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Map Projections Classification

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Abstract: Many books, textbooks and papers have been published in which the classification of map projections is based on auxiliary (developable) surfaces and projections are divided into conic, cylindrical and azimuthal projections. We argue that such a classification of map projections is unacceptable and give many reasons for that. Many authors wrote in more detail about the classification of map projections, and our intention is to give a new refined and rectified insight into the classification of map projections. Our approach can be included in map projection publications of general and thematic cartography. Doing this, misconceptions and unnecessary insistence on conceptuality instead of reality will be avoided.

Keywords: map projections; classification; developable surfaces; distortions; graticule; pseudograticule; projection aspect

1. Introduction

So far, many books and textbooks of general, thematic and even mathematical cartography have been published, as well as articles in journals, in which one of the classifications of map projections is based on auxiliary (developable) surfaces and projections are divided into conic, cylindrical and azimuthal projections. This classification is illustrated by images in which the surface of a cone and the surface of a cylinder or a plane touch or intersect the Earth's sphere (Figure 1).
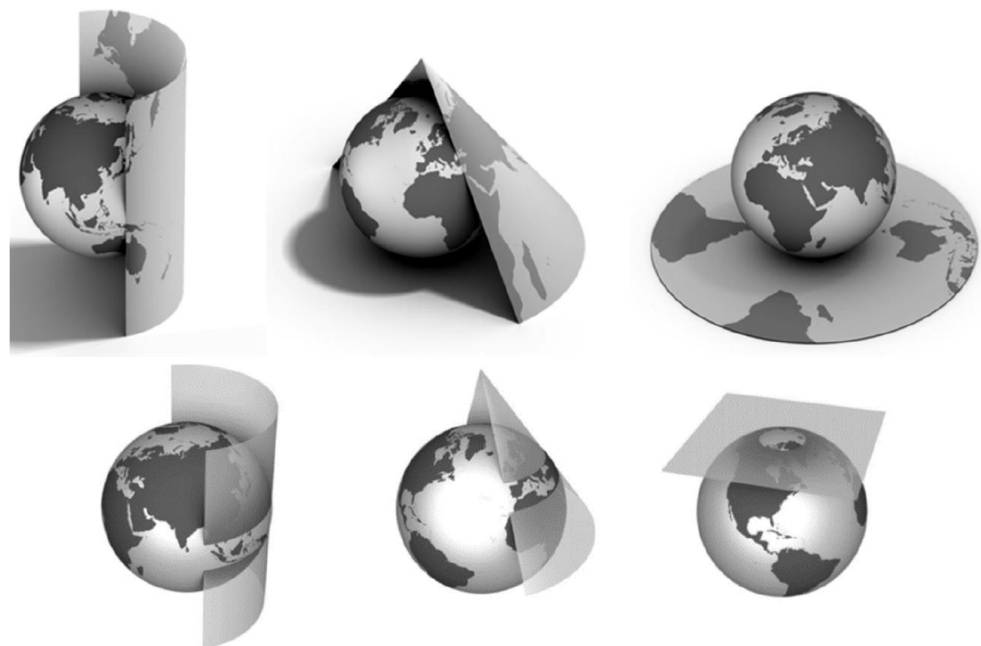
Figure 1. The three developable projection surfaces: cylinder, cone and plane. (Only half of the cylinder and cone are shown.) By permission of Charles Preppernau, the author of the image.

When searching for a term, many of us will first look for a solution on the Internet, and most often it will be Wikipedia. We can find this description there: "A fundamental projection classification is based on the type of projection surface onto which the globe is conceptually projected. The projections are described in terms of placing a gigantic surface in contact with the Earth, followed by an implied scaling operation. These surfaces are cylindrical (e.g., Mercator), conic (e.g., Albers), and plane (e.g., stereographic). Many mathematical projections, however, do not neatly fit into any of these three conceptual projection methods." [1]. The same or very similar approach can be found in many references, e.g., in [2–12].

There are several reasons why this classification of map projections is not recommended.

1.1. History

- The authors of the oldest known cylindrical and conic projections did not define their projections using indirect, developable surfaces. For example, Mercator, in the 16th century, did not use a cylindrical surface to define the projection now called Mercator's cylindrical projection, or Lambert, in the 18th century, did not use a conical surface to define the projection now called Lambert's conformal conic projection. On the contrary, after deriving the equations of that projection, Lambert [13] mentioned that the map made in that projection could be folded into a cone.
- It seems that developable surfaces related to map projections appear for the first time in the second half of the 19th century. D'Avezac-Macaya [4] referred to "les projections perspectives, les developements de surfaces osculatrices ou pénétrantes", that is, tangent or secant surfaces, specifically "les développements cylindriques" and "cóniques", and the catch-all "les systèmes conventionneles". Snyder [3] did not find any earlier cartographic references to developable surfaces. Let us mention the famous older authors who dealt with map projections, not to mention developable surfaces: Lambert [13], Euler [14], Lagrange [15], Gauss [16] and Tissot [17].

1.2. Conceptuality

- One of the most common classifications, and the one principally used by Snyder [3], is based on association, at least conceptually, to a developable surface.
- Map projections are mappings of a curved surface, most often a sphere or ellipsoid, into a plane. The introduction of developable surfaces as intermediate surfaces and the interpretation according to which a sphere or ellipsoid is first mapped to such a surface which then develops into a plane is a fabrication that generally does not correspond to reality. It is usually justified by the term *conceptual* [2] and explained by the claim that map projections are easier to interpret in this way.
- "The reference globe and developable surfaces are conceptual aids that help illustrate the projection process, but they are not used to create projections today. Rather, the field of mathematics is utilized to create projections, and so it is important to understand some of the basic mathematical manipulations used to project the Earth onto a map." [9]. The first part of the first sentence is questionable because developable surfaces are not used in the projection process, in general. The rest of the quotation we fully accept. As everybody can see, there are no developable surfaces in the whole chapter 8.3 of the cited book.
- "The classification of projections according to developable surfaces (cylinder, cone, and plane) is useful for the comprehension of selecting projections and their parameters. However, while developable surfaces are a useful conceptual tool, it needs to be emphasized that most map projections cannot be constructed geometrically but are instead defined mathematically" [18]. It remains unclear why Šavrič et al. [18] mentioned the classification of map projections using developable surfaces when they are not used in their *Projection Wizard*.
- "While the concept of developable surfaces provides a nice way to visualize the basics of map projections, as stated before, the transformation is really driven by

mathematical equations" [10]. The basics of map projections are not connected to developable surfaces at all.

1.3. Prior Warning

- This approach to the classification of map projections is common. Map projections are typically classified according to the geometric surface from which they are derived: cylinder, cone or plane. However, such an approach is only correct at first glance. In fact, the opposite is true. Cylindrical projections are not called so because of the mapping on a cylindrical surface but because the map made in such a projection can be bent into a cylinder. Similarly, conic projections are not called so because of mapping to a conical surface but because a map made in a conic projection can be bent into a cone. More than 100 years ago, in a paper on map projections, Close and Clarke [19] wrote: "Conical projections are those in which the parallels are represented by concentric circles and the meridians by equally spaced radii. There is *no necessary connexion* between a conical projection and any touching or secant cone. *The name conical* is given to the group embraced by the above definition, because as is obvious, a projection so drawn can be round to form a cone." In a well-known paper on the nomenclature and classification of map projections, Lee [20] explained: "*Cylindric*: projections in which the meridians are represented as a system of equidistant parallel straight lines, and the parallels by a system of parallel straight lines at right angles to the meridians.
  - *Conic*: projections in which the meridians are represented as...
  - *Azimuthal*: projections in which the meridians are represented as...

No reference has been made in the above definitions to cylinders, cones or planes. The projections are termed cylindric or conic because they can be regarded as developed on a cylinder or cone, as the case may be, but it is as well to dispense with picturing cylinders and cones, since they have given rise to much *misunderstanding*."

1.4. Isometry

- Authors who nowadays describe map projections in great detail with the help of developable surfaces may not even be aware of the fact that, in this way, they introduce double mappings into the theory of map projections. First, the Earth's sphere is mapped to the auxiliary developable surface, and then it is transformed in some way, e.g., by development, into a map in the plane. Double mappings have their role in the theory of map projections in some special cases, not in general. One should know that developing into a plane is isometry, i.e., such a mapping that preserves distances.
- The use of development surfaces in the definition of map projection is justified only for a small number of projections, projections in which the mapping is real and not conceptual to the corresponding developable surface. Namely, in addition to cylindrical and conic projections, there are many others, such as azimuthal, pseudocylindrical, pseudoconic, conditional, etc., which cannot be interpreted by mapping to a cylinder or cone surface. There are some attempts in that direction. For example, it is said that a pseudocylindrical projection is a mapping to a pseudocylinder but without saying what a pseudocylinder is [7]. Or that such a projection is interpreted as a mapping to an oval surface, without noticing that it is not a developable surface [8].
- In the context of the classification of projections into conical, cylindrical and azimuthal/plane, it is unnatural to use a plane as a developable surface. Namely, developing is isometry, and thus we see no purpose in developing plane into plane.
- Developing the auxiliary surface into a plane (Figure 2) preserves distances (isometry). This would then mean that the two selected parallels as standard parallels in all normal aspect projections are mapped to parallels that are at the same distance from each other. Is that possible? Of course not.
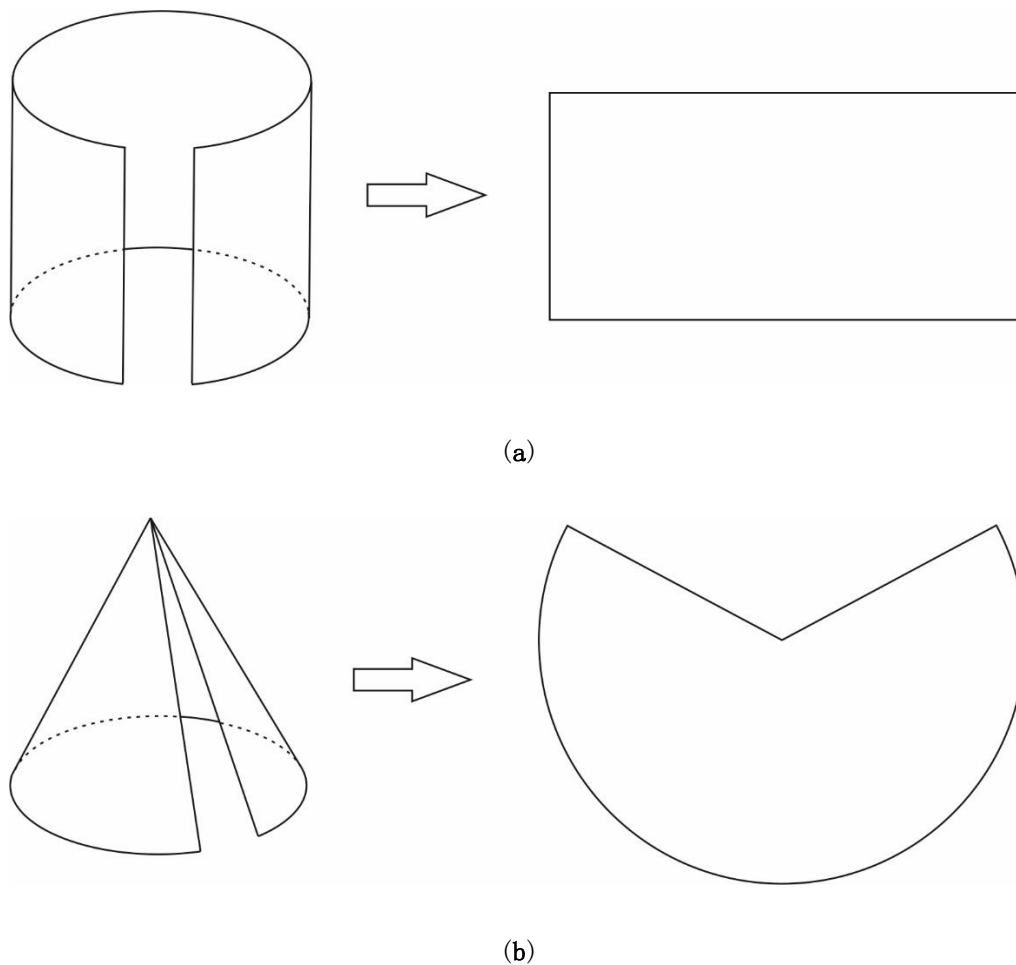
Figure 2. (**a**) Cylindrical surface cut from base to base and development of the cylindrical surface. (**b**) Conical surface cut from base to apex and development of the conical surface.

1.5. Standard vs. Secant Parallels

- Authors who perceive map projections as mappings with the help of an auxiliary surface regularly distinguish the case of touching and cutting. For them, as a rule, the cross-sectional curve is also a curve without deformations. They take that fact for granted, without proof. However, it has been shown that standard parallels and secant parallels are two different concepts and that they generally do not coincide [21–23].
- "Where the ellipsoid and the map projection surface touch, in this case, intersect, there is no distortion. However, between the standard lines the map is under the ellipsoid and outside of them the map is above it." [5,6]. From this quote, it is clear that van Sickle did not distinguish between secant and standard parallels.
- The use of developable surfaces leads to secant projections, i.e., mappings on the auxiliary developable surface that intersects the mapped surface and thence the conclusion that azimuthal projections can have at most one standard parallel and conic at most two. This is wrong because there are azimuthal and conic projections with several standard parallels [24,25].
- It is generally accepted that a secant projection is a mapping on an auxiliary surface that intersects a sphere or an ellipsoid and that the intersections of the secant conic and cylindrical projection are without deformations (Figure 3). Thus, for example, in ESRI's dictionary [11], we were able to read several years ago: "Secant projection: A projection whose surface intersects the surface of a globe. A secant conic or cylindrical projection, for example, is recessed into a globe, intersecting it at two circles. At the lines of intersection, the projection is free from distortion." This definition is not

good. First, the intersection of two second-order surfaces is a fourth-order curve that generally does not consist of two circles. However, this shortcoming could be corrected by specifying the mutual position of the surface of the cone or cylinder and the sphere. However, the claim that the line of intersection of two surfaces is free of deformation cannot be an integral part of the definition because, in addition to being a claim, it is generally *erroneous*. However, it is so ingrained that it is almost regularly taken as true without evidence. The cited webpage is no longer available, but ESRI retains the term secant projection within the definitions of cylindrical and conical projections [11].
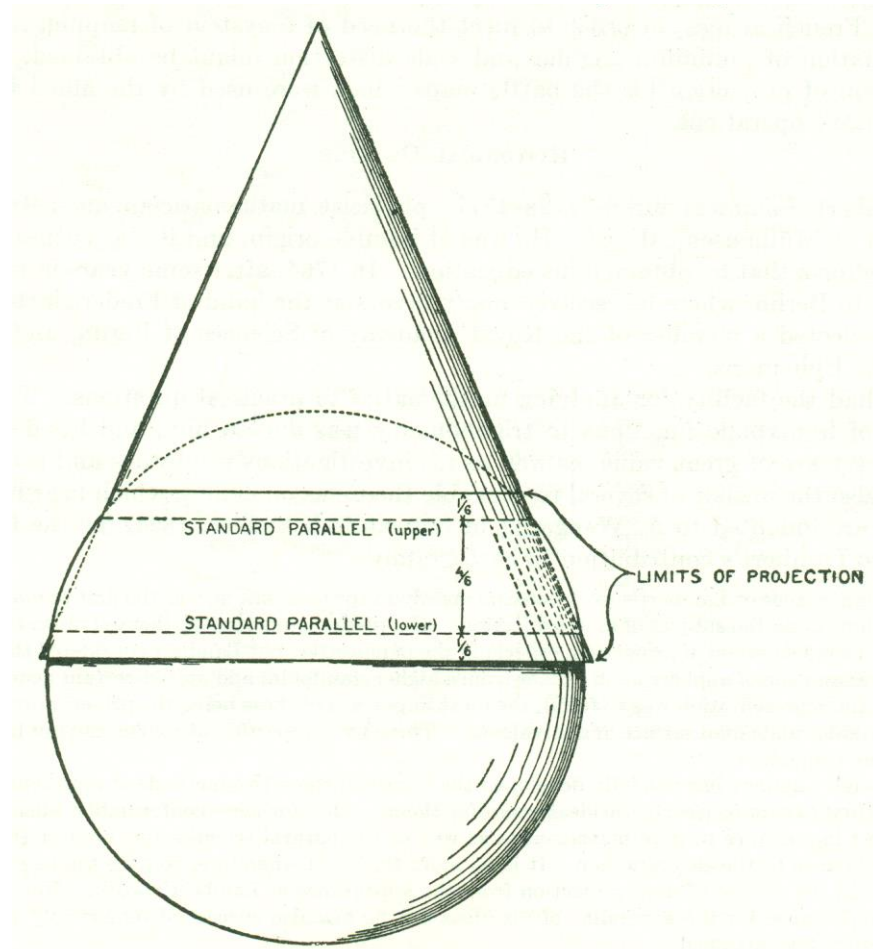

Figure 3. Illustration of standard parallels as secant parallels. Source: Deetz and Adams [26], page 80. This is a misguided approach accompanied by the caption: "Diagram illustrating the intersection of a cone and sphere along two standard parallels". In other words, for Deetz and Adams (and many others) standard parallels and secant parallels are identical. The proof is missing, and this is a fake.

1.6. Conclusions

- Instead of a conceptual approach that is also wrong in some parts, it is necessary to return to reality and not run away from mathematics. Let us just recall that the study of map projections was not so long ago called *mathematical cartography*.
- Explaining the mapping of a sphere or ellipsoid to the surface of a cylinder or cone is a convenient story, which, unfortunately, while avoiding the mathematical background of the process, leads to erroneous claims which, most likely, the authors are not aware of.
- It is possible to communicate perfectly without the introduction of auxiliary surfaces. A good example of this are the books by Bugayevskiy and Snyder [27], Bugayevskiy [28] and Kessler and Battersby [29].

D'Avezac de Castera-Macaya [4], Deetz and Adams [26], Maurer [30], Urmayev [31], Lee [20], Kavrayskiy [32], Tobler [33], Solov'ev [34], Richardus and Adler [35], Maling [36], Konusova [37], Starostin et al. [38], Vakhrameeva et al. [39], Snyder [40], Bugayevskiy and Snyder [27] and Kessler and Battersby [29], for example, wrote in more detail about the classification of map projections. Therefore, it is not our intention to give some new insights into the classification of projections but to write a text that can be included in the chapters on map projections in textbooks of general and thematic cartography and that everything is correct.

2. Map Projections Classification

There is an infinite number of different map projections, and several hundred are known [3]. To make them easier to study and therefore select for practical application, we divide them into groups.

As a basis for the division of map projections are usually taken:

1. Properties of mapping or types of distortions;
2. The shape of a (pseudo)graticule;
3. Projection aspect, i.e., position of the axis of projection in relation to the axis of rotation of the sphere.

2.1. Classification of Map Projections According to Types of Distortions

According to the types of distortions, map projections are divided into these four groups: conformal (equiangular), equivalent (equal-area), equidistant in a certain direction (equal-length) and compromise. Let us define the local linear scale factor as usual:

$$c = \frac{ds\prime}{ds} \tag{1}$$

where ds' is a differential of arc length in the plane of projection, and ds is the appropriate differential of arc length on the original surface, usually a sphere or ellipsoid. Analogously, the local area scale factor is

$$p = \frac{dp\prime}{dp} \tag{2}$$

where dp' is a differential of an area in the plane of projection, and dp is the appropriate differential of the area on the original surface.

In conformal projections, the distortion of the angles is equal to zero, and the linear scale at a point is equal in all directions, i.e., it does not depend on the azimuth. Because angle distortion in the conformal projections is zero and the scale does not depend on the azimuth at each point, the similarity of the corresponding infinitesimal figures on the ellipsoid and in the plane is maintained, hence their name conformal, i.e., similar in form.

In equal-area projections, the equality of the area of the corresponding figures on the ellipsoid and in the projection is preserved. Therefore, in equal-area projections, the local area scale factor is equal to one in the entire map field.

In equidistant projections, the local linear scale factor is equal to one along one of the main directions, i.e., the directions along which the linear scale has the largest and smallest value. Equidistant projections are in the middle between conformal and equal-area projections by types and magnitudes of distortions. Namely, in equidistant projections, area distortions are smaller than in conformal projections, and angle distortions are smaller than in equal-area projections.

The group of compromise projections includes all those projections that are neither conformal nor equivalent nor equidistant.

2.2. Classification of Projections According to the Shape of the Pseudograticule

The network of pseudomeridians and pseudoparallels (pseudograticule) is obtained by rotating the network of meridians and parallels (graticule) to any other position

(Figures 4 and 5). It is obvious that if there is no mentioned rotation, then the pseudo-graticule coincides with the graticule.
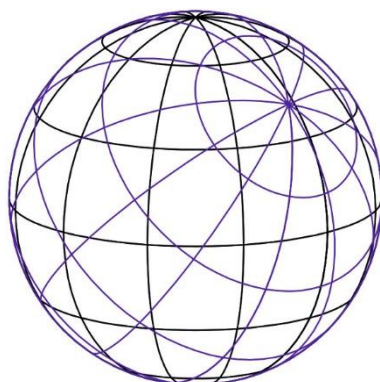

Figure 4. Graticule (black) and pseudograticule (blue) on a sphere.
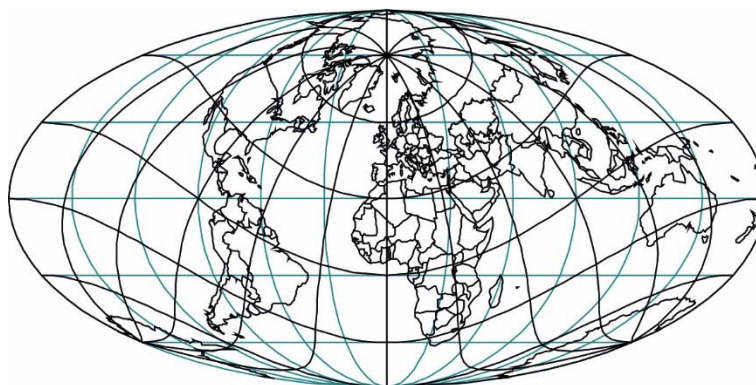

Figure 5. Image of the graticule (black) and the pseudograticule (blue) in the oblique Mollweide projection.

The pseudograticule has the important property that, regardless of the position of the axis, i.e., the aspect of the projection mapped into the plane, it retains the characteristic shape for a particular group of projections and thus allows a unique definition of each group of projections. The following are definitions of the most important groups of projections using a pseudograticule [41,42].

- *Cylindrical projections* are projections in which pseudomeridians are shown in mutually parallel straight lines and pseudoparallels in mutually parallel straight lines perpendicular to images of pseudomeridians (Figure 6a). Cylindrical projections are not mappings onto a cylindrical surface in general. A map made in a cylindrical projection can be folded into a cylindrical surface.
- *Conic projections* in the narrower sense are projections in which pseudomeridians are represented by straight lines that intersect at one point and pseudoparallels by concentric circular arcs, with the angle between any two pseudomeridians being less than the difference of the corresponding pseudogeographic longitudes (Figure 6b). Conic projections are not mappings onto a conical surface in general. A map made in a conic projection can be folded into a conical surface.
- *Azimuthal projections* are projections in which pseudomeridians are shown as straight lines that intersect at one point and pseudoparallels by concentric circles, with the angle between any two pseudomeridians being equal to the difference of the corresponding pseudogeographic longitudes (Figure 6c). Azimuthal projections are planar projections, but this is not their special characteristic because all map projections are mappings onto a plane.

- *Pseudocylindrical projections* are projections in which pseudomeridians are represented by curves symmetrical with respect to the middle pseudomeridian mapped as a straight line and pseudoparallels as mutually parallel straight lines perpendicular to the image of the central pseudomeridian (Figure 6d).
- *Pseudoconic projections* are projections in which pseudomeridians are represented by curves symmetric with respect to the middle pseudomeridian mapped as a straight line and pseudoparallels as arcs of concentric circles (Figure 6e).
- *Polyconic projections* are projections in which pseudomeridians are mapped as curves symmetric with respect to the central pseudomeridian which is mapped as a straight line, and pseudoparallels are mapped as eccentric circles whose centers are on the central pseudomeridian (Figure 6f).
- *Circular projections* are projections in which pseudomeridians are mapped as arcs of circles symmetrical in relation to the central pseudomeridian which is mapped as a straight line, and pseudoparallels are mapped as arcs of circles whose centers are on the central pseudomeridian (Figure 6g).
- *Other projections* are projections in which pseudomeridians and pseudoparallels are mapped as more complex curves (Figure 6h).
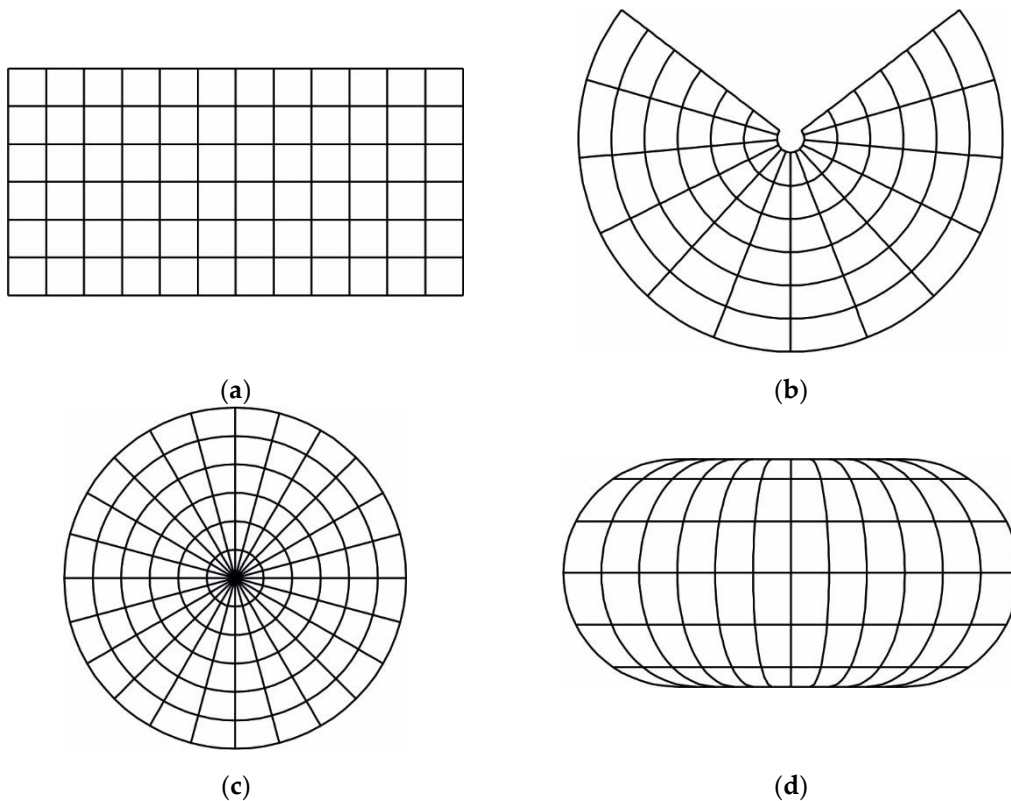
(a)

(b)

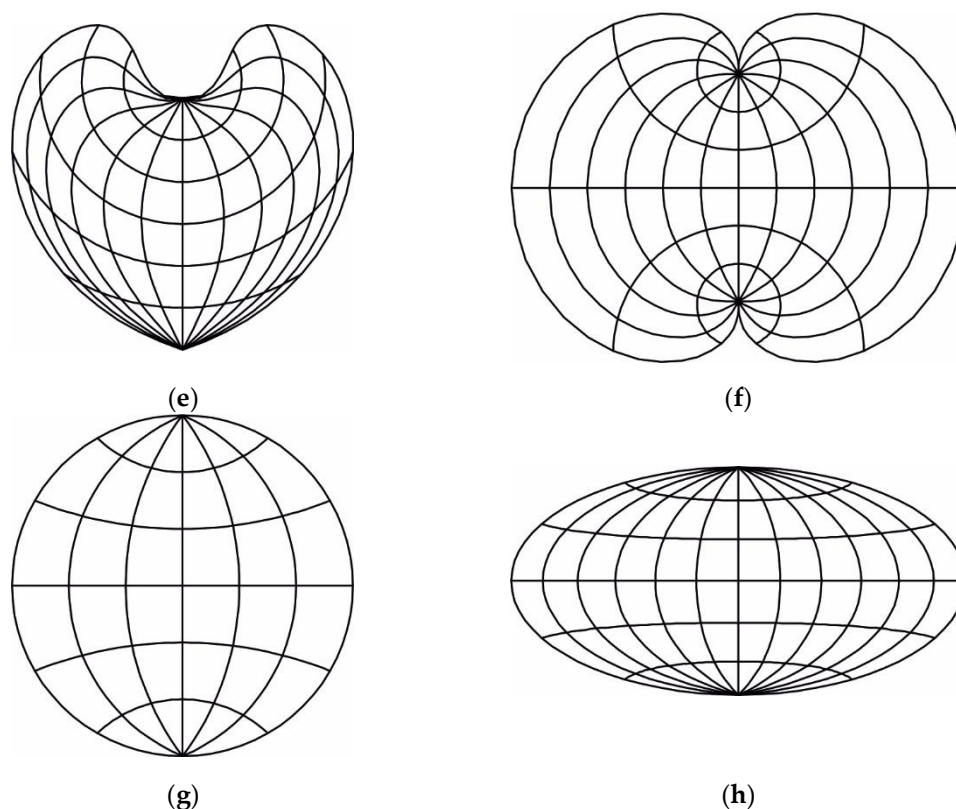**(c)**

(d)

Figure 6. *Cont.*

Figure 6. Classification of projections according to the shape of the pseudograticule. It is also the graticule in normal aspect projections (see next section): (**a**) cylindrical; (**b**) conic; (**c**) azimuthal; (**d**) pseudocylindrical; (**e**) pseudoconic; (**f**) polyconic; (**g**) circular; (**h**) other.

2.3. Classification of Projections According to the Aspect of Projection

The classification of projections according to the position of the axis of projection in relation to the axis of rotation of the sphere is applied primarily to the mapping of the sphere. Snyder [40] (p. 29) wrote: "While it is fairly straightforward to apply a suitable transformation to the sphere, transformation is much more difficult on the ellipsoid because of the constantly changing curvature. Transformation has been applied to the ellipsoid, however, in important cases under certain limiting conditions." There is another possibility to map the ellipsoid to a sphere and then the sphere to a plane. In this case, this classification also applies to ellipsoid mapping.

The straight line passing through the poles of the pseudograticule is the axis of projection. The position of the axis of the projection in relation to the axis of rotation of the sphere parameterized by geographical parameterization is called the aspect of the projection. The aspect can be normal, transverse or oblique.

- The *normal aspect* is the aspect in which the axis of projection coincides with the axis of the sphere, and the pseudograticule coincides with the graticule (Figure 7a).
- The *transverse aspect* is the aspect in which the axis of projection is perpendicular to the axis of the sphere (Figure 7b).
- The *oblique aspect* is an aspect that is neither normal nor transverse (Figure 7c).
- It should be emphasized that instead of the normal, transverse and oblique aspect of map projection, the terms normal, transverse and oblique projection are also used. The pseudograticule is also called in the literature the network of verticals and almucantars, and the image of that network in the projection plane is a normal cartographic network, i.e., a network that has a simpler shape in a given group of projections than any other network [26].
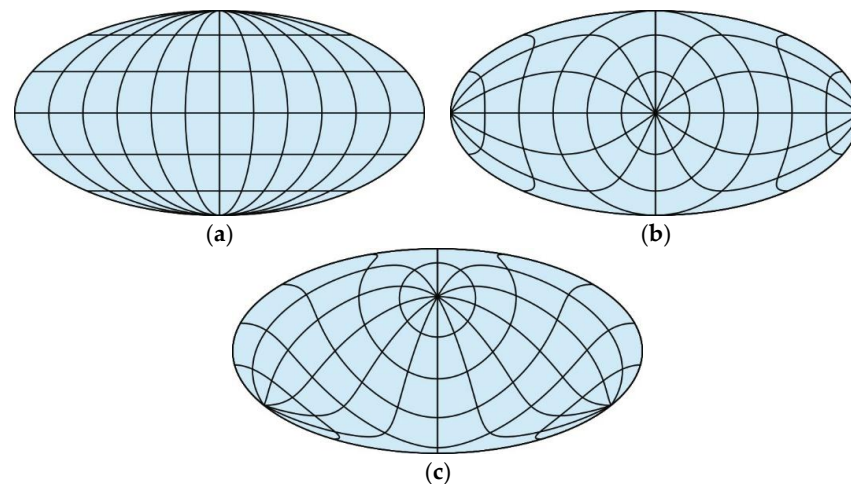
Figure 7. Image of the graticule in the Mollweide (**a**) normal aspect projection, (**b**) transverse aspect projection, (**c**) oblique aspect projection.

Most often, the aspect of projection is explained on maps of the world, but it should be emphasized that maps of an area, such as maps of North America (Figure 8), can be in all three aspects [43].
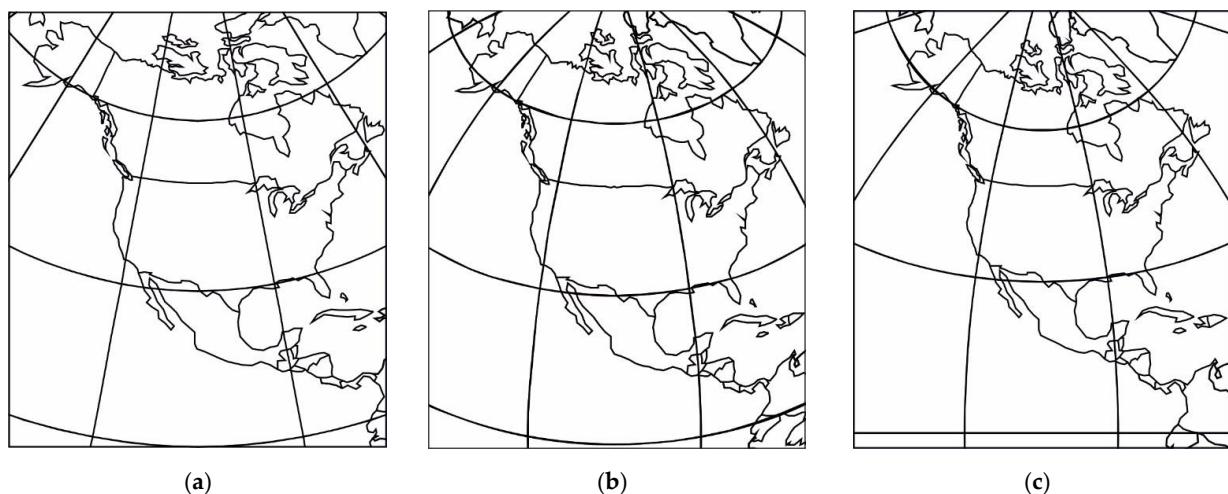

Figure 8. North America in (**a**) normal conic, (**b**) oblique azimuthal and (**c**) transverse cylindrical projection.

3. Conclusions

It is not wise to use intermediate developable surfaces in the interpretation of map projections in general because:

- Most map projections in their definition do not have an auxiliary surface.
- The application of the auxiliary/intermediate developable surface can lead to the wrong conclusion about the distortion distribution—standard parallels (parallels with zero distortion) and secant parallels generally do not coincide.
- It is not recommended to classify map projections based on the developable surfaces because such a classification (1) cannot fit many map projections, (2) implicitly or explicitly gives a high weight to the developable surfaces in the theory of map projection which they do not deserve, (3) misleads that standard parallels and secant parallels are one and the same, which is not true in general, and (4) states the wrong conclusion that there are other developable surfaces (pseudocylinders, ovals) by which pseudocylindrical and other map projections can be defined.

- Cylindrical, conical and azimuthal projections belong to the classification of map projections according to the shape of the graticule in normal aspect projections. In such a classification, naturally enter pseudocylindrical, pseudoconical, polyconic and other projections.

Furthermore, there is no need to use developable surfaces in map projection classification. Map projections can be classified into several classes or groups according to different criteria:

1. According to kinds of distortions: conformal, equal-area, equidistant and other;
2. According to aspect: normal, transversal, oblique;
3. According to the form of the graticule in normal aspect: cylindrical, conic, azimuthal, pseudoconic, pseudocylindrical, polyconic, circular and others.

In map projections theory, we do not need conceptuality. We only need reality.

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