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Two-Tier Marketplace with Multi-Resource Bidding and Strategic Pricing for Multi-QoS Services

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Abstract: Fog computing introduces a new dimension to the network edge by pooling diverse resources (e.g., processing power, memory, and bandwidth). However, allocating resources from heterogeneous fog nodes often faces limited capacity. To overcome these limitations, integrating fog nodes with cloud resources is crucial, ensuring that Service Providers (SPs) have adequate resources to deliver their services efficiently. In this paper, we propose a game-theoretic model to describe the competition among non-cooperative SPs as they bid for resources from both fog and cloud environments, managed by an Infrastructure Provider (InP), to offer paid services to their end-users. In our game model, each SP bids for the resources it requires, determining its willingness to pay based on its specific service demands and quality requirements. Resource allocation prioritizes the fog environment, which offers local access with lower latency but limited capacity. When fog resources are insufficient, the remaining demand is fulfilled by cloud resources, which provide virtually unlimited capacity. However, this approach has a weakness in that some SPs may struggle to fully utilize the resources allocated in the Nash equilibrium-balanced cloud solution. Specifically, under a nondiscriminatory pricing scheme, the Nash equilibrium may enable certain SPs to acquire more resources, granting them a significant advantage in utilizing fog resources. This leads to unfairness among SPs competing for fog resources. To address this issue, we propose a price differentiation mechanism among SPs to ensure a fair allocation of resources at the Nash equilibrium in the fog environment. We establish the existence and uniqueness of the Nash equilibrium and analyze its key properties. The effectiveness of the proposed model is validated through simulations using Amazon EC2 instances, where we investigate the impact of various parameters on market equilibrium. The results show that SPs may experience profit reductions as they invest to attract end-users and enhance their quality of service QoS. Furthermore, unequal access to resources can lead to an imbalance in competition, negatively affecting the fairness of resource distribution. The results demonstrate that the proposed model is coherent and that it offers valuable information on the allocation of resources, pricing strategies, and QoS management in cloud- and fog-based environments.

Keywords: resource allocation; cloud; fog; QoS; bidding; nondiscriminatory mechanism; price differentiation; fairness

Academic Editor: Konstantinos Serfes

Received: 15 February 2025
Revised: 9 April 2025
Accepted: 15 April 2025
Published: 21 April 2025

Citation: Habli, S., El-Azouzi, R., Sabir, E., Datar, M., Elbiaze, H., & Sadik, M. (2025). Two-Tier Marketplace with Multi-Resource Bidding and Strategic Pricing for Multi-QoS Services. *Games*, *16*(2), 20. https://doi.org/10.3390/g16020020

1. Introduction

In recent years, an explosion in data traffic over communications networks has been driven by the rapid growth of cloud computing and the ubiquity of mobile devices. Web and mobile applications have experienced exceptional growth, both in quality and quantity, thanks to the strong support provided by companies such as Google, Microsoft, Amazon, and others through cloud services, which use cloud computing to eliminate hardware bottlenecks for consumers and businesses. With the emergence of artificial intelligence and the evolution of 5G/6G (Bega et al., 2017), this trend is set to continue, fueled by a more competitive market among Service Providers (SPs) . Indeed, the opening up of telecom networks to competition has been successful overall, driving prices down. However, recent developments in autonomous driving and Industry 4.0, to name but a few, require highly reliable connections with excellent connectivity and very low latency. For this reason, fog computing (FC), also known as edge computing (EC), is an emerging technology that complements cloud computing by bringing computing power closer to the network edge, where data are being generated. This helps to ensure faster data processing and a guaranteed level of computing capacity (Nadeem & Saeed, 2016). In addition, artificial intelligence has been incorporated into most applications to analyze and generate useful information to offer the best user experience while using these applications (Krishnan et al., 2019). These applications create complex end-user profiles, requiring various resources to satisfy their specific constraints. Therefore, efficient resource allocation becomes crucial for SPs to meet the performance goals of these applications. Moreover, the resource allocation problem is more challenging when multiple types of resources and competing SPs have different characteristics and preferences (Nguyen et al., 2019). However, we face a major challenge with FC technology due to limited resources for competing services with diverse characteristics and needs, necessitating the design of a mechanism that allows resource sharing, ensuring fairness.

Related Works

In this section, we provide an overview of the related works, focusing on the competition among SPs in the context of fog and cloud computing, pricing strategies, and resource allocation mechanisms. To facilitate the comparison, we summarize the key features and methodologies of the existing studies in Table 1.

Table 1. Comparison of existing work on SP competition and resource allocation.

Study	**Focus Area**	**Methodology**	**Key Contributions**
(Nguyen et al., 2019)	Resource Allocation in FC	Game Theory	Introduced a game-theoretic model for competitive resource allocation in fog computing environments.
(Luong et al., 2017)	Pricing Strategies	Non-Cooperative Game Theory	Examined pricing and Quality of Service (QoS) strategies for Service Providers SPs competing in cloud and fog markets.
(Baslam et al., 2011)	Fair Resource Allocation	Nash Equilibrium	Investigated fairness in resource allocation among SPs using Nash equilibrium concepts.
(Habli et al., 2023)	Pricing and QoS	Pricing Strategy Models	Analyzed pricing and QoS decisions in the telco market, including cloud computing services.

Table 1. *Cont.*

Study	Focus Area	Methodology	Key Contributions
(Nadeem & Saeed, 2016)	Fog Computing (FC)	Technological Framework	Described Fog Computing as a complement to cloud computing, providing faster data processing and lower latency by bringing computing closer to the edge.
(Krishnan et al., 2019)	AI in Resource Allocation	Algorithmic Approaches	Discussed the use of artificial intelligence for analyzing and generating useful information to optimize user experience in applications.
(Zhang et al., 2018)	Fog Computing in Smart Cities	System Design	Explored how fog computing supports smart city applications by reducing latency and improving real-time decision-making for urban management.
(Bonomi et al., 2012)	Latency in Fog Computing	Empirical Analysis	Investigated the challenges of reducing latency in fog computing networks, offering insights into performance improvements.
(Losada et al., 2021)	Energy Efficiency in Fog Computing	Optimization Models	Addressed energy consumption issues in fog computing and proposed optimization models to enhance energy efficiency in resource allocation.
(Elsayed et al., 2023)	Fog Computing in Intelligent Transportation	Case Study	Showed how fog computing improves traffic flow and congestion management by processing data closer to their source.
(Alraddady et al., 2022)	Performance in Fog Computing Networks	Experimental Analysis	Examined the impact of fog computing on performance, focusing on reduced data transmission and resource optimization.
(Bogucka et al., 2023)	Real-Time Applications in Fog	Model Development	Developed models for real-time applications in autonomous vehicles, emphasizing fog computing's role in meeting stringent time constraints.
(Bachiega et al., 2023)	Resource Constraints in Fog Computing	Theoretical Analysis	Analyzed the resource constraints in fog computing and proposed methods to address the heterogeneity of resources in fog environments.
Our work	SP Competition in fog–cloud environment	Non-Cooperative Game Theory	Investigated SPs' competition in resource allocation, addressing QoS and market dynamics under resource constraints. Focused on the impact of price differentiation for fairer fog resource distribution.

Despite the tremendous potential of the cloud, many challenges remain to be addressed, such as latency and bandwidth limitations (Jie et al., 2020). Fog computing offers a valuable complement to cloud infrastructure fog by processing data closer to the user, enabling faster service, improving user experience, reducing latency, and enhancing privacy (Jie et al., 2019; Zhang et al., 2018). From a practical standpoint, fog computing provides benefits such as reduced latency (Bonomi et al., 2012), heightened sensitivity, lower energy consumption (Losada et al., 2021), enhanced safety (Samara et al., 2021; Yi et al., 2015), and improved reliability (Khan et al., 2017). Serving as a complement and extension to cloud computing, fog computing has the potential to ease network traffic and improve performance, cost-effectiveness, and time constraints. Fog computing enables real-time data processing at the network's edge, facilitating immediate decision-making in intelligent transportation systems. This capability helps to optimize traffic flow and reduce congestion in urban areas (Elsayed et al., 2023). On the one hand, by processing data closer to their source, fog computing reduces latency and decreases the load on centralized cloud servers. This approach enhances system performance and can lead to cost savings by minimizing data transmission and cloud resource usage (Alraddady et al., 2022). On the other hand, fog computing's proximity to end-users allows it to meet stringent time constraints required by applications such as autonomous vehicles and industrial automation, where rapid response times are critical (Bogucka et al., 2023). It holds promise for future applications in data transmission and distributed computing. However, challenges such as resource constraints and heterogeneity persist (Bachiega et al., 2023). Fog is less powerful than the cloud regarding computing capabilities and storage capacity. Therefore, overcoming latency, storage, and bandwidth limitations requires a shift toward combined resource allocation in a cloud–fog architecture that can support a wider range of Internet of Things (IoT) applications.

The primary objective of this work is to analyze competition among SPs with the emergence of new applications that require diverse resources from the Infrastructure Provider (InP) while imposing stringent Quality of Service (QoS) constraints. To address this challenge, we propose a novel market-based framework that balances the needs of each SP while investigating the impact of competition on prices and market sharing among SPs (Baslam et al., 2011; Luong et al., 2017; Nguyen et al., 2019). Specifically, we present a comprehensive model of market power and illustrate how SPs exert it at various levels of competition. In such an environment, an SP must employ a reliable demand model to determine the optimal pricing strategy given the prices and QoS levels offered by competing SPs. The relationship between demand and price for each SP may be modeled in various ways, with one critical consideration being the behavior of the total market size as a function of price changes and service offerings. Typically, the total market size decreases as aggregate service prices increase.

In this paper, we focus on pricing strategies under scenarios where the total market size demonstrates high or moderate sensitivity to price fluctuations. We assume that the attractiveness of an SP's service is a linear function of its price, QoS, and the prices and QoS levels of its competitors (El-Azouzi et al., 2003). Consequently, the demand for a given SP is represented as a function that depends on the price and QoS vectors of all the SPs. In previous work (Habli et al., 2023), we explored the interplay between pricing strategies and QoS considerations in the telco market, particularly for cloud computing facilities. Here, we model the interactions among SPs as a non-cooperative game, analyze the Nash equilibrium in price choices, and examine how varying levels of competition influence equilibrium prices and market shares in the context of cloud and fog facilities.

Our main contributions are as follows:

- We present a framework to model the interactions among competing SPs, with a focus on price, QoS, and the resource demands of each SP as the primary factors. In this paper, QoS is characterized as a combination of resources, including random access Memory (RAM), Central Processing Unit (CPU) cycles, and Bandwidth (BW).
- We investigate how SPs set their prices strategically to maximize profit while competing in the market and analyze the influence of pricing strategies on resource allocation and market dynamics.
- We develop mathematical models to capture and analyze the market dynamics of pricing competition and QoS management within the marketplace.
- We derive conditions for the existence and uniqueness of the Nash equilibrium, compute it explicitly, and characterize its properties to study resource allocation strategies under competitive settings.

- We introduce a price differentiation mechanism among SPs to address the imbalances in the allocation of fog resources and achieve a more equitable allocation of resources at the Nash equilibrium. This mechanism improves fairness, particularly for SPs with limited access to fog resources, without compromising strategic behavior.
- We provide extensive numerical results to validate the proposed model's consistency and explore how various parameters influence market prices. For validation, we use Amazon EC2 instances as benchmarks for resource allocation in the proposed services.

The remainder of this paper is structured as follows: Section 2 introduces the system model and discusses its key components, providing a foundational understanding of the problem. Section 3 delves into allocating resources from cloud and fog computing environments, deriving equilibrium conditions. This section also investigates the existence and uniqueness of equilibria, offering insights into the strategic behavior of SPs at equilibrium. In Section 4, we present numerical results that validate the theoretical models developed in the previous sections and compare the performance under different scenarios. Finally, Section 5 concludes the paper by summarizing the findings and proposing potential directions for future research.

2. System Model

We consider a two-level marketplace, where, at the upper level, a set of SPs $\mathcal{K} = \{1, 2, \cdots, K\}$ leases resources from Infrastructure Providers (InPs) to deliver various heterogeneous vertical applications (services), such as IoT, Virtual/Augmented reality, cloud computing, and more. At the lower level, the SPs (sellers) use the leased resources to compete to attract the maximum number of end-users (buyers).

Specifically, we assume that each InP has a network comprising cloud computing (CC) or fog computing (FC) facilities, or both, as depicted in Figure 1. This represents a comprehensive infrastructure configuration designed to provide a variety of computing, storage, and networking services. The cloud and fog facilities encompass physical resources such as RAM, CPU, BW, radio resources, etc., which are leased to SPs offering various services to end-users. Let $\mathcal{R} = \{1, \cdots, r, \cdots, R\}$ be the set of R distinct resource types. For clarity, the main notations are listed below in Table 2.

The SPs can access the resources from either FC or CC facilities. However, fog facilities have limited resources, while the cloud offers virtually unlimited capacity. Consequently, each SP needs to combine the resources allocated by both the fog and the cloud to ensure sufficient resources to deliver services of a certain quality to its end-users. The fog facility's resources have a predefined capacity, denoted by $\mathbf{C} = (C_1, \cdots, C_R)$, which represents the total quantities of various resources available in the fog. Any unmet requirements are fulfilled by acquiring additional resources from the cloud.

The services provided by an SP are characterized by two essential factors: the price charged and the QoS delivered. Thus, each SP k establishes two defining parameters for its offered services: $(p_k, q_k) \in \mathbb{R}_+^2$, where p_k represents the cost per unit of demand imposed by SP k on its end-users, and q_k embodies the QoS it guarantees. In this paper, we consider that the QoS offered by each SP is fixed.

Each SP $k \in \mathcal{K}$ leases resources from the InP and competes to attract users by offering better QoS at lower prices. We consider that each SP k experiences a demand that can be defined by function $D_k : \mathbb{R}_+^{2K} \to \mathbb{R}_+$, which depends on its own service parameters, p_k and q_k, as well as on the prices $p_{-k} = (p_j)_{j \neq k}$ and the QoS $q_{-k} = (q_j)_{j \neq k}$ of its competitors. Thus, each demand function D_k is influenced by the entire price vector $\mathbf{p} = (p_1, \cdots, p_K)$ and QoS vector $\mathbf{q} = (q_1, \cdots, q_K)$. In general, demand functions can take on several functional forms, each representing different interactions among the SPs. In this work, we restrict our analysis to a linear demand model, which captures the average behavior of a telecommunication market with stochastic demand generated by end-users. This linear model reflects the condition where demand increases as price decreases or as QoS improves.

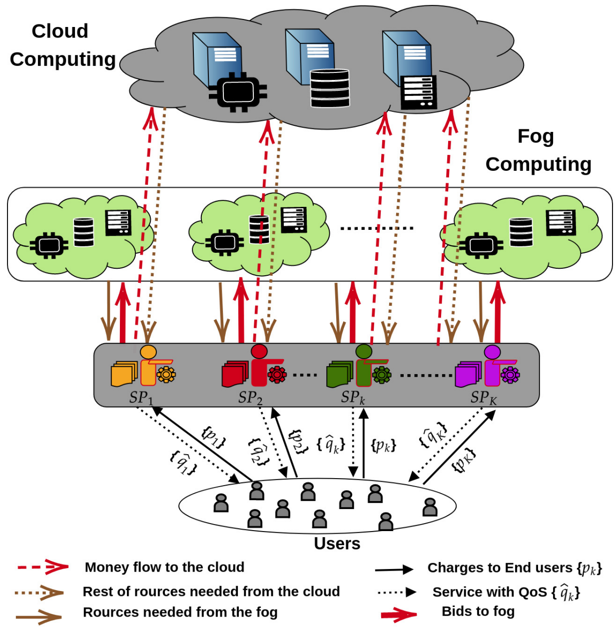

Figure 1. System architecture among SPs, their end-users, edge, and cloud computing.

Table 2. Main notations used in this paper.

$\mathcal{K}$	set of SPs.
InP	Infrastructure Provider
$\mathcal{R}$	set of resources available in the InP.
$\mathbf{p}$	vector of prices SPs charge for their services.
$\mathbf{p_{-k}}$	vector price of all SPs except SP k.
$\mathbf{q}$	vector of QoS offered by the SPs.
C_r	capacity of resource type r.
x_k^r	total amount of resource type r allocated from the InP to SP k, $k \in \mathcal{K}, r \in \mathcal{R}$.
$\bar{x}_k^r$	amount of resource type r allocated from the fog to SP k, $k \in \mathcal{K}, r \in \mathcal{R}$.
$\tilde{x}_k^r$	amount of resources allocated from the cloud to SP k, $k \in \mathcal{K}$.
h_k^r	minimum amount of resource type r SP k needs from the cloud to provide a unit service rate.
$\bar{h}_k^r$	minimum amount of resource type r SP k needs from the fog to provide a unit service rate.
$\mathbf{x}_k$	vector of total resources SP k needs from the InP.
$\bar{\mathbf{x}}_k$	vector of resources SP k needs from the fog.
$\tilde{\mathbf{x}}_k$	vector of resources SP k needs from the cloud.
$f_r(\mathbf{p}, \mathbf{w})$	cost for utilizing a resource type r from the InP.
w_k^r	bidding of SP k for resource type r.
w_{-k}^r	$= (W_r - w_k^r)$.
$\mathbf{w_k}$	vector of bids of SP k to request resources from the fog.
W_r	sum of the bidding of all SPs for resource type r.
ζ_r^{cloud}	price per unit of each resource type r each SP pays to the cloud.
ζ_r^{fog}	price per unit of each resource type r SP k pays to the fog.

2.1. SP Demand Model

We consider the demand function of each SP k to be linear and continuous with respect to all price and QoS parameters. Specifically, the demand function is formulated as

$$D_k(\mathbf{p}) = d_0 - b_k p_k + \sum_{\substack{j \neq k \\ j \in \mathcal{K}}} b_k^j p_j + \beta_k q_k - \sum_{\substack{j \neq k \\ j \in \mathcal{K}}} \beta_k^j q_j, \tag{1}$$

where b_k, b_k^j, β_k, and β_k^j are positive constants. To ensure consistent and realistic market behavior, we impose the following assumptions: $b_k > b_k^j$ for all $j \neq k$, indicating that each provider's price sensitivity is stronger for its own price than for those of others. $\beta_k > \beta_k^j$ for all $j \neq k$, reflecting that a provider's sensitivity to its service quality is stronger than to that of competitors. These assumptions are necessary to maintain the structure of competitive market interactions, where players are generally more sensitive to their own decisions (pricing and QoS) than to the decisions of others.

In this model, the demand function D_k reflects the sensitivity of demand to both price and QoS factors. Specifically, the demand for each SP k decreases as its price p_k increases. Conversely, it increases with higher prices p_j charged by its competitors. Similarly, an improvement in its own QoS q_k boosts demand, while better QoS offered by competitors $q_j, j \neq k$ reduces it. This framework underscores the dynamic interplay between competition, pricing strategies, and QoS in determining the demand for each SP's services.

2.2. QoS Model

In practical scenarios, QoS encompasses various metrics such as delay, completion time, packet loss, and jitter. However, our model emphasizes that QoS primarily hinges on allocating resources to an SP to deliver services to end-users with specific requirements. In this context, we adopt a model in which the QoS of SP k is determined by the combination of resources allocated to run a particular service. Thus, the amount of resources needed to achieve a certain QoS level q_k for SP k depends on the demand it faces. Higher demand and desired QoS levels necessitate greater resource allocation from the InP. In this paper, we specifically adopt the Leontief function to define the QoS offered by each SP. The QoS of each SP can be expressed as the minimum ratio of resource allocation to resource requirements across all necessary resources.

Since resource types in fog computing complement the same resource types in the cloud, we need to calculate the minimum required resources in both the cloud and fog for each SP. For cloud resources, the QoS offered by each SP is given by

$$g_k(\tilde{x}_k^r) = \min_{\mathcal{R}} \left\{ \frac{\tilde{x}_k^r}{h_k^r} \right\}, \forall r \in \mathcal{R}. \tag{2}$$

Thus, the minimum resources that can be allocated to an SP $k \in \mathcal{K}$ to support QoS for one unit of service is given by $\tilde{x}_k^r = \hat{q}_k h_k^r$. For fog resources, the QoS is given by

$$g_k(\bar{x}_k^r) = \min_{\mathcal{R}} \left\{ \frac{\bar{x}_k^r}{\bar{h}_k^r} \right\}, \forall r \in \mathcal{R}. \tag{3}$$

Thus, the minimum resource type r needed by each SP k to support the QoS for one unit of service is

$$g_k(\tilde{x}_k^r, \bar{x}_k^r) = \min_{\mathcal{R}} \left\{ \frac{\tilde{x}_k^r}{h_k^r}, \frac{\bar{x}_k^r}{\bar{h}_k^r} \right\}, \forall r \in \mathcal{R}. \tag{4}$$

We used Leontief function since the resource types are perfect complements (Sun et al., 2019), meaning that obtaining a resource type r in excess does not yield a higher utility. In the above equations, the value h_k^r denotes the minimum amount of resource type r required by SP k to provide a unit service rate to its end-users. For illustration purposes, we assume SP k requires 0.3 units of BW, 0.2 units of RAM, and 0.2 units of CPU to support a unit QoS rate. If SP k receives from the InP (cloud and fog facilities) 0.6 units of BW, 0.8 units of RAM, and 0.2 units of CPU, then SP k can support one unit of QoS for its single end-user, as shown by

$$g_k(x_k^r) = \min\left\{\frac{0.6}{0.3}, \frac{0.8}{0.2}, \frac{0.2}{0.2}\right\} = 1$$

In this work, we define, for each SP k, a resource allocation $\mathbf{x_k} = (x_k^1, \cdots, x_k^r, \cdots, x_k^R) \in \mathcal{X}_k$, which represents the total need from each resource type r. Here, $\mathcal{X}_k \subset \mathbb{R}_+^R$ is a convex and compact set such that $0 \in \mathcal{X}_k$, indicating that zero resources is a possible allocation.

For an SP $k \in \mathcal{K}$, since it utilizes both cloud and fog resources, its total resource allocation for a specific type of resource $r \in \mathcal{R}$ is divided between the fog and cloud environments. A portion of the demand is supported by fog computing, while the remaining demand is fulfilled by cloud computing, depending on the availability of resource r in the fog. The total resource allocation for resource type $r \in \mathcal{R}$ is then given by

$$x_k^r = \bar{x}_k^r + \tilde{x}_k^r \tag{5}$$

Let us assume that the resources obtained in the fog, which are not known in advance, cover $\alpha_r D_k(\mathbf{p})$ of its demand, where $\alpha_r \in [0,1]$. Then, we have

$$\bar{x}_k^r = \hat{q}_k \bar{h}_k^r \alpha_r D_k(\mathbf{p}) \tag{6}$$

Hence, the remaining demand that is not served by fog resources, $(1-\alpha_r)D_k(\mathbf{p})$, is fulfilled by the resources available in the cloud.

2.3. Resources in the Fog

For the multiple resources available in the fog, we propose employing the Kelly mechanism (Kelly, 1997) as a solution for resource allocation within fog facilities. This mechanism enables the equitable distribution of resources among SPs through optimized pricing strategies, ensuring efficient allocation in the fog computing environment. Our work focuses on utilizing this mechanism to establish an optimal pricing structure between SPs and fog facilities. The constrained capacity of the fog facility leads to two-sided competition: one aspect to attract end-users based on the pricing set by the SPs and the other to procure resources based on the fees charged by the fog for each resource type in $\mathcal{R}$. We delve into the interactions among SPs under the Kelly mechanism when resources are allocated to them by the fog. Subsequently, we formally define the Kelly mechanism in this context.

Kelly Mechanism

In Kelly's pricing mechanism, each player (i.e., SP) bids on each type of resource $r \in \mathcal{R}$. SPs compete for a share of divisible resources, and each SP k submits a bid ($w_k^r \geq 0$), which is equal to the fee paid by each SP k to the fog for obtaining a share of $\bar{x}_k^r$ of the resource type r. Once all SPs have placed their bids, the total bidding at the fog for each resource type r is $W_r = \sum_{j=1}^{K} w_j^r, \forall r \in \mathcal{R}$. The price per unit resource that each SP pays to the fog for utilizing a resource r is

$$\zeta_r^{fog} = \frac{W_r}{C_r} \tag{7}$$

where C_r is the capacity of the resource type r.

The resource share vector $\bar{\mathbf{x}}_k$ of each SP k is proportional to its bidding vector $\mathbf{w}_k$. Mathematically, we note the bid vector as $\mathbf{w}_k = (w_k^1, \cdots, w_k^r, \cdots, w_k^R)$; thus, the resource allocation vector $\bar{\mathbf{x}}_k = (\bar{x}_k^1, \cdots, \bar{x}_k^r, \cdots, x_k^R)$ is defined by

$$\bar{x}_k^r = \begin{cases} \frac{w_k^r}{W_r} C_r, & \text{if } w_k^r > 0 \\ 0, & \text{if } w_k^r = 0 \end{cases} \tag{8}$$

2.4. Resources Allocated from the Cloud

For SPs in $\mathcal{K}$ utilizing resources purchased from fog and cloud computing, their cost depends on the amount of resources obtained from the fog through the Kelly mechanism. Let $\bar{x}_k^r$, $r \in \mathcal{R}$ be the amount of resource type r obtained in the fog. Thus, from (6), we have

$$\alpha_r = \frac{\bar{x}_k^r}{\hat{q}_k \bar{h}_k^r D_k(\mathbf{p})} \tag{9}$$

Then, the remaining resources needed from the cloud are given by

$$\begin{aligned} \tilde{x}_k^r &= \hat{q}_k h_k^r (1 - \alpha_r) D_k(\mathbf{p}) && (10) \\ &= \hat{q}_k h_k^r D_k(\mathbf{p}) - \frac{h_k^r}{\bar{h}_k^r} \bar{x}_k^r && (11) \\ &= \hat{q}_k h_k^r D_k(\mathbf{p}) - \frac{h_k^r}{\bar{h}_k^r} \frac{w_k^r}{W_r} C_r && (12) \end{aligned}$$

2.5. Utility of SPs

The utility function of SP $k \in \mathcal{K}$ is the difference between the revenue from its end-users and the total cost incurred by the utilization of resources in $\mathcal{R}$:

$$U_k(\mathbf{p}, \mathbf{w}) = p_k D_k(\mathbf{p}) - \sum_{r \in \mathcal{R}} f_r(\mathbf{p}, \mathbf{w}) \tag{13}$$

The total cost each SP $k \in \mathcal{K}$ needs to pay to the cloud and fog computing, from (7), (8), and (12), is given by

$$\begin{aligned} f_r(\mathbf{p}, \mathbf{w}) &= \zeta_r^{cloud} \tilde{x}_k^r + \zeta_r^{fog} \bar{x}_k^r && (14) \\ &= \zeta_r^{cloud} \left(\hat{q}_k h_k^r D_k(\mathbf{p}) - \frac{h_k^r}{\bar{h}_k^r} \frac{w_k^r}{W_r} C_r \right) + w_k^r && (15) \\ &= \zeta_r^{cloud} \hat{q}_k h_k^r D_k(\mathbf{p}) + w_k^r \left(1 - \zeta_r^{cloud} \frac{h_k^r}{\bar{h}_k^r} \frac{C_r}{W_r} \right) && (16) \end{aligned}$$

Thus, the utility function of each SP k defined in (13) becomes

$$U_k(\mathbf{p}, \mathbf{w}) = D_k(\mathbf{p}) \left(p_k - \hat{q}_k \sum_{r \in \mathcal{R}} \zeta_r^{cloud} h_k^r \right) - \sum_{r \in \mathcal{R}} w_k^r \left(1 - \zeta_r^{cloud} \frac{h_k^r}{\bar{h}_k^r} \frac{C_r}{W_r} \right) \tag{17}$$

2.6. Game and SP Strategies

In the Kelly mechanism, each SP places a bid on each resource type $r \in \mathcal{R}$ while playing the best response. We assume that all SPs are rational and aim to maximize their utilities. In this scenario, the decision of each SP k depends not only on its price and its bidding vector, $(p_k, \mathbf{w_k})$, where $\forall k \in \mathcal{K}$, $\mathbf{w_k} = (w_k^1, \cdots, w_k^r, \cdots, w_k^R)$ denotes the bid to request different types of resources from the fog and p_k represents the price each SP k

charges to its end-users. It also depends on the price and bidding vectors of its competitors $(\mathbf{p}_{-k}, \mathbf{w}_{-k})$; the utility of each SP in $\mathcal{K}$ depends on its price-bidding strategy $(p_k, \mathbf{w_k})$, as well as the price-bidding strategies of its competitors $(\mathbf{p}_{-k}, \mathbf{w}_{-k})$. We define the price-bidding strategy of each SP k as

$$\mathcal{S}_k = \{(p_k, \mathbf{w_k}) : 0 \leq l(\hat{q}) \leq p_k \leq p^{max}; \quad w_k^r \geq 0\},$$

and define the strategy space by

$$\mathcal{S} = \mathcal{S}_1 \times \mathcal{S}_2 \times \mathcal{S}_3 \times \cdots \times \mathcal{S}_K,$$

where $p_k^{min} = l(\hat{q})$ allows explicitly eliminating the strategies that will result in negative utility for SP k. The fact that there are maximum prices (p^{max}) ensures that, above reasonable prices, the demand will be zero (whatever the prices of the competitors and the fixed QoS $\mathbf{q}$). Finally, $\forall k \in \mathcal{K}$ and $\forall r \in \mathcal{R}$, the bids, w_k^r, are non-negative.

Kelly's pricing mechanism induced a non-cooperative game denoted as

$$\mathcal{G}_{(\mathbf{p},\mathbf{w})} \triangleq \langle \mathcal{K}, (p_k, \mathbf{w_k}) \in \mathcal{S}_k, (U_k)_{k \in \mathcal{K}} \rangle$$

- set of players $\mathcal{K}$;
- players' respective strategy spaces $\mathcal{S}_k = \{(p_k, \mathbf{w_k}) : 0 \leq l(\hat{q}) \leq p_k \leq p^{max}; \quad w_k^r \geq 0\}$;
- $\mathcal{S} = \mathcal{S}_1 \times \mathcal{S}_2 \times \mathcal{S}_3 \times \cdots \times \mathcal{S}_K$ defines the joint action space for the game;
- players' utility functions $(U_k(\mathbf{p}, \mathbf{w}))_{k \in \mathcal{K}}$.

Definition 1 (Nash Equilibrium in Prices and Bidding)**.** *Let $U_k(\mathbf{p}, \mathbf{w})$ be the utility of SP k when the vectors of prices and bids set simultaneously by all the SPs are given by $(\mathbf{p}, \mathbf{w})$. Then, a two-parameter Nash equilibrium in $(\mathbf{p}, \mathbf{w})$ is the vector couple $(\mathbf{p}^*, \mathbf{w}^*)$ that solves the following system:*

$$U_k(\mathbf{p}^*, \mathbf{w}^*) \quad = \max_{(p_k, \mathbf{w}_k) \in \mathcal{S}_k} U_k(p_k, \mathbf{w}_k, \mathbf{p}^*_{-k}, \mathbf{w}^*_{-k}), k \in \mathcal{K}.$$

For this scenario, we establish the existence and uniqueness of the Nash equilibrium solution of the game $\mathcal{G}_{(\mathbf{p},\mathbf{w})}$ when the vector of QoS levels of all SPs is fixed.

3. Equilibrium Analysis

We establish the existence and uniqueness of the Nash equilibrium of the game $\mathcal{G}_{\mathbf{p},\mathbf{w}}$. We study the interactions among SPs under the pricing and bidding strategies. Let

$$\delta_k^r = \frac{\bar{h}_k^r}{h_k^r}, r \in \mathcal{R} \tag{18}$$

The variable δ_k^r represents the ratio of the reference resource complement in the fog ($\bar{h}_k^r$) to the current resource complement in the cloud (h_k^r) for each SP k, where $r \in \mathcal{R}$ indicates the type of resource. This ratio, δ_k^r, is essential for modeling price differentiation between fog and cloud computing environments. Specifically, it reflects how the resource complement in the fog compares to that in the cloud, which has important implications for resource allocation strategies in competitive environments.

To illustrate the role of δ_k^r, consider a service that requires a CPU for task processing, BW for transmitting requests and receiving responses, and storage space to hold task requests. When the QoS includes a deadline for task execution, the proximity of fog resources offers a significant advantage over cloud resources. In such cases, fog resources provide lower latency, which directly reduces the BW needed to meet QoS constraints compared to cloud resources, which are farther from the end-user and require higher BW to compensate for increased latency.

Thus, δ_k^r helps to quantify the difference in resource utilization between fog and cloud, and it is a key factor in determining pricing strategies for SPs competing to attract end-users while managing resource costs. This differentiation plays a crucial role in the optimal allocation of resources in fog and cloud environments.

For each $r \in \mathcal{R}$, we define $\sigma_r : \mathcal{K} \to \mathcal{K}$ a mapping function such that $\delta^r_{\sigma_r(1)} \leq \delta^r_{\sigma_r(2)} \leq \cdots \leq \delta^r_{\sigma_r(K)}$. From (17) and (18), the utility of each SP $\in \mathcal{K}$ becomes

$$U_k(\mathbf{p}, \mathbf{w}) = D_k(\mathbf{p})\left(p_k - \hat{q}_k \sum_{r \in \mathcal{R}} \zeta_r^{cloud} h_k^r\right) - \sum_{r \in \mathcal{R}} w_k^r \left(1 - \frac{\zeta_r^{cloud}}{\delta_k^r} \frac{C_r}{W_r}\right) \tag{19}$$

The following theorem establishes the existence of a Nash equilibrium of the game $\mathcal{G}_{\mathbf{p},\mathbf{w}}$.

Theorem 1. *There exists a Nash equilibrium of the non-cooperative pricing game* $\mathcal{G}_{\mathbf{p},\mathbf{w}}$.

The detailed proof is provided in Appendix A.1.

We now study the uniqueness of the Nash equilibrium of the game $\mathcal{G}_{\mathbf{p},\mathbf{w}}$. We present the necessary elements and conditions required to establish the proof of the uniqueness of the Nash equilibrium in this non-cooperative game. The analysis will focus on the strategic interactions among SPs and the resource allocation mechanism, considering both the price and bidding strategies involved.

Definition 2 (Diagonally Dominant Matrix)**.** *A square matrix* $\mathbf{B}$ *is diagonally dominant if* $\sum_{j \neq k, j \in \mathcal{K}} |\mathbf{B}_{kj}| \leq |\mathbf{B}_{kk}|$ *for all* $k \in \mathcal{K}$.

The following lemma gives a sufficient condition for a symmetric matrix to be negative definite.

Lemma 1 (Gershgorin Circle Theorem (Horn & Johnson, 2012))**.** *If a symmetric matrix is strictly diagonally dominant and has strictly negative diagonal entries, it is negative definite.*

We shall make the following assumption, which will be used to show the uniqueness of the Nash equilibrium.

Assumption 1. *The constants* b_k *and* b_k^j *satisfy* $\sum_{j \in \mathcal{K}, j \neq k} b_k^j \leq b_k$.

Assumption 1 implies that the influence of an SP's own price on its observed demand is significantly greater than the influence of other SPs' prices. This condition could then be interpreted as the presence of customer loyalty or the reputation of the SPs.

Assumption 2. *We assume that the demand function of each SP k satisfies*

$$D_k\left(p^{max} - \hat{q}_k \sum_{r \in \mathcal{R}} \zeta_r^{Cloud} h_k^r, \mathbf{0_{-k}}\right) \leq 0, \forall k \in \mathcal{K} \tag{20}$$

where $\hat{q}_k \sum_{r \in \mathcal{R}} \zeta_r^{Cloud} h_k^r = p_k^{min}$, *and* $\mathbf{0_{-k}}$ *is the* $K-1$ *dimensional null vector.*

Assumption 2 requires that, if we extrapolate the demand of an SP linearly out of the feasible region and allow its competitors' prices to be zero, then the demand would be non-positive.

Definition 3 (Diagonal Strict Concavity (Rosen, 1965))**.** *A game with profiles of strategies z and profiles of utility function U is called diagonally stricty concave (DSC) for a given vector r if for every distinct $\bar{z}$ and $\hat{z}$,*

$$[g(\bar{z}, r) - g(\hat{z}, r)](\bar{z} - \hat{z})' < 0, \tag{21}$$

with g the concatenation of the weighted gradients of the players' utility functions, and z defines the price or bidding strategy of the game $\mathcal{G}_{(\mathbf{p},\mathbf{w})}$,

$$g(z, r) = \Big[r_1 \nabla_1 U_1(z), r_2 \nabla_2 U_2(z), \ldots, r_k \nabla_K U_K(z)\Big], \tag{22}$$

where $\nabla_k U_k(z)$ denotes the gradient of utility of player k with respect to its own price strategy p_k or bidding strategy w_k.

The following lemma gives a sufficient condition for a symmetric matrix to be negative definite.

Lemma 2. *A symmetric real matrix $[\mathbf{B}]$ is negative definite if $Y^T[B]Y < 0$, $\forall Y \in \mathbb{R}^K, Y \neq 0$ where Y is the column vector:*

$$\mathbf{Y} = \begin{bmatrix} y_1 & \ldots & y_k & \ldots & y_K \end{bmatrix}^T$$

First, we start to show the uniqueness of the Nash equilibrium and its characterization of the game $\mathcal{G}_{(\mathbf{p},\mathbf{w})}$ when $\delta_k^r = \delta^r$, $\forall k \in \mathcal{K}$, where δ^r is a real positive value.

Theorem 2. *There exists a unique Nash equilibrium of the non-cooperative game $\mathcal{G}_{(\mathbf{p},\mathbf{w})}$, with*

$$p^* = A^{-1}(I - V)^{-1} J, \tag{23}$$

$$\text{and} \qquad (w_k^r)^* = \frac{(K-1)\zeta_r^{cloud} C_r}{K^2 \delta^r}, k \in \mathcal{K}, \tag{24}$$

where $A = diag(2b_1, \ldots, 2b_k)$, $V_{kj} = \frac{b_k^j}{2b_k}$ $k \neq j$, $V_{kk} = 0$ $\forall k \in \mathcal{K}$, and $J_k = d_0 + \left(\beta_k + b_k \sum_{r \in \mathcal{R}} \zeta_r^{cloud} h_k^r\right) \hat{q}_k - \sum_{\substack{j \neq k \\ j \in \mathcal{K}}} \beta_k^j \hat{q}_j$.

The proof is detailed in Appendix A.2.

Now, we will deal with the case where δ_k^r values are different and depend on the SP as SPs offer different services and the correlation between resource types is not the same for each SP.

Theorem 3. *There exists a unique Nash equilibrium of the non-cooperative game $\mathcal{G}_{(\mathbf{p}-,\mathbf{w})}$, with p^* given by (23) and*

$$(w_k^r)^* = \begin{cases} \frac{(k^*-1)\zeta_r^{cloud} C_r}{\sum_{k'=1}^{k^*} \delta_{k'}^r} \left(1 - \frac{(k^*-1)\delta_k^r}{\sum_{k'=1}^{k^*} \delta_{k'}^r}\right) & \text{if } k \in \mathcal{B}^* \\ 0 & \text{otherwise} \end{cases} \tag{25}$$

where $k_r^* = max_k\{k \mid \sum_{k'=1}^{k} \delta_{\sigma_r(k')}^r - (k-1)\delta_{\sigma_r(k)}^r > 0\}$ and $\mathcal{B}^* = \{\sigma(1), \ldots, \sigma(k_r^*)\}$.

The proof is detailed in Appendix A.3.

One of the main issues with this solution is that some SPs cannot use the resources available in Nash's balanced cloud. The Nash balanced solution tends to allocate more resources to SPs, with a significant advantage in utilizing resources in the fog. This results in a lack of fairness among SPs utilizing the resource in the fog.

Rather than implementing a nondiscriminatory mechanism, we consider a price differentiation among SPs. Our motivation regarding price differentiation is to achieve a fairer allocation in the fog at the Nash equilibrium. In particular, for each SP k and resource r in the fog, let v_k^r be the price per unit associated with SP k and thus the total cost for an offer w_k^r is $v_k^r w_k^r$. Consequently, the utility function of SP k under the generalized Kelly mechanism becomes

$$U_k(\mathbf{p}, \mathbf{w}) = D_k(\mathbf{p}) \left(p_k - \hat{q}_k \sum_{r \in \mathcal{R}} \zeta_r^{cloud} h_k^r \right) - \sum_{r \in \mathcal{R}} w_k^r \left(v_k^r - \frac{\zeta_r^{cloud}}{\delta_k^r} \frac{C_r}{W_r} \right) \quad (26)$$

Let $\mathcal{G}^d_{(\mathbf{p}-,\mathbf{w})}$ denote the new non-cooperative game under generalized Kelly mechanism.

Proposition 1. *There exists a unique Nash equilibrium of the game $\mathcal{G}^d_{(\mathbf{p}-,\mathbf{w})}$. If the price differentiation is designed such that $v_k^r \delta_r^k = \delta^r$, where δ^r is a real positive value, then the NE is given by (23) and (24).*

The proof of this proposition is in Appendix A.4.

Proposition 1 implies that, if InPs charge SPs that benefit more by utilizing resources in the fog, the Nash equilibrium results in a fair allocation among SPs in the fog. Price differentiation, therefore, enables InP to scale the price paid by an SP k for each resource type r according to δ_k^r. The higher this value, the higher the price paid by SP k. In this way, the InP can use this pricing to target a specific objective according to its preferences. If its objective is to guarantee fair sharing, according to Proposition 1, the price paid by SP k is the inverse of its δ_k^r.

4. Numerical Investigations and Discussion

In this section, we provide the numerical results to validate the theoretical findings presented in the previous sections. We analyze the interactions among the SPs within a game-theoretic framework, specifically investigating how these interactions manifest at the Nash equilibrium of the game $\mathcal{G}_{\mathbf{p},\mathbf{w}}$. To delve deeper into these interactions, we assume that each SP requires three distinct types of resources, RAM, CPU, and BW, from the fog and cloud computing environments (InP) to deliver their services effectively. These resources are critical for the operation and performance of the services provided by each SP. Our analysis examines how varying parameters impact several key metrics, including

- Price: the cost that each SP charges for its services.
- Equilibrium cost: the overall expenditure incurred by the SPs in procuring the necessary resources.
- Market share (MS): the proportion of the market each SP captures based on its service offerings and resource utilization.
- Market size: the total demand for services within the market.

We set up two distinct case scenarios to simulate and analyze these effects. Each scenario reflects different combinations of resources required by the SPs to operate their services. For our simulations, we utilize Amazon EC2 instances *Amazon EC2 Instance Types* (n.d.), as detailed in Table 3. The specific instance used by each SP is as follows:

- SP_1: Requires resources equivalent to the Amazon EC2 instance type r4.8xlarge .
- SP_2: Requires resources equivalent to the Amazon EC2 instance type r4.16xlarge.
- SP_3: Requires resources equivalent to the Amazon EC2 instance type c5.18xlarge.

Table 3. API instances from Amazon EC2.

API Name	Bandwidth (BW) (Gbps)	Central Processing Unit (CPU)	Memory (RAM) (GB)	Instance Type
r4.8xlarge	10.00	32.00	244.00	Memory optimized
c5.18xlarge	25.00	72.00	144.00	Compute-optimized
r4.16xlarge	25.00	64.00	488.00	Memory optimized

Each instance type provides a different balance of RAM, CPU, and BW, influencing the SPs' operational characteristics and competitive dynamics. By varying each parameter and observing its effects, we aim to provide insights into how resource requirements and market conditions affect the strategic decisions of SPs and the overall equilibrium of the market.

We assume that SP_1 and SP_3 have a higher reputation than SP_2 in all the scenarios. SP_2 focuses on enhancing customer satisfaction and attracting additional subscribers.

The demand function for each SP $i = 1, 2, 3$ with $i \neq j, i \neq k$ is given by

$$D_i(\mathbf{p}) = d_0 - b_i p_i + b_i^j p_j + b_i^k p + \beta_i q_i - \beta_i^j q_j - \beta_i^k q_k \tag{27}$$

with $d_0 = \begin{bmatrix} 350 & 350 & 350 \end{bmatrix}$, $b_i = \begin{bmatrix} 14 & 10 & 20 \end{bmatrix}$, $\beta_i = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$, $q_i = \begin{bmatrix} 2 & 2 & 4 \end{bmatrix}$, $b_i^j = \begin{bmatrix} 0 & 9 & 5 \\ 4 & 0 & 4 \\ 6 & 12 & 0 \end{bmatrix}$, and $\beta_i^j = \begin{bmatrix} 0 & 3 & 1 \\ 5 & 0 & 1 \\ 2 & 2 & 0 \end{bmatrix}$.

The InP charges a uniform price per unit of resource to each SP, $\zeta_r^{cloud} = 0.02$ for each resource type $r \in \{\text{RAM}, \text{CPU}, \text{BW}\}$. The capacity of the fog is $C_r = \begin{bmatrix} 200 & 30 & 10 \end{bmatrix}$.

The market share of each SP i is calculated as

$$MSE_i = \frac{D_i}{MSE}, \quad i = 1, 2, 3. \tag{28}$$

where D_i represents the demand for SP i, and MSE is the equilibrium market size, defined as

$$MSE = \sum_{i=1}^{3} D_i, \tag{29}$$

We perform simulations to examine the behavior of the SPs under these conditions, focusing on how resource allocation and pricing changes affect each SP's strategic decisions, market dynamics, and market share. We investigate the effect on overall market share and resource allocation strategies by systematically varying specific parameters in each scenario.

4.1. Case of 2 SPs

We focus on the interaction between two SPs, SP_1 and SP_2, and analyze the Nash equilibrium behavior of the game under different scenarios, varying a parameter in each case, and highlighting its effects on demand, price, market share, and market size.

4.1.1. Scenario 1: The Impact of Demand Sensitivity to Price

In this scenario, we investigate the impact of b_2 on market behavior and pricing strategies. b_2 denotes the demand responsiveness of SP_2 to its price; SP_2 focuses on customer loyalty through targeted advertising and attractive subscription offers. Figure 2 illustrates how SP_2's demand responsiveness to its price b_2 affects demand, pricing, utility, market share, and market size for both SPs. Figure 2a shows that the quantity demanded for SP_1 declines sharply. D_1 begins to decline rapidly, while D_2 increases at the beginning, with the two demands intersecting at $b_2 = 25$. After this point, D_1 decreases more gradually, while D_2 starts to decline linearly. To counterbalance the loss while increasing b_2, SP_2 rapidly reduces its price, as illustrated in Figure 2b; to remain operational and competitive in the market, SP_2 adopts an aggressive approach, significantly lowering its prices to attract end-users and offset its market losses. This strategy also pressures SP_1 to make slight price reductions, as reflected by the intersection at $b_2 = 25$ in Figure 2a. While SP_2 manages to retain a market share by offering services at lower prices and consequently earning reduced profits, SP_1 sustains its services with more moderate pricing, leveraging its strong reputation and loyal customer base.

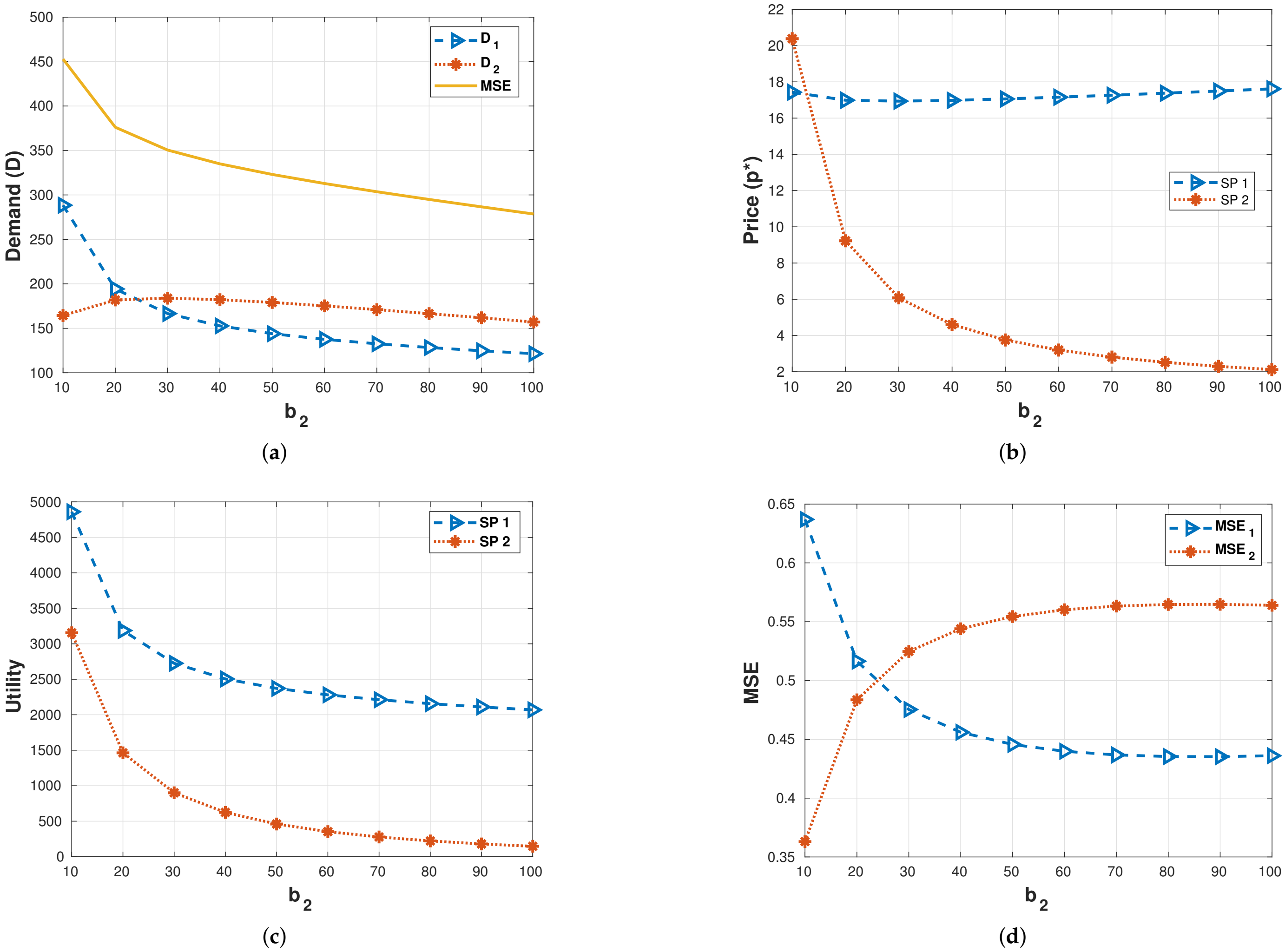

Figure 2. The impact of b_2 on demand, price, utility, and market share for SP_1 and SP_2. (**a**) Demand. (**b**) Price. (**c**) Utility. (**d**) Market share. SPs support r4.8xlarge and r4.16xlarge, respectively.

Figure 2c shows that the utility of SP_1 decreases gradually as b_2 increases. In contrast, SP_2's utility declines more sharply with b_2. As b_2 approaches 100, the utilities of both SPs converge. This suggests that b_2 significantly impacts the competitive dynamics between the two SPs, reducing the utility gap as it increases. SP_2 is losing profit while investing in attracting end-users. The interaction between the SPs occurs when both hold a market share, as illustrated in Figure 2d; the market experiences a sharp decline before eventually stabilizing, ultimately concluding with a decrease in SP_1's market share and an increase in SP_2's market control.

4.1.2. Scenario 2: The Effect of QoS

We examine how q_2 impacts the market behavior and pricing strategies; SP_2 offers higher quality to attract more subscribers. SP_1 maintains the same QoS. As illustrated in Figure 3a, the quantity demanded from SP_2 decreases slightly as q_2 increases; however, the quantity demanded from SP_1 experiences a slight increase. The MSE is depicted as a flat line, indicating minimal deviation across q_2. In Figure 3b, the price of p_2^* increases linearly as q_2 rises, while SP_1's price remains relatively constant at a lower value. This indicates that SP_2's pricing strategy is sensitive to changes in q_2, whereas SP_1 maintains a consistent pricing approach regardless of q_2.

In Figure 3c, the utility of SP_1 increases with q_2, whereas the utility of SP_2 decreases. In Figure 3d, MSE_1 increases slightly as q_2 grows, while MSE_2 decreases. This demonstrates that q_2 has an opposing effect on the two SPs, benefiting SP_1 in terms of utility and market share while negatively impacting SP_2.

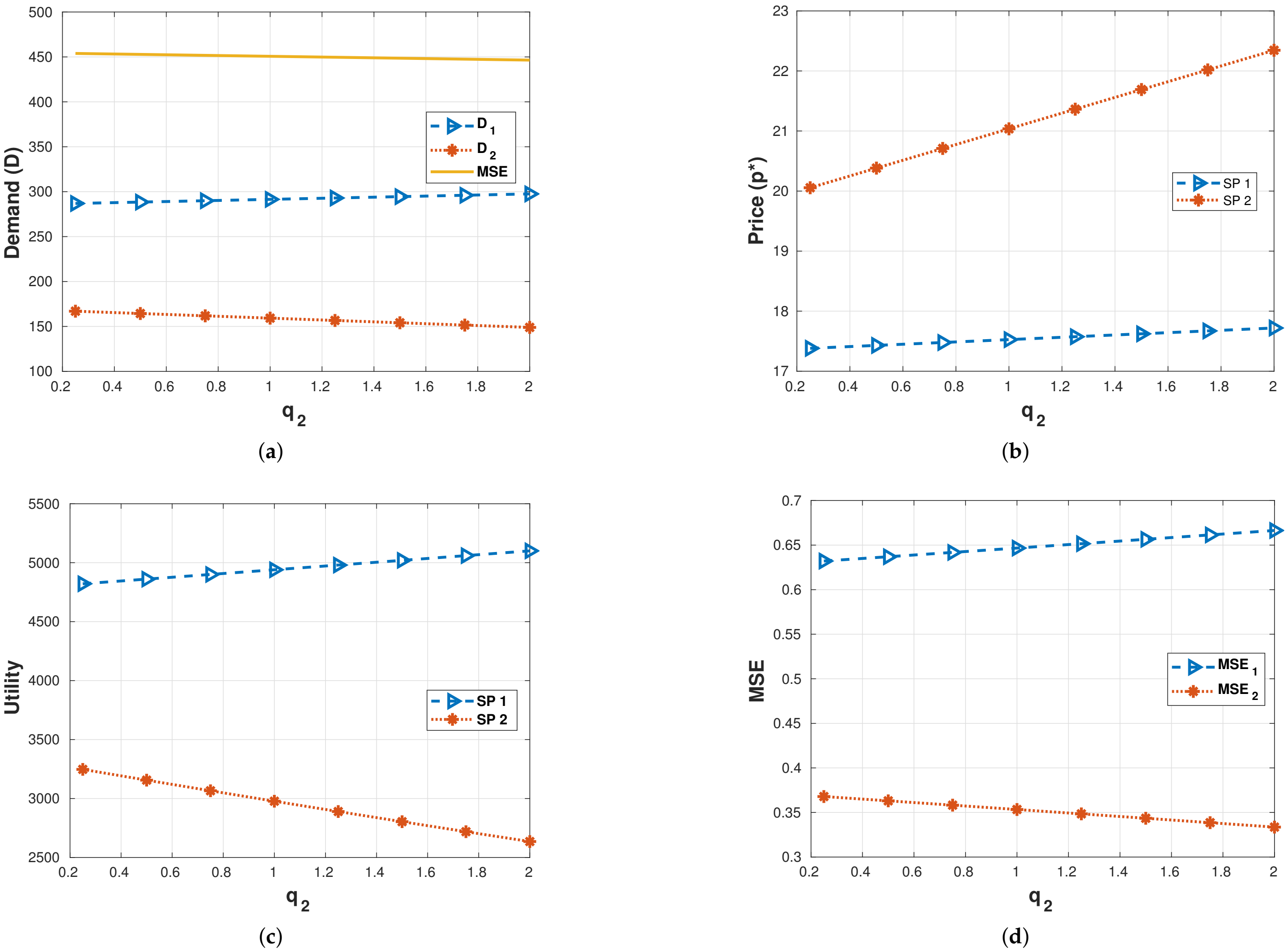

Figure 3. The impact of q_2 on demand, price, utility, and market share for SP_1 and SP_2. SPs support r4.8xlarge and r4.16xlarge, respectively. (**a**) Demand. (**b**) Price. (**c**) Utility. (**d**) Market share.

The analysis of the impact of q_2 reveals contrasting effects on utility, market share, and pricing dynamics for both SPs. As q_2 increases, SP_1 gains utility and slightly increases its market share, while SP_2 experiences a decline in both utility and market share. SP_2's prices rise with q_2, whereas SP_1's prices remain stable. Overall, q_2 enhances SP_1's competitiveness while disadvantaging SP_2.

4.2. Case of 3 SPs

Now, we examine the competition among the three SPs, SP_1, SP_2, and SP_3, by analyzing the Nash equilibrium behavior of the game across various scenarios. Each scenario involves varying a specific parameter to explore its impact on demand, price, market share, and market size.

4.2.1. Scenario of Demand Responsiveness

We study the effect of b_2 on each SP's pricing strategy, focusing on attracting end-users and fostering loyalty through competitive subscriptions. Figure 4 illustrates how b_2 influences demand, pricing, utility, market share, and overall market size for all the SPs. As shown in Figure 4a, the quantity demanded from SP_1 initially decreases sharply, while D_2 increases at first. The demands intersect at $b_2 = 20$, after which D_1 gradually declines and D_2 decreases linearly. SP_3 exhibits a similar trend to SP_1, experiencing a decline in quantity demanded that eventually stabilizes. The interaction among the SPs occurs when all three hold market share, as depicted in Figure 4a, where the overall market experiences a sharp decline before stabilizing.

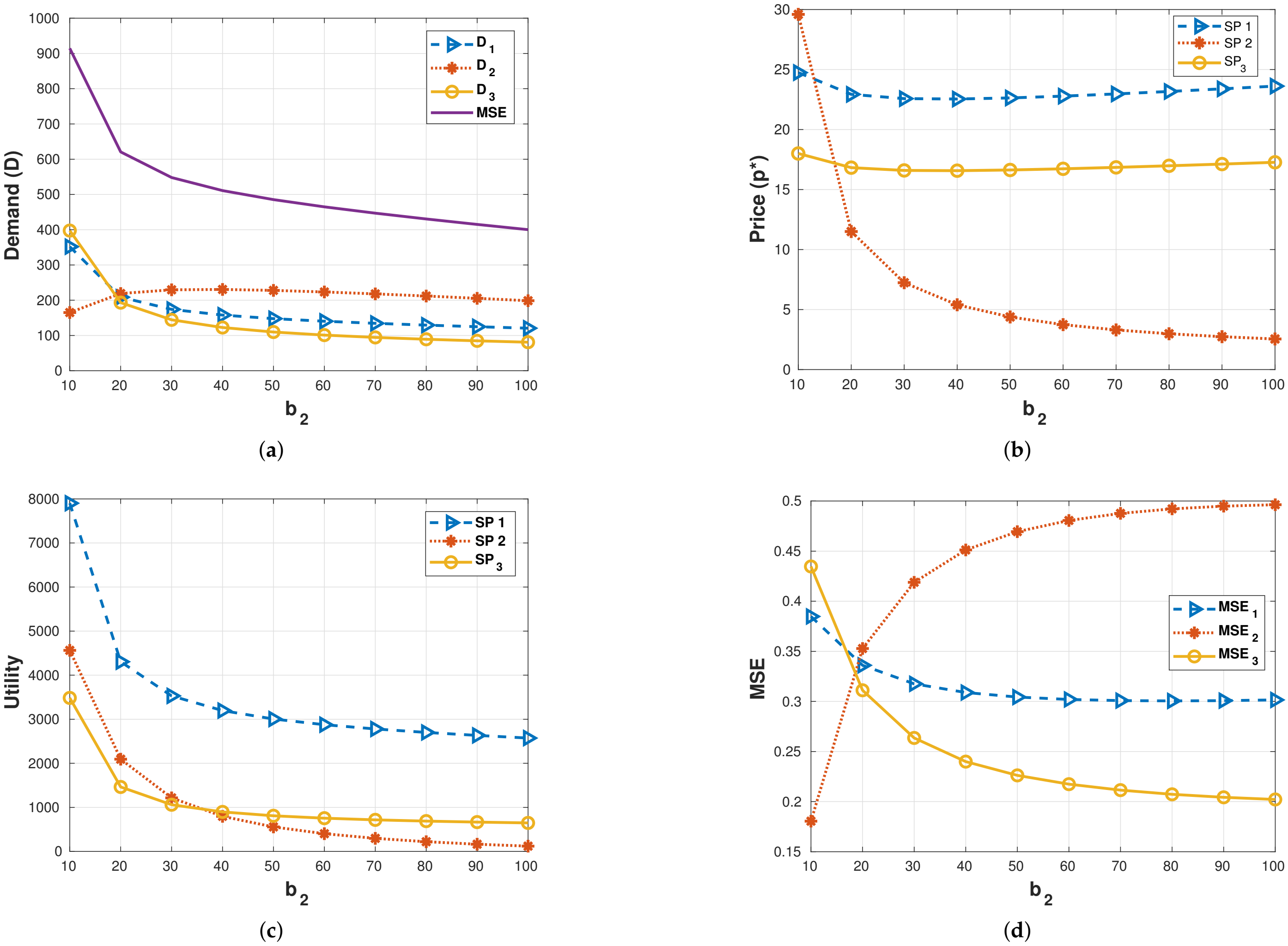

Figure 4. The impact of b_2 on demand, price, utility, and market share with SP_1, SP_2, and SP_3. SPs support r4.8xlarge, c5.18xlarge, and r4.16xlarge, respectively. (**a**) Demand. (**b**) Price. (**c**) Utility. (**d**) Market share.

To counterbalance its losses while increasing b_2, SP_2 adopts an aggressive pricing strategy, as shown in Figure 4b. It significantly reduces prices to attract end-users and mitigate its market losses. This approach exerts pressure on SP_1 and SP_3, forcing them to lower their prices slightly, reflected by the intersection at $b_2 = 20$ in Figure 4a. Although SP_2 retains a portion of the market by offering lower prices, SP_1 and SP_3 sustain their services with more stable pricing, capitalizing on their strong reputations and loyal customer bases to remain competitive.

Figure 4c demonstrates how the utility of each SP decreases as b_2 increases. SP_1 maintains a higher utility compared to SP_2 and SP_3, which converge toward lower values. This emphasizes the sensitivity of utility to b_2, particularly for SP_2, and SP_3. As illustrated in Figure 4d, SP_1 and SP_3's market shares decline, leading to a further reduction in their market shares, while the MSE of SP_2 increases steadily, allowing it to gain increased control over the market as b_2 increases.

4.2.2. Scenario: Impact of QoS

We investigate how q_2 influences market dynamics and pricing strategies, with SP_2 improving QoS to attract additional subscribers, while SP_1 and SP_3 keep their QoS constant. The figures demonstrate the effect of q_2 on each SP's demand, utility, and pricing strategies. Figure 5a depicts the quantity demanded from the SPs as a function of q_2, along with the MSE. The quantity demanded from SP_1 remains relatively stable, indicating minimal sensitivity to changes in q_2. In contrast, D_2 consistently exhibits low demand, suggesting limited responsiveness to q_2. Meanwhile, D_3 shows a steady increase in demand, highlighting a positive correlation with q_2. The MSE remains constant across all the values of q_2, indicating stable model performance. Figure 5b illustrates the pricing strategies (p^*) of the SPs as a function of q_2. The price of SP_2 increases linearly with q_2, indicating that SP_2 adjusts its pricing proportionally as its service quality improves. In contrast, the price of SP_1 remains constant across all the values of q_2, showing no dependence on the changing QoS q_2. Similarly, SP_3 maintains a consistent price at a lower level, suggesting it does not adapt its pricing strategy to q_2. The plot demonstrates contrasting approaches to pricing among the three SPs, with SP_2 adopting a dynamic strategy, while SP_1 and SP_3 maintain fixed pricing structures.

As shown in Figure 5c, the SP_1 maintains a high and constant utility, demonstrating stability and independence from q_2. SP_2's utility declines linearly as q_2 increases, indicating a diminishing return or increased costs associated with quality improvements. In contrast, SP_3 experiences a gradual increase in utility, overtaking SP_2 around $q_2 = 7$, suggesting SP_3 benefits more as q_2 grows, possibly due to an optimal allocation or pricing strategy. As illustrated in Figure 5d, SP_1's MSE remains constant, unaffected by variations in q_2. SP_2's MSE steadily decreases with higher q_2, signaling a decline in market share. Meanwhile, SP_3's MSE consistently increases, suggesting an improving market performance as q_2 grows. This dual trend in utility and MSE highlights differing strategies and impacts among the SPs in response to changes in QoS.

The influence of q_2 on utility, market share, and pricing dynamics varies significantly between the SPs. As q_2 increases, SP_1 and SP_3 experience rises in utility and either maintain or slightly improve their market share. Conversely, SP_2 suffers declines in both utility and market share. This disparity is further reflected in pricing trends: SP_2 raises its prices as q_2 grows, while its competitors, SP_1 and SP_3, keep their prices steady. The increase in q_2, to attract more subscribers, stengthens the competitive edge of SP_1 and SP_3 while puttig SP_2 at a disadvantage.

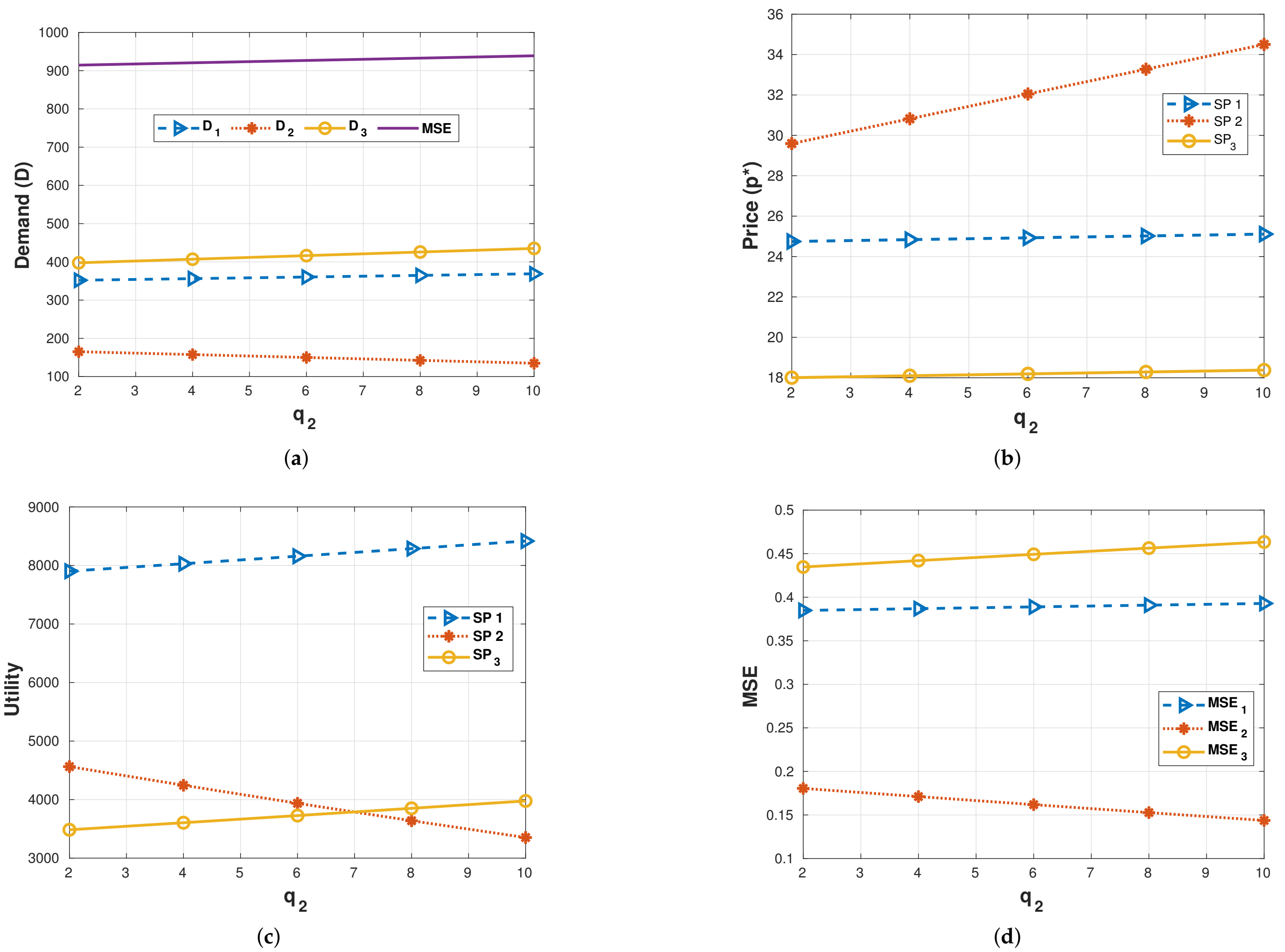

Figure 5. The impact of q_2 on demand, price, utility, and market share with SP_1, SP_2, and SP_3. SPs support r4.8xlarge, c5.18xlarge, and r4.16xlarge, respectively. (**a**) Demand. (**b**) Price. (**c**) Utility. (**d**) Market share.

4.2.3. Scenario of Minimum Amount of Resources

In this study, we investigate the influence of $\bar{h}_1^{RAM}$ on the equilibrium prices. Specifically, we examine how variations in $\bar{h}_1^{RAM}$ impact equilibrium prices, which provide insights into how SPs adjust their pricing strategies in response to changes in resource constraints. To conduct this analysis, we simulate various scenarios with different values of $\bar{h}_1^{RAM}$ while keeping $\bar{h}_k^{RAM}$ for $k = 2, 3$ constant. This approach enables us to isolate the effect of $\bar{h}_1^{RAM}$ on pricing, equilibrium cost, demand, and market share, and understand how changes in resource requirements for SP_1 influence the overall market dynamics and pricing strategies of all the players.

Figure 6a illustrates the variations in bidding weights w_k, associated with RAM, CPU, and BW, as a function of $\bar{h}_1^{\mathrm{RAM}}$. Among the RAM bidding weights ($w_1^{\mathrm{RAM}}, w_2^{\mathrm{RAM}}$, and w_3^{RAM}), w_1^{RAM} decreases significantly as $\bar{h}_1^{\mathrm{RAM}}$ increases, indicating a rapid reduction in bidding priority for this resource as its capacity grows. In contrast, w_2^{RAM} and w_3^{RAM} remain relatively stable, suggesting less sensitivity to $\bar{h}_1^{\mathrm{RAM}}$. The CPU bidding weights ($w_1^{\mathrm{CPU}}, w_2^{\mathrm{CPU}}$, and w_3^{CPU}) remain consistent and at low values close to 0.1, showing minimal fluctuation and emphasizing steady bidding behavior for CPU resources. Meanwhile, the BW biddings remain the lowest, starting at values below 0.1 and decreasing slightly as $\bar{h}_1^{\mathrm{RAM}}$ increases. This indicates that BW resources receive the least bidding priority, and this priority diminishes further as RAM capacity grows. Overall, the system prioritizes RAM bids over CPU and BW, as evidenced by the consistently higher RAM bidding weights. If balanced bidding among resources is desired, adjustments may be required to ensure that CPU and BW are not overshadowed by RAM. To achieve a fairer resource allocation, further analysis could explore the implications of declining bids for RAM and assess their potential impact on the overall efficiency of resource allocation.

Figure 6b illustrates that the price per unit resource in the fog environment, (ζ_{fog}), remains stable for CPU and BW across the range of $\bar{h}_1^{\mathrm{RAM}}$, while the price for RAM decreases before eventually converging.

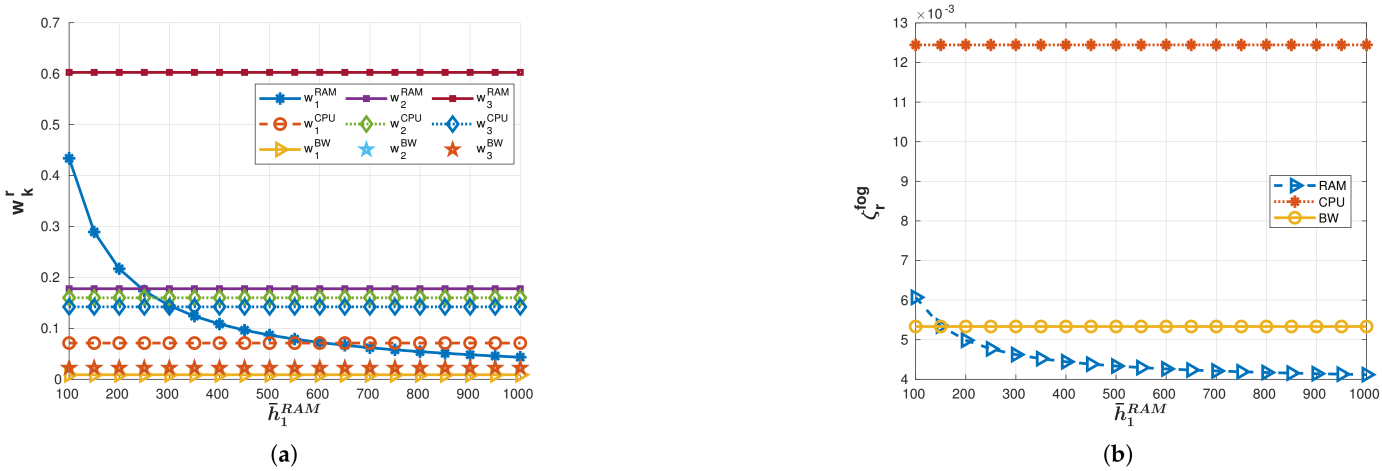

Figure 6. The impact of $\bar{h}_1^{RAM}$ on bidding and price per unit resource ζ_r^{fog}, with SP_1, SP_2, and SP_3. SPs support r4.8xlarge, c5.18xlarge, and r4.16xlarge, respectively. (**a**) Bidding. (**b**) Price per unit resource ζ_r^{fog}.

4.3. Complexity and Strengths of the Proposed Model

The proposed model is based on a game-theoretic framework that captures the competitive dynamics among non-cooperating SPs bidding for resources from fog and cloud environments, provided by an InP. The complexity of the model arises from the interaction between multiple SPs, each with different pricing and QoS strategies. The SP demand is influenced not only by their prices and QoS but also by the strategies of their competitors, making the problem non-linear and strategic.

The superiority of this model lies in its ability to account for the real-world dynamics of resource allocation in fog and cloud environments. The fog environment provides local access with lower latency but limited resources, while the cloud offers unlimited resources with potentially higher latency. Our model addresses this challenge by allowing for a dynamic and competitive bidding process, where SPs' strategies evolve over time as they adapt to competitors' available resources and pricing strategies.

One of the key innovations in this model is the introduction of price differentiation among SPs to achieve a fairer allocation of resources in the fog environment. Unlike traditional approaches that use nondiscriminatory allocation methods, our model considers unequal access to resources and adjusts the pricing strategies to achieve a fairer distribution. This price differentiation mechanism ensures that even SPs with limited fog resource utilization can maintain fair access to these resources, which would otherwise be dominated by larger players. Additionally, the Nash equilibrium in this model ensures that no SP can improve its outcome by unilaterally changing its strategy, providing a stable and predictable market outcome. This equilibrium concept guarantees that the resource allocation, pricing strategies, and QoS management will stabilize over time, even as the competition between SPs intensifies.

The model has been validated through simulations using Amazon EC2 instances. The results demonstrate that our approach effectively balances market forces and ensures a more equitable distribution of resources while considering the unique characteristics of fog and cloud environments.

This approach provides valuable insights into SPs' strategic behavior and offers a more robust and fair mechanism for resource allocation.

4.4. Managerial Insights

Based on the results and practical implications of the proposed model, the following managerial insights are drawn:

- *Resource Allocation Strategy:* SPs should strategically bid for resources from the fog environment, considering both their pricing and QoS and their competitors' strategies. Efficient resource allocation from the fog can lead to lower latency and higher user satisfaction. It is essential to monitor and adjust pricing strategies dynamically to maintain a competitive edge in a rapidly evolving market.
- *Price Differentiation for Fairer Allocation:* SPs with limited access to fog resources can utilize price differentiation to improve fairness in the allocation process. By adopting this pricing strategy, smaller SPs can gain better access to fog resources, which otherwise may be dominated by larger players. This leads to more balanced competition and ensures a fairer distribution of resources in the fog environment.
- *Importance of Competitive Bidding:* SPs should focus on evolving their bidding strategies over time to adapt to the competitive dynamics. Since fog resources are limited, the demand for these resources can be highly competitive. Thus, SPs need to continuously reassess their pricing and QoS strategies to ensure optimal resource utilization and to avoid losing out to competitors.
- *Nash Equilibrium as a Stabilizing Force:* The Nash equilibrium provides a stable and predictable market outcome. SPs should understand that their pricing and QoS strategies cannot be improved unilaterally when in equilibrium. This concept assures them that, by adopting optimal strategies, resource allocation, pricing, and QoS will stabilize, even in highly competitive environments.
- *Scalability and Adaptability:* The ability of SPs to scale their operations by transitioning from fog to cloud resources is essential. The cloud provides unlimited resources but with potentially higher latency. Managers should take this into account when planning service offerings, ensuring that they can leverage the advantages of both environments depending on the demand and QoS requirements.
- *Cost Optimization:* SPs must weigh the cost and performance trade-offs. SPs can better manage operational costs by adopting a dynamic pricing strategy that adjusts based on resource availability and competition. Optimal resource allocation from the fog environment can help to reduce costs and improve performance, thus offering a better value proposition to end-users.

5. Conclusions

We addressed the problem of allocating multiple resources, where SPs lease diverse resources, such as RAM, CPU, and BW, from both cloud and fog environments to deliver their services and satisfy user requirements. In this paper, we have introduced a game-theoretic model that describes the competition between non-cooperating SPs as they bid for resources in fog and cloud environments, managed by an InP, to offer paid services to end-users. The demand of each SP is influenced not only by its pricing and QoS but also by the strategies employed by its competitors. The InP allocates resources starting from the fog, which offers lower latency but limited capacity, and then supplements with cloud resources when the fog's capacity is reached. This bidding mechanism drives competition among SPs, motivating them to optimize both their pricing and QoS strategies. Through the establishment of a Nash equilibrium, we have ensured that the resulting allocation is stable, with no SP able to improve its outcome by altering its strategy alone. We validated the model via simulations using Amazon EC2 instances, which revealed valuable insights into the interplay of market prices, resource allocation, and QoS management in these environments. However, our findings also highlight a challenge in the Nash equilibrium where SPs with stronger access to fog resources gain a disproportionate share of resources, leading to fairness concerns. The parameters of the model, such as resource availability, bidding, QoS, and demand's sensitivity to price, directly affect these dynamics. For example, $\bar{h}_1^{\text{RAM}}$ determines the SP's ability to access fog resources. If these parameters are set to favor SPs with greater fog access, other SPs may be disadvantaged, reducing fairness in resource distribution. In light of this, we have proposed price differentiation among SPs as a method to foster a fairer allocation in the fog while maintaining equilibrium stability. These insights address fairness issues and contribute to the broader understanding of how price strategies can effectively balance competition and resource utilization in cloud and fog environments. Furthermore, analyzing the impact of various parameters, such as bidding and resource constraints, reveals whether they provide an advantage or disadvantage for SPs.

For further research, we aim to explore the role of multiple QoS attributes, such as bandwidth, latency, and reliability, in shaping the competitiveness of SPs. By incorporating these additional factors, we can investigate how they influence customer preferences and market share among SPs. These attributes not only affect the attractiveness of services but also influence SPs' bidding strategies, providing a richer perspective on how SPs compete in complex environments. Analyzing these attributes will offer a deeper understanding of the trade-offs involved in service provision and help to refine pricing strategies in fog and cloud systems, ensuring they align with end-users' expectations and enhance the overall service experience. In addition, we plan to explore the application of learning algorithms and machine learning techniques to improve decision-making processes for SPs. By leveraging machine learning models, we can better predict user behavior, optimize resource allocation, and refine pricing strategies in real time. These techniques will enable SPs to adapt to dynamic market conditions and customer preferences more effectively, offering a significant advantage in competitive environments. This approach will also allow for the continuous improvement of SPs' strategies by learning from past interactions and adjusting in response to new data.

Author Contributions: Conceptualization, S.H., R.E.-A. and M.D.; methodology, S.H., R.E.-A. and M.D.; software, S.H.; validation, R.E.-A., E.S. and M.D.; formal analysis, R.E.-A.; writing—original draft preparation, S.H.; writing—review and editing, R.E.-A. and M.D.; supervision, R.E.-A., E.S., H.E. and M.S.; funding acquisition, R.E.-A. and H.E. All authors have read and agreed to the published version of the manuscript.

Funding: This work was partially supported by the National Center for Scientific and Technical Research (CNRST), Morocco, and the SLIME project (43765TA), funded by PHC Toubkal 2020. Additionally, the authors acknowledge the financial support provided by Professor Elbiaze Halima during the internship of Samira Habli in LATECE, at the University of Quebec at Montreal (UQAM). The authors are grateful for their contributions.

Data Availability Statement: The original contributions presented in this study are included in the article. Further inquiries can be directed to the corresponding authors.

Acknowledgments: The authors express their gratitude to the anonymous reviewers and the academic editor for their useful comments, suggestions, and observations aimed at improving the paper.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

Appendix A.1

Proof of Theorem 1. We consider two resource types (r, s), and we establish the existence of a Nash equilibrium. The utility function defined in (19) is continuous, increasing, and concave. At the same time, the strategy space of each SP is convex and compact for the variables p_k and w_k^r, $\forall k \in \mathcal{K}, \forall r \in \mathcal{R}$.

Then, we have

$$\frac{\partial U_k(\mathbf{p},\mathbf{w})}{\partial p_k} = d_0 - 2b_k p_k + \sum_{\substack{j \neq k \\ j \in \mathcal{K}}} b_k^j p_j - \sum_{\substack{j \neq k \\ j \in \mathcal{K}}} \beta_k^j q_j + \left(\beta_k + b_k \sum_{r \in \mathcal{R}} \zeta_r^{cloud} h_k^r \right) \hat{q}_k \tag{A1}$$

$$\frac{\partial U_k(\mathbf{p},\mathbf{w})}{\partial w_k^r} = -\left(1 - \left(\frac{w_{-k}^r}{(W_r)^2} \right) \frac{\zeta_r^{cloud}}{\delta_k^r} C_r \right), \quad \text{with } w_{-k}^r = (W_r - w_k^r) \tag{A2}$$

$$\frac{\partial^2 U_k(\mathbf{p},\mathbf{w})}{\partial p_k^2} = -2b_k, \quad \forall k \in \mathcal{K} \tag{A3}$$

$$\frac{\partial^2 U_k(\mathbf{p},\mathbf{w})}{\partial p_k p_j} = b_k^j, \quad j \neq k \tag{A4}$$

$$\frac{\partial^2 U_k(\mathbf{p},\mathbf{w})}{\partial^2 w_k^r} = -\left(\frac{2w_{-k}^r}{(W_r)^3} \right) \frac{\zeta_r^{cloud}}{\delta_k^r} C_r, \quad \forall k \in \mathcal{K}, \forall r \in \mathcal{R} \tag{A5}$$

$$\frac{\partial^2 U_k(\mathbf{p},\mathbf{w})}{\partial w_k^r \partial p_k} = \frac{\partial^2 U_k}{\partial p_k \partial w_k^r} = 0, \quad \forall k \in \mathcal{K}, \forall r \in \mathcal{R} \tag{A6}$$

Similarly, we can conclude the derivatives for $U_j, \forall j \in \mathcal{K}$. We have $\frac{\partial^2 U_k}{\partial p_k^2} = -2b_k < 0$ and $\frac{\partial^2 U_k}{\partial^2 w_k^r} = -\left(\frac{2w_{-k}^r}{(W_r)^3} \right) \frac{\zeta_r^{cloud}}{\delta_k^r} C_r < 0$; this shows that the utility is concave in (p_k, w_k); the utility of each player is concave in its decisions and the action set is compact; thus, the existence of Nash equilibrium follows from Thm. 1 (Rosen, 1965). □

Appendix A.2

Proof of Theorem 2. Define $H_r \triangleq N_r + N_r^T$, where N is a square $2K \times 2K$ matrix, defined as follows:

$$[\mathbf{N}_r]_{i,j} = \frac{\partial^2 U_k}{\partial p_j \partial p_k}, [\mathbf{N}_r]_{i+K,j+K} = \frac{\partial^2 U_k}{\partial w_j^r \partial w_k^r}, k, j \in \mathcal{K}.$$

By the diagonal strict concavity theorem (Rosen, 1965), it suffices to show $H_r \prec 0$ for each resource type $r \in \mathcal{R}$. The elements of the Hessian matrix $\mathbf{N}$ of utility profiles $U = [U_1, \ldots, U_K]$ are given by

$$\frac{\partial^2 U_k}{\partial p_j \partial p_k} = \begin{cases} -2b_k & \text{if } j = k \\ b_k^j & \text{if } j \neq k, \end{cases} \tag{A7}$$

$$\frac{\partial^2 U_k}{\partial w_j^r \partial w_k^r} = \begin{cases} -\frac{2w_{-i}^r}{(W_r)^3} \frac{\zeta_r^{cloud}}{\delta_k^r} C_r & \text{if } j = k \\ \frac{(w_k^r - w_{-i}^r)}{(W_r)^3} \frac{\zeta_r^{cloud}}{\delta_k^r} C_r & \text{if } j \neq k \end{cases} \tag{A8}$$

and

$$\frac{\partial^2 U_k}{\partial p_j \partial w_k^r} = \frac{\partial^2 U_k}{\partial w_j^r \partial p_k} = 0, \forall k, j \in \mathcal{K}. \tag{A9}$$

Hence,

$$H_r = \begin{bmatrix} \mathbf{Q} & 0 \\ 0 & \mathbf{P} \end{bmatrix} \tag{A10}$$

where Q and P are $K \times K$ matrices, defined as follows:

$$\mathbf{Q} = \begin{bmatrix} -4b_1 & \cdots & b_1^K + b_K^1 \\ \vdots & \ddots & \vdots \\ b_K^1 + b_1^K & \cdots & -4b_K \end{bmatrix}, \tag{A11}$$

and

$$\mathbf{P} = \frac{\zeta_r^{cloud} C_r}{(W_r)^3} \cdot \begin{bmatrix} -\frac{4w_{-1}^r}{\delta_1^r} & \cdots & \frac{\left(w_1^r - w_{-1}^r\right)}{\delta_1^r} + \frac{\left(w_K^r - w_{-K}^r\right)}{\delta_K^r} \\ \vdots & \ddots & \vdots \\ \frac{\left(w_K^r - w_{-K}^r\right)}{\delta_K^r} + \frac{\left(w_1^r - w_{-1}^r\right)}{\delta_1^r} & \cdots & -\frac{4w_{-K}^r}{\delta_K^r} \end{bmatrix} \tag{A12}$$

Under Assumption 1, we have

$$\sum_{j \neq k, j \in \mathcal{K}} (b_k^j + c_j^k) \leq 4b_k, \qquad \forall k \in \mathcal{K}. \tag{A13}$$

Applying Lemma 1, the matrix $\mathbf{Q}$ is negative definite. Now, we show that the matrix $\mathbf{P}$ is also negative definite. For any $z \in \mathbb{R}^N - \{0\}$,

$$\frac{\delta^r (W_r)^3 z^T \mathbf{P} z}{\zeta_r^{cloud} C_r} = \sum_{j=1}^{N} \left[-4w_{-j}^r z_j^2 + 2\left(w_j^r - w_{-j}^r\right) z_j \sum_{k \neq j} z_k \right] \tag{A14}$$

$$= \sum_{j=1}^{N} \left[-4\left(W_r - w_j^r\right) z_j^2 + 2\left(2w_j^r - W_r\right) z_j \sum_{k \neq j} z_k \right] \tag{A15}$$

$$= -2W_r \sum_{i=1}^{N} z_k^2 - 2W_r \sum_{i=1}^{N} \sum_{k=1}^{N} z_k z_k + 4 \sum_{i=1}^{N} \sum_{k=1}^{N} w_k^r z_k z_k \tag{A16}$$

$$= -2 \sum_{j=1}^{N} w_j^r \left[\sum_{i=1}^{N} z_k^2 + \left(\sum_{i=1}^{N} z_k \right)^2 - 2z_j \sum_{i=1}^{N} z_k \right] \tag{A17}$$

$$= -2 \sum_{j=1}^{N} w_j^r \left[\sum_{i \neq j} z_k^2 + \left(\sum_{i \neq j} z_k \right)^2 \right] \tag{A18}$$

$$< 0, \tag{A19}$$

which shows that matrix H_r is negative definite, and, according to the Rosen paper (Rosen, 1965), the game $\mathcal{G}_{(\mathbf{p},\mathbf{w})}$ admits a unique Nash equilibrium.

Next, we shall determine the closed form of the unique Nash equilibrium p^* and w^*. We have

$$\frac{\partial U_k(\mathbf{p})}{\partial p_k} = d_0 - 2b_k p_k + \left(\beta_k + b_k \sum_{r \in \mathcal{R}} \zeta_r^{cloud} h_k^r \right) \hat{q}_k + \sum_{\substack{j \neq k \\ j \in \mathcal{K}}} b_k^j p_j - \sum_{\substack{j \neq k \\ j \in \mathcal{K}}} \beta_k^j \hat{q}_j = 0 \tag{A20}$$

It follows that

$$2b_k p_k - \sum_{\substack{j \neq k \\ j \in \mathcal{K}}} b_k^j p_j = d_0 + \left(\beta_k + b_k \sum_{r \in \mathcal{R}} \zeta_r^{cloud} h_k^r\right) \hat{q}_k - \sum_{\substack{j \neq k \\ j \in \mathcal{K}}} \beta_k^j \hat{q}_j \tag{A21}$$

In other words, p_k^* is the unique solution to the previous linear equation (optimal condition). We can conclude that p^* is a solution to the following linear system:

$$\mathbf{S} \cdot \mathbf{p} = \mathbf{J} \tag{A22}$$

where $\forall k \in \mathcal{K}$, $\mathbf{S}_{kk} = 2b_k$, $\mathbf{S}_{kj} = -b_k^j$ for $k \neq j$, and

$$\mathbf{J}_k = d_0 + \left(\beta_k + b_k \sum_{r \in \mathcal{R}} \zeta_r^{cloud} h_k^r\right) \hat{q}_k - \sum_{\substack{j \neq k \\ j \in \mathcal{K}}} \beta_k^j \hat{q}_j.$$

Let us show that the linear system of equations in p admits a unique solution, meaning a unique Nash equilibrium exists. To do so, we have

$$\mathbf{S} = \begin{bmatrix} 2b_1 & -b_1^2 & \cdots & -b_1^K \\ -b_2^1 & 2b_2 & \cdots & -b_2^K \\ \vdots & \vdots & \ddots & \vdots \\ -b_K^1 & -b_K^2 & \cdots & 2b_K \end{bmatrix} = \mathbf{A}(\mathbf{I} - \mathbf{V}), \tag{A23}$$

where $\mathbf{I}$ denotes the identity matrix,

$$\mathbf{A} = (2b_1, \cdots, 2b_K),$$

$$\text{and,} \qquad \mathbf{V} = \begin{bmatrix} 0 & \frac{b_1^2}{2b_1} & \cdots & \frac{b_1^K}{2b_1} \\ \frac{b_2^1}{2b_2} & 0 & \cdots & \frac{b_2^K}{2b_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{b_K^1}{2b_K} & \frac{b_K^2}{2b_K} & \cdots & 0 \end{bmatrix}$$

Under Assumption 1, we note that matrix $\mathbf{V}$ is sub-stochastic as $\frac{\sum_{j \in \mathcal{K}, j \neq k} b_k^j}{2b_k} < 1$. Thus, $\mathbf{S}$ is invertible, with $\mathbf{S}^{-1} = \mathbf{A}^{-1}(\mathbf{I} - \mathbf{V})^{-1}$, and $\forall k \in \mathcal{K}$:

$$p_k^*(\hat{\mathbf{q}}) = \left[\mathbf{S}_{\mathbf{kk}}^{-1}\left(\beta_k + b_k \sum_{r \in \mathcal{R}} \zeta_r^{cloud} h_k^r\right) - \sum_{\substack{j \neq k \\ j \in \mathcal{K}}} \mathbf{S}_{\mathbf{kj}}^{-1} \beta_j^k\right] \hat{q}_k + \sum_{j=1}^{K} \mathbf{S}_{\mathbf{kj}}^{-1} d_0 + \sum_{j \neq k} \left[\mathbf{S}_{\mathbf{kj}}^{-1}\left(\beta_j + b_j \sum_{r \in \mathcal{R}} \zeta_r^{cloud} h_j^r\right) - \sum_{\substack{l \neq k \\ l \in \mathcal{K}}} \mathbf{S}_{\mathbf{kl}}^{-1} \beta_l^j\right] \hat{q}_j \tag{A24}$$

This completes the derivation of the closed form of the equilibrium price vector. For the closed form of the bidding, as the Nash equilibrium is unique, it is sufficient to show that the bidding w^* is given by (24). The KKT condition of the bidding is given by

$$\begin{aligned} -\left(1 - \frac{(W_r - w_k^r)}{(W_r)^2} \frac{\zeta_r^{cloud} C_r}{\delta_k^r}\right) &= 0, && \text{if } w_k^r > 0 \\ &\leq 0, && \text{if } w_k^r = 0 \end{aligned} \tag{A25}$$

Now, let us show that the solution w^* given by (24) satisfies the KKT conditions (A25). We have

$$\frac{W_r^* - (w_k^r)^*}{(W_r^*)^2} = \frac{\frac{(K-1)\zeta_r^{cloud}C_r}{K\delta^r} - \frac{(K-1)\zeta_r^{cloud}C_r}{K^2\delta^r}}{\left(\frac{(K-1)\zeta_r^{cloud}C_r}{K\delta^r}\right)^2} = \frac{\delta^r}{\zeta_r^{cloud}C_r} \tag{A26}$$

Thus,

$$\frac{(W_r^* - (w_k^r)^*)}{(W_r^*)^2}\frac{\zeta_r^{cloud}C_r}{\delta_k^r} = 1 \tag{A27}$$

which shows that w^* satisfies the KKT conditions (A25) and completes the proof of Theorem 2. □

Appendix A.3

Proof of Theorem 3. Since the matrix $\mathbf{Q}$ is negative definite, then, for any Nash equilibrium (p, w) of the game $\mathcal{G}_{(\mathbf{p}-,\mathbf{w})}$, $p = p^*$, where p^* is given by (23). So, it remains to show that bidding given by (25) is the unique Nash equilibrium. First, we can easily verify that the solution w^* given by (25) satisfies the KKT conditions (A25), and then $(p^*, w)^*$ is a Nash equilibrium of the game $\mathcal{G}_{(\mathbf{p}-,\mathbf{w})}$. Assume there is another Nash equilibrium $\varpi \neq w^*$, which satisfies (25). Then, by the necessity of the KKT conditions (A25), there exists a set $\mathcal{B} \neq \mathcal{B}^*$ such that ϖ is given by (25) in which the set $\mathcal{B}^*$ is replaced by $\mathcal{B}$. Thus, to have positive biding, the set $\mathcal{B}$ satisfies the following condition

$$\sum_{k' \in \mathcal{B}} \delta_k^r - (\mid A \mid -1)\delta_k^r > 0, \quad \forall k \in \mathcal{B} \tag{A28}$$

Since $\mathcal{B} \neq \mathcal{B}^*$ and w^* is Nash equilibrium, then there exists an SP $\hat{k}$ such that $\hat{k} \in \mathcal{B}^*$ and $\hat{k} \notin \mathcal{B}$. Hence, there exists an SP $\bar{k} \in \mathcal{B}$ such that $\delta_{\hat{k}}^r \leq \delta_{\bar{k}}^r$. We have

$$\sum_{k' \in \mathcal{B} \cup \{\hat{k}\}} \delta_{k'}^r - \mid \mathcal{B} \mid \delta_{\hat{k}}^r = \sum_{k' \in \mathcal{B}} \delta_{k'}^r + \delta_{\hat{k}}^r - \mid \mathcal{B} \mid \delta_{\hat{k}}^r \tag{A29}$$

$$= \sum_{k' \in \mathcal{B}} \delta_{k'}^r - (\mid \mathcal{B} \mid -1)\delta_{\hat{k}}^r \tag{A30}$$

$$> \sum_{k' \in \mathcal{B}} \delta_{k'}^r - (\mid \mathcal{B} \mid -1)\delta_{\bar{k}}^r \tag{A31}$$

$$> 0 \tag{A32}$$

where the third inequality is due to the fact that $\delta_{\hat{k}}^r \leq \delta_{\bar{k}}^r$ and the last inequality follows from (A28). Therefore, the SP $\hat{k}$ can improve its utility by making a positive bidding, which shows that ϖ is not a Nash equilibrium. □

Appendix A.4

Proof of Proposition 1. To show the uniqueness, just set $\bar{\delta}_k^r = v_k^r \delta_r^k$, and the proof is similar to the proof of Theorem 3. For the second part of the theorem, if the price differentiation is chosen such that $v_k^r \delta_r^k = \delta^r$, the KKT conditions (A25) become

$$-\left(v_k^r - \frac{(W_r - w_k^r)}{(W_r)^2}\frac{\zeta_r^{cloud}C_r}{\delta_k^r}\right) \begin{cases} = 0, & \text{if } w_k^r > 0 \\ \leq 0, & \text{if } w_k^r = 0 \end{cases} \tag{A33}$$

Thus,

$$-\left(1-\frac{\left(W_r-w_k^r\right)}{\left(W_r\right)^2}\frac{\zeta_r^{cloud}C_r}{\delta^r}\right) \begin{cases} =0, & \text{if } w_k^r>0 \\ \leq 0, & \text{if } w_k^r=0 \end{cases} \quad \text{(A34)}$$

We obtain the same KKT conditions in the case where $\delta_k^r = \delta^r$, and, therefore, the Nash equilibrium is given by (23) and (24). □

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