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Delay to Deal: Bargaining with Indivisibility and Round-Dependent Transfer

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Abstract: We examine a bargaining game in which players cannot make arbitrary offers. Instead, players alternately decide whether to accept or delay, and are rewarded with an indivisible portion and a perishable transfer that depends on the round. Our analysis demonstrates that when the initial transfer is large enough, the subgame perfect Nash equilibrium consists of a finite number of rounds of delay before an agreement is reached. The equilibrium delay is longer when the players are more patient, and when the transfer is initially higher and depreciates slower. Nevertheless, the game's chaotic characteristic makes it arduous to forecast the exact number of delayed rounds or which player will make the ultimate decision. This game can be applied to many social scenarios, particularly those with exogenous costs.

Keywords: bargaining; delay; indivisibility; subgame perfect Nash equilibrium

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1. Introduction

A bargaining game is one in which two or more players bargain over how to divide a certain amount of gains. Usually, one player makes an offer and the other decides whether to accept or not. In a classic bargaining game, rational players would immediately reach a deal [1]. However, this scenario seldom arises in reality. Instead, people may observe a delay before participants reach a deal in a bargaining game. For example, Backus et al. [2] finds that only around one-third of field sequential bargaining interactions end immediately. Several studies have provided explanations for bargaining delays: such delays may arise due to incomplete information [3–5], multiple equilibria [6], a large number of players [7], knowledge limitations [8], or the learning processes of players [9].

However, it is important not to overlook the fact that participants in bargaining games may not always have the freedom to decide on the value split. This can significantly reduce the possibility of an immediate deal and contribute to a delay in bargaining. The inability may stem from the fact that the division is determined by the third party, or by the nature. For example, the centipede game [10–12] is a classic bargaining game where players cannot freely make an offer and delaying is profitable. In the centipede game, the payoffs are set up so that if one player delays and her opponent accepts the offer in the next round, the former will receive less than accepting on that round, but after an additional switch, the potential payoff is higher. Thus, even though a player has an incentive to accept in each round, they would be better off waiting. Also, in most cooperative bargaining rather than non-cooperative bargaining, the payoff split is exogenously given [13,14]. Perry and Reny [15] raised a case where in the bargaining game, the split is arbitrary but the timing to make an offer is not. There are also some practical instances, such as, Chaves and Varas [16], who note that bargaining in security involves types of payments, which may turn to a non-arbitrary amount of cut in offer.

Based on these considerations, this study offers an alternative explanation for the observed delay in certain social bargaining games. We suggest a bargaining game where payoffs are comprised of an indivisible part and a round-dependent transfer, and players are not fully in control of how the payoff is distributed. Our analysis demonstrates that when the initial transfer is large enough, the subgame perfect Nash equilibrium consists of a finite number of rounds of delay before an agreement is reached. The equilibrium delay is longer when the players are more patient, and when the transfer is initially higher and depreciates slower. However, for high-valued and slowly depreciating transfers where players exhibit a greater sense of patience, it becomes challenging to predict the precise number of delayed rounds or the identity of the party who will make the ultimate decision, due to the non-linear characteristic of the property.

The contributions of this paper are mainly the following. First, we enrich the study of bargaining games, especially in situations where players are not free to decide on the distribution of gains. Second, under the rationality assumption and the non-arbitrary distribution of gains, we give an alternative explanation for the delayed arrival equilibrium. Finally, this explanation can provide insight into many situations that involve moral and fairness considerations, and may contribute to the understanding of practical issues in social psychology and international political science.

The rest of this paper is organized as follows. Section 2 presents the motivation for such strategy-restricted bargaining with psychological and political stories. Section 3 establishes and analyzes the formal model. Section 4 discusses the equilibrium result, and Section 5 concludes.

2. Motivation

Although it may seem counter-intuitive to play a bargaining game where participants cannot choose the division of payoffs, this often occurs in real-world applications. We begin with two anecdotes to illustrate the model. The first story is from a Chinese tale, "Kong Rong Giving Away Pears" [17]. In certain cultures, selfishness is viewed as a transgression of social morality. Those who demonstrate selfishness in negotiations may evoke distaste from the opposing party. Under this premise, let us examine a game in which two identical players haggle over a pear that cannot be divided. Following Zheng et al. [18], the winner obtains the pear, but endures a social loss since they did not attempt to relinquish the fruit to their competitor. The loser does not receive a physical reward, but earns social recognition morally. However, unlike the typical bargaining game in which players alternate making offers and deciding whether to accept them, our game imposes restrictions on the amount of "social recognition" or "favor" transferred to the other, as established by social norms [19,20]. If the "favor" is low, the player simply keeps the pear and ignores the naysaying of the other; otherwise, the player gives the pear to the other. If they are mutually aware that the transfer will decrease over time, they can wait for the exogenous transfer adjustment until it reaches an acceptable equilibrium. For instance, after simulating generosity and exchanging the pear for numerous rounds, retaining the pear will not be perceived as impolite compared to keeping it at the onset, and eventually, someone will concede to keeping the pear. This mechanism may offer an explanation for the observation of generosity in practical situations, but the effects of generosity- or altruism-induced delay are not permanent.

Another story comes from the indivisible issue in political bargaining [21–23]. Two countries negotiate over disputed territory that could potentially lead to war, but the international community intervenes. Neither side is willing to share the land due to domestic political pressure. However, if a country were to initiate a war (assuming a certain victory), it would face international sanctions and could incur greater losses than the territory gained, with international benefits bestowed upon the opposing nation [24]. In the event that international interests weigh more heavily than the potential gains from war, no country would engage in invasion and peace is maintained. However, if international interests decrease, the two countries may still engage in war later on [25].

The games in these two stories have some similarities. First, the game is on the infinite horizon, but the strategy set is finite. The direct benefits are indivisible, and players cannot adjust the transfer payment arbitrarily by changing their strategies. They can only select between "accept the pear" and "give up the pear", or "invade the territory" and "wait".

Second, the game features constant-sum payoffs, which is similar to the constant-sum centipede game [26]. Players face the trade-off between delaying for higher payoffs, and the risk that the counterpart will end the game. Third, when the initial transfer is high, the bargain exhibits finite rounds of delay before the transfer depreciates enough and players reach a deal. Fourth, there is no uncertainty from any sources, and all subgames can be perfectly isolated, which is different from Selten [27]. In the following section, the model will be formally established and the pure-strategy subgame perfect Nash equilibrium will be analyzed.

3. Model

3.1. Model Setup

Let us consider the following bargaining game: two rational, self-interested players bargain over an indivisible good with normalized value 1. In round 1, player 1 chooses to either accept (A) or delay (D), leaving the choice to player 2. If player 1 accepts, she obtains the value but has to pay a transfer a to player 2. Thus, player 1's payoff is $1-a$, and player 2's is a. On the other hand, if player 1 delays, in round 2, player 2 chooses between A and D, and vice versa. Suppose the value of the good does not depreciate, but the value of the transfer does at the factor p: player 2's acceptance yields $(ap, 1-ap)$. Similarly, in round 3, player 1's acceptance yields $(1-ap^2, ap^2)$, and in round 4, player 2's acceptance delivers $(ap^3, 1-ap^3)\ldots$ Denote π_k and π_{-k} for the payoffs of the decision-maker and the other player when the game ends in round k, then $(\pi_k, \pi_{-k}) = (1-p^{k-1}a, p^{k-1}a)$. Figure 1 shows the extensive form of the bargaining game.

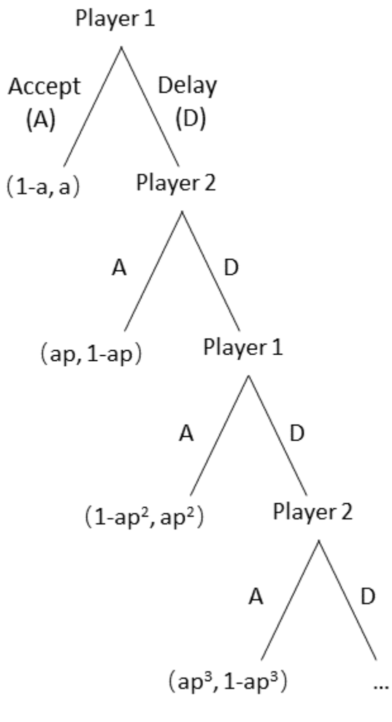

Figure 1. Extensive-form game. Notes: This figure shows the extensive-form game in this study. In each round, a player chooses between A and D; if D is selected, the next round begins, and the other player makes the choice. Otherwise, if A is selected in round k, the player who made the decision obtains $\pi_k = 1 - p^{k-1}a$, and the other player obtains $\pi_{-k} = p^{k-1}a$. Payoffs are subject to time discounts δ^{k-1} in round k.

We will show that a subgame perfect Nash equilibrium exists under certain conditions, with both players employing strategy D for several rounds and strategy A for the remaining rounds. The equilibrium implies that the bargaining ends after a finite number of rounds, but not immediately, which differs from Rubinstein [1].

3.2. Strategy Analysis

We first analyze the subgame starting at round k. Note that the necessary and sufficient condition for reaching round k is that D has been chosen from round 1 to $k-1$. Thus, a necessary condition for a subgame Nash equilibrium at round k to become a subgame perfect equilibrium at round 1 is that D is chosen in all decisions from rounds 1 to $k-1$.

For the decision-maker in round k's subgame ($k = 1, 2, 3, \ldots$), she makes a choice between A and D. If A is chosen, the game ends with payoff π_k (valued at round k) for her; if D is chosen but the opponent chooses A in the next round, the decision maker in the current round will obtain the discounted value $\delta\pi_{-(k+1)}$; if D is chosen and the opponent also chooses D in the next round, the game goes to round $k+2$, and she can guarantee herself a payoff of at least $\delta^2\pi_{k+2}$. Therefore, the condition that makes D a weakly dominant strategy is that

$$\begin{cases} \pi_k \leq \delta\pi_{-(k+1)} & \text{(1a)} \\ \pi_k \leq \delta^2\pi_{(k+2)} & \text{(1b)} \end{cases}$$

which solves

$$\begin{cases} p^{k-1}a \geq \dfrac{1}{1+\delta p} & \text{(2a)} \\ p^{k-1}a \geq \dfrac{1}{1+\delta p}\dfrac{1-\delta^2}{1-\delta p} & \text{(2b)} \end{cases}$$

When $0 < p < \delta \leq 1$, that is, the transfer value a depreciates faster than the discount rate, we have $1 - \delta^2 \leq 1 - \delta p$, so condition (2a) is binding. As long as (2a) holds in round k, that is, $p^{k-1}(1+\delta p)a \geq 1$, the current decision-maker will choose D, and the delay occurs. On the contrary, if $p^{k-1}(1+\delta p)a < 1$, the decision-maker will immediately end the game. Therefore, the subgame perfect Nash equilibrium is that, if $a < \frac{1}{1+\delta p}$, the game ends immediately in the first round; otherwise, both players choose to delay until round k, where

$$k = \left\lceil 1 - \frac{\ln\left[(1+\delta p)a\right]}{\ln p} \right\rceil$$

and whoever makes the decision in round k will accept the split and will end the bargaining.

Remarkably, the driving force of the delay is purely from the depreciation of the exogenous transfer. Players are sufficiently patient (relative to the time discount) to wait for the adjustment of the transfer to make a deal, which is different from the centipede game [26]. Meanwhile, the result implies that, for a greater value of initial transfer a and transfer depreciation rate p, k is also greater, that is, the game is expected to end after a longer delay. Also, the parity of k can answer who made the final choice, obtaining the good and paying the transfer. This question is particularly important if the transfer is a worth-based, imaginary utility, for example, the moral value in the story of Kong Rong, but is less crucial if the transfer is substantial [18]. We find that when p is close to 1, the parity of k exhibits some chaotic behavior, which may come from the non-linearity of the payoffs, and the fact that $\lim_{p \to 1^-} k = +\infty$. Therefore, it is difficult to predict who will win the bargain.

When $0 < \delta < p < 1$, that is, the transfer value a depreciates slower than the discount rate, condition (2b) is binding, which means that the driving force for ending the bargaining is not the fear of the other player's ending the game, but the fact that continuing the delay is no longer profitable. Notably, when the condition (2b) is violated, condition (2a) may still hold, that is, delaying for one round (and no more) is still profitable.

Suppose that in round k, condition (2b) is violated. If condition (2a) is also violated, the decision-maker will immediately end the game; if (2a) holds, the player will choose D only when she expects that in the next round, both condition (2a) and (2b) are violated, so that the other player will let the game end in round $k+1$ rather than kicking the ball back. The following conditions capture the parameters for this situation:

$$\begin{cases} \pi_k \leq \delta \pi_{-(k+1)} & (3a) \\ \pi_k > \delta^2 \pi_{(k+2)} & (3b) \\ \pi_{k+1} > \delta \pi_{-(k+2)} & (3c) \\ \pi_{k+1} > \delta^2 \pi_{(k+3)} & (3d) \end{cases}$$

Condition (3d) is guaranteed by (3b). If $p < \frac{1}{1+\delta-\delta^2}$[1], (3c) is guaranteed by (3b), so the decision-maker in round k delays the game for another round, and ultimately, the game ends at round $k+1$, where

$$k = \left\lceil 1 - \frac{\ln\left[(1+\delta p)(1-\delta p)a/(1-\delta^2)\right]}{\ln p} \right\rceil$$

Otherwise, if $\frac{1}{1+\delta-\delta^2} < p < 1$, (3c) is violated, and the decision-maker in round k will not bring the game to round $k+1$. Instead, she chooses A and finishes the game.

Lastly, for the singular cases $p = \delta$ or $p = \frac{1}{1+\delta-\delta^2}$, the corresponding conditions at round k hold with equality. Therefore, if $a \geq \frac{1}{1+\delta p}\frac{1-\delta^2}{1-\delta p}$, the game delays till round k, and the decision-making player is indifferent between A and D. Thus, the whole game can either ends in round k or round $k+1$.

3.3. Equilibrium

Therefore, in summary, for the discount rate $0 < p < 1$, the transfer depreciation rate $0 < \delta \leq 1$, and the initial transfer a, the game has an subgame perfect Nash equilibrium stated as follows.

(1) When $0 < p < \delta \leq 1$, that is, the transfer value a depreciates faster than the discount rate, both players choose to delay until round $k = \bar{k} \equiv \left\lceil 1 - \frac{\ln[(1+\delta p)a]}{\ln p} \right\rceil$.

(2) When $0 < \delta < p < \frac{1}{1+\delta-\delta^2} < 1$, that is, the transfer value a depreciates slower than the discount rate, but not so slow, both players choose to delay until round $k = \overline{k'} + 1$, where $\overline{k'} \equiv \left\lceil 1 - \frac{\ln[(1+\delta p)(1-\delta p)a/(1-\delta^2)]}{\ln p} \right\rceil$.

(3) When $0 < \delta < \frac{1}{1+\delta-\delta^2} < p < 1$, that is, the transfer value a depreciates much slower than the discount rate, both players choose to delay until round $k = \overline{k'}$.

(4) For the singular cases, when $0 < \delta = p < \frac{1}{1+\delta-\delta^2} < 1$, or $0 < \delta < \frac{1}{1+\delta-\delta^2} = p < 1$, both players choose to delay until round $k = \overline{k'}$, or $k = \overline{k'} + 1$, which derives indifferent payoffs.

4. Discussion

The equilibrium in Section 3 implies several characteristics of the game.

First, the trade-off between delaying and accepting is the desire of delay the game for higher payoff and the risk of being terminated by the other player. Depending on the relative size of time-discount factor δ and the transfer depreciation rate p, the driving forces may differ. When $p < \delta$, that is, players are patient relative to the transfer depreciation, the main concern is to delay the deal, waiting for the nature to adjust the transfer. However, when $p > \delta$, that is, players are impatient relative to the transfer depreciation, their main concern is not letting the counterpart terminate the game.

Second, as long as the initial transfer is sufficiently large, that is,

$$a > \begin{cases} \frac{1}{1+\delta p} & \text{if } p < \delta \\ \frac{1}{1+\delta p}\frac{1-\delta^2}{1-\delta p} & \text{if } p \geq \delta \end{cases}$$

there is no immediate deal as an equilibrium. Instead, after finite rounds of delay, the game will end in round $2 \leq k < +\infty$. However, it is difficult to predict the outcome, because the number of rounds is sensitive to the preference parameters and the initial value and depreciation rate of the transfer. As an example, we made a grid search by varying p from 0 to 1, and varying δ from 0 to p, and Figure 2 shows the results. The light- and dark-colored area corresponds to the parameters with which players 1 and 2 chose the ultimate A, respectively. We find that when p is close to 1, the parity of equilibrium rounds k exhibits some chaotic behavior, which may come from the non-linearity of the payoffs, and the fact that $\lim_{p \to 1^-} k^* = +\infty$. Therefore, it is difficult to predict who will win the bargain.

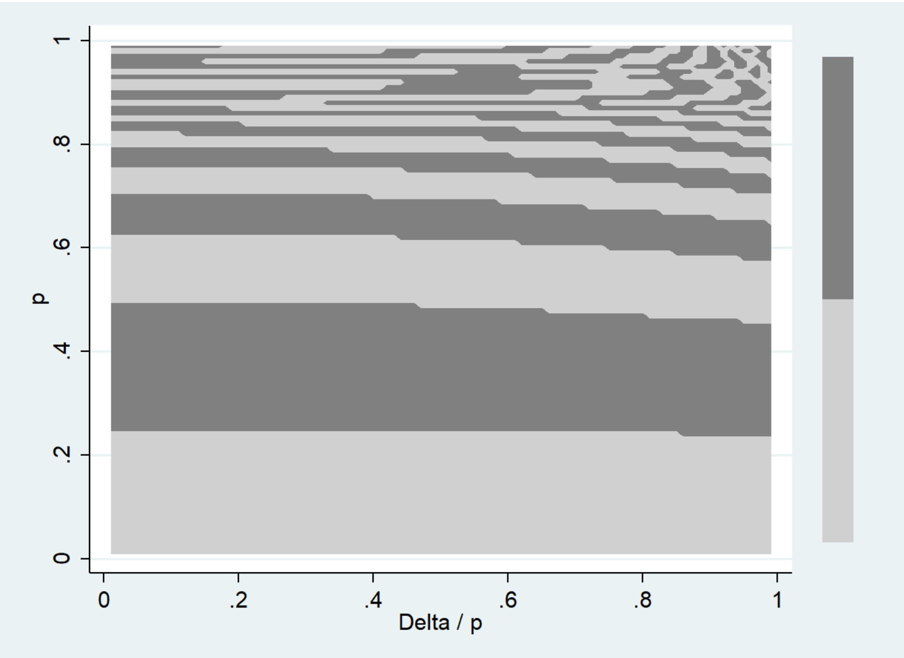

Figure 2. Grid simulation for who ultimately ends the game. Notes: This figure shows a grid simulation for the player who ultimately ends the game with different parameters. With the parameters in the light-colored area, the ultimate "A" is made by player 1, while player 2 makes the final choice in the dark-colored area. Parameters: $a = 4$, $p \in (0, 1)$, and $\delta / p \in (0, 1)$. The step length is set to 0.01.

Third, in general, for the number of rounds delayed k^*, taking the first derivatives, we obtain $\partial k^* / \partial a \geq 0$, $\partial k^* / \partial p \geq 0$, and $\partial k^* / \partial \delta \geq 0$, indicating that the equilibrium delay is longer when the players are more patient, and when the transfer is initially higher and depreciates slower. However, this is not always the case: as the payoff of the winner (who obtain the fixed value but pays the transfer) is not necessarily higher than that of the loser (who obtains the transfer only), sometimes players may strategically wait for their opponents to end the game. Therefore, while patient players may wait longer in a general bargaining game, in some rare cases, given a and p, impatient players may delay the agreement one round longer than that of patient players, expecting to receive the transfer rather than the fixed value of the pie.

Finally, this paper cannot guarantee the completeness of the Nash equilibrium. Beyond the delay-to-deal equilibrium discussed in this paper, there may exist other equilibria with pure or mixed strategies. Thus, whether delays occur in a real game experiment is also governed by other factors such as equilibrium selection.

5. Conclusions

We propose a bargaining game in which players alternately decide between accepting and delaying, and payoffs consist of an indivisible part and a round-dependent, perishable transfer. We show that when the initial transfer is sufficiently large, the subgame perfect

Nash equilibrium consists of a finite number of rounds of delay before making a deal. Moreover, for high-valued and slowly depreciating transfers where players exhibit a greater sense of patience, it becomes challenging to predict the precise number of delayed rounds or the identity of the party who will make the ultimate decision, owing to the non-linear characteristic of the property. This model can offer an alternative explanation for the finite-round delays in certain Kong-Rong-style bargaining games, where the offer maker cannot freely determine the payoff split. It also has the potential to shed light on real-world cases in politics, economics, and psychology, and could serve as a basis for future bargaining theory research.

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Note

[1] Note that, for $f(\delta) \equiv \frac{1}{1+\delta-\delta^2}$, we have $f(\delta) \geq \delta$, and $f(\delta) \in [0.8, 1]$ for $0 < \delta < 1$. For $p \leq 0.8$, this condition is always satisfied.

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