

Article

Mapping Approaches for Assessing Regional Soil Liquefaction Potential: A Comparative Assessment

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Abstract

Seismic liquefaction poses a significant threat to infrastructure and human safety, making accurate regional-scale hazard assessment essential for effective disaster mitigation. This study systematically compares simple kriging (SK), ordinary kriging (OK), and sequential Gaussian simulation (SGS) for liquefaction potential mapping using a case study of the Chi-Chi earthquake. Results show that among the eight theoretical semivariogram models, the exponential model was identified as the optimal semivariogram model, showing the best fit and the lowest validation errors. SK and OK yield nearly identical LPI estimates with high accuracy and low computational cost, but oversmooth high-risk zones and underestimate spatial uncertainty. In contrast, SGS captures spatial heterogeneity and extreme values, with variance and coefficient of variation maps revealing higher and more heterogeneous uncertainties, which offers a more comprehensive basis for probabilistic risk assessment. However, these advantages came at a substantially higher computational cost. The choice of method should therefore depend on the engineering objective: kriging for rapid, large-scale trend estimation, and SGS for detailed probabilistic risk evaluation where capturing extreme values and uncertainty is critical.

Keywords: liquefaction mapping; kriging; random field; semivariogram; uncertainty



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1. Introduction

Soil liquefaction triggered by earthquakes is a major geohazard of critical concern in geotechnical earthquake engineering. Historical seismic events have repeatedly demonstrated that liquefaction can cause severe damage to infrastructure and foundations, posing significant threats to human safety and resulting in large economic losses [1]. Therefore, accurate assessment of liquefaction potential and associated risks is essential for effective disaster prevention and mitigation.

Liquefaction assessment approaches can be broadly categorized into two complementary paradigms: location-specific liquefaction potential analysis and regional liquefaction potential mapping. Location-specific analyses aim to determine whether a given soil unit at a particular site is susceptible to liquefaction under a specific seismic loading, typically

producing a binary or probabilistic outcome for that individual location. In recent years, significant advances have been made in this category using data-driven and probabilistic methods [2–5]. However, such location-specific analyses are inherently confined to individual borehole or CPT positions and cannot provide spatially continuous hazard information across a site. In contrast, regional liquefaction potential mapping serves as a fundamental tool for seismic safety evaluation at the site or regional scale. It provides spatially continuous, probability-based estimates that integrate liquefaction susceptibility and seismic input to delineate high-risk zones and support the design of site-specific ground improvement measures [6–8].

Early liquefaction mapping efforts were primarily grounded in geological interpretation, relying on limited subsurface data to achieve a qualitative to semi-quantitative characterization of hazard [9,10]. For instance, Broughton et al. [11] developed geological liquefaction susceptibility maps and qualitatively verified them using borehole logs, while Baise et al. [12] introduced the liquefaction potential index (LPI) for regional mapping by associating estimated liquefied extents with geological units. With the development of geostatistical approaches, Liu et al. [13] employed random field models to map liquefaction potential at both sampled and unsampled locations, and Bahari et al. [14] generated liquefaction hazard maps using deterministic and reliability-based analyses. More recently, Chen et al. [15] integrated multiscale random field and geostatistical models into a CPT-based framework to account for soil spatial variability, with a case study in the Marina District of San Francisco. Juang et al. [7] further established a high-resolution 3D synthetic soil model to compare the performance of random field methods under varying data availability using information theory criteria. At broader scales, Bozzoni et al. [16] proposed a GIS-based framework coupling data-driven and knowledge-driven techniques for continental-scale liquefaction risk mapping across Europe. Xu et al. [17] conducted a three-dimensional liquefaction assessment for the Yellow River Subaquatic Delta using in-situ CPT data, compared inverse distance weighting and full-cokriging methods in predicting liquefaction vulnerability index, and verified that full-cokriging with cone tip resistance and sleeve friction achieves higher accuracy than ordinary kriging.

Collectively, kriging-based interpolation and random field-based simulation have become the dominant approaches for regional liquefaction mapping, particularly under conditions of limited field investigation. However, most existing studies have concentrated on the deployment of individual techniques within specific case settings, with comparatively little attention devoted to head-to-head comparisons that investigate their respective theoretical underpinnings, estimation accuracy, and representation of spatial uncertainty. Furthermore, the trade-offs between smooth trend estimation, extreme-value preservation, and computational expense are significant and practical, as they fundamentally govern the fitness of each method for distinct engineering decision contexts.

To address these gaps, this study presents a systematic comparison of three liquefaction potential mapping methods: simple kriging (SK), ordinary kriging (OK), and sequential Gaussian simulation (SGS). First, the theoretical frameworks of the semivariogram, kriging, and random field approaches are introduced. Second, the optimal semivariogram model is selected and its influence on liquefaction mapping is evaluated. Third, using the optimal model, liquefaction risk maps are generated for a case study, and the methods are compared in terms of accuracy, spatial variability representation, and computational efficiency. By clarifying the strengths and limitations of each approach, this study aims to provide practical guidance for selecting appropriate mapping techniques to support risk-informed design and mitigation of liquefaction hazards.

2. Assessment of Liquefaction Mapping Approaches

2.1. Semivariogram

Spatial interpolation relies on the structural analysis of spatial correlations among sample data, which is based on the semivariogram theory. Semivariograms characterize key attributes of regionalized variables, and the LPI was selected as the regionalized variable in this study.

The semivariogram function, $\gamma(h)$, was originally defined by Matheron [18] as half the mean squared difference between attribute values at two spatial points separated by a lag distance h , and its calculation is given by:

$$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} (Z(x_i) - Z(x_i + h))^2 \quad (1)$$

The empirical semivariogram provides a description of how the data correlate with distance, where $N(h)$ is the set of all pairwise Euclidean distance h , $Z(x_i)$ and $Z(x_i + h)$ are data values at spatial locations x_i and $x_i + h$, respectively. In essence, the semivariogram is a core function that describes the spatial dependence of the stochastic process $Z(x_i)$ for spatially distributed random fields. A scalar form of the vector lag distance, denoted as h , is calculated using the following equation:

$$h = \sqrt{\left(\frac{h_1}{a_1}\right)^2 + \left(\frac{h_2}{a_2}\right)^2 + \left(\frac{h_3}{a_3}\right)^2} \quad (2)$$

The vector lag distance h is a standardized metric that accounts for spatial anisotropy, incorporating both magnitude parameters (the range parameters a_1, a_2, a_3) and directional parameters (the principal directions of spatial continuity h_1, h_2, h_3) [19].

Under the condition of second-order stationarity (where statistical properties are invariant with spatial location and dependent only on the lag vector h), the covariance function $C(h)$ has a direct mathematical relationship with the semivariogram $\gamma(h)$, expressed as:

$$C(h) = \sigma^2 - \gamma(h) \quad (3)$$

To fit the empirical semivariogram and capture its inherent spatial structural characteristics, eight theoretical semivariogram models were adopted in this study: the spherical, pentaspherical, bounded linear, circular, exponential, Gaussian, stable, and Whittle models. These models are designated as M1 to M8, with their formulations given in Table A1 [20,21]. This study primarily investigates the influence of different theoretical semivariogram model selections on the accuracy of regional soil liquefaction potential assessment.

2.2. Kriging-Based Approaches

Kriging is a local estimation technique that uses the best linear unbiased estimator (BLUE) to predict unknown spatial-temporal variable values, considering both spatial distance and variation degree between known data points. Its core lies in determining weight coefficients to ensure unbiased estimation and minimize estimation variance [22,23]. Representative kriging methods include simple kriging (SK), ordinary kriging (OK), block kriging (BK), indicator kriging (IK), universal kriging (UK), and co-kriging (CK), each with distinct assumptions and applications [24].

This study adopts the widely used simple kriging (SK) and ordinary kriging (OK). SK requires variables to be second-order stationary with a known mean, whereas OK has broader applicability and can be used when the variable mean is unknown. Since the mean of the dataset is known in this study, both SK and OK are suitable for the analysis.

The spatial variance structure of SK can be characterized by the covariance or variance function (see Equation (3)). To ensure unbiasedness and minimal estimation error variance, weights are solved via the following system of linear equations [25]:

$$\sum_{i=1}^n \lambda_i C(x_i - x_j) = C(x_0 - x_j) \quad (4)$$

The matrix form of Equation (4) is:

$$\begin{bmatrix} C_{11} & \dots & C_{1n} \\ \vdots & & \vdots \\ C_{n1} & \dots & C_{nn} \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_n \end{bmatrix} = \begin{bmatrix} C_{01} \\ \vdots \\ C_{0n} \end{bmatrix} \quad (5)$$

where $C_{ij} = C(x_i - x_j)$ representing the value of the covariance between x_i and x_j ; C_{ii} is the variance value. Once the theoretical semivariogram $\gamma(h)$ is characterized, it will be plugged into Equation (3) to calculate $C(h)$. Thus, the weights λ_i could be drawn by Equation (5) and these generated values are substituted into Equations (6) and (7) to find the estimated value $Z_{SK}^*(x_0)$ and estimated variance σ_{SK}^2 .

$$Z_{SK}^*(x_0) = m + \sum_{i=1}^n \lambda_i [Z(x_i) - m] \quad (6)$$

$$\sigma_{SK}^2 = \sigma^2 - \sum_{i=1}^n \lambda_i C(x_0 - x_i) \quad (7)$$

OK is the most robust and widely used kriging estimator. OK requires that all locations x and lag distances h satisfy two assumptions: the mean exists and is independent of location x ; and the variable has a finite variance for any h . To ensure unbiased estimation with minimum estimation variance, the weights must be determined using the following system of linear equations.

$$\sum_{i=1}^n \lambda_i \gamma(x_i - x_j) + \mu = \gamma(x_j - x_0) \quad (8)$$

The matrix form of Equation (8) is:

$$\begin{bmatrix} \gamma_{11} & \dots & \gamma_{1n} & 1 \\ \vdots & & \vdots & \vdots \\ \gamma_{n1} & \dots & \gamma_{nn} & 1 \\ 1 & \dots & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \vdots \\ \lambda_n \\ \mu \end{bmatrix} = \begin{bmatrix} \gamma_{01} \\ \vdots \\ \gamma_{0n} \\ 1 \end{bmatrix} \quad (9)$$

Once the theoretical semivariogram model is selected, the corresponding values between two sample points can be obtained. λ_i denotes the weight. Owing to $\sum_{i=1}^n \lambda_i = 1$, a Lagrange multiplier μ must be introduced when minimizing the estimation variance.

Using the computed weights and Lagrange multiplier, the unknown value and estimation variance at location x_0 can be calculated by the following formulas.

$$Z^*(x_0) = \sum_{i=1}^n \lambda_i Z(x_i) \quad (10)$$

$$\sigma_{OK}^2 = \sum_{i=1}^n \lambda_i \gamma(x_0 - x_i) + \mu \quad (11)$$

From the above, the OK method is essentially a weighted average of the sample data to obtain the estimated value and estimation variance at the site, thereby deriving the probability distribution at each unsampled location.

2.3. Random Field-Based Approaches

Sequential Gaussian simulation (SGS) is a widely used stochastic simulation technique in geostatistics that generates realizations from a multi-Gaussian random field. It serves as an effective tool for spatial prediction of geohazards [7,8]. To satisfy the multi-Gaussian assumption underlying the simulation, the measured data at the calibration points were usually transformed to a standard normal distribution using a normal-score mapping method [26]. In this method, realizations of non-Gaussian variables (such as lognormally distributed LPIs) are generated conditionally from a standard Gaussian distribution, and then back-transformed to the original scale. As demonstrated in earlier work [27], this procedure offers a reasonable approximation and maintains the spatial dependence structure effectively.

Mathematically, the essence of SGS lies in constructing a realization through an iterative procedure. The algorithmic framework of SGS can be succinctly summarized as:

$$(Z_n|Z_p = z) \sim N\left(\Sigma_{np} \cdot \Sigma_{pp}^{-1} \cdot z, \sigma_n^2 - \Sigma_{np} \cdot \Sigma_{pp}^{-1} \cdot \Sigma_{pn}\right) \quad (12)$$

where the subscripts n and p denote “next” and “previous”, respectively; Z_p is the vector of known data; Z_n is the data vector at the unsampled location n . Σ is the covariance matrix of adjacent measurements. The simulated value is then randomly drawn from a Gaussian distribution, with its mean and variance computed via SK. Subsequently, this simulated datum is added to the vector of known data Z_p , for the next sequential step.

The random field model uses the covariance matrix Σ to represent the spatial dependence of the measured parameters. The covariance between variables at two separate locations is given by:

$$\Sigma = \text{COV}[Z_i, Z_j] = \rho_{Z_i, Z_j} \cdot \sigma_{Z_i} \cdot \sigma_{Z_j} \quad (13)$$

where ρ_{Z_i, Z_j} is the correlation between the random variables Z_i and Z_j with standard deviations of σ_{Z_i} and σ_{Z_j} , respectively. The correlation ρ refers to the similarity of spatial measurements and is calculated as:

$$\rho(h) = 1 - \frac{\gamma(h)}{\text{COV}(0)} \quad (14)$$

By substituting the optimal semivariogram $\gamma(h)$ into Equations (13) and (14), the unknown value Z_n at location n can be derived from Equation (12). The generated value is then treated as known conditioning data for the subsequent simulation. This procedure is repeated along a random path until all unsampled locations have been estimated. The completion of all mesh simulations constitutes a single realization.

For each realization, a random path was generated using a random permutation function to visit all unsampled grid nodes sequentially. This stochastic path helps to minimize potential artifacts associated with systematic visitation orders. Multiple realizations are generated using Monte Carlo simulations (MCSs).

2.4. Assessment Criteria

To evaluate the effectiveness of the quantitative methods used in this study, several assessment criteria are adopted, as defined below.

Mean absolute percentage error (MAPE) is one of the most common metrics for measuring model forecasting accuracy. A lower MAPE value indicates better predictive performance.

$$\text{MAPE} = \frac{100\%}{n} \sum_{i=1}^n \left| \frac{\hat{y}_i - y_i}{y_i} \right| \quad (15)$$

where n is the number of points to be estimated, $\hat{y}_i$ is the estimated value for the i th observation in the data set, and y_i is the true value.

Root mean square error (RMSE) measures the average distance between the predicted values and the actual values in the dataset. A lower RMSE indicates a better fit of the model to the data.

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \quad (16)$$

Bias factor (BF) is commonly used to account for the effect of differences in failure criteria on the reliability of the evaluation.

$$\text{BF} = \frac{1}{n} \sum_{i=1}^n \frac{\hat{y}_i}{y_i} \quad (17)$$

Coefficient of variation (CV) is commonly used to compare data dispersion between distinct series of data.

$$\text{CV} = \frac{\sigma}{\mu} \quad (18)$$

where σ is the standard deviation, and μ is mean.

Finally, the error rate at the validation points is calculated using the following equation:

$$\text{error rate} = \frac{|\hat{y}_i - y_i|}{y_i} \quad (19)$$

3. Case Study

The present study uses a large-scale land reclamation project near the Chang-Hwa Coastal Industrial Park in Taiwan as a case study [28,29]. The reclaimed land is predominantly composed of silt to fine sand, derived from sediment deposits originating from waterways and the seabed. The study site covers an area of 160 hectares (2000 m × 800 m). For analysis, the site was discretized into 16,000 grids, each with a uniform size of 10 m × 10 m, as shown in Figure 1.

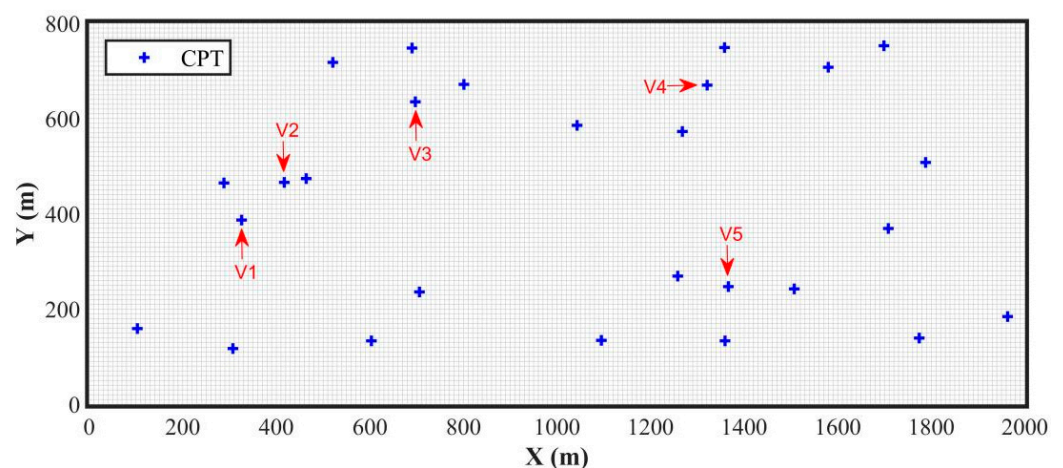


Figure 1. Study site (element size = 10 m).

Liquefaction occurred at the study site during the Chi-Chi earthquake ($M_w = 7.6$, focal depth 7.0 km) on 21 September 1999. To assess the liquefaction potential at the site, a comprehensive evaluation was conducted using both kriging-based and random field-based approaches. The liquefaction potential index (LPI), initially proposed by Iwasaki et al. [30], provides a useful tool for describing liquefaction hazard at a regional scale and was therefore adopted in this study for liquefaction mapping. A total of 27 cone penetration tests (CPTs) were conducted at the study site, and the CPT-based liquefaction model proposed by Robertson & Wride [31] was adopted for LPI calculation.

The distribution of the 27 CPTs is illustrated in Figure 1. The corresponding LPI values for these 27 CPTs were determined and recorded in Table 1. Among these data points, 22 were designated as calibration samples (C1–C22), and the remaining five as validation samples (V1–V5).

Table 1. Locations and LPI values of the 27 CPT samples.

No.	X (m)	Y (m)	LPI	Type
1	103	159	0.25	C1
2	288	465	2.30	C2
3	307	117	0.37	C3
4	464	474	0.79	C4
5	521	718	0.45	C5
6	603	133	1.50	C6
7	690	748	0.64	C7
8	706	236	3.83	C8
9	801	672	0.69	C9
10	1043	586	0.85	C10
11	1095	134	0.41	C11
12	1258	269	5.70	C12
13	1268	573	5.82	C13
14	1358	749	1.12	C14
15	1359	133	6.16	C15
16	1507	242	1.68	C16
17	1580	708	1.82	C17
18	1699	753	1.14	C18
19	1708	369	2.72	C19
20	1774	139	1.38	C20
21	1788	508	1.80	C21
22	1963	184	7.71	C22
23	326	387	0.68	V1
24	417	466	2.57	V2
25	697	635	0.48	V3
26	1321	670	0.88	V4
27	1366	247	2.04	V5

Note: type = Ci stands for calibration sample, type = Vi stands for verification sample.

4. Results

4.1. Assessment of the Semivariogram Model

The choice of theoretical semivariogram model significantly influences the subsequent generation of liquefaction hazard maps. Therefore, the performance of various semivariogram models was first evaluated and validated. Eight theoretical semivariogram models (M1–M8) in Table A1 were tested to identify the most suitable model for the study site, and the fitted semivariogram curves for the estimated LPI values under the SK framework were constructed.

As shown in Figure 2, M3 exhibits the poorest fit to the computed semivariogram values. The remaining theoretical models yield similar fitting results, with differences primarily observed at lag distances of less than 100 m. A comparative analysis reveals that model M5 provides the closest agreement with the computed values.

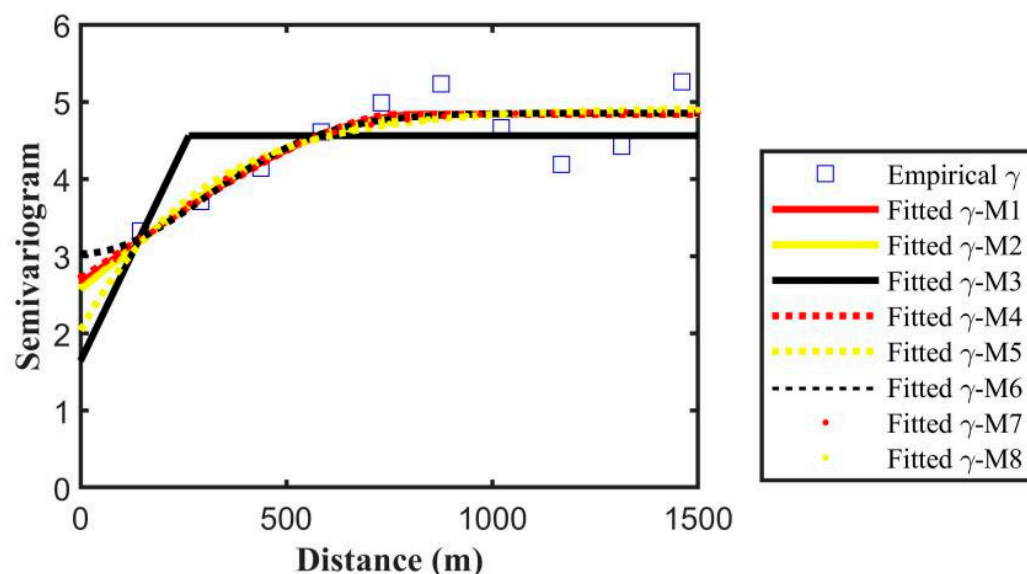


Figure 2. Comparison of theoretical semivariogram models.

To further compare the influence of the eight theoretical semivariogram models on the accuracy of the generated liquefaction maps, LPI values from the 22 calibration samples were used to generate LPI estimation maps via the SK method. The error rates defined in Equation (19) at the five validation points were then calculated using the measured LPI values as true value. The results are presented in Figure 3, where the error bars indicate the range between the maximum and minimum error rates.

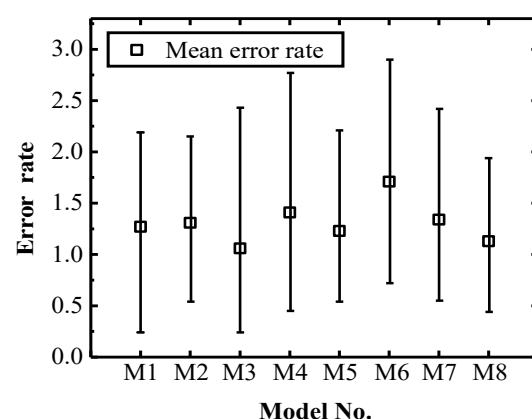


Figure 3. Comparison of error rate of M1–M8.

As shown in Figure 3, model M5 (the exponential model) exhibits a relatively low average error rate and a narrow error range. Consequently, the exponential model is identified as the optimal theoretical semivariogram model and is adopted for subsequent comparisons of different liquefaction mapping approaches.

4.2. Comparison of Liquefaction Mapping Approaches

The LPI of the study site after the Chi-Chi earthquake was evaluated using kriging methods (SK and OK) and the random field-based method (SGS), and the characteristics

of the three approaches were compared. The exponential model was adopted as the semivariogram model for all three methods. Following the finding of a previous study [8], the LPI data at the study site follows a lognormal distribution. For SGS, Each MCS generated a realization of the $\ln(\text{LPI})$ field, and the final liquefaction hazard map for the entire site was obtained by back-transforming to LPI fields across all grids. A parametric study indicated that 1000 MCSs were sufficient to achieve a stable random field, as the COV of the LPI estimates stabilized beyond this threshold.

The statistical properties of the results obtained from the three methods are summarized in Table 2. In this study, the computational time for SK and OK is 5 min each, while SGS with 1000 realizations (denoted as SGS-1000) requires approximately 1410 min. To reflect the stochastic nature of SGS, the statistical results from single and 500 realizations are also presented. In addition, the MAPE, RMSE, and BF values were calculated for each method using the measured LPI values at the 27 CPT locations and the corresponding estimated values at the grid centers where these CPTs are located.

Table 2. Statistics of estimated LPI values by liquefaction mapping approaches.

Item	Number	Min	Max	Median	Mean	Variance	MAPE	RMSE	BF
Samples	27	0.25	7.71	1.38	2.07	4.11	/	/	/
SK	16,000	0.26	7.67	1.81	2.06	1.54	0.02	0.05	0.99
OK	16,000	0.26	7.67	1.82	2.07	1.54	0.02	0.05	0.99
SGS-1	16,000	0.02	45.48	1.79	2.04	1.60	0.11	0.35	0.90
SGS-500	16,000	0.12	11.23	1.45	2.03	2.75	0.25	0.75	0.83
SGS-1000	16,000	0.12	13.70	1.60	2.02	2.62	0.18	0.54	0.86

Note: SGS-1 denotes SGS with 1 realization.

Comparing the results in Table 2, both kriging methods (SK and OK) produce LPI estimate ranges and means that are similar to the measured values. Based on the MAPE, RMSE, and BF values, both methods yield small estimation errors at the measurement locations. Moreover, their computational formulas are straightforward, making them suitable for rapid and robust estimation of overall trends. However, the median LPI values obtained from the kriging methods are notably higher than those of the measured samples, and the variance is significantly reduced, which may fail to capture the true spatial variability or may underestimate the extreme liquefaction potential in high-risk areas. In contrast, a single SGS realization generates extreme values considerably higher than those from kriging, thus better capturing high liquefaction risks. The averaged results from multiple realizations are more stable than those from a single realization while retaining higher variability than kriging. Therefore, SGS with multiple realizations represents a compromise choice in engineering practice that balances accuracy and risk assessment.

Figure 4 presents the spatial distribution of the estimated mean LPI values obtained by different approaches. SK and OK (Figure 4a,b) produce highly similar and smoothed maps, where high-LPI zones appear continuous but lack local variability, reflecting the inherent smoothing effect of kriging. The average of 1000 SGS realizations (Figure 4c) retains more spatial details, although still influenced by the averaging process. In contrast, the single SGS realization (Figure 4d) exhibits strong local variability and scattered high-LPI hotspots, with a wider range of LPI values. This demonstrates its capability to capture the inherent spatial heterogeneity and extreme values of liquefaction potential.

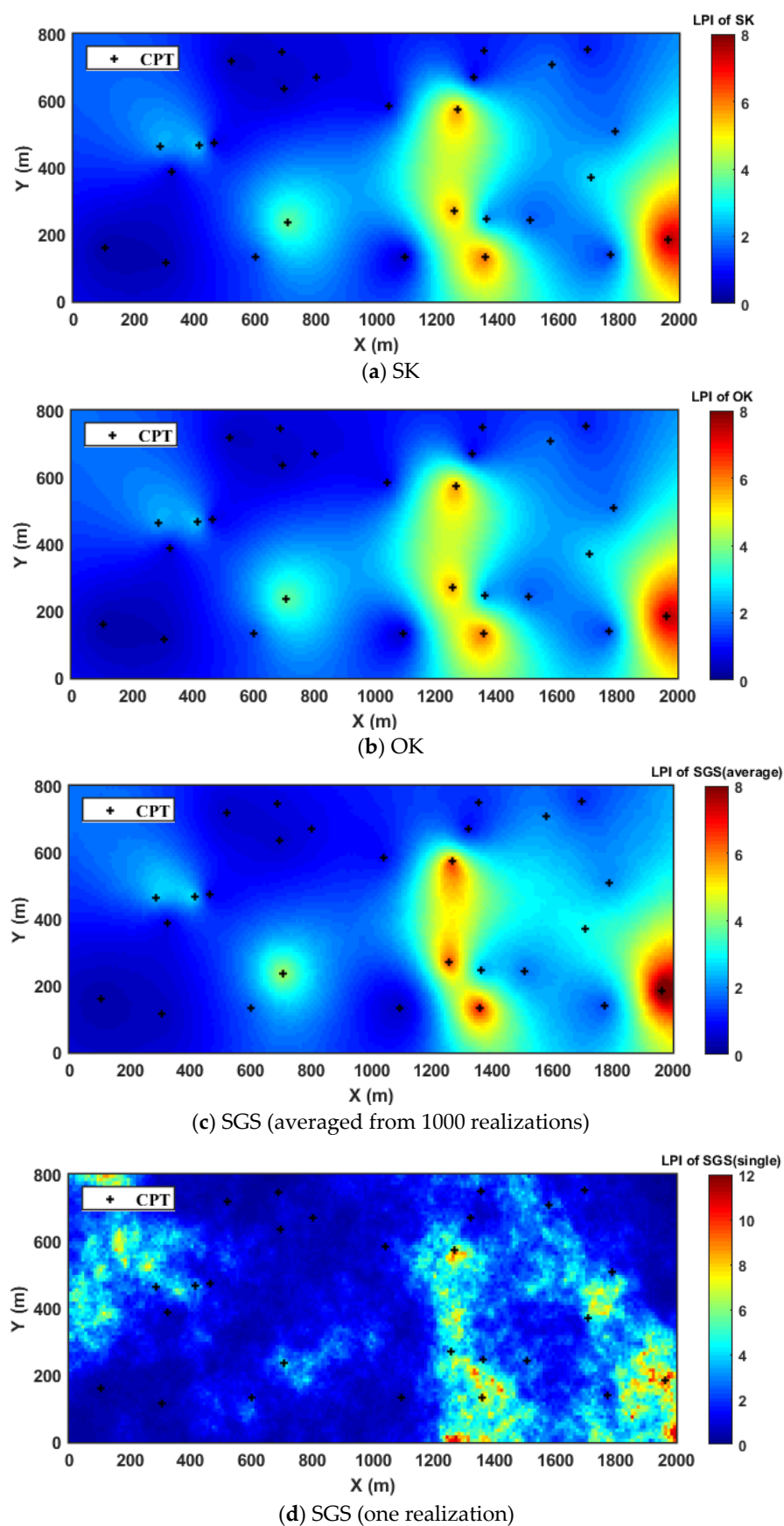


Figure 4. Estimated mean LPI values by different liquefaction mapping approaches.

Correspondingly, uncertainty characterizations of the three mapping approaches are shown in Figures 5 and 6, representing the variance and CV maps, respectively. In the context of liquefaction risk mapping, the variance of estimated LPI values reflects the magnitude of uncertainty at each location, with higher variance indicating less confidence in the predicted LPI value. The CV, defined as the ratio of the standard deviation to the mean, provides a normalized measure of relative uncertainty. A high CV in areas with low mean LPI indicates that, although the expected LPI is low, the potential for an unexpectedly high LPI value cannot be dismissed.

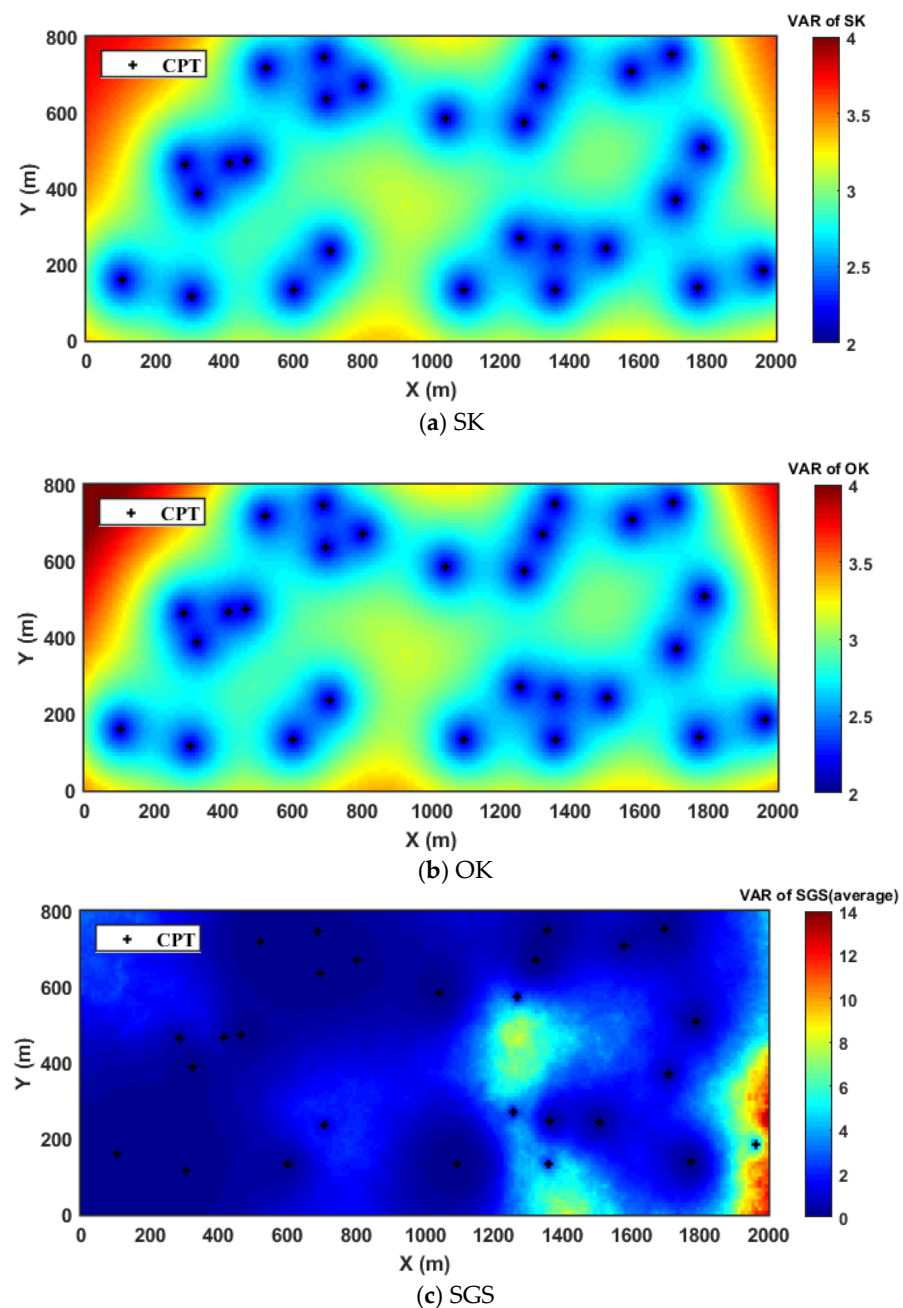


Figure 5. Variance maps of estimated LPI values obtained by the three methods.

As shown in Figures 5 and 6, kriging methods produce low, smoothly distributed variance, with minimum values concentrated around the measured CPT locations and gradually increasing with distance from them. This simplified pattern substantially underestimates the true spatial uncertainty, particularly within high-LPI zones. In contrast,

the SGS-based variance and CV maps exhibit considerably higher and spatially heterogeneous uncertainties. Notably, the CV map from SGS identifies extensive high-uncertainty zones even in areas with low mean LPI values, revealing hidden liquefaction risks that are overlooked by kriging.

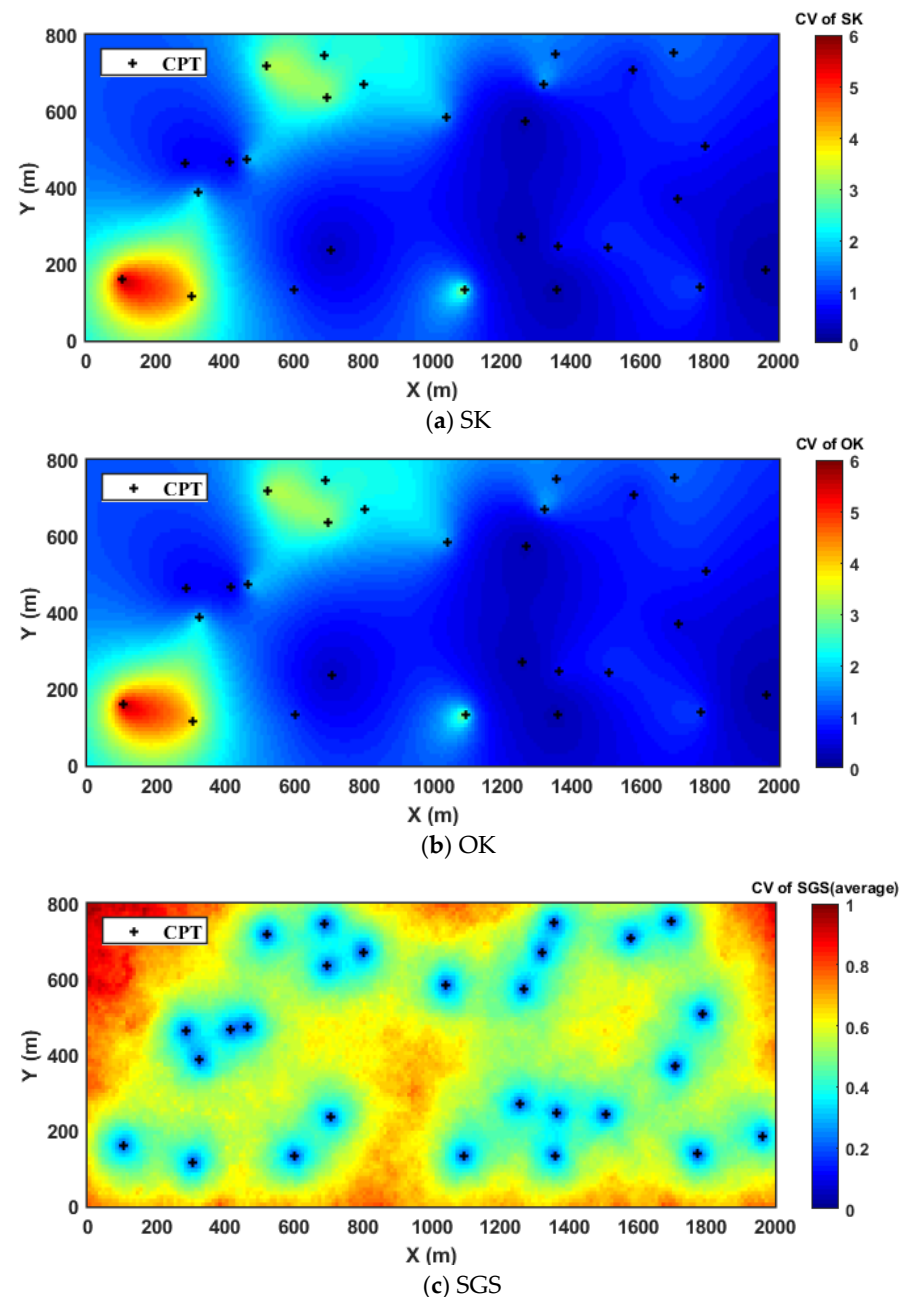


Figure 6. CV maps of estimated LPI values obtained by the three methods.

Overall, kriging methods provide accurate mean estimates with low computational cost but fail to capture spatial variability and uncertainty, whereas SGS offers a more comprehensive representation of both extreme values and uncertainty at the expense of higher computational effort. These findings highlight the importance of selecting appropriate mapping approaches for regional liquefaction risk assessment, depending on whether the goal is trend estimation or detailed probabilistic risk evaluation.

Figure 7 compares the LPI estimates obtained by SK and OK. The 1:1 scatter plot (Figure 7a) shows that the results from SK and OK are almost identical, with all points closely aligned along the 1:1 line, indicating a high degree of consistency between the

two methods. The difference map (Figure 7b) further reveals that the discrepancies are negligible, with most areas exhibiting near-zero differences. Minor positive differences ($LPI_{OK} > LPI_{SK}$) are observed only at the edges of the study area, which can be attributed to the different mean assumptions inherent to the two methods. Given that the mean LPI is known in this study, both SK and OK yield comparable results and are equally suitable for liquefaction potential assessment.

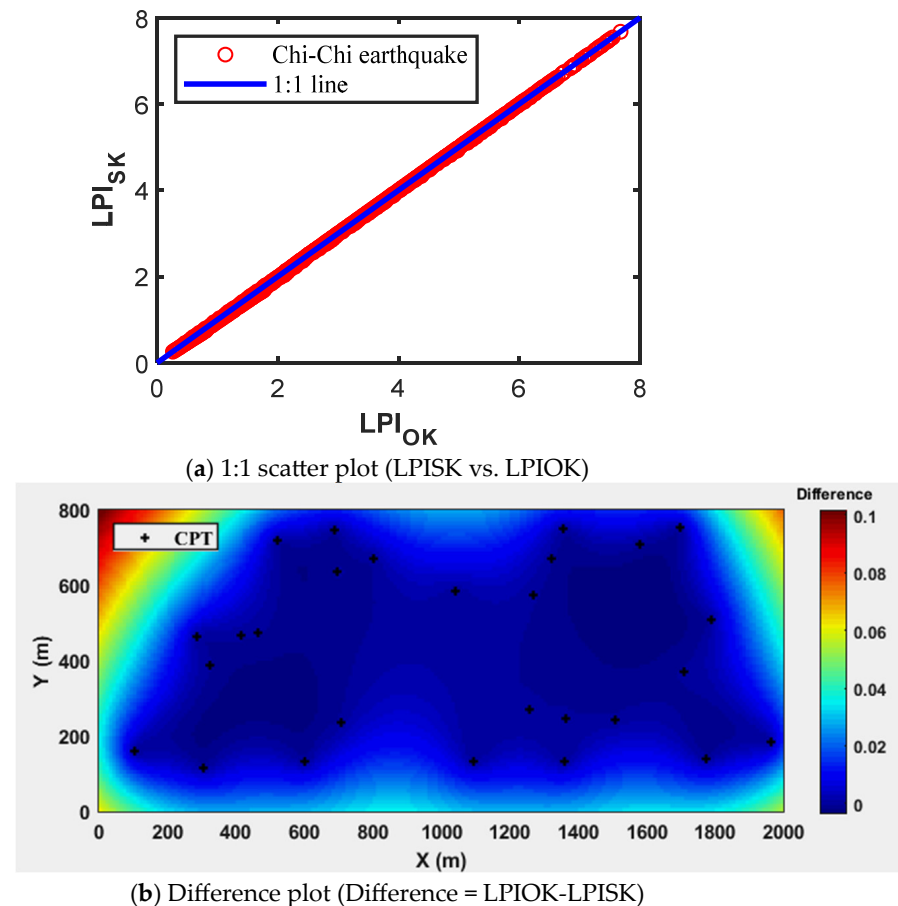


Figure 7. Comparison of SK and OK method.

Figure 8 compares the LPI estimates obtained by SK and SGS. The 1:1 scatter plot (Figure 8a) shows that the points deviate significantly from the 1:1 line, particularly in the high-LPI range, indicating that SGS yields higher values than SK in high-risk zones. The difference map (Figure 8b) further reveals that the discrepancies are substantial and spatially heterogeneous. SGS produces higher LPI values in the central-eastern part of the site (red zones), where liquefaction risk is high, while lower values are observed in some localized areas (blue zones). These differences highlight that SK tends to smooth out spatial variability and underestimate extreme liquefaction potential, whereas SGS is capable of capturing inherent spatial heterogeneity and extreme values, thereby providing a more comprehensive representation of liquefaction risk for detailed assessment.

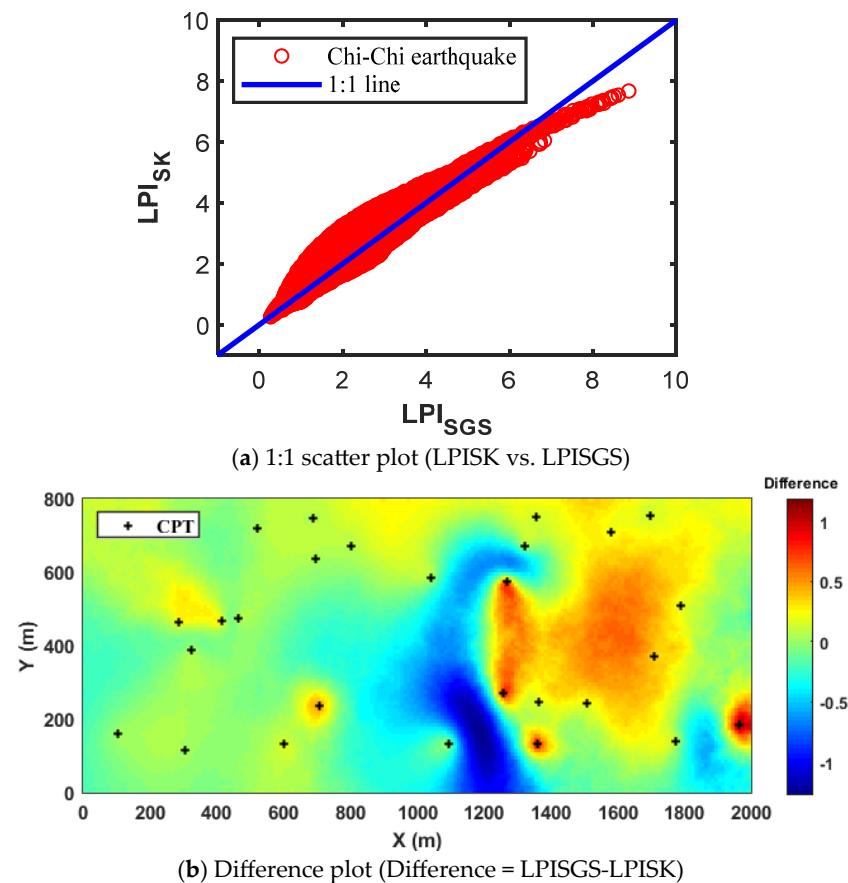


Figure 8. Comparison of SK and SGS method.

5. Conclusions

This study systematically evaluated and compared the performance of three liquefaction potential mapping methods, namely simple kriging (SK), ordinary kriging (OK), and sequential Gaussian simulation (SGS), based on liquefaction potential index (LPI) data. A case study from the Chi-Chi earthquake was adopted for illustration. The main conclusions are as follows:

(1) Among the eight commonly used theoretical semivariogram models, the exponential model (M5) was identified as the most suitable for the study area, showing the best fit to the empirical semivariogram and the lowest validation error rates.

(2) Both SK and OK produced nearly identical LPI estimates with high accuracy (MAPE = 0.02, RMSE = 0.05, BF = 0.99) and low computational cost (5 min), but significantly reduced variance and oversmoothed high-risk zones, failing to capture spatial heterogeneity and extreme liquefaction potential.

(3) In contrast to kriging, SGS captures inherent spatial variability and extreme values, with variance and coefficient of variation maps revealing substantially higher and spatially heterogeneous uncertainties. This provides a more comprehensive basis for probabilistic risk assessment compared to the smoothed estimates from kriging. However, these advantages came at a significantly higher computational cost (approximately 1410 min for 1000 realizations).

This study highlights the critical trade-offs among accuracy, computational efficiency, and the representation of spatial variability in liquefaction mapping. Kriging methods are suitable for rapid, large-scale trend assessments due to their low computational cost and high accuracy at measured points. SGS, by contrast, is strongly recommended for detailed probabilistic risk evaluations to capture extreme values and uncertainty. Future

work should explore the applicability of these methods to other geologic settings and seismic scenarios, as well as the integration of additional site-specific parameters to further improve prediction accuracy.

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Appendix A

Table A1. Calculation formulas for the theoretical semivariogram models.

Model No.	Model	Equation
M1	Spherical model	$\gamma(h) = \begin{cases} C_0 + C_1 \left(\frac{3h}{2a} - \frac{h^3}{2a^3} \right) & 0 \leq h \leq a \\ C_0 + C_1 & h > a \end{cases}$
M2	Pentasppherical model	$\gamma(h) = \begin{cases} C_0 + C_1 \left(\frac{15h}{8a} - \frac{5h^3}{4a^3} + \frac{3h^5}{8a^5} \right) & 0 \leq h \leq a \\ C_0 + C_1 & h > a \end{cases}$
M3	Bounded linear model	$\gamma(h) = \begin{cases} C_0 + C_1 \frac{h}{a} & 0 \leq h \leq a \\ C_0 + C_1 & h > a \end{cases}$
M4	Circular model	$\gamma(h) = \begin{cases} C_0 + C_1 \left(1 - \frac{2}{\pi} \cos^{-1} \left(\frac{h}{a} \right) + \frac{2h}{\pi a} \sqrt{1 - \frac{h^2}{a^2}} \right) & 0 \leq h \leq a \\ C_0 + C_1 & h > a \end{cases}$
M5	Exponential model	$\gamma(h) = C_0 + C_1 \left(1 - e^{-\frac{h}{a}} \right)$
M6	Gaussian model	$\gamma(h) = C_0 + C_1 \left(1 - e^{-\frac{h^2}{a^2}} \right)$
M7	Stable model	$\gamma(h) = C_0 + C_1 \left(1 - e^{-\frac{h^\alpha}{a^\alpha}} \right)$
M8	Whittle model	$\gamma(h) = C_0 + C_1 \left(1 - \frac{h}{a} K_\nu \left(\frac{h}{a} \right) \right)$

Note: C_0 denotes the nugget, representing the discontinuity of the function from the origin between the very short sample distance ($h \approx 0$); C_1 is the arch height, and $C_0 + C_1$ is the sill, representing the variance of the random field; the distance h is calculated by Equation (2); a is the variation distance or correlation scale; K_ν is the modified Bessel function of second kind that has a smoothness parameter ν , and ν can vary in the range of 0 (very rough) to infinity (very smooth), which is equal to 1 in the whittle model.

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