Collusive Stability with Relative Performance and Network Externalities

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Abstract: In this paper, we aim to investigate the collusive stability in the presence of network externalities among firms with relative performance in the firm’s objective functions. We demonstrate that collusive stability is increasing (decreasing) in the degree of relative performance, product substitutability and network effect when the network effect is sufficiently large (small). A competition agency might need to provide different guidance for anti-competitive regulation in the network industry.

Keywords: relative performance; network externalities; stability of collusion

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1. Introduction

In recent years, oligopolistic market structures have become prevalent in many industries, such as the telecommunications and mobile phone markets. Concurrently, relative performance and network effects play an important role in industrial competition. The decision-making of the board of directors not only cares about the performance of the firm itself but also pays attention to the performance of competitors as well; meanwhile, commodity production between firms also affects consumers’ willingness to pay prices due to network effects, affecting firms’ competitive and cooperative behaviors.

Clarifying the conditions under which firms can easily collude will assist anti-monopoly agencies in monitoring anti-competitive behaviors more effectively. Friedman [1] was the first to show that cooperation could be achieved in an infinitely repeated game by using trigger strategies that switch forever to the stage-game Nash equilibrium following any deviation, while Deneckere [2] provided findings about the ability to collude in repeated Cournot and Bertrand duopolies; collusion between firms becomes unsustainable as firms produce closer substitutes for the commodity.

On the collusion problem in the traditional literature, most settings of the firms’ objective functions are considering their profits. In the business operation model with managerial delegation, several schemes of managerial compensation have been considered by Vickers [6], Fershtman and Judd [7], Miller and Pazgal [8] and Jansen et al. [9,10], among others. In an oligopolistic product market, shareholders strategically use information on rival firms’ performances when designing management-incentive contracts (for example, Joh [11]; Chowdhury and Gürtler [12]; Bloomfield et al. [13]). Bloomfield et al. [13] examined whether the potential for costly sabotage acted as a deterrent to firms’ use of relative performance evaluation (RPE) in CEO pay plans and exploited illegal cartel membership as a source of variation in the potential for costly sabotage, while documenting that firms...
are more likely to use RPE if they are currently cartel members. Matsumura and Matsushima [14] showed that when the competition becomes more intensive among firms with relative performance in the firm’s objective functions in an oligopoly market with homogenous goods, it destabilizes the collusion. Yang and Zeng [15] examined the collusive effect of cross-holding with asymmetry in cost functions across firms in an infinitely repeated Cournot duopoly game and found that cross-holding may either facilitate or hinder collusion. Sun and Wang [16] analyzed upstream firms’ collusive sustainability when downstream firms adopt the relative-performance delegation in an infinitely repeated Cournot or Bertrand game and demonstrated that relative performance delegation makes managers’ actions more aggressive and that leads to more difficulty in sustaining upstream collusion compared with sales-revenue delegation regardless of the competition modes.

Another stream of work in the literature has studied the stability of collusion in terms of network externalities. In a differentiated oligopoly with network externalities, Song and Wang [17] demonstrated that the closer the substitutes of products were, the more stable the collusion under relatively strong network externalities. Choi and Lee [19] further showed that if network externalities are weak (strong), collusion in quantities (prices) is more stable than in prices (quantities). Lee et al. [21] showed that larger network externalities lead to less collusive incentive for an inefficient firm, while for an efficient firm, this depends on the efficiency gap. An increase in network externalities will destabilize the downstream collusion when the cost asymmetry is large and the network externalities are relatively weak.

In this paper, we aim to investigate the collusive stability in the presence of network externalities among firms with relative performance in the firm’s objective functions. There are two departures of our framework from that of Matsumura and Matsushima [14]: one is network externality, and the other is product differentiation. Network externalities affect the competitiveness of the market, playing an important role in the stability of collusion. We show that the collusive stability is increasing (decreasing) in the degree of relative performance, product substitutability and network effect when the network effect is sufficiently large (small). Our finding is different from Matsumura and Matsushima [14] in that the comparative strength of the positive and negative effects of competitiveness on critical discount factors is conditional on the parameters’ values; of note, since an increase in the network effect will weaken the competition among firms and the effectiveness of punishment, it will destabilize the collusion.

The rest of this paper is arranged as follows. The basic model is provided in Section 2. Section 3 reports the main results of this paper. Section 4 provides the concluding remarks.

2. Basic Model

Suppose that there are two firms producing differentiated goods with constant production marginal cost, c. Following Hoernig [22], Bhattacharjee and Pal [23] and Song and Wang [17], the utility function of the representative consumer is given by the following:

\[
U(x_1, x_2; y_1, y_2) = \frac{a(x_1 + x_2)}{1 - \gamma} - \frac{(x_1^2 + 2\gamma x_1 x_2 + x_2^2)}{2(1 - \gamma^2)} + n\left(\frac{(y_1 + y_2) x_1 + (y_2 + \gamma y_1) x_2}{1 - \gamma} - \frac{y_1^2 + 2\beta y_1 y_2 + y_2^2}{2(1 - \gamma^2)}\right) + m
\]

where \(x_i\) denotes the quantity of firm \(i\)’s production; \(y_j\) denotes the consumer’s expectation regarding firm \(i\)’s total sales and \(\alpha > 0\). \(m\) is the numeraire that denotes the consumption of all other goods, while parameters \(\gamma\) and \(n\) indicate the degree of product differentiation and strength of network effects, respectively. A greater \(\gamma\) corresponds to a lesser differentiation of product. We assume \(\gamma \in (0, 1)\) and \(n \in (0, 2)\). \(n < 2\) ensures that the output is always positive. It is easy to check, as follows:

\[
\frac{\partial}{\partial x_i} \left( \frac{\partial U}{\partial x_i} \right) = n \frac{\partial U}{\partial x_i} = \frac{\gamma n}{1 - \gamma^2}
\]

(1)
Hence, a higher magnitude of $n$ indicates a stronger positive network effect generated from not only one’s own production but also the rival’s production.

The inverse demand function for goods $i$ is as follows:

$$p_i(x_i, x_j; y_i, y_j) = \frac{\alpha}{1 - \gamma} - \frac{x_i + \gamma x_j}{1 - \gamma^2} + \frac{n(y_i + \gamma y_j)}{1 - \gamma^2}, \quad i, j = 1, 2, i \neq j$$

where $p_i$ denotes the price of good $i$. A stronger network effect can be seen from Equation (2), as it clearly shows a higher willingness to pay (price) when the expectations of the network sizes increase.

The payoff of firm $i$ ($i = 1, 2$) is considered by relative performance and is given by the following:

$$U_i = \pi_i - \lambda \pi_j, \quad i \neq j$$

where $\pi_i$ is the profit of firm $i$,

$$\pi_i = p_i x_i - c x_i, \quad i \neq j$$

Assuming $\frac{\alpha}{1 - \gamma} > c > 0$, the consumers’ reservation price is larger than the marginal cost and $\lambda \in (-1, 1)$, indicating the weight of relative performance for firm $i$’s management. Relative performance schemes of managerial compensation have been considered by Miller and Pazgal [8] and Jansen et al. [9, 10], among others. In Jansen et al. [10], the weight on the rival’s profit ranges between zero and one, whereas in Miller and Pazgal [8], it may range between minus one and one. Matsumura and Matsushima [14] interpret $\lambda$ as a parameter indicating the severity of competition, and it may range between minus one and one. $\lambda = 0$ indicates the standard Cournot case, $\lambda = 1$ the perfectly competitive case and $\lambda = -1$ the monopoly case.

For profit accounting under cross-shareholding, Zhang and Zhang [29] and Yang and Zeng [15] applied the direct financial interests approach proposed by Reynold and Snapp [30]. Zhang and Zhang [29] studied rivalry between strategic alliances and determined that one natural interpretation for this formulation was cross-shareholding in equity alliances, where the weight on the rival’s profit ranges between zero and one. Yang and Zeng [15] restricted the weight on the rival’s profit ranges between zero and half due to the assumption that the shareholding firm only receives financial interests in the invested firm. In our framework, a negative $\lambda$ represents the profit of the invested firm taking into account the shareholding firm; a positive $\lambda$ represents the owner putting more weight on the relative performance on his management.

By letting $\delta \in (0, 1)$ denote the discount factor between periods in an infinitely repeated game, which implies that the value today of a dollar has to be received one period later, we examine the effects of $\gamma$, $n$, $\lambda$ on the stability of the collusion. The grim-trigger strategy of Friedman [1] is used to analyze the punishment phrase.

In each period, firms and consumers make choices and take actions in a three-stage game:

Stage 1: Given the last period’s performance, each firm $i$ decides whether to collude or not.

Stage 2: If collusion is agreed on, firms choose production jointly to maximize aggregate profits. If not, each firm chooses its production independently.

Stage 3: Consumers make expectations of firms’ production rationally and decide on the consumption of both goods.

Following Katz and Shapiro [31], our equilibrium concept is that of fulfilled expectation equilibrium, where each firm chooses its output level under the assumption that the consumer’s expectations about the sizes of the networks are given and the actual output level of the other firms is fixed. In the equilibrium, we have the following:

$$x_1 = y_1, \quad x_2 = y_2$$
Firstly, if the firms choose to collude in the output decision, the joint payoff \((U_I)\) is as follows:
\[
U_I = (\pi_i - \lambda \pi_j) + (\pi_j - \lambda \pi_i) = (1 - \lambda)(\pi_i + \pi_j)
\]  
(6)

The joint payoff is maximized when
\[
x_1^C = x_2^C = \frac{\alpha - c(1 - \gamma)}{2 - n}
\]  
(7)

The collusion payoff for each firm is
\[
U_I^C = \frac{(1 - \lambda)(\alpha - c(1 - \gamma))^2}{(1 - \gamma)(2 - n)^2}
\]  
(8)

where the superscript ‘C’ denotes the outcome under collusion.

Secondly, given the cooperative output of firm 2, firm 1 chooses to deviate the cooperative output and maximizes its payoff \(U_1\). The first-order condition is
\[
\frac{\alpha}{1 - \gamma} - c - \frac{2x_1 + \gamma(1 - \lambda)x_2 - n(y_1 + \gamma y_2)}{1 - \gamma^2} = 0
\]  
(9)

Substituting rationality condition \(x_1 = y_1\) and \(x_2 = y_2\) into the first-order condition, we obtain the following reaction function
\[
x_1 = \frac{(1 + \gamma)[\alpha - c(1 - \gamma)] + \gamma(1 - n - \lambda)x_2}{2 - n}
\]  
(10)

Substituting cooperative production of firm 2, \(x_2^C = \frac{\alpha - c(1 - \gamma)}{2 - n}\), we obtain the following:
\[
x_1^D = \frac{(\alpha - c(1 - \gamma))(2 - n + \gamma(1 + \lambda))}{(2 - n)^2}
\]  
(11)

where the superscript ‘D’ denotes deviation from collusion. The resulting deviation payoff is
\[
U_1^D = \frac{(\alpha - c(1 - \gamma))^2(\gamma^2(1 - \lambda)^2 + (4 - 2n)(1 + \gamma)(1 + \lambda) + (n^2 - 2n)(1 + \gamma)^2(1 - \gamma^2)\lambda)}{(2 - n)^3(1 - \gamma^2)}
\]  
(12)

Thirdly, if firm 1 chooses to deviate from the cooperative output, the grim-trigger strategy is employed, and the firms compete in the Cournot fashion after the deviant period of game. By solving the reaction function (10) and using rational expectation condition,
\[
x_1^E = x_2^E = \frac{(\alpha - c(1 - \gamma))(1 + \gamma)}{2 - n(1 + \gamma) + \gamma(1 - \lambda)}
\]  
(13)

where ‘E’ denotes the Cournot–Nash equilibrium. The non-cooperative profit and payoff of the firms are as follows:
\[
\pi_1^E = \pi_2^E = \frac{(\alpha - c(1 - \gamma))(1 + \gamma)(1 - \gamma \lambda)}{(1 - \gamma)(2 + \gamma(1 - \lambda) - n(1 + \gamma))^2}
\]  
(14)

\[
U_1^E = U_2^E = \frac{(1 + \lambda)(\alpha - c(1 - \gamma))^2(1 + \gamma)(1 - \gamma \lambda)}{(1 - \gamma)(2 + \gamma(1 - \lambda) - n(1 + \gamma))^2}
\]  
(15)

3. Sustainability of Collusion

Assume that once defection occurs between firms, the opponent will adopt a grim-trigger strategy. Given the deviation payoff \(U_1^D\), the Cournot–Nash competition payoff
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\( U^E_i \), and the collusion payoff \( U^C_i \), the tacit collusion is sustainable if and only if the present value of collusion is not less than the one of deviation, that is

\[
\frac{U^C_i}{(1 - \delta)} \geq U^D_i + \frac{\delta U^E_i}{(1 - \delta)}
\]  

(16)

substituting \( U^C_i \), \( U^D_i \) and \( U^E_i \) into inequality (16), and letting \( \delta^* \) satisfying Equation (17) with equality. As Friedman [1] pointed out, this critical discount factor for the cartel maintains collusion between the two firms. The tacit collusion is sustainable if and only if \( \delta \geq \delta^* \). We obtain the following:

\[
\delta^* = \frac{U^D_i - U^C_i}{U^C_i - U^E_i} = \frac{(2-n)(1+\gamma) - \gamma(1+\lambda)}{(2-n)\gamma(1-n) - \gamma(1-\lambda) - \gamma^2(2-2n)(1-\lambda^2) + \gamma^3(1-n-\lambda)^2(1-(1-n)^2\lambda)}
\]

(17)

Here, it can be seen that when \( \delta > \delta^* \), the discount rate of the firm is relatively large, indicating that the current utility is preferred to the future utility and the firm is prone to betrayal. Collusion stability in terms of critical discount factor \( \delta^* \) with respect to \( \gamma \) \( (n, \lambda) \) is summarized in the following Proposition 1.

**Proposition 1.** The sign of \( \frac{\partial \delta^*}{\partial \lambda} \) is conditional on the values of \( \gamma, n \) and \( \lambda \), and \( \delta^* \) is decreasing in \( \lambda \) when \( n \) is sufficiently large \( (n > \pi = \frac{2+\gamma-\gamma^2}{1+\gamma}) \).

**Proof.** From (17), we have \( \frac{\partial \delta^*}{\partial \lambda} < 0, \frac{\partial \delta^*}{\partial n} < 0, \frac{\partial \delta^*}{\partial \pi} < 0 \), if \( n > \pi = \frac{2+\gamma-\gamma^2}{1+\gamma} \). □

The influence of the weight of relative performance on the discount factor depends on the effect of the network externalities. Our finding is different from Matsumura and Matsushima’s [14] work. Matsumura and Matsushima [14] demonstrated that \( \delta^* \) is increasing in \( \lambda \), and so, an increase in \( \lambda \) causes a greater instability in collusive behavior. As is analyzed in the previous literature, an increase in the relative performance parameter \( \lambda \), on the one hand, leads to a less cooperative relationship between firms and increases their competition while improving the effectiveness of punishment (in this case, the original Cournot–Nash competition), which will stabilize the collusion. On the other hand, it leads to more gains from deviation by cutting down its rival’s profit, which will destabilize the collusion. In Matsumura and Matsushima [14], the former destabilizing effect is always dominated by the latter, so the stability of collusion improves with \( \lambda \). But, introducing \( \gamma \) and \( n \) into our model, the comparative strength of the positive and negative effects of \( \lambda \) on \( \delta^* \) are conditional on the strength of network effects.

In our framework, when \( n \) is relatively large \( (n > \pi) \), this indicates a high degree of network externality, which implies a less competitive market environment; hence, the stabilizing effect of \( \lambda \) on \( \delta^* \) is strengthened. Due to that \( \frac{\partial \delta^*}{\partial \pi} < 0 \), the collusion is sustainable and more stable; however, if \( n \) is relatively small \( (n < \pi) \), the collusion becomes less stable with a rising degree of relative performance, which is consistent with Matsumura and Matsushima’s [14] findings.

Song and Wang [17] pointed out that the impact of product differentiation on collusion stability is non-monotonic and depends crucially on the degree of network externalities. In our framework, when \( n \) is relatively large \( (n > \pi > 1) \), indicating a low degree of product differentiation (an increase in \( \gamma \), then this implies a more competitive market environment, so the stabilizing effect of \( \gamma \) on \( \delta^* \) is strengthened. Therefore, \( \frac{\partial \delta^*}{\partial \pi} < 0 \), the collusion is sustainable and more stable. Our analytical results confirm the robustness of Song and Wang [17].
4. Concluding Remarks

In this paper, we show that collusive stability is increasing in the degrees of relative performance, product substitutability and network effect when the industry-wide network effect is sufficiently large. The policy implication of our findings is that a competition agency might need to provide differing guidance strategies for anti-competitive or pro-competitive regulations in the network industry.

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Notes
1. For infinitely repeated games, see Abreu [3], Aramendia and Wen [4,5], among others.
2. Pal and Scimitore [18] demonstrated that the relationship between collusion sustainability and market concentration in a homogenous oligopoly with network externalities depended on the strength of network externalities.
3. Basak and Petrakis [20] recently showed from the viewpoint of social desirability that as long as the cost of entry is high, entry is socially insufficient if network goods are completely incompatible, the goods are sufficiently differentiated, and the level of network externalities is low.
4. The same utility function is also used by many works; for example, see Naskar and Pal [24], Pal [25,26], Toshimitsu [27,28], and Choi and Lee [19].

References

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