Seismic Behavior of Retaining Walls: A Critical Review of Analytical and Field Performance Studies

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Abstract: Given the abundance and importance of earth retention structures, the problem of seismic earth pressure has attracted not only the research community but also industry and government establishments. The dynamic response, even in the case of the simplest retaining wall, presents a complex problem of soil–structure interaction, encompassing a multitude of competing and complementary factors. This article presents a thorough and critical evaluation of notable analytical and field studies related to the dynamic earth pressures acting on retaining walls. Despite numerous studies spanning nearly a century regarding seismically induced lateral earth pressures, there remains a noticeable disparity between theoretical understanding and the actual field performance of retaining structures during seismic events. This review underscores the necessity for a more meticulous examination of dynamic analysis techniques and the existing design methodologies for retaining structures.

Keywords: seismic earth pressure; retaining wall; dynamic soil–structure interaction; critical review; analytical methods; observed field performance

1. Introduction

Retaining structures play a vital role in various aspects of infrastructure, including transportation networks, ports, lifelines, and other constructed facilities. Consequently, the examination of these structures has been of paramount importance for geotechnical engineers for several decades, particularly in seismically active regions. In the event of an earthquake, both the retaining structure and the soil it retains experience acceleration, leading to the imposition of corresponding inertial forces. When combined with the existing gravitational forces, these forces result in additional loading on the retaining structure. It is crucial to highlight that seismic inertial forces differ from gravitational forces in that they cyclically fluctuate in amplitude and direction and are transient. These distinctions have significant implications for seismic design considerations of a retaining structure.

Shaking-induced damage to retaining structures can result in consequences of moderate (e.g., minor cracks) to devastating (e.g., complete collapse and/or loss of life) scale. The dynamic behavior of retaining structures presents a challenging soil–structure interaction problem that encompasses a wide range of competing and interrelated factors. Wall movements and dynamic earth pressures are contingent upon the response of the foundation soil, the characteristics of the backfill, their mutual interactions, the inertial and flexural reactions of the wall, and the specific nature of the seismic inputs. Therefore, to safeguard both people and critical infrastructure, the development of earthquake-resistant earth-retaining systems is of utmost importance [1].

Over the years, researchers have shown considerable interest in addressing the issue of seismic earth pressures on retaining structures. The groundbreaking contribution of Okabe [2] and Mononobe and Matsuo [3], commonly referred to as the Mononobe–Okabe (M-O) method [2,3], based on a pseudostatic approach, is the prevailing and widely adopted approach for calculating earthquake-induced lateral earth pressures on retaining walls.
Subsequently, several alternative analytical approaches were suggested to assess active and passive earth thrust, including pseudo-dynamic methods [4–14], closed-form stress plasticity solutions [15,16], upper-bound [17–19] and lower-bound [20,21] limit analysis approaches, the continuum mechanics approach [22,23], and the method of slices [24–26]. However, a common limitation in most of these analytical studies is the assumption of a rigid (nonyielding) retaining wall, harmonic motion, or linear amplification of backfill acceleration. In reality, retaining walls undergo deformation, seismic motion is nonharmonic, and amplification is nonlinear [27]. Over time, efforts have been made to address these limitations, with some analytical studies featuring the nonlinear amplification of backfill acceleration under horizontal harmonic motions [28–30].

In contrast, the earthquake performance of various retaining wall types has generally been satisfactory, indicating that the M-O method tends to overestimate the total seismic active thrust, particularly under higher seismic loads [31,32]. Despite numerous investigations carried out in the past nine decades on the topic of seismic earth pressures, there appears to be a noticeable disparity between theoretical understanding and the actual observed behavior of retaining structures during significant seismic events [31–34]. Currently, there exists no comprehensive and categorized review article concerning the seismic performance of retaining walls. Considering the significance of addressing seismic earth pressures when designing retaining structures in regions prone to earthquakes, it is valuable to conduct a review to further our understanding of the subject through a combination of significant analytical studies and the actual observed behavior of retaining structures during major seismic events. This article presents a review of significant analytical and field performance studies on the seismic performance of retaining walls. This review shows that there is a need for further careful consideration of dynamic analysis and current design approaches for retaining structures. In this regard, further physical model testing results are needed in addition to the development of a large database of actual design and field performance data on modern retaining structures.

2. Analytical Studies

Different techniques are employed to analyze seismic earth pressures on retaining structures, and they are typically categorized into three main approaches: limit–state, elastic, and hybrid (elastoplastic) methods [35]. The limit–state approach focuses on the balance of a soil wedge constrained by a moving retaining wall, which leads to potential failure conditions within the soil. Conversely, elastic methods assume minimal wall movements and rely on solutions derived from the equilibrium equation of a linear elastic continuum. Similarly, hybrid methods blend both elastic and plastic elements, incorporating equilibrium equation solutions while considering a range of wall displacements and the soil’s hysteretic behavior. Neither limit–state nor elastic approaches give a true picture of actual physical problems with significant limitations, resulting in conservative estimations of earth pressure in general. However, these methods are advantageous for their speed in providing solutions. Conversely, hybrid methods excel in accurately simulating actual situations and can effectively replicate experimental data. Nevertheless, their predictive capacity is restricted due to their more intricate formulation [36].

Traditionally, retaining walls are conventionally divided into two categories for analytical purposes: “yielding” walls, capable of undergoing sufficient movement to generate minimum active and/or maximum passive earth pressures, and “nonyielding” walls, which lack the necessary movement to mobilize the shear strength of the backfill soil. Consequently, the conditions limiting the development of minimum active or maximum passive earth pressures cannot be achieved. Importantly, in this context, the term “yielding” pertains to the permanent displacement of the wall caused by an earthquake, and it does not imply that stresses within the structural system were exceeded. Typically, free-standing gravity or cantilever walls are classified as yielding walls, while building basement walls constrained at the top and bottom, as well as massive gravity walls founded on rock, are often categorized as nonyielding walls [1].
Despite several limitations, limit–state techniques, notably recognized by the well-known M–O [2,3] method and its extension by Seed and Whitman [37], persist as the prevailing practices in geotechnical engineering for assessing seismic earth pressures acting on yielding walls. Elastic methods are frequently used to analyze nonyielding walls as an alternative to limit–state methods. The next sections provide a comprehensive examination of the limit–state, elastic, and hybrid approaches, with the aim of acquainting the reader with the essential assumptions of these analytical frameworks.

2.1. Limit–State Methods

The limit–state approaches employ a pseudostatic analysis and assume that the soil behaves in a fully plastic manner. Within pseudostatic analysis, two primary types of solutions are commonly utilized: stress-based solutions and kinematic solutions [15]. These solutions are derived from established theories, including the Rankine [38] theory for stress-based solutions and the Coulomb [39] theory for kinematic solutions. Stress-based solutions consider the stress condition within the soil to fulfill both equilibrium equations and boundary prerequisites, in addition to satisfying a predetermined Mohr–Coulomb failure criterion. Conversely, kinematic solutions center around the configuration and motion of the soil mass without directly addressing the stress condition within the soil. Many analyses have been proposed using stress-based solutions, including [15,25,40–42], and kinematic solutions, including [1–3,16,17,37,43].

Okabe [2], along with Mononobe and Matsuo [3], devised a method commonly referred to as the Mononobe–Okabe (M–O) method [2,3]. This method builds upon Coulomb’s [39] original formulations to determine seismic earth pressures acting on retaining structures. It involves the application of earthquake forces to the soil using pseudostatic horizontal accelerations denoted as $k_h g$ and vertical accelerations denoted as $k_v g$, where $k_h$ and $k_v$ stand for the horizontal and vertical seismic coefficients, respectively, and $g$ stands for the acceleration due to gravity $(9.81 \text{ m/s}^2)$. The forces under consideration in the Mononobe–Okabe [2,3] analysis are depicted in Figure 1a. Based on the Coulomb [39] theory, the total seismic active thrust, $P_{ae}$, encompasses both static and seismic force components and can be expressed by Equation (1).

$$P_{ae} = \frac{1}{2} K_{ae} \gamma H^2 (1 - k_v)$$

(1)

where the seismic active earth pressure coefficient, $K_{ae}$, is given by

$$K_{ae} = \frac{\cos^2(\phi - \theta - \beta)}{\cos \theta \cos^2 \beta \cos(\delta + \beta + \theta)} \left(1 + \sqrt{\frac{\sin(\phi + \delta) \sin(\phi - \theta - i)}{\cos(\delta + \beta + \theta) \cos(i - \beta)}}\right)$$

(2)

where $\gamma$ is the unit weight of the soil, $H$ is the height of the wall, $\phi$ is the angle of internal friction of the soil, $\beta$ is the angle of earthquake friction, $\delta$ is the slope of the wall relative to the vertical, $i$ is the slope of the backfill, and $\theta = \tan^{-1}\left[\frac{k_h}{1 + k_v}\right]$ is the seismic inertia angle. However, importantly, the M–O method estimates the resulting dynamic thrust applied to the wall, but it does not provide a seismic pressure distribution analogous to that of the Coulomb theory. The authors of this method also suggested that the location of $P_{ae}$ should be at a distance of $\frac{H}{2}$ from the base of the wall, which aligns with the position of the static active force. One limitation of the M–O method becomes apparent when Equation (2) fails to converge in scenarios where $\theta > 90^\circ$. This challenge can become more pronounced, particularly when dealing with common soil friction angles (i.e., $\phi \geq 35^\circ$), and it becomes especially problematic when the peak ground acceleration (PGA) exceeds 0.7 g [44]. Later, Kapila [45] proposed a set of analogous equations for seismic passive pressure using the same general M–O approach.

Shortly thereafter, Prakash and Basavanna [46] found that the seismic pressure distribution varies with $\phi$, $\delta$, and $\theta$ and is not hydrostatic, as is often assumed. They proposed that $P_{ae}$ acts at a distance $h_a$ from the base of the wall, where $h_a = C_{na} \times \frac{H}{2}$, and $C_{na}(> 1)$ is dependent on $k_h$. 
Seed and Whitman [37] further extended the M-O analysis and suggested that the increase in dynamic pressure is more pronounced in the upper part of the wall. This led to the interpretation of an “inverted–triangle” distribution of the seismic pressure, where the resulting seismic thrust acts between 0.5 and 0.67H above the wall’s base. To address the convergence problem of the original M-O Equation (2), Seed and Whitman [37] proposed a modification by segregating the total lateral earth pressure into static and dynamic components:

\[
P_{ae} = P_a + \Delta P_{ae} = \frac{1}{2} \gamma H^2 K_a + \frac{1}{2} \gamma H^2 \Delta K_{ae} = \frac{1}{2} \gamma H^2 (K_a + \Delta K_{ae})
\]

where \(K_a\) is the coefficient of static earth pressure, and \(\Delta K_{ae} \approx 0.75 k_h\) is the dynamic increment for a vertical wall \((\beta = 0)\) with horizontal backfill slope \((i = 0)\), and \(\phi = 35^\circ\). They also proposed the use of 85% of the PGA in designing retaining walls because the PGA occurs only momentarily and has a small effect on the wall displacements. The force diagram used in the Seed and Whitman [37] analysis is shown in Figure 1b. Additionally, Seed and Whitman [37] proposed that retaining walls designed to meet static requirements will inevitably be able to withstand strong ground vibrations, potentially negating the necessity for special seismic provisions in many cases. According to Mikola and Sitar [47], retaining walls can sustain an additional resulting dynamic earth pressure up to PGAs of 0.3 g and 0.6 g for static safety factors of 1.5 and 1.2, respectively.

Figure 1. Force diagrams used in the (a) Mononobe–Okabe [2,3] analysis and (b) Seed and Whitman [37] analysis. Adapted from Paultre [48].

In all the aforementioned studies, wall inertia was not considered, and there was no effort to predict the resulting seismic wall displacements. This issue was initially addressed by Richards and Elms [49], who introduced the utilization of the Newmark [50] procedure to estimate seismic displacements of gravity walls. Their method enabled the design of retaining walls with controlled displacements and, in combination with the original M-O equation, laid the groundwork for contemporary design guidelines. Whitman and Liao [51] subsequently introduced a structured approach to handle uncertainties in the original method of Richards and Elms [49], offering suggestions for a more enhanced and cost-effective design strategy. Later, Steedman and Zeng [52] and Zeng and Steedman [53] introduced a displacement-centered approach for analyzing gravity walls utilizing the Newmark [50] method. Other researchers [54–56] proposed modifications to pseudostatic methods by introducing the “intermediate wedge concept” to explore the correlation between dynamic pressure and wall displacements.

Whitman [57] incorporated the influence of permissible displacement by relating the wall–soil system more logically with the Newmark [50] approach, by updating the seismic coefficient \((k_h)\) selection. However, this procedure was specifically designed for gravity
walls. Bray et al. [58] took this idea further and refined the approach by incorporating a deformable sliding mass, indicating potential incoherence within the backfill, and drawing from a significantly broader dataset. Li et al. [19] conducted an upper-bound limit analysis to determine the yield acceleration for the translational failure of gravity walls and deduced that wall roughness significantly affected the yield acceleration. Pain et al. [59] proposed a methodology that emphasized the change in the location of the resultant dynamic pressure after each time step, which was not accounted for by Zeng and Steedman [53]. Pain et al. [59] concluded that the rotational displacement of retaining walls depends on various factors, such as the input motion characteristics, wall geometry, and properties of the backfill and wall material. They observed significant influences of the soil friction angle, wall friction angle, and wall inclination angle on the rotational displacement.

Candia and Sitar [60] proposed that the general form of the coefficient of the seismic active pressure for cohesive soils with a homogeneous surcharge can be represented as

\[ K_{ae} = N_{aq} + \frac{2\mu}{\gamma H} N_{aq} + \frac{2c}{\gamma H} N_{ac} \]  

(4)

where \( N_{aq} \), \( N_{aq} \), and \( N_{ac} \) represent dimensionless factors for earth pressure that must be optimized to evaluate the maximum thrust. Prakash and Saran [61] and Saran and Prakash [62] proposed a general solution for dynamic pressures acting on retaining walls with \( c - \phi \) soil. Their solution considered surface cracks and wall adhesion; however, they disregarded the vertical seismic coefficient. Subsequently, Das and Puri [63] incorporated inclined backfills and vertical seismic coefficients into their solution. These methods indicated the presence of various failure surfaces due to the independent improvement of coefficients \( N_{aq} \), \( N_{aq} \), and \( N_{ac} \). Chen and Liu [17] employed the upper-bound theorem of limit analysis to calculate dynamic earth pressures. They considered translational wall movements and composite failure surfaces (including both linear and log-spiral surfaces), yielding results consistent with Okabe’s [2] analysis for cohesionless soils. Figure 2 illustrates a comparison of various methods, where \( \bar{c} = \frac{c}{\gamma H} \). It is evident that an increase in cohesion results in notable reductions in the overall seismic pressure. It is important to observe that, for cohesionless soils and medium seismicity \( (k_h < 0.4) \), these approaches converge to Okabe’s solution. Additionally, it is noteworthy that Chen and Liu’s solution does not become indefinite at high accelerations, and Das and Puri’s solution for the dynamic load increment remains independent of cohesion. One significant limitation of these methods is the absence of experimental data at high acceleration levels, necessitating the use of numerical solutions to accurately determine the critical earth pressure coefficient in such scenarios [60].

![Figure 2](image-url)
Subba Rao and Choudhury [64] calculated seismic passive earth pressure by providing a comprehensive solution that considers cohesive backfill and composite failure surfaces (both planar and log spiral). Lancellotta [21] developed a technique for computing seismic passive earth pressure on retaining walls using a lower-bound limit analysis approach. Additionally, the author addressed the constraints of force-based approaches and limit equilibrium methods, which often involve a curved surface resembling a logarithmic spiral.

Shukla et al. [65] and Ghosh et al. [66] developed closed-form solutions of Equation (4) for scenarios involving smooth vertical walls with flat backfills with and without surcharge. Richards and Shi [67], however, determined the lateral seismic loads on retaining walls by solving the inertial equations applicable to free-field conditions in homogeneous $c - \phi$ soils. Their findings closely align with Okabe’s [2] theory and the design recommendations of the National Cooperative Highway Research Program (NCHRP 611) report proposed by Anderson et al. [68], as shown in Figure 3. In all of these methods, it was assumed that the adhesion at the wall–soil interface was equivalent to the cohesion of the backfill, leading to conservative estimates of dynamic forces. Shukla and Bathurst [69] conducted an analytical investigation aimed at calculating the seismic active pressure generated by $c - \phi$ soil. They considered various factors such as backfill tension cracks, backfill surcharge, horizontal and vertical pseudostatic coefficients, and adhesion and friction along rigid retaining walls. Iskander et al. [41] extended the Rankine [38] solution to accommodate inertial forces when predicting seismic active earth pressures behind stiff walls that support sloped $c - \phi$ backfill. Unlike kinematic solutions [61,69], this method considers both the wall’s inclination and the slope of the backfill. It also offers insights into the dynamic distribution of earth pressure behind the wall, including information about the length of tension cracks. However, it should be noted that this approach disregards soil–wall adhesion.

Shamsabadi et al. [25] proposed a new analytical model called the log-spiral–Rankine (LSR) model for estimating active and passive seismic earth pressures to address the limitations of existing analytical models. The proposed model utilizes a composite failure surface and the method of slices to estimate interslice shear forces. This approach yields active earth pressures consistent with those of standard trial wedge methods and analytical solutions such as Okabe [2]. Krabbenhoft [70] established earth pressure coefficients through upper- and lower-bound finite element limit analysis, considering a spectrum of seismic coefficients and soil–wall interface friction angles. These coefficients exhibit a validated error margin of no more than $\pm 1\%$ and are suitable for application within a
limit equilibrium framework for designing different embedded retaining structures. The research asserts that these novel earth pressure coefficients surpass the existing ones in terms of accuracy and conservatism, making them preferable for designing retaining walls under both static and seismic conditions.

For active and passive conditions, Mylonakis et al. [15] developed a mathematical solution for calculating the combined static and dynamic earth pressures acting on retaining walls with cohesionless backfills. They divided the backfill into two distinct zones with varying stress patterns: one closer to the soil surface and the other near the wall. The analysis assumed that the soil resists yielding based on the Mohr–Coulomb criterion, considering the joint action of static and dynamic forces [15]. However, the physical meaning of this method is limited, particularly when applied to dynamic problems, although it remains mathematically valid. The method assumes a linear earth pressure distribution, and when there is no surcharge load, the point of application of the resultant thrust is positioned at a height of \( \frac{h}{2} \) above the base. A comparison between this proposed method and the numerical outcomes, as well as the results from the M-O method, indicates that the solution tends to overestimate active pressure while underestimating passive pressure [15].

Despite having several drawbacks, limit–state approaches characteristically known through the M-O method [2,3] and its extension by Seed and Whitman [37] remain widely utilized by engineers in routine design tasks due to their straightforward, closed-form mathematical expressions. However, importantly, these methods provide approximate estimations of seismic earth pressure. Additionally, the outcomes of the pseudostatic analyses employed in these traditional methods are significantly dependent on \( k_b \). Selecting an appropriate value for \( k_b \) is challenging, as it varies based on the region’s seismic activity, the significance of facilities, local geological conditions, and soil characteristics. Adding to the complexity, different nations may recommend different values for \( k_b \). Pseudostatic methods assume earthquake loadings as constant, disregarding their cyclic nature, their changes in magnitude and direction over time, and their limited duration. This oversimplification can lead to an overestimation of seismic failure risk and result in conservative designs [71]. Furthermore, pseudostatic methods do not consider the frequency contents of the input motion, which can significantly affect the magnitude and distribution of seismic earth pressures. Moreover, these solutions assume that the retaining wall is rigid and capable of yielding to induce an active stress state in the backfill. Consequently, these methods are not applicable to walls that do not meet these assumptions. Most limit–state methods rely on kinematic solutions, which do not allow for the direct determination of the distribution of earth pressures. Subsequently, capturing the experimental location of the resultant dynamic thrust becomes a challenging task. Additionally, limit–state methods do not consider how the displacement of the retaining structure might affect the development of dynamic earth pressures, an aspect that could have significance in certain situations.

In addition, the M-O method suffers from the negative root problem. Numerically, this occurs when \( \theta < \phi - i \), where \( \theta = \tan^{-1}\left[\frac{k_b}{1 - k_b}\right] \) is the seismic inertia angle, \( \phi \) is the angle of internal friction of the soil, and \( i \) is the slope of the backfill. Beyond this point, the calculated force becomes undefined because of a negative value within a square root in the denominator (Equation (2)). When this point is reached, the values of \( K_{ae} \) or \( K_{pe} \) become infinite, yielding earth pressures that are physically implausible. While this is primarily a concern with very high horizontal accelerations, it is relevant in regions with high seismic activity, where such loading conditions are plausible. This negative root problem still exists even with a zero backfill slope. As with the M-O method, Lancellotta’s [21] approach also suffers from the same negative root problem for the same parameter values. This can be more easily understood through a Mohr circle diagram (recall Rankine’s theory). In static conditions, the mobilized shear strength of soils equals the peak strength. However, under dynamic conditions, only a fraction of the soil’s shear strength is mobilized. For instance, in the active state, it holds that \( K_{ae} \geq K_{pe} \). Consequently, a Mohr circle representing the dynamic active state of stress with \( c_\text{a} = K_{ae} \gamma z \) (assuming \( k_v = 0 \)) and \( \sigma_1 = \gamma z \) is situated within the corresponding circle representing static conditions with \( c_\text{s} = K_{pe} \gamma z \) and \( \sigma_3 = \gamma z \).
The latter represents a state of failure touching the Coulomb’s failure criterion, whereas the Mohr circle depicting dynamic conditions does not. This holds true for the passive state as well since $K_{pe} < K_p$ [22]. The fundamental issue with these methods is their attempt “to extend” a seismic circle to meet a static criterion. Importantly, as known, seismic excitation produces its own $c$ and $\phi$ values. Other limit equilibrium solutions [15,61,63,67] face a similar limitation.

Another commonly used method is the so-called Newmark sliding–block method [50]. However, retaining walls with complex geometries and earthquake excitations with high PGAs cannot be analyzed by this method. Neither the M-O approach [2,3] nor its extension by Seed and Whitman [37] takes wall inertia into account, and these methods are only applicable to dry cohesionless soils. Using Newmark’s [50] method, Richards and Elms [49] observed that the inertia of gravity walls can be on par with the dynamic earth pressure calculated using the M-O method [1]. In both the M-O [2,3] and the Richards and Elms [49] approaches, it is assumed that there is no phase difference between the wall inertia and dynamic earth thrust. Seed and Whitman [37] concluded that when employing the M-O approach to estimate the overall active thrust for typical wall designs, vertical accelerations can be neglected. However, Das and Puri [63] found that increasing the backfill slope increases the dynamic active force. Additionally, when the horizontal seismic coefficient is minimal, the vertical seismic coefficient can have a considerable effect on the dynamic active force. Furthermore, Anderson et al. [68] showed that the M-O method is invalid for steep backfill soil slopes as the planar failure surface approaches the backfill slope, resulting in the formation of an infinite mass of active failure wedge.

Mylonakis et al. [15] introduced stress solutions in closed form for cohesionless soil and explored various aspects of soil–wall system behaviors under seismic conditions as an alternative to the M-O method. However, the solution was found to be conservative compared to that of the M-O method, particularly under certain conditions, such as high PGA, high friction angles, steep backfills, and negative wall inclinations. Notably, the Mylonakis [15] solution was validated with analytical results only, and a uniform acceleration was considered for the backfill. Despite widely recognized limitations, the limit–state methods, particularly those that employ kinematic solutions, remain widely applied in engineering practice due to their simplicity. In general, contemporary design procedures utilizing limit–state methods tend to overestimate the seismic pressure acting on retaining structures. This overestimation does not necessarily stem from inherent conservatism but rather results from a cautious selection of seismic demand inputs.

2.2. Elastic Methods

Elastic approaches typically apply the principles of elasticity to assess the seismic behavior of nonyielding walls, such as basement walls, by modeling the soil as a viscoelastic continuum. To model the backfill–wall interaction accurately, proper boundary conditions were considered. Elastic methods have also been extended to analyze yielding walls. However, their applicability in such cases is constrained because even a minor deflection of the wall could trigger a failure state in the soil, which contradicts the fundamental assumption of an elastic response.

Matsuo and Ohara [72] introduced an approximate technique for estimating dynamic pressures on a stiff retaining wall with a semi-infinite soil medium under harmonic excitations while considering restricted vertical displacements. However, the accuracy of their approach could not be validated. Shortly thereafter, Scott [73] developed a simple one-dimensional model to assess the seismic response of retaining walls. In their model, a shear beam with Winkler springs was used at the soil–wall interface to estimate the seismic response of semi-infinite and bounded backfills. However, Veletsos and Younan [35] later showed that their solution was incorrect.

Wood [74] developed a solution for analyzing the behavior of nonyielding rigid walls that retain backfill with finite lengths. Their method involved studying an elastic soil layer with rigid walls and a rigid base, subjected to harmonic base excitations (Figure 4a). To
determine the total earth pressures during an earthquake, one must sum the dynamic earth pressures derived from this method with the static earth pressures. The assumption of wall rigidity implies that the static earth pressure component is considered the at-rest earth pressure. Given the complexity and limitations of the proposed solution, Wood [74] also presented an approximate static solution to offer a simpler method for engineers to estimate the maximum dynamic thrust exerted on nonyielding rigid walls in practical situations. They showed that the resultant dynamic pressure on the boundary wall, \( Q_b \), varies with Poisson’s ratio, \( \nu \), and the ratio of the distance between the side boundaries to the wall height, \( \frac{L}{H} \), as shown in Figure 4b. This resultant dynamic pressure acts at a height of approximately 0.55–0.60 times the height of the wall (\( H \)) above the base, aligning with the findings of Seed and Whitman [37]. Wood’s analysis further indicated that for rigid walls, the resultant dynamic pressure can be as much as double the prediction from the M-O method. Wood’s investigation also considered the frequency ratio \( \Omega = \frac{f}{f_s} \), where \( f \) is the motion frequency and \( f_s \) is the fundamental frequency of the backfill. They determined that motions with frequency ratios (\( \Omega < 0.5 \)) had minimal dynamic amplification effects, meaning these lower-frequency motions did not significantly influence the system’s dynamic response.

![Figure 4](image)

**Figure 4.** (a) Wood’s [74] proposed model for rigid walls and (b) dynamic pressure increment on rigid walls obtained by Wood [74]. Adapted from Candia and Sitar [60].

Nazarian and Hadjian [75] proposed elastic dynamic solutions that offer guidelines for estimating backfill pressures in situations where there are minor horizontal displacements. Additionally, they devised solutions to evaluate dynamic pore water pressures. Arias et al. [76] employed a simplified model of an elastic medium to derive comparatively straightforward analytical formulas for the pressures applied to walls induced by both sinusoidal and earthquake excitations. However, their solutions were limited to situations involving rigid walls.

Veletsos and Younan [35] focused on understanding the behavior of a semi-infinite homogeneous layer of viscoelastic material with massless rigid walls when subjected to harmonic and earthquake excitations. Their findings, when considering no vertical stress, aligned with Wood’s [74] approach. They also demonstrated that the distribution of earth pressure on the rigid wall exhibited a consistent increase, starting from zero at the wall’s base and reaching a maximum value at its top, thus validating Seed and Whitman’s [37] inverted–triangle interpretation. Additionally, Veletsos and Younan [77] extended their study to consider the rotation of massless, rigid walls using a torsional spring boundary condition, a concept also explored by Wood [74].

Veletsos and Younan [78] further expanded their analysis to include the effects of both wall flexibility (characterized by \( d_w = \frac{GH^3}{D_w} \)), where \( D_w = \frac{E_w t_w^3}{12(1-\nu_w^2)} \), and base flexibility (characterized by \( d_b \)). These parameters played a critical role in accurately depicting the seismic behavior of the wall-backfill system, as neglecting them would result in unrealistically high seismic earth pressures. The analysis method assumed that when the soil
medium is shaken horizontally, no vertical stresses develop within it. Additionally, the method presumed a perfect bond between the wall and the soil. Veletsos and Younan [78] presented their solution primarily for scenarios where the dominant frequencies of excitations are significantly smaller than the fundamental frequency of the wall–soil system (pseudostatic case).

Psarropoulos et al. [79] utilized finite element analyses to evaluate the accuracy of the methodologies proposed by Veletsos and Younan [35,77,78], whereas Giarlelis and Mylonakis [80] provided further comparisons to the experimental data. Psarropoulos et al. [79] employed numerical simulations to illustrate that elastic solutions, which are applicable to yielding walls, are not as suitable for nonyielding walls because even slight wall movements can trigger a soil failure state, contradicting the assumptions of elastic theory. Psarropoulos et al. [79] also observed that as the flexibility of the retaining wall and the rotational restraint at the base increase, the amplification factor of structural forces decreases. This observation stands in contrast to the conclusions drawn from the Veletsos and Younan [78] solution.

Richards et al. [81] further extended the work of Veletsos and Younan [35,77,78] by using a model in which the soil is represented as a sequence of springs, and they also incorporated plastic deformation in this soil–spring model. The authors concluded that the resultant magnitude of seismic wall pressure can be determined by examining the stress distribution within the free field, aligning with the principles of the M-O approach. Additionally, the authors noted that the distribution of earth pressure is influenced not only by the variation in stiffness within the backfill but also by the specific mode of deformation exhibited by the retaining wall.

Shortly thereafter, Li [82] introduced a comprehensive solution that focused on the relationship between the rotational movement of rigid walls and the flexibility of the foundation. Wu and Finn [83,84] took a similar approach, presenting a closed-form solution and providing design charts tailored specifically for assessing dynamic earth pressures acting on unyielding walls during earthquake events. Their research encompassed both uniform and nonuniform backfills. Expanding on the work of Veletsos and Younan [78], Jung et al. [85] proposed their solution by incorporating horizontal wall translation. The authors acknowledged that while their assumptions impose certain limitations on the solution’s applicability, their closed-form solution still offers a rational basis for investigating the effects of different parameters.

Kloukinas et al. [86] proposed Equation (5), a more versatile and simplified solution for evaluating the response of rigid walls on elastic foundations. They used techniques such as variable separation and Ritz functions to develop this solution, which aimed to offer a more generalized approach to the problem.

\[ Q_b = -\frac{16\psi}{\pi} \rho \bar{X} g H^2 \]  

(5)

where \( \psi = \frac{2}{\sqrt{(1-\nu)(2-\nu)}} \) is a compressibility factor. A comparison of all the above studies is shown in Figure 5 for a rigid wall. When considering typical Poisson’s ratio values, the increment in dynamic loads exhibits a linear relationship with the input acceleration and can be approximated as \( Q_b \approx \rho \bar{X} g H^2 = k_b g H^2 \) applied at \( 0.6H \) [60].

Brandenberg et al. [87] introduced a linear elastic solution that accounts for both base translation and shearing at the interface between the soil and the U-shaped basement walls. They observed that the resulting earth pressure increases monotonically from the base to a peak at the surface, thus validating the findings of Veletsos and Younan [77]. Brandenberg et al. [88] later extended their linear elastic solution to address the case of rigid walls retaining inhomogeneous backfill soils and resting on a rigid base. Shortly thereafter, Durante et al. [89] investigated the behavior of flexible walls retaining both homogeneous and inhomogeneous soil under seismic loading and resting on a rigid base. They employed numerical modeling using the pseudostatic approach.
Elastic methods generally provide upper-bound estimates of dynamic earth pressures, suggesting that the seismic thrusts predicted by these methods are often higher than those obtained using the M-O method. However, elastic methods have limitations in capturing the dynamic behavior of the soil and its potential for failure. Earlier studies that focused on nonyielding rigid walls often did not adequately account for the radiation-damping capacity of the medium (the soil’s ability to dissipate energy through wave propagation) and the backfill’s ability to transfer forces to the base through horizontal shearing. Neglecting these factors could lead to incomplete descriptions of the wall–soil system behavior. Despite these limitations, Wood’s [74] method remains widely used in practice for analyzing rigid walls under seismic conditions. However, Wood’s method does not incorporate the impact of wave propagation within the soil in its analysis.

In the 1990s, Veletsos and Younan [35,77,78,90] made significant contributions to elastic solutions for analyzing seismic earth pressures. They demonstrated that elastic solutions could yield reasonable and experimentally verifiable results for seismic earth pressures. Their work considered factors such as wall flexibility and foundation rotation stiffness. However, the solution presented by Veletsos and Younan [78] has certain limitations. The assumption of complete contact at the soil–wall interface, while useful in certain situations, gives rise to tensile stress on the wall. Additionally, the analysis does not consider the impact of horizontal translational displacement. The retaining wall was considered to have no mass, and linear elastic behavior of the soil layer was assumed. These simplifications were recognized as shortcomings, as they did not fully capture the realistic seismic response of a soil–wall system.

2.3. Hybrid Methods

Steedman and Zeng [4] addressed the limitations of limit–state methods by proposing a simple pseudo-dynamic approach. This method was developed based on insights from centrifuge tests. The authors considered both the finite shear wave velocity of the backfill and the input motion frequency. The pseudo-dynamic method represents a hybrid approach that bridges the gap between plasticity-based and elasticity-based methods. It extends the M-O analysis and considers constant amplitude sinusoidal horizontal acceleration in the backfill (Figure 6). The magnitude of the total earth pressure predicted by this method closely resembles the M-O predictions. However, the point of application of the dynamic thrust is higher than \( \frac{H}{4} \) above the base of the wall and depends on the motion frequency and soil properties [4]. The authors also noted that the phase difference of the backfill acceleration impacts the seismic pressure distribution but not the total dynamic thrust. Kloukinas et al. [16] presented an improved method extending the work of Steedman and

Figure 5. Comparison of elastic approaches by Matsuo and Ohara [72], Wood [74], Veletsos and Younan [35], and Kloukinas et al. [86]: (a) increment in the dynamic load coefficient and (b) resultant earth pressure location. Adapted from Candia and Sitar [60].
Zeng [4] by incorporating nonuniform acceleration in the backfill and aimed to satisfy the stress-boundary conditions of the problem more effectively. However, the solution over-predicts the active pressure and under-predicts the passive pressure. Both Steedman and Zeng [4] and Kloukinas et al. [16] obtained similar results, i.e., that higher-frequency excitations result in reduced total earth pressure coefficients.

Choudhury and Nimbalkar [5] made modifications to the pseudo-dynamic technique, adapting it to calculate the distribution of dynamic passive pressure behind a vertical wall considering various factors such as soil and wall friction angles, horizontal and vertical seismic accelerations, shaking duration, and primary and shear wave velocities. Continuing their work, Choudhury and his coworkers [6,91] calculated the seismic active thrust while considering shear and primary wave propagation within the backfill. They introduced temporal variation for rigid walls by accounting for harmonic horizontal and vertical seismic accelerations. Shortly thereafter, Choudhury and Nimbalkar [92] calculated the rotational displacement of a vertical gravity wall utilizing the Zeng and Steedman [53] method. Ghosh [8] applied the pseudo-dynamic approach to estimate the dynamic active thrust on inclined retaining walls. Subsequently, Basha and Babu [11] calculated the rotational displacements of gravity walls under passive conditions.

Basha and Babu [93] developed an approach to calculate seismic passive pressure coefficients, utilizing a compound failure surface that combined a log spiral and a planar failure surface. This approach was built upon the modified pseudo-dynamic method [5,6], which considered phase changes in primary and shear waves. Kolathayar and Ghosh [9] explored the dynamic active pressure distribution behind a rigid cantilever wall with a bilinear backface using the pseudo-dynamic approach. Using the horizontal slices method and applying the pseudo-dynamic approach, Ghanbari and Ahmadabadi [12] calculated the seismic active thrust behind inclined retaining walls. Ghosh [7,10] and Wang et al. [13] applied the pseudo-dynamic method to calculate dynamic pressure on inclined retaining walls retaining noncohesive backfill soil. Ghosh and Sharma [94] conducted further investigations into the impact of backfill inclination on seismic active thrust. Choudhury et al. [95] introduced an innovative dynamic method that considers the complete spectrum of seismic waves, including primary, shear, and Rayleigh waves, when calculating seismic active earth pressure on rigid retaining walls. This approach revealed a notably expanded seismic influence zone compared to those of existing pseudostatic and pseudo-dynamic techniques, underscoring the substantial impact of Rayleigh waves.

Candia and Sitar [60] presented a solution that considers the dynamic behavior of a gravity wall supporting a viscoelastic backfill as depicted in Figure 7. The solution was based on the wave equation, which is a mathematical equation that describes the propaga-
tion of waves through a medium. Unlike some previous methods, such as the Steedman and Zeng [4] and the M-O method, which considered a constant amplitude acceleration and a uniform acceleration respectively, the wave equation-based solution considered an acceleration with a decreasing amplitude toward the surface. This solution incorporated the principle of energy dissipation within the soil and upheld the vital condition of zero stress at the surface, which is an important aspect of accurately modeling the behavior of soil under dynamic loading. Notably, the solution anticipated significant amplification of the resultant earth pressure at the resonant frequencies of the backfill. Furthermore, the study delved into the effects of input motion frequency and depth on the bedrock [36].

![Figure 7. Force diagram adapted from the Candia and Sitar [60] analysis.](image)

Bellezza [14] introduced a novel pseudo-dynamic method, which relies on a standing shear wave within a viscoelastic backfill situated above a rigid base and subjected to harmonic loading. Considering other assumptions from the conventional pseudo-dynamic method, such as the absence of water, a homogeneous backfill, and a planar failure surface, this new approach derived closed-form expressions in dimensionless form. These expressions represent the horizontal inertia force, seismic active thrust, active pressure distribution, and overturning moment, all as functions of the normalized frequency of the shear wave and the damping ratio. Pantelidis [22] introduced a method based on continuum mechanics to obtain earth pressure coefficients suitable for \( c - \phi \) soils and applicable to both horizontal and vertical pseudostatic conditions. These coefficients were derived for any soil state between the at-rest state and the active or passive state. Nimbalkar et al. [96] proposed a more rational and advanced approach to analyze the seismic response of rigid retaining walls. Their solution, based on the pseudo-dynamic approach, focused on calculating the seismic active thrust and the nonlinear distribution of active earth pressure along the wall height under the translation mode. The authors considered the propagation of both shear and primary waves through the backfill soil and the retaining wall, enhancing the accuracy of their analysis.

The inadequacy of pseudostatic analysis in addressing the dynamic nature of earthquakes and its failure to consider time effects are widely acknowledged. In an attempt to address this limitation, Steedman and Zeng [4] introduced a straightforward pseudo-
dynamic analysis for seismic active soil thrust. This method incorporates phase difference and amplification effects within the dry backfill behind a vertical retaining wall subjected to horizontal acceleration that only varies across the wall face. Nevertheless, Steedman and Zeng’s pseudo-dynamic solution overlooks energy dissipation in the soil and disregards the zero-shear stress boundary condition at the surface. Consequently, the coefficient of the total earth pressure decreases steadily with increasing input frequency, failing to capture crucial aspects of the dynamic soil response.

Over time, the Steedman and Zeng [4] method was expanded by Choudhury and Nimbalkar [6], Ghosh [10], and Bellezza [14]. Choudhury and Nimbalkar [6] incorporated vertical acceleration, examining the effects of factors such as the shear resistance angle and soil–wall friction angle on the seismic active pressure distribution. Ghosh [10] proposed a solution for calculating seismic active thrust behind a battered retaining wall with a dry, cohesionless, inclined backfill. Bellezza [14] introduced a novel pseudo-dynamic approach established on the basis of more reasonable viscoelastic soil behavior. However, this solution also ignores vertical acceleration and is applicable only to dry backfill, akin to the Steedman and Zeng [4] approach. Furthermore, the pseudo-dynamic approach was broadened to estimate seismic passive pressure (Choudhury and Nimbalkar [5]; Ghosh [7]). This identical framework was then used to predict seismic displacements (Choudhury and Nimbalkar [92,97]) and formulate the design of retaining structures, including those with reinforced backfill (Ahmad and Choudhury [98–100]; Choudhury and Ahmad [101]; Nimbalkar et al. [102]; Nimbalkar and Choudhury [103]).

Although pseudo-dynamic methods have been widely used for analyzing retaining structures under seismic conditions, they have notable limitations. Pseudo-dynamic methods often assume the retaining walls to be rigid and hence neglect their seismic response. They also ignore the phase difference between the dynamic earth thrusts and the wall inertia. These methods assume a rigid connection between the wall and its foundation. Pseudo-dynamic methods commonly assume that the seismic acceleration is a uniform sinusoidal wave. In reality, accelerograms can be complex, with multiple amplitudes and a broad spectrum of frequency components. These methods also do not consider the impact of the foundation layer on the development of dynamic earth pressures. Additionally, damping properties are not typically considered for backfill in pseudo-dynamic methods. These limitations can influence the accuracy of the results obtained from these methods. Table 1 presents a summary of significant analytical methods that have practical use.

Table 1. Summary of analytical approaches for seismic earth pressures and wall displacements (only simplified practical methods are included).

<table>
<thead>
<tr>
<th>Method</th>
<th>Equation</th>
<th>Notes</th>
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<tr>
<td><strong>Limit–State Based Methods</strong></td>
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<tr>
<td>Okabe (1924) [2]; Mononobe and Matsuo (1929) [3],</td>
<td>[ K_{ae} = \frac{\cos\theta \cos^2(\phi - \theta - \beta)}{\cos\theta \cos^2(\beta \cos(\delta + \beta) + \theta) \left(1 + \frac{\sin(\phi + \delta) \sin(\phi - \theta - \beta)}{\cos(\phi + \delta) \cos(\theta + \beta)}\right)^2} ]</td>
<td>The M-O method is most widely used. Initially designed for gravity walls retaining cohesionless soil. ( P_{ae} ) acts at ( \frac{1}{4} H ) (H is the height of the wall). ( K_{ae} ) does not converge when ( \theta &lt; \phi - i ).</td>
</tr>
<tr>
<td>Seed and Whitman (1970) [37],</td>
<td>[ K_{ae} = K_t + \frac{3}{4} k_b ]</td>
<td>An extension of the M-O approach. Other aspects of the problem are comparable to the M-O approach, with the exception of a dynamic component acting at 0.6H.</td>
</tr>
<tr>
<td>For vertical walls and horizontal dry backfill</td>
<td></td>
<td></td>
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<tr>
<td>Richards and Elms (1979) [49],</td>
<td>Permanent wall displacement,</td>
<td>Using the Newmark [50] procedure, a method for calculating seismic-induced permanent wall displacements was proposed. Wall inertia was considered. This provides the basis for all modern design guidelines.</td>
</tr>
<tr>
<td>Permanent wall displacement,</td>
<td>[ d_{perm} = 0.087 \frac{V^2}{A_g} \left[ \frac{N}{A} \right]^{-4} ]</td>
<td></td>
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<tr>
<td>( N = a_h = k_{bg} ) (Horizontal acceleration)</td>
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Table 1. Cont.

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<thead>
<tr>
<th>Method</th>
<th>Description</th>
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<tbody>
<tr>
<td>Zhang et al. (1998) [54,55],</td>
<td>Based on the “intermediate wedge concept”, dynamic earth pressures under any state can be determined. The strain increment ratio is an important parameter in this concept, typically determined by cyclic triaxial testing.</td>
</tr>
<tr>
<td>Zeng and Steedman (2000) [53],</td>
<td>Using the Newmark [50] approach, a rotating block method was devised to determine the seismic rotational displacement of gravity walls. No inertia effects were considered.</td>
</tr>
<tr>
<td>Mylonakis (2007) [15],</td>
<td>An alternative closed-form stress plasticity solution for the M-O method for calculating total (static + dynamic) earth pressures on retaining walls with cohesionless backfills. Uniform backfill was considered in the analysis.</td>
</tr>
<tr>
<td>Lancellotta (2007) [21],</td>
<td>Lancellotta [21] developed a technique for computing seismic passive earth pressure on retaining walls using a lower-bound limit analysis approach. Wall roughness was considered. However, as with the M-O approach, this method also presents the negative root problem.</td>
</tr>
<tr>
<td>Anderson et al. (2008) [68],</td>
<td>To provide guidelines in practical design problems, a chart method for applying the M-O method to cohesive soils was developed. The use of seismic coefficient charts was suggested. However, as with the M-O approach, this method is also limited to non-homogeneous soils and intricate back-slope geometry.</td>
</tr>
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### Elastic-Based Methods

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
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<tbody>
<tr>
<td>Wood (1973) [74],</td>
<td>For rigid walls that do not yield, and with finite backfill length subjected to harmonic base motions, a solution was developed. The resultant dynamic pressure acts at 0.55–0.6H above the wall base.</td>
</tr>
<tr>
<td>Kloukinas et al. (2012) [86],</td>
<td>A more flexible and versatile solution for assessing how rigid walls behave on an elastic stratum. The effect of wall flexibility was not considered.</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
\Delta M_{ae} &= \gamma H^2 k_b F_m \\
Q_b &= \gamma H^2 k_b F_P \\
\end{align*}
\]
Table 1. Cont.

Veletsos and Younan (1994, 1997) \[35, 77, 78\],

\[ Q_b = -\frac{16\psi_0}{\pi^2} pX_h H^2 \sum_{n=1}^{\infty} \frac{1}{(2n-1)^3} \sqrt{\frac{1 + \hat{i} \phi}{1 - (\hat{\phi}_n^2 + \hat{i} \phi)^n}} e^{\hat{\theta} t} \]

\[ \sigma_{wv}(\theta) = -(\sigma_1 + \sigma_2) \psi \psi \psi X_h H \]

\[ d_{\theta} = \frac{R_\theta}{G_H} \]

\[ d_w = \frac{D_w}{D_w} \]

\[ D_w = \frac{F_{wB}^3}{12(1 - v_w^2)} \]

Richards et al. (1999) \[81\],

For massless rigid walls, the harmonic and earthquake response of a semi-infinite uniform layer of viscoelastic material was investigated. The rotation of rigid walls around its base was further considered. The parabolic shear modulus distribution of the backfill was considered. The effect of the wall and base flexibility were also taken into account.

\[ \sigma_{xw} = K_0 \gamma z + \frac{1}{6} k_x C_2 H \sqrt{\frac{z}{H}} \left[-1 + 5 \frac{z}{H} - 4 \frac{z}{H} \sqrt{\frac{z}{H}} \right] \]

A straightforward kinematic approach for determining seismic earth pressure. It utilizes the Mohr–Coulomb failure criterion. The shear modulus value is idealized. Interestingly, the actual magnitude of the shear modulus has no impact on the distribution of active earth pressure.

**Hybrid Methods**

Steedman and Zeng (1990) \[4\],

\[ P_{nt} = \frac{Q_b(t) \cos(\alpha - \phi) + W \sin(\alpha - \phi)}{\cos(\delta + \phi - \alpha)} \]

A simple pseudo-dynamic analysis was proposed based on insights from centrifuge tests. The shear wave velocity of the backfill and the input motion frequency are considered. Seismic thrust is higher than \( \frac{2}{3} \) above the wall’s base, and it depends on the motion frequency and soil properties. Phase difference of acceleration in the backfill does not influence the seismic thrust.

Choudhury and Nimbalkar (2005) \[5\],

\[ P_{pc} = \frac{W \sin(\alpha + \phi) - Q_b(t) \cos(\alpha + \phi) - Q_v(t) \sin(\alpha + \phi)}{\cos(\delta + \phi - \alpha)} \]

Made modifications to the pseudo-dynamic technique, adapting it to calculate the distribution of dynamic passive pressure behind a vertical wall considering various factors.

Choudhury and Nimbalkar (2008) \[92\],

Rotational acceleration,

\[ \alpha = \left[ \left( P_{ae}(t) \cos(\delta) \right) h + \frac{W_c}{8} a_{y_c} y_c - W_v x_c + Q_{mv} y_c + Q_{cv} x_c - \left[ P_{ae}(t) \sin(\delta) \right] b_w \right] \]

Rotational displacement of a vertical gravity wall calculated utilizing the Zeng and Steedman \[53\] method. Wall–soil inertia effect, velocities of the primary and shear waves, and other factors are considered.

Pantelidis (2019) \[22\],

\[ k_{ae}^{c-R} = \left(1 - \sin(\phi') + \frac{k_h}{1 - k_v} \tan(\phi') \right) - \frac{1}{1 - k_v} \frac{2 \gamma}{\gamma z} \tan(45° - \frac{\phi'}{2}) \]

\[ k_{ae}^{c-R} = \left(1 + \sin(\phi') + \frac{k_h}{1 - k_v} \tan(\phi') \right) - \frac{1}{1 - k_v} \frac{2 \gamma}{\gamma z} \tan(45° - \frac{\phi'}{2}) \]

\[ k_{pc}^{c-R} = \left(1 - 2 \frac{k_h}{1 - k_v} \tan(\phi') + \frac{1}{1 - k_v} \frac{2 \gamma}{\gamma z} \tan(45° + \frac{\phi'}{2}) \right) \]

Based on continuum mechanics approach, Pantelidis \[22\] obtained earth pressure coefficients for \( c - \phi \) soils and horizontal and vertical pseudostatic conditions. These coefficients were derived for any soil state between the at-rest state and the active or passive state.

3. Field Performance Studies

Numerous investigations conducted after earthquakes have identified instances of retaining structure failures. Often, these failures can be directly attributed to two main
factors: poorly constructed nonengineered walls and soil-related failures [31–34,104–107]. Additionally, most of these failure cases occurred in marine environments.

Seed and Whitman [37] reported that retaining structures built with backfills that do not liquefy typically have ample strength to endure substantial earthquakes, often without the need for specific design modifications. This conclusion is consistent with observations made during significant earthquakes such as the 1989 Loma Prieta earthquake, the 1999 earthquake in Turkey, the 2010 earthquake in Chile, the 2011 earthquake in Japan, and the 2014 earthquake in Chile. This section examines selected cases of retaining structure failures and their satisfactory performance with nonliquefiable backfill.

After the 1971 San Fernando earthquake, Clough and Fragaszy [108] found that U-shaped floodway channel walls originally designed for static loads with a factor of safety of 1.3 performed well with PGA ≤ 0.4 g and sustained damage above 0.5 g. The observed damage included tilting of the walls around the toe and considerable wall–slab connection yielding. Clough and Fragaszy [108] suggested that the M-O theory used with 70% PGA gives consistent results with the observed performance.

After the 1989 Loma Prieta earthquake, Benuska [109] and Whitman [110] documented that basement walls and mechanically stabilized earth (MSE) walls showed no signs of damage. Similarly, following the 1994 Northridge earthquake, Stewart et al. [111], Hall [112], and Holmes and Somers [113] reported that basement walls remained unaffected, while Lew et al. [114] found that temporary deep excavations remained undamaged.

In the destructive 1995 Kobe earthquake, Koseki et al. [115], Gazetas et al. [116], and Lew et al. [117] documented failures and significant deformations in embankment and waterfront walls. However, the basement walls remained undamaged. Iida et al. [118], Yoshida [119], and Lew et al. [117] reported that several subway stations were damaged in Kobe, with the Dakai subway station suffering a complete collapse. This collapse was attributed to a combination of inadequate structural design and the possible liquefaction of the backfill, rather than seismic earth pressures.

In the 1999 Chi-Chi Taiwan earthquake, certain types of retaining walls performed well, while others experienced failures [120–122]. Ling et al. [120] found the performance of concrete walls and flexible reinforced soil walls to be good. However, several gravity walls and geosynthetic-reinforced walls (excluding cantilever walls), sustained damage during the earthquake [121,122]. The authors attributed the failures to the topography (steep slopes and hills) and soil conditions in Taiwan, and to the poor construction and design. However, the failures were not due to a significant increase in dynamic earth pressures.

After the 1999 Düzce Turkey earthquake, Rathje et al. [123] noted that retaining structures did not experience substantial damage. However, Gur et al. [124] documented damage to the semi-basement walls of a multistory school building caused by poor structural design. Bray et al. [125] reported satisfactory performance of several retaining structures subjected to high PGA in the 2010 Chile earthquake.

Sitar et al. [44] reported no substantial damage or collapse of retaining structures following the 2008 Wenchuan earthquake in China, the 2010 Chile earthquake, and the 2011 Tohoku earthquake in Japan. Following the 2010 Chile earthquake, Verdugo et al. [126] reported minor damage to some basement walls, mainly due to construction defects.

Rollins et al. [127] noted instances of retaining structure failures in the 2012 Samara, Costa Rica, earthquake. However, these failures were primarily attributed to issues related to construction quality and soil conditions rather than seismic earth pressures. Nikolaou et al. [128] reported minor damage to concrete gravity and cantilever walls, while failures were observed in nonengineered stone walls and walls located on steep terrain during the 2014 Cephalonia, Greece, earthquake.

Rollins et al. [129] reported varying degrees of damage to retaining structures after the 2014 Iquique, Chile, earthquake. Importantly, the observed failures were not primarily linked to an increase in dynamic earth pressure. For instance, cantilever walls failed due to poor construction practices; masonry walls failed due to a lack of reinforcement; MSE walls failed because of the use of corroded reinforcing strips in the backfill; and the failure
of quay walls was attributed to the potential liquefaction of backfill. Notably, the authors reported no damage to basement walls.

According to Candia et al. [130], the overall performance of retaining structures following the 2014 Iquique, Chile, earthquake was deemed satisfactory. However, retaining walls with inadequate footing dimensions were severely damaged at some locations. Likewise, MSE walls failed because of the corrosion of reinforcement strips used in the backfill. Notably, there were no reports of damage to basement walls or temporarily braced excavations.

After the 2015 Gorkha Nepal earthquake, Hashash et al. [131] documented minimal damage to retaining structures. De Pascale et al. [132] noted that retaining structures with nonliquefiable backfill remained intact during the 2015 Illapel, Chile earthquake. Nonetheless, they reported a gravity wall failure near the waterfront, which was attributed to possible liquefaction below the footing.

Lanzo et al. [133] reported significant damage to gravity walls and stone masonry walls during three major earthquake events from August to November 2016 in central Italy. The authors observed that the failures were mainly due to poor construction practices and soil liquefaction behind and beneath the walls. Dashti and Ganapati [134] reported the performance of retaining walls located along a major highway during the 2021 Haiti earthquake. They attributed the failure of all retaining walls to the impact of boulder debris from rockfalls.

Based on the observed field performance, it can be inferred that retaining structures generally exhibit satisfactory performance during earthquakes, and the need for extensive seismic design provisions may be reconsidered. The rare failures of retaining structures were due to either liquefaction of saturated backfill or poor construction and design practices.

4. Conclusions

This paper presents a comprehensive review of analytical and field performance studies for assessing seismic earth pressures acting on retaining structures. There is a varied range of backfill conditions and wall geometries that must be addressed efficiently. However, this review focused on previous research highlighting significant works relevant to this study.

The analytical approaches discussed above have: (1) Laid the foundation for the formulation of commonly utilized design methods for both yielding and nonyielding retaining walls, such as the Mononobe–Okabe [2,3] and Richards–Elms [49] methods; (2) Showcased the feasibility and legitimacy of integrating elastic solutions that account for the rotational flexibility of walls and foundations; (3) Made it feasible to develop hybrid-type solutions by combining plasticity-type and elasticity-type solutions, thereby providing a more comprehensive and rational approach to the problem of retaining wall design; (4) Indicated a lack of consensus regarding the exact location of the resultant seismic earth thrust behind retaining walls; (5) Assumed that there is no phase difference between the wall inertia and the resultant seismic earth thrust.

Observations from significant earthquakes have revealed that retaining structures featuring nonliquefiable backfills exhibited excellent performance. Additionally, the limited evidence of damage or failures related to seismic earth pressures further supports these findings. Even retaining walls solely designed for static loadings performed well during severe ground motions. This implies that, in some circumstances, significant seismic design considerations for retaining structures may not be needed.

There seems to be a consensus that, for low PGAs of up to 0.4 g, static loading and inertial forces should be the primary considerations for gravity wall design. However, for greater PGA, the dynamic earth pressure increment should be considered as an upper-bound utilizing the Seed and Whitman [37] approach. The observed field performance of undamaged retaining walls supports this consideration, in addition to the findings of experimental studies and numerical analyses.
To address the challenges involved in the dynamic soil–structure interaction problem, researchers have often resorted to simplified assumptions and idealizations in their analytical methods. These simplifications often lead to conservative estimates of seismic earth pressures, as they may not fully capture the actual seismic behavior of the backfill–wall systems. Given the significance of seismic earth pressure in the design of retaining structures in seismically prone regions, it is imperative to develop a standardized approach in this field of study.

The observed field performance of retaining structures during major earthquakes has provided invaluable insights that can be utilized for: (a) Further refining our understanding of dynamic soil–structure interaction mechanisms, thus contributing to the development of more accurate analytical models; (b) Assessing the accuracy and reliability of available analytical models, thereby identifying their strengths and limitations; (c) Evaluating the reliability of the experiments and validating their findings, thus ensuring that they are appropriately capturing the essential aspects of soil–wall interaction during earthquakes; (d) Developing updated design guidelines and codes that better account for the complexities of dynamic soil–structure interaction.

This review clearly demonstrates the necessity for a more thorough evaluation of dynamic analysis and current retaining structure design methodologies. In this regard, further experimentation is needed, along with the establishment of an extensive field performance database for these structures. Such a database would facilitate the evaluation of the way these structures perform under varied ranges of backfill conditions and wall geometries. Generating this database is vital for advancing state-of-the-art methods and harnessing advanced analytical tools in conjunction with contemporary performance-based design practices. The authors would also like to mention that a follow-up review paper is under preparation, focusing on experimental and numerical findings, to further our understanding of the subject of seismic behavior of retaining walls.

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