(Q_1)]\}^2}{9\Delta Q} \cdot K. \quad \text{(A5)}$$

From (A5):

$$\pi_2 = \frac{[\Delta Q]^2 K}{9\Delta Q} \cdot [2b - a - b + 2a - 2C_1'(Q_1)]^2. \quad \text{(A6)}$$

From (A6):

$$\pi_2 = \frac{\Delta Q \cdot K}{9} \cdot [b + a - 2C_1'(Q_1)]^2. \quad \text{(A7)}$$

From (A7), since $\Delta Q = \underline{Q}_2 - Q_1$ and $b > a = C_1'(Q_1)$, we have:

$$\frac{\partial \pi_2}{\partial \underline{Q}_2} = \frac{K}{9} \cdot [b + a - 2C_1'(Q_1)]^2 > 0 \quad \text{(A8)}$$

by contradiction. □

Proof of Proposition 4. (i) Inserting (4) and (5) into the profit per unit of firms 1 and 2, respectively, yields:

$$p_1 - C_1(Q_1) = \frac{(b-2a)\Delta Q - C_1(Q_1) + C_2(\bar{Q}_2)}{3}. \quad \text{(A9)}$$

$$p_2 - C_2(\bar{Q}_2) = \frac{(2b-a)\Delta Q + C_1(Q_1) - C_2(\bar{Q}_2)}{3}. \quad \text{(A10)}$$

Inserting (1) into the market share of firms 1 and 2, respectively, yields:

$$\Delta \underline{t} = \bar{t} - a = \frac{(b-2a)\Delta Q - C_1(Q_1) + C_2(\bar{Q}_2)}{3\Delta Q}. \quad \text{(A11)}$$

$$\Delta \bar{t} = b - \bar{t} = \frac{(2b-a)\Delta Q + C_1(Q_1) - C_2(\bar{Q}_2)}{3\Delta Q}. \quad \text{(A12)}$$

This is automatically proven by (A9), (A10) and (A11), (A12), respectively.
(ii) Inserting $a = C_1'(Q_1)$ into (A4) yields:

$$b = \frac{C_2(\bar{Q}_2) - C_1(Q_1)}{\Delta Q}. \quad \text{(A13)}$$

From (1) and (A13), and since $\Delta \bar{t} = b - \bar{t} > 0$, we have:

$$\frac{C_2(\bar{Q}_2) - C_1(Q_1)}{\Delta Q} > \frac{p_2 - p_1}{\Delta Q}. \quad \text{(A14)}$$

From (A14), we have:

$$p_1 - C_1(Q_1) > p_2 - C_2(\bar{Q}_2). \tag{A15}$$

From (A15) and part i of this proposition, it is automatically proven that

$$\Delta \underline{t} > \Delta \bar{t}. \tag{A16}$$

From (A15) and (A16), it is automatically proven that

$$\pi_1 > \pi_2. \tag{A17}$$

(iii) Denote $\bar{Q}_2 = Min\{\bar{q}_2, \bar{Q}\}$.
From (4), part i of this proposition, and since in equilibrium $p_1 \leq C_2(Q_1)$, we have:

$$p_1 = \frac{(b-2a)(\bar{Q}_2 - Q_1) + 2C_1(Q_1) + C_2(\bar{Q}_2)}{3} \leq C_2(Q_1). \tag{A18}$$

From (A18), we have:

$$b \leq 2a - \frac{3C_2(Q_1) - 2C_1(Q_1) - C_2(\bar{Q}_2)}{\bar{Q}_2 - Q_1}. \tag{A19}$$

□

An Explanation for Example 1

Inserting the values of Example 1 into (A4) yields:

$$(3.125 - Q_1)(-\frac{10}{8} + 2 \cdot \frac{9}{8} - 2 \cdot 2 \cdot 0.5Q_1) = -3.125 + 0.5Q_1^2. \tag{A20}$$

From (A20):

$$6Q_1^2 - 29Q_1 + 25 = 0. \tag{A21}$$

Solving (A21) generates two critical values of Q_1: $\ddot{Q}_1 = 3.7 > 3.125 = \bar{Q}$, which yields a minimum profit, and $\dddot{Q}_1 = 1.123$, which yields a maximum profit.

Notes

1 In the monopoly case, we will use the results of Mussa and Rosen (1978) [3] in order to compare them to our results, under the duopolistic case.

2 Assumption 1 states that net income is non-negative for all the consumers, since for the case where $p > t \cdot Q$, there is no consumer t who would choose the pair (Q, p) for every $t \in T$. Therefore, $e_t \geq t \cdot \bar{Q} \geq t \cdot Q \geq p$ implies that $y_t = e_t - p \geq 0$.

3 The intuition is that the marginal utility of consumer a , who can buy $\underline{q}_1$, cannot be less than the corresponding slope of cost function—$C_1^{'}(\underline{q}_1)$. Similarly, the marginal utility of consumer b who buys $\bar{q}_2$ cannot be below the corresponding slope of the cost function—$C_2^{'}(\bar{q}_2)$.

4 Assumptions 4 and 5 indicate that there will be an intersection of production costs. Intuitively speaking, this intersection leads to a set of qualities where each firm maintains a superior technology over the other. Otherwise, there will be no equilibrium with more than one active firm.

5 If $p > b\bar{Q}$, the consumer surplus of b and all other potential consumers of this quality price pair will be negative, which means that none of the consumers will choose this pair.

6 By contrast, if we have two quality price pairs $(\tilde{Q}, \tilde{p})$ and $(\hat{Q}, \hat{p})$, where $\tilde{Q} \geq \hat{Q}$ and $\tilde{p} < \hat{p}$, none of the consumers will choose the pair $(\hat{Q}, \hat{p})$.

7 This includes the possibility of not buying at all.

8 This result is contrasts with an equilibrium with a monopoly, in which $CS(a) = 0$. The intuition is that in the duopoly case, it is not beneficial for firm 1 to increase its price until $CS(a) = 0$ due to the "burden" of competition because of the existence of firm 2. See also in the following Example 1.

9 This equation also holds in the case of a monopoly when the market is fully served.

[10] See also Example 1.

[11] This result contrasts with the monopoly case, where the profit per unit increases with the quality index.

[12] See also Shaked and Sutton (1982) [4], who assumed that there were no production costs, and accordingly had $b \leq 2a$.

[13] The rationale why the value of the quality offered by firm 1 is 1.12 is proven in the last subsection of Appendix A. Note that in this example, there are numerical rounding offs.

[14] See also work by Shitovitz et al. (1989) [24], who derived similar results in the case of a monopoly with a finite number of consumers.

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