



Article

Cryptocurrencies Intraday High-Frequency Volatility Spillover Effects Using Univariate and Multivariate GARCH Models

Apostolos Ampountolas ^{1,2}

¹ School of Hospitality Administration, Boston University, Boston, MA 02215, USA; aampount@bu.edu or apostolos.ampountolas@brunel.ac.uk

² Department of Mathematics, College of Engineering, Design and Physical Sciences, Brunel University London, Uxbridge UB8 3PH, UK

Abstract: Over the past years, cryptocurrencies have drawn substantial attention from the media while attracting many investors. Since then, cryptocurrency prices have experienced high fluctuations. In this paper, we forecast the high-frequency 1 min volatility of four widely traded cryptocurrencies, i.e., Bitcoin, Ethereum, Litecoin, and Ripple, by modeling volatility to select the best model. We propose various generalized autoregressive conditional heteroscedasticity (GARCH) family models, including an sGARCH(1,1), GJR-GARCH(1,1), TGARCH(1,1), EGARCH(1,1), which we compare to a multivariate DCC-GARCH(1,1) model to forecast the intraday price volatility. We evaluate the results under the MSE and MAE loss functions. Statistical analyses demonstrate that the univariate GJR-GARCH model (1,1) shows a superior predictive accuracy at all horizons, followed closely by the TGARCH(1,1), which are the best models for modeling the volatility process on out-of-sample data and have more accurately indicated the asymmetric incidence of shocks in the cryptocurrency market. The study determines evidence of bidirectional shock transmission effects between the cryptocurrency pairs. Hence, the multivariate DCC-GARCH model can identify the cryptocurrency market's cross-market volatility shocks and volatility transmissions. In addition, we introduce a comparison of the models using the improvement rate (IR) metric for comparing models. As a result, we compare the different forecasting models to the chosen benchmarking model to confirm the improvement trends for the model's predictions.

Keywords: intraday volatility; bitcoin price forecasting; cryptocurrency; GARCH models; GJR-GARCH model; volatility forecasting; EGARCH model; TGARCH model; DCC-GARCH model; Ethereum; Litecoin; Ripple



Citation: Ampountolas, Apostolos. 2022. Cryptocurrencies Intraday High-Frequency Volatility Spillover Effects Using Univariate and Multivariate GARCH Models.

International Journal of Financial Studies 10: 51. <https://doi.org/10.3390/ijfs10030051>

Academic Editors: Sahbi Farhani, Muhammad Ali Nasir and Sabri Boubaker

Received: 29 May 2022

Accepted: 30 June 2022

Published: 8 July 2022

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Interest in cryptocurrencies has drawn considerable attention, including various stakeholders, namely, investors, analysts, and academic researchers. Cryptocurrencies are a digital form of currency that uses an online distributed ledger technology known as the blockchain, which is based on the security of an algorithm for online payments without a central bank involvement or control of supply (Corbet et al. 2019). The current development of the numerous available cryptocurrencies is due to the rising applications of the blockchain technology. This rising popularity, probably due to the decentralized nature and the increased trading volume and capitalization in the market with the prospect to achieve high returns in a short term of days, weeks, or months (Kristoufek 2013), creates a need to understand and manage the risk of the cryptocurrency's environment, which plays a crucial role in the existence of financial institutions (Dyhrberg 2016). Furthermore, from financial institutions to large corporations, different institutions consider developing a digital currency, with recent examples from the Swiss National Bank (SNB), Bank of England, or other central banks. They have announced setting up task forces to explore the opportunity of creating a central bank digital currency (CBDC) (McDonald 2021; Revill and

Wilson 2022), or the emergence of the Libra coin (Diem) and digital wallet from Facebook. At the same time, the cryptocurrencies' notable price growths reinforce regulatory, ethical, and lawful challenges to central financial institutions due to the significant speculative component and the high volatility of traditional currencies (Cheah and Fry 2015; Fry and Cheah 2016).

Cryptocurrencies research has evolved as a popular topic in current academic research, and previous studies have revealed that the cryptocurrency market is extremely risky, with frequent jumps creating highly volatile outliers, see Akhtaruzzaman et al. (2022); Briere et al. (2015); Trucíos (2019). Nevertheless, due to the urgency of empirical proof, there is still a strong need to understand and manage the volatility risk of the cryptocurrency market aggregate, conceive new evidence to quantify this risk in a variation of cryptocurrencies, and at the same time, recognize gaps in the existing empirical literature. It is without question that modeling volatility is critical for risk management assessment. Moreover, there is a need to evaluate their volatility compared to traditional instruments due to the highly volatile environment. Many volatility models have been developed since the seminal paper by Engle (1982), which developed the autoregressive conditional heteroscedastic (ARCH) class of models. Bollerslev (1986), following Engle's previous work, generalized the ARCH model and presented the GARCH model. Since then, the generalized autoregressive conditional heteroscedasticity (GARCH) models have evolved to the most utilized models for estimating volatility. Thus, it is essential to find the most appropriate technique for estimating cryptocurrencies volatility. The question is becoming increasingly essential as the cryptocurrency marketplace grows, with an expanding quantity of currencies, platforms, and buyers and a growing acceptance.

Our study contributes to the recent growth of literature as follows: the paper benefits from this literature but also differs in several ways. First, we examine the importance of high-frequency intervals to capture intraday dynamics using 1 min efficiency in this heavily dynamic market. This differentiates our study from previous studies, which, with some exceptions, e.g., Katsiampa et al. (2019), evaluate cryptocurrencies volatility linkages using daily data. Therefore, we analyze some statistical properties of GARCH models to find the best model or set of models for modeling the volatility of four popular cryptocurrencies, i.e., Bitcoin, Ethereum, Ripple, and Litecoin. The specified forecasting models allow the estimation of the multistep-ahead forecast and a better use of the data's information. Bauwens et al. (2010) argued that standard GARCH models might generate limited results if the time series of data exhibits structural breaks. Chu et al. (2017) endorsed this theory, suggesting an integrated GARCH model as a good fit for evaluating cryptocurrency volatility. The study extends the previous referred studies by evaluating a range of four cryptocurrencies' price volatilities by employing various GARCH models such as the standard GARCH (Bollerslev 1986), and asymmetric models such as the EGARCH (Nelson 1991), GJR-GARCH (Glosten et al. 1993), the asymmetric threshold generalized autoregressive conditional heteroscedastic (TGARCH) (Zakoian 1994) models and distributional assumptions.

Secondly, we assess the out-of-sample forecasting performances of cryptocurrency market volatility dynamics that remain available for experimentation at multiple horizons. As such, the study examines which conditional heteroscedasticity model and distributional assumptions can robustly define the cryptocurrencies' price volatility. Then, we examine high-frequency 1 min data using the dynamic conditional correlation-GARCH (DCC-GARCH) model to estimate volatility returns between the cryptocurrencies with the expectation that it will provide a better approach for the stakeholders. Wang and Ngene (2020) used intraday data to explore any dynamic linkages of cryptocurrencies while implementing an asymmetric DCC model (ADCC). Similarly, Urquhart and Zhang (2019) used the ADCC models to investigate the volatility dynamics between Bitcoin and currency markets and Tiwari et al. (2019) a copula-ADCC model for studying time-varying correlations between cryptocurrency and the stock market, respectively. Therefore, this study

contributes to the literature by evaluating the interlink within the four cryptocurrencies and significant volatility dynamics.

We investigate the relative out-of-sample predicting performance over a wide range of time horizons. In order to find the best set of models, we undertake pairwise model comparisons using the Diebold and Mariano test (Diebold and Mariano 1995) and the Harvey–Leybourne–Newbold (HLN) test (Harvey et al. 1997). The model results are assessed using two loss functions: mean absolute error (MAE) and mean square error (MSE).

This study, motivated by the cryptocurrency's price instabilities, assesses the volatility of four popular cryptocurrencies by using several GARCH-type models. The loss function results demonstrate that all models perform well overall forecasting time horizons, albeit the GJR-GARCH model appears to be the only one that cannot be outperformed at almost any confidence level. Hence, we summarize our findings as follows: (i) the skew GED distribution is generally the preferred distribution; (ii) each GARCH model exhibits higher accuracy, particularly in volatility predictions over longer time horizons; (iii) the GJR-GARCH model yields a higher accuracy almost in every time horizon in both loss functions, particularly at the longer, i.e., 60, 30, 15 min time horizons, respectively; (v) the TGARCH model, a particular case of GJR-GARCH model, generates good accuracy initially on longer horizons of 60 and 30 min and then surprisingly at the 1 min horizon; (iv) finally, the sGARCH performs with a good accuracy at longer horizons, i.e., 60 min and 30 min followed by the EGARCH in a similar accuracy performance. Finally, we further estimate the dynamic conditional high-frequency correlation using the generalized dynamic conditional correlation (DCC-GARCH) model.

The following section discusses a short overview of the relevant literature while explaining the model estimation. The paper proceeds with an analysis of the study results, highlighting the employed techniques and adding a further discussion in Section 3. Finally, Section 4 summarizes the paper's conclusions and directions for future research.

2. Research Background

Several academic studies have studied the elements affecting cryptocurrencies' price and volatility. The most general models for the exchange rates of traditional currencies are based on generalized autoregressive conditional heteroscedasticity (GARCH) models. Similarly, the available academic work initially focuses mainly on the GARCH modeling of Bitcoin, the first and the most popular implementation for cryptocurrencies (see, Dyhrberg (2016); Katsiampa (2017); Lahmiri et al. (2018); Liu et al. (2017); Trucíos (2019)). Before looking at GARCH models, it is essential to disclose that GARCH models have been used to predict the variance of a time series also in other industries and not only in the financial sectors because of the extensive extensions to the initial GARCH model (Ampountolas 2021). On the other side, there lacks a comprehensive study towards the intraday dynamics of cryptocurrencies, although there is considerable work on fitting GARCH-type models to the exchange rates of alternative cryptocurrencies. Specifically, GARCH family models are utilized to evaluate the varying volatility of cryptocurrencies. Recently, Bouoiyour et al. (2016); Katsiampa (2017); Kim et al. (2021); Trucíos (2019) have compared some GARCH-type models. Kim et al. (2021) compared GARCH models and stochastic volatility to evaluate the volatility of nine cryptocurrencies. Their results indicated that the stochastic volatility method generated better forecasting results than the GARCH models for longer forecast horizons. Trucíos (2019) investigated daily volatility forecasting of Bitcoin of an out-of-sample by using several GARCH models. The study identified that the best model for forecasting the volatility was the AV-GARCH. At the same time, Katsiampa (2017) evaluated several GARCH models resulting in the AR-CGARCH model as a better fit than the other GARCH models to explain Bitcoin price volatility. However, Charles and Darné (2019) replicated the previous study using six GARCH-type models and obtained partially different results from Katsiampa (2017). The results identified that GARCH-type models described by short memory, asymmetric effects, or long-run and short-run movements appear inappropriate for modeling the Bitcoin returns. Lahmiri et al. (2018) examined

various Bitcoin markets' nonlinear volatility patterns, and the empirical results identified that the fractionally integrated GARCH (FIGARCH) framework captured the presence of long-range memory, independently of distributional inference. [Hyun et al. \(2019\)](#) study examined dependence relationships of five cryptocurrencies by employing a neural network autoregression model and machine learning techniques. Their results showed a copula directional dependence to be higher from Bitcoin to Litecoin and from Ethereum to the other four cryptocurrencies, which indicated that a returns shock of one cryptocurrency influenced to a great extent the other four. Accordingly, they showed that the fitting of GARCH models to cryptocurrencies return was insufficient for the neural network model. The specific Bitcoin studies and later the various currencies compared to Bitcoin volatility studies have employed a single conditional heteroscedasticity model. [Chu et al. \(2017\)](#) study fitted twelve GARCH models to assess the most popular cryptocurrencies and fit the best model. Their results showed that the IGARCH and GJR-GARCH models fitted best to assess volatility.

In this context, our contribution extends the above works, considering widely estimated asymmetric univariate models of conditional volatility or multivariate models as described below.

2.1. Model Specifications

2.1.1. GARCH Model

The most general models for the exchange rates of traditional currencies are based on generalized autoregressive conditional heteroscedasticity (GARCH) models. The GARCH model is a continuation of the autoregressive conditional heteroscedasticity (ARCH) model that supports the conditional variance to change over time as a function of past errors leaving the unconditional variance constant ([Bollerslev 1986](#)). The ARCH introduced by [Engle \(1982\)](#) and the GARCH family of models proposed by [Bollerslev \(1986\)](#) have been extensively utilized in the field of financial modeling to model the volatility of financial returns. Similarly, the available academic work focuses mainly on the GARCH modeling of Bitcoin, the first and the most popular cryptocurrency. The financial returns are subject to severe shocks (e.g., coronavirus), creating outliers (see [Figures 1–3](#)). These shocks may challenge the employed volatility estimation models such as the GARCH-type models.

Because there is a consensus that volatility is more constant than the price on the financial markets, much research has been devoted to studying volatility models. As such, we define the mathematical definition of volatility as follows:

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (\hat{r} - r_i)^2} \quad (1)$$

where σ is the volatility, \hat{r} is the mean log return, r_i is the log return at time i , and n is the number of log return observations.

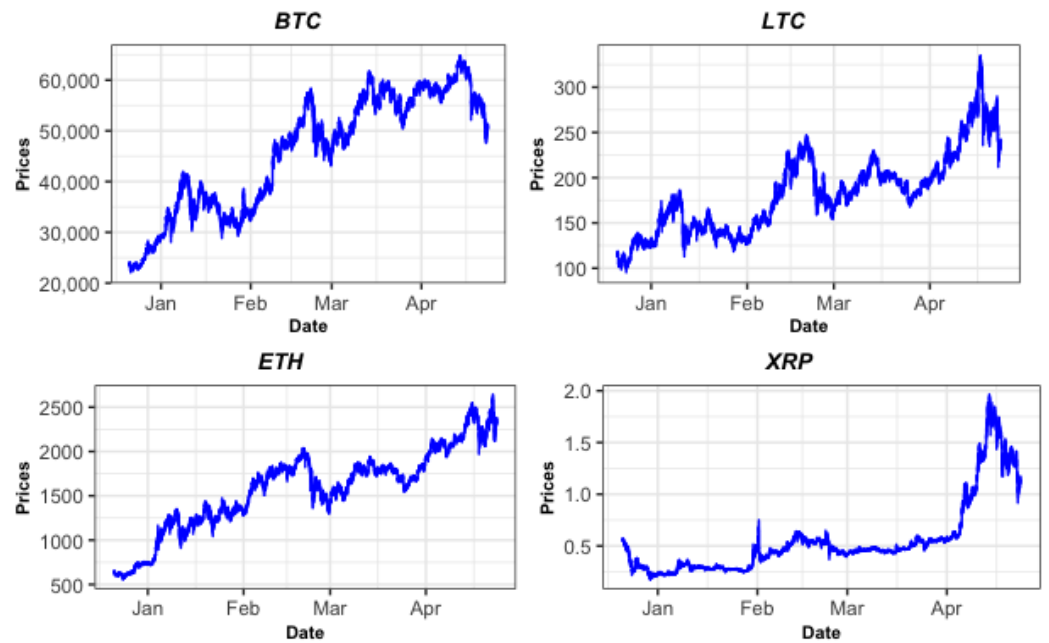


Figure 1. Cryptocurrencies' sample prices. Note: Data observation time period: 21 December 2020 to 23 April 2021.

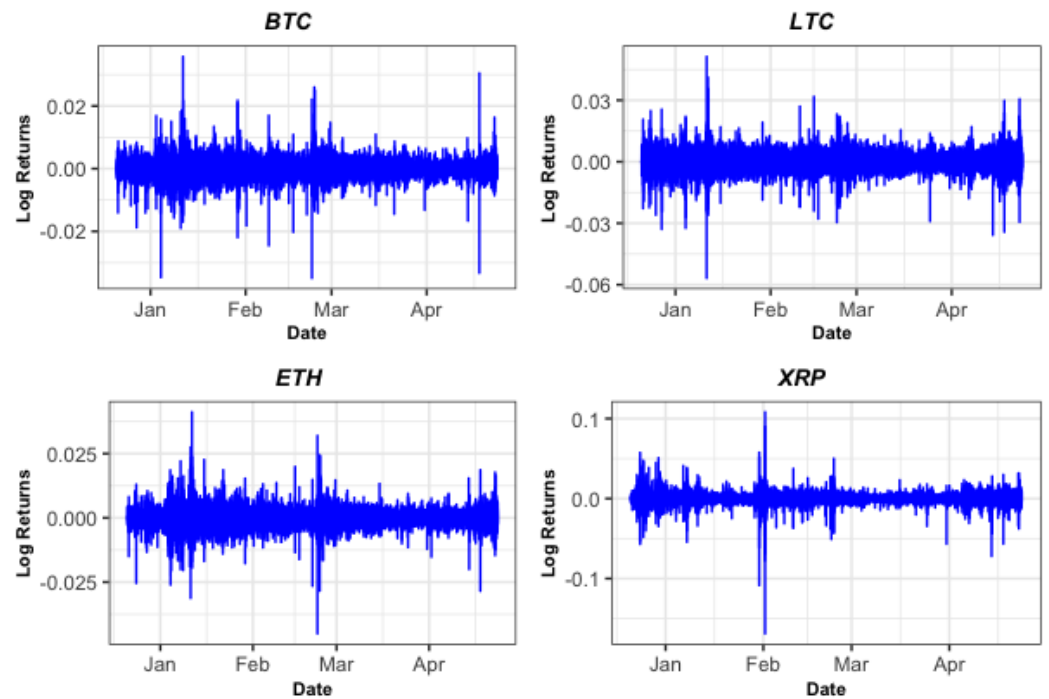


Figure 2. One-minute log returns (r_t) of all coins. Note: Data observation time period: 2020 to 23 April 2021.

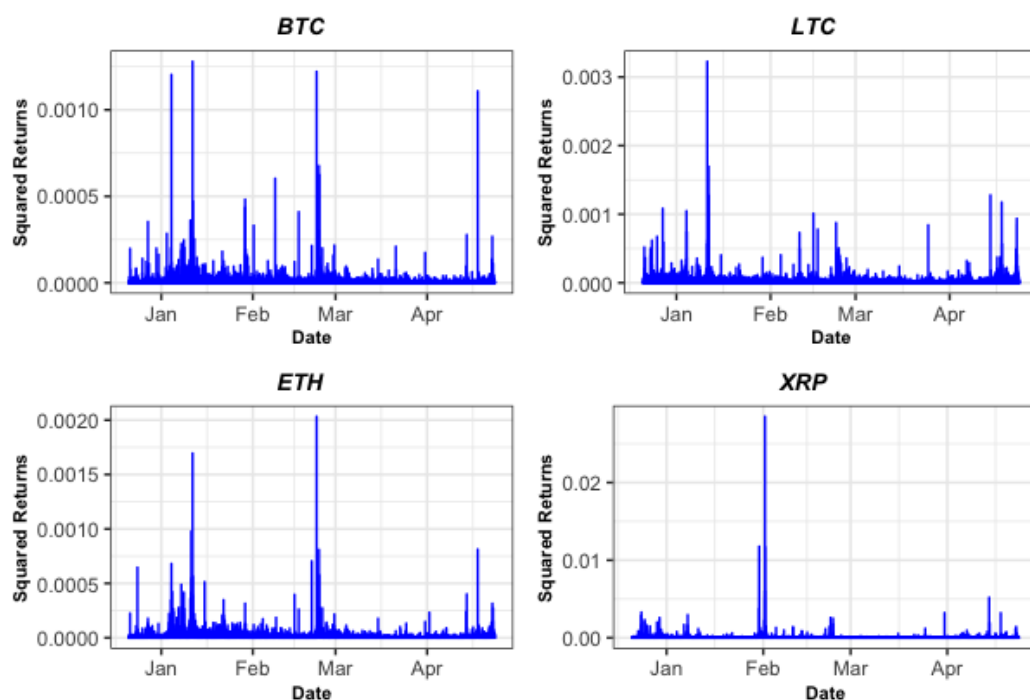


Figure 3. One-minute squared returns (r_t) of all coins. Note: Data observation time period: 21 December 2020 to 23 April 2021.

Normal, skew normal, Student’s t, skew Student’s t, GED, skew GED, normal inverse Gaussian, generalized hyperbolic, and Johnson’s reparameterized SU innovation distributions are some of the error distributions for the parameterizations of the GARCH model distributions (Ghalanos 2018). The Student’s t, skew Student’s t, and skew GED were utilized to parameterize the distributions of the GARCH-type models in this research.

Positive and negative error terms have an asymmetric effect on volatility, as per the standard GARCH model, which implies that positive and negative news have the same impact on volatility. However, this assumption readily fails in the financial stock market because a declining tendency in the stock exchange market has a more significant effect on the volatility index than a positive change, and inversely. In this context, asymmetric GARCH models were established to adjust asset returns. Therefore, this study evaluated the cryptocurrency’s price volatility by employing the standard GARCH and asymmetric GARCH models such as the EGARCH, GJR-GARCH, and the TGARCH models described briefly in the following sections. We chose the best model (p and q) using the Akaike information criterion (AIC).

In the following, we present the models used in the study application.

2.1.2. Standard GARCH (sGARCH) Models

A standard GARCH model (Bollerslev 1986) extends the ARCH model and is given by:

$$\begin{cases} y_t = \mu_t + e_t, \\ e_t = \sqrt{h_t} \cdot \eta_t, \eta_t \sim iid(0, 1), \\ h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \gamma_1 h_{t-1} \end{cases} \quad (2)$$

where μ_t represents the time-varying conditional mean which is given by $\beta'x_t$. The β represents a $k \times 1$ vector of parameters that is to be estimated, and x_t is the $k \times 1$ vector of stochastic covariates. α_0, α_1 , and $\gamma_1 \geq 0$ are parameters that fulfill certain conditions, and $\alpha_1 + \gamma_1 \leq 1$.

2.1.3. EGARCH Model

To address the shortcomings of symmetric models, such as the leverage effect conditional heteroscedasticity, Nelson (1991) presented the exponential generalized autoregressive conditional heteroscedasticity (EGARCH) model, a precise parametric form. To achieve a non-negative result, he used the natural logarithm of conditional variance. The symmetric ARCH and GARCH models can replicate volatility clustering and leptokurtosis, but they do not account for the asymmetric relationship between asset returns and volatility variations necessary when working with financial time-series data. Specifically, the logarithm of EGARCH is defined as

$$\log(h_t) = \alpha_0 + \alpha_1 \left(\left| \frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}} \right| - \mathbb{E} \left| \frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}} \right| \right) + \zeta \frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}} + \gamma_1 \log(h_{t-1}) \tag{3}$$

where $\alpha_1 \left(\left| \frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}} \right| - \mathbb{E} \left| \frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}} \right| \right)$ denotes the magnitude effect and $\zeta \frac{\epsilon_{t-1}}{\sqrt{h_{t-1}}}$ the sign effect. As a result, if $\zeta < 0$, negative innovations cause a greater volatility than positive innovations of the same magnitude. The logarithmic modification of the volatility specification implies that the parameters in this model are not limited to positive values.

2.1.4. GJR-GARCH Model

The Glosten–Jagannathan–Runkle generalized autoregressive conditional heteroscedasticity model (GJR-GARCH) was introduced by Glosten et al. (1993) as an alternate technique to model volatility asymmetry. The model has become popular among researchers and it assumes a specific parametric form for this conditional heteroscedasticity. GJR-GARCH has the advantage of being easier to execute in reality than EGARCH because the variance is explicitly represented rather than using the natural logarithm (Choi 2002).

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \zeta \epsilon_{t-1}^2 \cdot I_{\{\epsilon_{t-1} < 0\}} + \gamma_1 h_{t-1} \tag{4}$$

where the effect of the unexpected return depends on its sign:

$$I_{\{\omega\}} = \begin{cases} 0 & \text{if } \epsilon_{t-1} \geq 0 \\ 1 & \text{if } \epsilon_{t-1} < 0 \end{cases} \tag{5}$$

The common restrictions on the parameters are $\omega, \alpha, \gamma, \beta > 0$. The GARCH model is a restricted version of the GJR-GARCH, with $\gamma = 0$. Testing the hypothesis that $\gamma_1 = 0$ thus provides a test of the asymmetric effect of current and past squared returns on volatility.

2.1.5. Threshold GARCH Model

To overcome the limitations of GARCH models, Zakoian (1994) proposed the TGARCH model, which is similar to the GJR-GARCH model and has been further developed by Rabemananjara and Zakoian (1993). However, it concentrates on conditional standard deviation (volatility) rather than a conditional variance. It has been utilized to investigate the effects of negative and positive returns on conditional volatility dynamics. The form of a threshold GARCH model is given:

$$\begin{aligned} \sigma_t &= \omega + \alpha^+ \epsilon_{t-1}^+ + \gamma^- \epsilon_{t-1}^- + \beta \sigma_{t-1} \\ &= \omega + \alpha I(\epsilon_{t-1} \geq 0) \epsilon_{t-1} + \gamma I(\epsilon_{t-1} < 0) \epsilon_{t-1} + \beta \sigma_{t-1} \end{aligned} \tag{6}$$

where,

$$I_{t-1} = \begin{cases} 1, & \text{if } \epsilon_{t-i} < 0, & \text{bad news} \\ 0, & \text{if } \epsilon_{t-i} \geq 0, & \text{good news} \end{cases} \tag{7}$$

where $\epsilon_t^+ = \epsilon_t$ if $\epsilon_t > 0$, $\epsilon_t^+ \equiv 0$ otherwise, $\epsilon_t^- \equiv \epsilon_t - \epsilon_t^+$.

The TGARCH model is identical to the GJR-GARCH model; however, it does not place any constraints on the positivity of the volatility coefficients.

2.1.6. DCC-GARCH Model

Various univariate GARCH models have previously been discussed. Correlation models indicate the decomposition of the conditional covariance matrix into conditional standard deviations and correlations. As a result, the following model is based on conditional variances and correlations rather than basic modeling. The constant conditional correlation (CCC-GARCH) model and its extensions, such as the dynamic conditional correlation (DCC-GARCH) model, are members of this class.

The most straightforward multivariate correlation model nested in the conditional correlation models is the constant conditional correlation (CCC-GARCH) model introduced by (Bollerslev 1990). Although the CCC-GARCH model is appealing in many ways, empirical studies have indicated that the assumption of constant conditional correlations is too restrictive. Therefore, Engle (2002) proposed the dynamic conditional correlation (DCC-GARCH) model, which has a dynamic conditional correlation structure and extends the CCC-GARCH with additional parameters. For the DCC-GARCH model, the GARCH parameters are initially estimated, followed by correlations. In the end, the parameterizations of the conditional correlation variance of the DCC-GARCH are provided as follows

$$H_t = D_t R_t D_t \tag{8}$$

Accordingly, the dynamic correlation model allows R to be time-varying, and the parameterizations of R hold the exact requirements as H_t represents the conditional covariance matrix. Moreover, the conditional variances must be unity. The matrix R_t is the conditional correlation matrix, and D_t is a diagonal matrix with time-varying parameterizations (Engle 2002). That is,

$$D_t = \text{diag}\{D_{1t}, D_{2t}, \dots, D_{kt}\} \quad R_t = [\rho_{ij,t}] \tag{9}$$

where $\rho_{ij,t}$ refers to the correlation between the i th and j th return series. Therefore, Engle took into account a dynamic matrix process that can be written as

$$\begin{aligned} Q_t &= \bar{Q} + \alpha(z_{t-1}z'_{t-1} - \bar{Q}) + \beta(Q_{t-1} - \bar{Q}) \\ &= \bar{Q}(1 - \alpha - \beta) + \alpha z_{t-1}z'_{t-1} + \beta Q_{t-1} \end{aligned} \tag{10}$$

where α is a positive and β a non-negative scalar parameter such that $\alpha + \beta < 1$ to ensure the stationarity and positive definiteness of Q_t , and \bar{Q} is the unconditional matrix of standardized errors (z_t) that is introduced into the equation using the covariance targeting part $\bar{Q}(1 - \alpha - \beta)$; Q_0 is positive definite (Ghalanos 2015). Although this method provides positive definiteness, it does not build accurate correlation matrices. The correlation matrix R is obtained by rescaling Q_t as follows:

$$R_t = \text{diag}(Q_t)^{-1/2} Q_t \text{diag}(Q_t)^{-1/2} \tag{11}$$

The DCC-GARCH model extends the CCC-GARCH model by adding more parameters.

2.2. Methodological Development

We accept that in cryptocurrency markets, returns r_t follow an autoregressive fractionally integrated moving average process (Granger and Joyeux 1980), as follows:

$$\Phi(L)(1 - L)^d(r_t - \mu_0) = \Theta(L)\epsilon_t \quad \text{where } \epsilon_t \sim \text{i.i.d.N}(0, \sigma_\epsilon^2). \tag{12}$$

where μ_0 is the mean of the process and the lag polynomials in Equation (12) are defined as: $\Phi(L) = (1 - \phi_1 L - \dots - \phi_p L^p)$ and $\Theta(L) = (1 - \theta_1 L - \dots - \theta_q L^q)$, where p and q are the autoregressive and moving average orders ($Lh_t = h_{t-1}$), respectively. Furthermore, the

lag operator is defined as $d \in (-1/2, 1/2)$, L and the fractional differencing operator as $(1 - L)^d$; see, e.g., Brockwell and Davis (1991) by the following equation:

$$(1 - L)^{-d} = 1 + \sum_{k=1}^{\infty} \frac{\Gamma(k + d)L^k}{\Gamma(d)\Gamma(k + 1)}, \quad (13)$$

where $\Gamma(\cdot)$ is the gamma function. Moreover, the innovation process, ϵ_t , is described as follows:

$$\epsilon_t = u_t \sigma_t, \quad (14)$$

where u_t denotes a set of independent identically distributed normal random variables with a mean of zero, a unit variance, and time-varying dynamics of σ_t .

3. Data and Results

The data set of this study contained daily historical high-frequency 1 min time series data of Bitcoin (BTC-USD) closing prices and other cryptocurrencies such as Litecoin (LTC-USD), Ethereum (ETH-USD), and Ripple (XRP-USD), to forecast the price volatility and evaluate the model. Each observation on the database contained the exchange date, time, symbol, open, high, low, close price, and volume for all transactions in coins and in USD. A total of 178,560 data set observations were used, covering the period from December 2020 to April 2021 (12/21/20 0:00 to 4/23/21 23:59 EDT). Note that the data set sample covers the period of the coronavirus, following a large upswing and downswing in the prices of cryptocurrencies. The data sets were obtained from "CryptoDataDownload.com (accessed on 4 May 2021),¹" and the prices are based on the EDT time zone. Table 1 provides an overview of the cryptocurrency trading volume and capitalization in the market.

Table 1. Capitalization of the cryptocurrencies.

Symbol	Name	Market Cap (USD)	Price (USD)	Circulating Supply
BTC	Bitcoin	\$704,141,870,936	\$37,019.80	18,939,768 BTC
ETH	Ethereum	\$295,477,305,134	\$2461.43	119,308,579 ETH
LTC	Litecoin	\$7,535,211,234	\$107.95	69,492,969 LTC
XRP	Ripple	\$29,254,109,616	\$0.6101	47,736,918,345 XRP

Note: Data are publicly available at <https://coinmarketcap.com>, accessed: 19 February 2022.

Figure 1 presents the historical price performance for the cryptocurrencies during the observed period. We divided the data set into two segments: a training set and a test set to evaluate the model's forecasting performance, following a ratio of 80:20, as suggested by Pindyck and Rubinfeld (1998). Therefore, we used 20% (35,712 of 1 min closing price observations) for validation purposes. Finally, because the assessed models required stationary data, we computed the log returns and ran the augmented Dickey–Fuller test to verify the nonexistence of unit root.

Cryptocurrency returns were calculated using the formula $r_t = \ln\left(\frac{P_t}{P_{t-1}}\right)$ and we let $\alpha_t = r_t - E_{t-1}$, where P_t was the closing price of the cryptocurrency on minute t . Table 2 reports the summary statistics for the cryptocurrencies. The standard deviation for Bitcoin (BTC) is 0.001541, and the largest return drop is -0.034947 , while the most significant return increase is 0.035773. For the remaining cryptocurrencies, the standard deviations are 0.001865, 0.002275, 0.003355 for Ethereum (ETH), Litecoin (LTC), and Ripple (XRP), respectively. The relatively high standard deviations characterize the high level of volatility, in addition to the minimum and maximum rates. Furthermore, all four cryptocurrencies present a positive average return. Table 2 shows that XRP is the only cryptocurrency that shows more robust positive returns during the evaluation period. It suggests that XRP has a more negligible correlation with the cryptocurrency market than other cryptocurrencies, implying that its systematic risk in the cryptocurrency market is low. As a result, it may draw more interest among investors trying to form a market portfolio.

Moreover, from the summary statistics (Table 2), we observe that all four cryptocurrencies' price 1 min returns have a positive performance, with ETH presenting the highest. Following the same pattern, we observe that BTC generates the lowest risk (smallest standard deviation), then ETH and LTC, while the largest is found in XRP. Moreover, the data set samples exhibit a moderate negative asymmetry, as all four cryptocurrencies are left-skewed, followed by a moderate kurtosis except for XRP, which has a very large excess kurtosis comparable to the other cryptocurrencies. Furthermore, we used the Hurst exponents $H(t)$ to determine the degree of persistence, a statistical measure used to categorize time series and infer the degree of difficulty in predicting and selecting an acceptable model for the time series. A Hurst index value between 0.5 and 1.0 indicates persistent behavior; the more significant the H value, the stronger the trend in the volatility process. The cryptocurrency log returns are close to 0.5 (BTC = 0.4921, ETH = 0.4920) or over (XRP = 0.5232). The GARCH models generally feature weak persistent behavior, especially over extended time horizons, when the time correlation drops and a simple uncorrelated Itô process is revealed (Carbone et al. 2004). Finally, the Jarque–Bera (JB) test (Jarque and Bera 1987) shows a general departure from normality for all cryptocurrencies price returns (Table 2, Part A—Descriptive Statistics).

In our analysis, we started with the Dickey–Fuller (ADF) and Phillips–Perron (PP) tests, both coefficient tests for detecting non seasonal unit roots (Dickey and Fuller (1979); Phillips and Perron (1988)), to determine the data stationarity. Both unit root tests rejected the null hypothesis for the examined cryptocurrencies returns at the 0.01 level. To confirm the results of the ADF test, we also implemented the KPSS stationarity test that accepts the null hypothesis for the cryptocurrency return rates (Table 2, Part B—Test Statistics).

Figure 1 displays the closing prices of the four cryptocurrencies in this study. We notice that Bitcoin, Ethereum, and Litecoin experienced significant growth until the end of February or the beginning of March when the first actions in response to the coronavirus were disclosed. Similarly, we notice that Bitcoin and Litecoin carry almost a similar pattern until the end of January; hence they could be correlated. Ripple follows a more stable pattern increase until the beginning of April, where it exhibits an incremental increase. Finally, the plots illustrate that, except for Ethereum, where we see a sharp decline and increase, all four cryptocurrencies' prices have gradually declined since the second half of April.

Furthermore, Figure 2 shows the corresponding 1 min log returns of the observed market price indices for all exchanges trading in the examined cryptocurrencies, while Figure 3 presents the associated squared returns. Similar to the previous information provided in the descriptive statistics table, the plots indicate that the log returns are moderately symmetrically distributed, with some picks during the observed period.

Table 3 reports the Pearson correlation coefficients for the cryptocurrency's pairs, which are positive and significant using 1 min returns data. Moreover, Ethereum and Litecoin report a high correlation with Bitcoin, 0.72269 and 0.54538, respectively. In addition, Ethereum shows a considerable moderate correlation with Litecoin (0.56353) and the lowest correlation with Ripple (0.40855). Similarly, Ripple shows the lowest correlations with Bitcoin and Litecoin, 0.41015 and 0.37183, respectively.

Table 2. Descriptive statistics and unit root tests of cryptocurrencies’ 1 min returns.

	Bitcoin—BTC	Ethereum—ETH	Litecoin—LTC	Ripple—XRP
A. Descriptive statistics				
Obs	178,560	178,560	178,560	178,560
Minimum	−0.034947	−0.045088	−0.056805	−0.168867
1-Quartile	−0.000711	−0.000800	−0.000779	−0.001183
Mean	0.000004	0.000007	0.000004	0.000004
Median	0.000000	0.000000	0.000000	0.000000
3-Quartile	0.000713	0.000822	0.000789	0.001156
Maximum	0.035773	0.041179	0.051331	0.108117
Stdev	0.001541	0.001865	0.002275	0.003355
Skewness	−0.022956	−0.075367	−0.099042	−1.384401
Kurtosis	19.791322	15.802376	19.215616	86.717473
Hurst exponent	0.492180	0.492006	0.463128	0.523267
VaR 5%	−0.002235	−0.002763	−0.003451	−0.004500
VaR 1%	−0.010735	−0.011320	−0.015664	−0.076805
JB <i>p</i> -value	<0.001	<0.001	<0.001	<0.001
B. Test Statistics				
ADF	−58.998 **	−59.065 **	−60.263 **	−57467 **
PP	−169567 **	−169173 **	−170385 **	−172599 **
KPSS	0.18957	0.12696	0.024004	0.21985

Note: ** Significant at the 0.01 level. JB: Jacque–Bera test statistics; ADF: augmented Dickey–Fuller statistics; PP: Phillips–Perron adjusted *t*-statistics of the lagged dependent variable; KPSS: KPSS test statistics using residuals from regressions.

Table 3. Correlation matrix of 1 min returns.

	Bitcoin—BTC	Ethereum—ETH	Litecoin—LTC
Bitcoin			
Ethereum	0.72296 ***		
Litecoin	0.54538 ***	0.56353 ***	
Ripple	0.41015 ***	0.40855 ***	0.37183 ***

Note: *** Significant at the 0.001 level.

3.1. Estimation Results

We partitioned the data set into two periods: training and evaluation. The parameters of the volatility models were assessed employing the training intraday 1 min observations. The models’ out-of-sample performances were evaluated using a rolling window scheme with a window size of 1440 1 min observations. The estimations were used for the subsequent *n* periods to generate one-step-ahead predictions. Each model led to a sequence of losses, $L_{k,t} \equiv L(\hat{\sigma}_t^2, h_{k,t}^2), t = 1, \dots, n$, defining the relative performance variables. As a result, model *k* generated a sequence of forecasts, $h_{k,1}^2, \dots, h_{k,n}^2$, compared to $\hat{\sigma}_1^2, \dots, \hat{\sigma}_n^2$, using a loss function *L* as follows:

$$X_{k,t} \equiv L_{0,t} - L_{k,t}, \quad k = 1, \dots, l, t = 1, \dots, n. \text{ (Hansen and Lunde 2005)}$$

3.2. Model Selection

This study’s GARCH models selection of the ((*p* = 1) and (*q* = 1)) were estimated using the method of maximum likelihood. Hansen and Lunde (2005) identified there was no evidence that other more sophisticated models could outperform a GARCH(1,1). A GARCH(1,1) model with constant mean could be expressed as follows: $y_t = \mu_t + e_t$ with $e_t \sim N(0, \sigma_t^2)$.

$$\begin{cases} y_t = \mu_t + e_t, \\ e_t = \sqrt{h_t} \cdot \eta_t, \eta_t \sim iid(0, 1), \\ h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \gamma_1 h_{t-1} \end{cases} \quad (15)$$

The discrimination was evaluated employing the Akaike information criterion (AIC) from Akaike (1981), defined by:

$$\text{AIC} = 2k - 2 \ln L(\hat{\Theta}), \quad (16)$$

where k denotes the number of unknown parameters, Θ the vector of unknown parameters, and $L(\hat{\Theta})$ their maximum likelihood (ML) estimates; the optimal model was found when minimizing the criteria. Moreover, AIC tends to favor larger models (Gilli et al. 2019). In addition to the AIC, we used the Hannan–Quinn criterion (HQC) (Hannan and Quinn 1979) defined by:

$$\text{HQC} = -2 \ln L(\hat{\Theta}) + 2k \ln(\ln n). \quad (17)$$

In Tables 4 and 5 we report the GARCH models evaluation. The optimal lag in the ARFIMA(p, d, q) is based primary on the AIC criterion and secondary to the HQC criterion; the smaller the values of the criteria, the better the fit. The TGARCH model provides the lowest AIC over all cryptocurrencies. Hence, based on the value of the AIC model selection criterion, this implies that, in general, TGARCH is superior among the GARCH family models. Moreover, Tables 4 and 5 present the ML estimates and asymptotic standard errors (in parenthesis) for the models sGARCH(1,1), EGARCH(1,1), GJR-GARCH(1,1), and TGARCH(1,1). It is worth noting that the GARCH and FIGARCH models produce quite comparable results, with the fractional differencing parameter close to one.

Table 4. Maximum likelihood estimates of the GARCH models.

BTC	sGARCH			EGARCH			GJRGARCH			TGARCH		
	sged	sstd	std	sged	sstd	std	sged	sstd	std	sged	sstd	std
μ	0.000004 *** (0.000000)	0.000001 (0.000002)	0.000005 ** (0.000002)	−0.000036 *** (0.000002)	0.000027 *** (0.000001)	0.000026 *** (0.000001)	0.000004 *** (0.000000)	0.000009 *** (0.000002)	0.000008 *** (0.000002)	0.000004 *** (0.000000)	0.000000 (0.000002)	0.000001 (0.000002)
ω	0.000000 (0.000000)	0.000000 (0.000000)	0.000000 (0.000000)	−0.199788 *** (0.000296)	−0.127159 *** (0.000086)	−0.128498 *** (0.000099)	0.000000 (0.000000)	0.000000 (0.000000)	0.000000 (0.000000)	0.000000 (0.000000)	0.000008 *** (0.000001)	0.000008 *** (0.000001)
α_1	0.050000 *** (0.000000)	0.070629 *** (0.002009)	0.056567 *** (0.001466)	−0.027195 *** (0.001304)	−0.022736 *** (0.001235)	−0.022642 *** (0.001235)	0.050000 *** (0.000000)	0.054460 *** (0.001369)	0.053428 *** (0.001243)	0.050000 *** (0.000000)	0.071472 *** (0.001217)	0.071333 *** (0.001209)
β_1	0.900000 *** (0.000005)	0.916645 *** (0.002002)	0.934692 *** (0.001450)	0.985003 *** (0.000002)	0.990558 *** (0.000022)	0.990458 *** (0.000022)	0.900000 *** (0.000006)	0.920309 *** (0.001706)	0.921220 *** (0.001626)	0.900000 *** (0.000005)	0.940540 *** (0.000941)	0.940661 *** (0.000948)
γ_1				0.143372 *** (0.000400)	0.127978 *** (0.000515)	0.128087 *** (0.000526)	0.050000 *** (0.000000)	0.028111 *** (0.001627)	0.028138 *** (0.001588)			
η_{11}										0.050000 *** (0.000000)	0.213108 *** (0.010034)	0.214260 *** (0.010001)
$Log(L)$	773619.2	946853.5	946946.3	945566.8	947018.3	947014.9	746828.2	947081.6	947056.3	702209	947359.8	947358
AIC	−8.665	−10.605	−10.606	−10.591	−10.607	−10.607	−8.3649	−10.608	−10.608	−7.8651	−10.611	−10.611
HQ	−8.6648	−10.605	−10.606	−10.591	−10.607	−10.607	−8.3647	−10.608	−10.607	−7.8649	−10.611	−10.611
ETH												
μ	0.000007 *** (0.000000)	0.000000 (0.000003)	0.000014 *** (0.000002)	0.000008 *** (0.000002)	−0.000045 *** (0.000002)	−0.000037 *** (0.000002)	0.000007 *** (0.000000)	−0.000003 (0.000003)	0.000021 *** (0.000002)	0.000007 *** (0.000000)	0.000000 (0.000001)	0.000000 (0.000001)
ω	0.000000 (0.000000)	0.000000 (0.000000)	0.000000 (0.000000)	−0.252306 *** (0.000314)	−0.189859 *** (0.000054)	−0.195798 *** (0.000227)	0.000000 (0.000000)	0.000000 (0.000000)	0.000000 (0.000000)	0.000000 (0.000000)	0.000000 (0.000000)	0.000000 (0.000000)
α_1	0.050000 *** (0.000000)	0.066440 *** (0.001338)	0.069004 *** (0.001272)	−0.034462 *** (0.001480)	−0.031997 *** (0.001399)	−0.031576 *** (0.001391)	0.050000 *** (0.000000)	0.053718 *** (0.001034)	0.051967 *** (0.000715)	0.050000 *** (0.000000)	0.089606 *** (0.001234)	0.087648 *** (0.001176)
β_1	0.900000 *** (0.000005)	0.928220 *** (0.001199)	0.924150 *** (0.001249)	0.980649 *** (0.000007)	0.985219 *** (0.000032)	0.984798 *** (0.000036)	0.900000 *** (0.000006)	0.917992 *** (0.001482)	0.926868 *** (0.001130)	0.900000 *** (0.000005)	0.933196 *** (0.000876)	0.934648 *** (0.000827)
γ_1				0.143349 *** (0.000423)	0.155793 *** (0.000996)	0.152671 *** (0.001001)	0.050000 *** (0.000000)	0.039489 *** (0.001635)	0.033988 *** (0.001618)			
η_{11}										0.050000 *** (0.000000)	0.167657 *** (0.008134)	0.172251 *** (0.008216)
$Log(L)$	745916	917369.2	917288.2	917453	917216.6	917137.9	719008.2	917515.1	917591.5	678231.9	917187.1	917188.2
AIC	−8.3547	−10.275	−10.274	−10.276	−10.273	−10.273	−8.0533	−10.277	−10.278	−7.5966	−10.273	−10.273
HQ	−8.3545	−10.275	−10.274	−10.276	−10.273	−10.272	−8.0531	−10.277	−10.277	−7.5964	−10.273	−10.273

Note: GARCH models were estimated with the Student's *t*, and skew GED distributions. Asymptotic standard errors are given in parentheses. AIC: Akaike's information criterion; HQ: Hannan–Quinn criterion. Mean model: BTC: ARFIMA(0,0,5) and ETH: (0,0,3), respectively. ** Significance at the 5% level, *** at the 1% level.

Table 5. Maximum likelihood estimates of the GARCH models.

	sGARCH			EGARCH			GJR GARCH			TGARCH		
	sged	sstd	std	sged	sstd	std	sged	sstd	std	sged	sstd	std
LTC												
μ	0.000004 *** (0.000000)	0.000002 (0.000002)	0.000003 * (0.000002)	0.000000 * (0.000000)	-0.000016 *** (0.000004)	0.000000 (0.000001)	0.000004 *** (0.000000)	-0.000001 (0.000002)	0.000004 ** (0.000002)	0.000004 *** (0.000000)	0.000000 (0.000001)	0.000000 (0.000001)
ω	0.000000 (0.000000)	0.000000 (0.000000)	0.000000 (0.000000)	-0.042510 *** (0.000038)	0.003790 *** (0.000139)	-0.124647 *** (0.000590)	0.000000 (0.000000)	0.000000 (0.000000)	0.000000 (0.000000)	0.000000 (0.000000)	0.000101 *** (0.000011)	0.000020 *** (0.000004)
α_1	0.050000 *** (0.000000)	0.059881 *** (0.000658)	0.060617 *** (0.000663)	0.069596 *** (0.002222)	0.042853 *** (0.009338)	-0.047306 *** (0.004832)	0.050000 *** (0.000000)	0.052327 *** (0.000366)	0.053982 *** (0.000360)	0.050000 *** (0.000000)	1.000000 *** (0.006372)	0.258998 *** (0.002287)
β_1	0.900000 *** (0.000005)	0.938900 *** (0.000670)	0.937879 *** (0.000681)	0.992120 *** (0.000003)	1.000000 *** (0.000001)	0.987396 *** (0.000014)	0.900000 *** (0.000006)	0.939018 *** (0.000671)	0.937384 *** (0.000690)	0.900000 *** (0.000005)	0.899687 *** (0.001938)	0.922289 *** (0.001787)
γ_1				0.339121 *** (0.001074)	0.459831 *** (0.010359)	0.602413 *** (0.008646)	0.050000 *** (0.000000)	0.014514 *** (0.001480)	0.013659 *** (0.001496)			
η_{11}										0.050000 *** (0.000000)	0.083975 *** (0.009137)	0.067701 *** (0.009897)
Log(L)	709188.6	888892.2	888860.8	1050125	891656.5	891469.1	682631.5	888907.3	888866.9	660488	893258.3	893294.8
AIC	-7.9433	-9.9561	-9.9558	-11.762	-9.9871	-9.985	-7.6458	-9.9563	-9.9558	-7.3978	-10.005	-10.005
HQ	-7.9431	-9.9559	-9.9556	-11.762	-9.9869	-9.9848	-7.6456	-9.9561	-9.9556	-7.3976	-10.005	-10.005
XRP												
μ	0.000004 (0.000004)	0.000006 * (0.000004)	0.000001 (0.000003)	-0.000023 *** (0.000001)	0.000000 (0.000001)	0.000000 (0.000001)	0.000004 (0.000019)	0.000002 (0.000004)	-0.000005 * (0.000003)	0.000004 *** (0.000000)	0.000000 (0.000003)	-0.000002 (0.000003)
ω	0.000000 ** (0.000000)	0.000000 (0.000000)	0.000000 (0.000000)	-0.274114 *** (0.000527)	-0.205692 *** (0.000087)	-0.198845 *** (0.000284)	0.000000 (0.000000)	0.000000 (0.000000)	0.000000 (0.000000)	0.000000 (0.000000)	0.000013 *** (0.000001)	0.000013 *** (0.000001)
α_1	0.050000 *** (0.000000)	0.068878 *** (0.001179)	0.074544 *** (0.001150)	-0.014142 *** (0.002111)	-0.010385 *** (0.001973)	-0.011185 *** (0.001910)	0.050000 *** (0.000001)	0.061845 *** (0.000548)	0.062147 *** (0.000622)	0.050000 *** (0.000000)	0.096955 *** (0.001677)	0.097085 *** (0.001668)
β_1	0.900000 *** (0.000003)	0.927172 *** (0.001102)	0.923875 *** (0.001034)	0.977363 *** (0.000004)	0.982719 *** (0.000054)	0.983362 *** (0.000026)	0.900000 *** (0.000008)	0.932527 *** (0.000954)	0.930476 *** (0.001035)	0.900000 *** (0.000001)	0.927709 *** (0.000952)	0.927631 *** (0.000953)
γ_1				0.241130 *** (0.001435)	0.255071 *** (0.001119)	0.241243 *** (0.002419)	0.050000 *** (0.000001)	0.008515 *** (0.001582)	0.009471 *** (0.001604)			
η_{11}										0.050000 *** (0.000000)	0.057707 *** (0.009731)	0.059406 *** (0.009682)
Log(L)	673665.1	845577.4	845630.5	845364.3	844559.4	844557.4	647702.6	845649	845613.5	612925.9	846029.6	846018.7
AIC	-7.5455	-9.471	-9.4716	-9.4686	-9.4596	-9.4596	-7.2547	-9.4718	-9.4714	-6.8651	-9.4761	-9.4759
HQ	-7.5454	-9.4709	-9.4715	-9.4685	-9.4595	-9.4595	-7.2545	-9.4717	-9.4713	-6.865	-9.4759	-9.4758

Note: GARCH models were estimated with the Student's t , and skew GED distributions. Asymptotic standard errors are given in parentheses. AIC: Akaike's information criterion; HQ: Hannan-Quinn criterion. Mean model: LTC: (5,0,0) and XRP: (0,0,0), respectively. * Significance at the 10% level, ** at the 5% level, *** at the 1% level.

3.3. Forecasting Volatility Accuracy

We evaluated the accuracy of the volatility models employing standard loss functions in terms of the mean absolute error (MAE) and mean square error (MSE). We generated volatility forecasts at horizons of 1 min, 5 min, 15 min, 30 min, and 60 min, for each model. The aim of assessing multiple forecast horizons was to understand how the predictive ability of the models improve within the examined horizons in such a high-frequency data set.

$$\text{Mean absolute error: } \text{MAE}_1 = n^{-1} \sum_{t=1}^n |\sigma_t - h_t|, \quad (18)$$

$$\text{Mean square Error: } \text{MSE}_1 = n^{-1} \sum_{t=1}^n (\sigma_t - h_t)^2. \quad (19)$$

where n is the number of observations in the testing data set, σ_t denotes the benchmark volatility value, and h_t refers to the predicted values at time t , respectively.

In addition, because we compared each model to the sGARCH model, which was considered the benchmark model, the improvement rate (IR) was added as a metric for comparing models (Parker et al. 2009). This way, we presented a direct benchmarking between the various forecasting models to the chosen benchmarking model:

$$\text{Improvement rate mean square error: } \text{IR}_{\text{MSE}} = \frac{\text{MSE}_{\text{Method}_a} - \text{MSE}_{\text{Method}_b}}{\text{MSE}_{\text{Method}_b}} \times 100\% \quad (20)$$

where IR_{MSE} expresses the improvement rate of the target model A over its benchmark model B.

Furthermore, we used the Diebold and Mariano (1995) predictive accuracy test to determine the statistical significance of gains in each forecast in terms of predicting accuracy (Diebold and Mariano 1995).

$$DM = \frac{\bar{d}}{\sqrt{\frac{2\pi\hat{f}_d(0)}{T}}} \quad (21)$$

where $2\pi\hat{f}_d(0)$ is a consistent estimate. Consider $2\pi\hat{f}_d(0) = \sum_{\tau=-(T-1)}^{(T-1)} w_\tau \gamma_d(\tau)$, where $\gamma_d(\tau) = \frac{1}{T} \sum_{t=|\tau|+1}^T (d_t - \bar{d})(d_{t-|\tau|} - \bar{d})$, assuming that $\gamma_d(\tau) = 0$ for $\tau \geq h$, we can use a rectangular lag window estimator by setting $w_\tau = 0$ for $\tau \geq h$ (Triacca 2016).

Although Diebold and Mariano's test is regarded as an essential tool for comparing forecasts, it loses accuracy as forecast horizons grow longer. As a result, Harvey et al. (1997) presented the Harvey–Leybourne–Newbold (HLN) test, a modified Diebold–Mariano test that applies to forecasts that are more than one step ahead and creates a more comprehensive approach for analyzing the variations in forecast distribution performance.

Our experimental results are presented in Table 6 for an h -step-ahead forecast. The log returns of Bitcoin, Ethereum, Litecoin, and Ripple were fitted to sGARCH (1,1), EGARCH (1,1), GJR-GARCH (1,1), and TGARCH(1,1) models. The Student's t -distribution, skew Student's t -distribution and skew generalized error distribution were chosen as the distributions for the innovation process. To provide a robust comparison, we used two loss functions and adaptive forecasting using a 1440 min rolling window for testing over a recursive one because it was more robust to time-varying parameters, but we reported the results using a 60 min rolling window evaluation. To assess the sensitivity of their prediction of the volatility of cryptocurrencies' high-frequency time series return data, we investigated five different horizons (h), namely 1, 5, 15, 30, and 60 min, encompassing short-term and long-term intervals. The out-of-sample forecasting evaluation demonstrates that the predicting performance is dependent on the forecasting horizon, as shown in Table 6. The forecast losses are significantly reduced under each metric over longer forecast time horizons for all four cryptocurrencies. According to the loss function analysis, the prediction results with a 60 min lag provide the highest degree of accuracy for all four

cryptocurrencies, followed by a 30 min lag with one exception, the ranking of the XRP prediction. Furthermore, the models do not generate greater forecasting accuracy or value to model price surges for investors concentrating on very short horizon realized variance, such as 1 min ahead realized variance.

More specifically, $h = 60$, $h = 30$, and $h = 15$ min exhibit higher accuracy in both loss functions. Hence, the 60 min out-of-sample MSEs (mean squared errors) of the BTC are 0.00000033, 0.00000244, 0.00000294, and 0.00000490 for GJR-GARCH, TGARCH, sGARCH, and EGARCH, respectively. Similarly, when the forecasting time horizon is the longest ($h = 60$), both loss functions, MSE and MAE, generate a more robust accuracy, for example, for the four cryptocurrencies with, for BTC (0.00000033, 0.00013856), ETH (0.00000126, 0.00025217), LTC (0.00000107, 0.00024752), and XRP (0.00000228, 0.00031409), respectively. As such, this tendency is exhibited in all other cryptocurrencies, regardless of the conditional volatility types.

As an outcome, the results show that among the four fitting GARCH models, the GJR-GARCH (1,1) model with the skew generalized error distribution (sged) outperforms the other GARCH models in nearly every time horizon, particularly in volatility predictions over longer time horizons. Our findings support the findings of [Kim et al. \(2021\)](#), who found that the forecasting error is more accurate for longer forecast time horizons. Another best-performing model is the TGARCH model. The TGARCH model exhibits better forecasting accuracy than the sGARCH and EGARCH models, especially in volatility forecasting over longer time horizons, i.e., $h = 60$ and $h = 30$ min, similar to the GJR-GARCH model. The comparison results in [Table 7](#) confirm the high predictive capacity of the GJR-GARCH model, followed by the TGARCH model, as all of the IR values of the model improve the predictions regardless of the distribution. Moreover, the results illustrate a statistically significant difference between the benchmark model sGARCH and the various forecast models based on the DM test and between the distribution of forecasts based on the HLN test.

Furthermore, the multivariate DCC-GARCH model was used to estimate pairwise models between the cryptocurrencies while employing the following pairs: Bitcoin to Ethereum, Ethereum to Litecoin, Litecoin to Ripple, and Bitcoin to Ripple. Generally, regardless of the cryptocurrencies pair studied, a cryptocurrency's volatility is highly influenced by its historical squared shocks and its historical volatility, as seen by statistically significant coefficient estimates mainly at the 1% level. According to the results of the DCC-GARCH model in [Table 8](#), we see evidence of significant cross-market effects between the variability of the returns of all cryptocurrencies pair, particularly regarding shock and volatility spillovers, respectively, among cryptocurrencies. The estimates of α_{12} , α_{21} , β_{12} , and β_{21} are statistically significant at the 1% level. This is due to the bidirectional transmission and volatility associations between each cryptocurrency pair in the pairwise models. The findings reveal that lagged shocks and volatility considerably and significantly influence existing conditional volatility in cryptocurrency (α_{11}) and own volatility spillovers (β_{11}).

Table 6. Out-of-sample evaluation of the volatility forecasts.

BTC		ETH				LTC				XRP						
GJR Minutes	MSE	Rank	MAE	Rank	MSE	Rank	MAE	Rank	MSE	Rank	MAE	Rank	MSE	Rank	MAE	Rank
1 min	0.00000743	17	0.00272521	18	0.00000485	7	0.00220320	10	0.00004644	14	0.00681453	15	0.00000742	8	0.00272411	11
5 min	0.00000173	4	0.00076404	4	0.00001100	14	0.00185668	7	0.00001082	9	0.00191696	6	0.00002454	12	0.00269211	9
15 min	0.00000120	3	0.00045877	3	0.00000490	8	0.00090568	3	0.00000393	5	0.00078603	3	0.00000892	10	0.00111919	3
30 min	0.00000060	2	0.00023282	2	0.00000245	2	0.00045977	2	0.00000211	2	0.00046304	2	0.00000447	6	0.00057457	2
60 min	0.00000033	1	0.00013856	1	0.00000126	1	0.00025217	1	0.00000107	1	0.00024752	1	0.00000228	1	0.00031409	1
sGARCH Minutes	MSE		MAE		MSE		MAE		MSE		MAE		MSE		MAE	
1 min	0.00000578	13	0.00240351	16	0.00000296	4	0.00172090	6	0.00003905	13	0.00624863	13	0.00000337	3	0.00183641	6
5 min	0.00000832	19	0.00257684	17	0.00002617	19	0.00465479	19	0.00005256	15	0.00660735	14	0.00005055	17	0.00649120	18
15 min	0.00000593	14	0.00202695	13	0.00001548	17	0.00304218	17	0.00002747	12	0.00454032	12	0.00006212	19	0.00638460	17
30 min	0.00000349	9	0.00144101	8	0.00001001	13	0.00236939	11	0.00001548	11	0.00310212	11	0.00003988	15	0.00491923	14
60 min	0.00000294	6	0.00138984	7	0.00000969	12	0.00246646	13	0.00001218	10	0.00275465	10	0.00003060	13	0.00441387	13
EGARCH Minutes	MSE		MAE		MSE		MAE		MSE		MAE		MSE		MAE	
1 min	0.00000780	18	0.00279231	19	0.00000677	10	0.00260150	14	0.00013980	20	0.01182353	20	0.00000404	5	0.00200991	7
5 min	0.00001015	20	0.00284840	20	0.00003212	20	0.00521955	20	0.00012304	19	0.01001655	19	0.00005326	18	0.00669586	20
15 min	0.00000688	16	0.00211564	14	0.00001818	18	0.00322996	18	0.00007135	18	0.00706960	16	0.00006373	20	0.00654225	19
30 min	0.00000478	10	0.00179136	10	0.00001267	15	0.00271726	15	0.00006382	16	0.00721222	17	0.00004247	16	0.00510121	16
60 min	0.00000490	11	0.00191453	11	0.00001428	16	0.00301768	16	0.00006664	17	0.00756150	18	0.00003789	14	0.00506421	15
TGARCH Minutes	MSE		MAE		MSE		MAE		MSE		MAE		MSE		MAE	
1 min	0.00000321	8	0.00179101	9	0.00000454	6	0.00213031	9	0.00000494	6	0.00222331	8	0.00000761	9	0.00275831	12
5 min	0.00000640	15	0.00219810	15	0.00000721	11	0.00237138	12	0.00000746	8	0.00241906	9	0.00000919	11	0.00269240	10
15 min	0.00000542	12	0.00193153	12	0.00000562	9	0.00196485	8	0.00000569	7	0.00198221	7	0.00000628	7	0.00208186	8
30 min	0.00000304	7	0.00127807	6	0.00000314	5	0.00129822	5	0.00000319	4	0.00131020	5	0.00000352	4	0.00138427	5
60 min	0.00000244	5	0.00118351	5	0.00000251	3	0.00120136	4	0.00000253	3	0.00120950	4	0.00000272	2	0.00125883	4

Note: BTC: Bitcoin; ETH: Ethereum; LTC: Litecoin; XRP: Ripple; generalized autoregressive conditional heteroscedasticity (sGARCH); exponential generalized autoregressive conditional heteroscedastic (EGARCH); Glosten–Jagannathan–Runkle GARCH (GJR GARCH); threshold generalized autoregressive conditional heteroscedastic (TGARCH); MAE: mean absolute error; MSE: mean square error.

Table 7. Comparison of forecasting methods improvement in terms of MSE.

60 Min	BTC	ETH	LTC	XRP
	IR_{MSE}	IR_{MSE}	IR_{MSE}	IR_{MSE}
GJR-GARCH	−0.887914	−0.870400	−0.911985	−0.925626
EGARCH	0.664820	0.474607	4.472185	0.238292
TGARCH	−0.169938	−0.741103	−0.792089	−0.911139

Therefore, historical news about Bitcoin shocks positively impacts the current conditional volatility of Ethereum, and similarly between the pair Ethereum to Litecoin, Litecoin to Ripple, and finally, Bitcoin to Ripple, on recent volatility. This is distinct from the study by [Katsiampa et al. \(2019\)](#) in which it was reported a negative impact between the pair Bitcoin and Litecoin and the pair Litecoin and Ethereum, although they used a multivariate BEKK model. Furthermore, we encountered evidence of a bidirectional shock transmission results between the cryptocurrency pairs because both estimates α_{12} and α_{21} were statistically significant at the 1% level. In addition, we discovered evidence of a two-way volatility spillover effect between all reported pairs such as Bitcoin and Ethereum, Bitcoin and Ripple, Ethereum and Litecoin, Litecoin and Ripple, since both β_{12} and β_{21} were statistically significant at the 1% level. These findings are in accordance with [Liu and Serletis \(2019\)](#), who observed substantial own-mean spillovers among Bitcoin, Ethereum, and Litecoin. The results raise the possibility of estimating current returns based on historical cryptocurrency returns in the short span. Nevertheless, the bidirectional shock spillover in each pair shows that the cryptocurrencies are interdependent.

The bivariate model revealed that Bitcoin caused a surge in conditional volatility, which had disruptive implications for other cryptocurrencies. We discovered that the coefficient estimate α_{11} for Litecoin to Ripple was less critical than α_{12} , implying that prior period shocks and disruption in cross-market volatility shocks had a more substantial impact on the conditional volatility of Litecoin than that of the effect of Litecoin's volatility shock on the current period volatility in the Litecoin market.

Table 8. DCC-GARCH model estimates.

	BTC to ETH	ETH to LTC	LTC to XRP	BTC to XRP
μ_1	0.000009 *** (0.000002)	0.000014 *** (0.000002)	0.000003 * (0.000002)	0.000008 * (0.000002)
μ_2	0.000011 *** (0.000002)	0.000003 * (0.000002)	−0.000004 (0.000003)	−0.000004 (0.000003)
ω_1	0.000000 (0.000000)	0.000000 (0.000000)	0.000000 (0.000000)	0.000000 (0.000000)
ω_2	0.000000 (0.000000)	0.000000 (0.000000)	0.000000 (0.000000)	0.000000 (0.000000)
α_{11}	0.077681 *** (0.000957)	0.069304 *** (0.00218)	0.066071 *** (0.001038)	0.072285 *** (0.002915)
α_{12}	0.069877 *** (0.002227)	0.066071 *** (0.001039)	0.070768 *** (0.001636)	0.070768 *** (0.001533)
β_{11}	0.911222 *** (0.000939)	0.923727 *** (0.002094)	0.932672 *** (0.000908)	0.917494 *** (0.002862)
β_{12}	0.922965 *** (0.002138)	0.932672 *** (0.000907)	0.92767 *** (0.001515)	0.92767 *** (0.001392)
ν_{11}	6.761541 *** (0.10571)	6.077415 *** (0.091198)	3.155579 *** (0.020067)	6.833 *** (0.09398)
ν_{12}	6.222508 *** (0.099647)	3.155579 *** (0.020122)	4.871043 *** (0.056171)	4.871043 *** (0.061898)
$Log(L)$	1907809	1783877	1688118	1774470
AIC	−21.369	−20.143	−19.062	−20.037
HQ	−21.368	−20.143	−19.061	−20.036

Note: statistically significant * $p = 0.10$ and *** $p = 0.001$, respectively.

Figure 4 illustrates the conditional correlations between the four pairs of cryptocurrencies. The charts show dynamic conditional correlations between the cryptocurrencies, with

positive and negative values for the correlations. This result is consistent with the findings in the study of Wang and Ngene (2020).

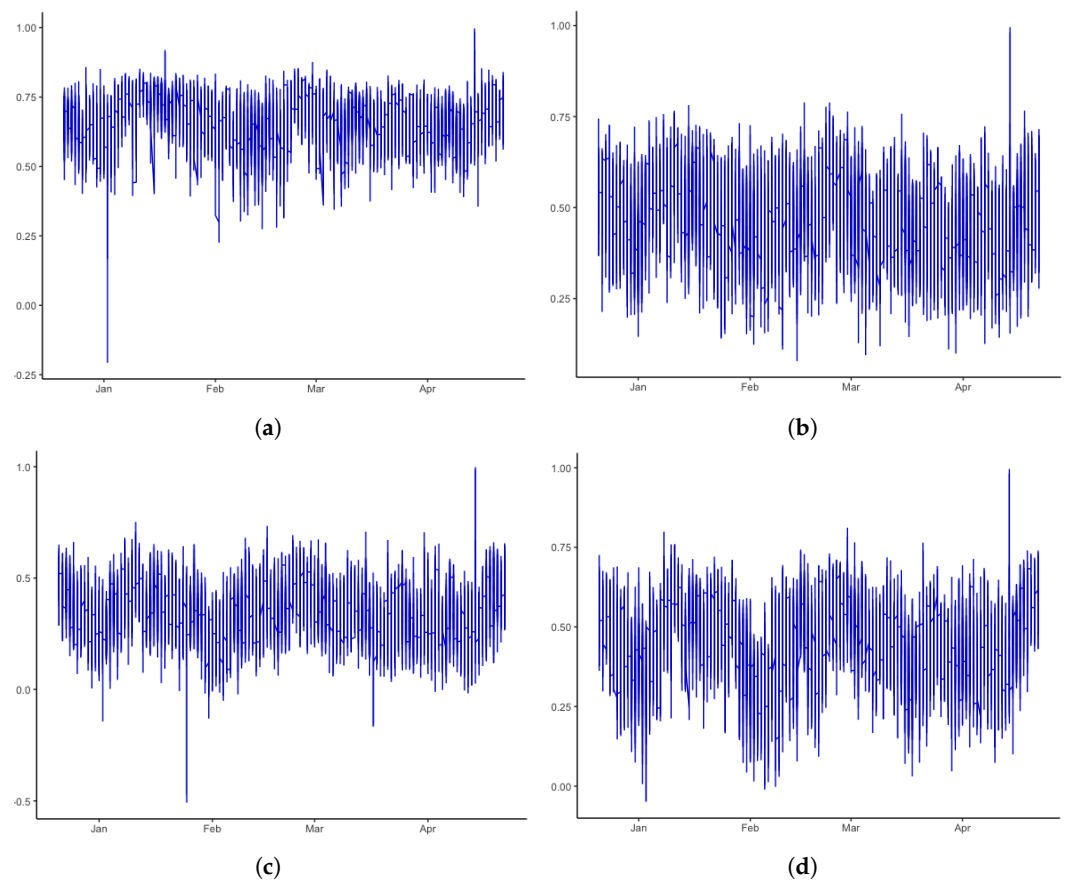


Figure 4. Conditional correlation between the cryptocurrencies. (a) Conditional correlation between BTC and ETH; (b) conditional correlation between ETH and LTC; (c) conditional correlation between LTC and XRP; (d) conditional correlation between BTC and XRP.

4. Conclusions

Both investors and managers must understand the volatility of the most recognized cryptocurrencies. This research offered a comprehensive empirical examination of the comparable forecasting model performance by evaluating the volatility dynamics of key cryptocurrencies, such as Bitcoin, Ethereum, Litecoin, and Ripple, undertaking a systematic analysis of their price changes. Using multiple GARCH-family models, we compared the ability of many volatility models to forecast conditional variance in an out-of-sample context. Hence, this article explored the conditional volatility dynamics of cryptocurrencies and, in addition, the conditional correlations between pairs of cryptocurrencies using four pairwise multivariate DCC models for Bitcoin–Ethereum, Ethereum–Litecoin, Litecoin–Ripple, and Bitcoin–Ripple. The out-of-sample volatility models' performance was measured using two loss functions, where realized variance was used to estimate the unobserved conditional variance. Our research showed that cryptocurrencies such as Bitcoin, Ethereum, Litecoin, and Ripple exhibited a high volatility, particularly at intraday pricing. This tendency became more prominent as the forecasting horizons increased. The GJR-GARCH(1, 1) model outperformed the other GARCH models regarding volatility forecasting accuracy when dealing with highly volatile financial data, such as cryptocurrencies. As a result, these findings will aid portfolio and risk management and assist others in making better-informed judgments about financial investments and the possible benefits and drawbacks of using cryptocurrencies. Therefore, it is a good option for portfolio risk managers who want to enter or invest in digital marketplaces.

We observed that the past shocks and volatility of a cryptocurrency significantly impacted its current conditional variance. This study contributed by expanding and enriching previous GARCH models and asymmetry terms to create superior extensions of the GARCH-family model to predict and anticipate volatility. Our findings demonstrated that the GJR-GARCH model had a greater volatility predicting accuracy during more prolonged periods. According to out-of-sample data based on two loss function criteria, the asymmetry GJR-GARCH model can significantly outperform the other GARCH models. Furthermore, the multivariate DCC-GARCH model was employed to estimate pairwise models between the cryptocurrencies.

As such, the study uncovered evidence of bidirectional shock transmission effects between the cryptocurrency pairs. In addition, we found bidirectional volatility spillover effects between all four cryptocurrency pairs. In the end, Figure 4 exhibited how conditional correlations varied over time, primarily with positive correlations prevailing.

Cryptocurrencies, such as Bitcoin, Ethereum, Litecoin, and Ripple, have expanded more in recent years than others. However, the suggested models still have some limitations. There still is extensive debate about whether cryptocurrencies should be classified as currencies or investment vehicles. Many governments are increasingly strengthening regulations and policies relating to cryptocurrencies, or their central banks are building their digital currency as part of a plan to govern cryptocurrency exchanges and digital currencies. Furthermore, rather than long-term effects, the introduction of short-term asymmetry and high volatility effects is credited with a significant improvement in the predictive capacity of volatility models.

In a future study, we might use other methods such as a neural-network-based volatility model to compare the various GARCH models with recent cryptocurrency time series data using an extended number of different coins. For example, we might combine recurrent neural networks and long short-term memory models with more traditional forecasting time series models to assess the cryptocurrency's returns volatility. In addition, we might consider conducting an analysis by applying alternative models, e.g., conditional autoregressive range models, DCC-CARR, and DCC-RGARCH (Chou et al. 2009; Fiszeder et al. 2019) to receive more accurate estimates of variances and correlations.

If stakeholders want to avoid the negative implications of infectious shocks, they must closely monitor other cryptocurrencies and closely follow up on any changes in Bitcoin. These findings provide investors and policymakers with helpful portfolio diversification and risk management information. Finally, the results support previous studies showing an inter-relationship among the cryptocurrency markets.

Funding: This research received no external funding.

Conflicts of Interest: The authors declare no conflict of interest.

Note

¹ Data are publicly available at <https://www.cryptodatadownload.com/data/> (accessed on 4 May 2021).

References

- Akaike, Hirotugu. 1981. Likelihood of a model and information criteria. *Journal of Econometrics* 16: 3–14. [CrossRef]
- Akhtaruzzaman, Md, Sabri Boubaker, Duc Khuong Nguyen, and Molla Ramizur Rahman. 2022. Systemic risk-sharing framework of cryptocurrencies in the COVID-19 crisis. *Finance Research Letters* 47: 102787. [CrossRef] [PubMed]
- Ampountolas, Apostolos. 2021. Modeling and Forecasting Daily Hotel Demand: A Comparison Based on SARIMAX, Neural Networks, and GARCH Models. *Forecasting* 3: 580–95. [CrossRef]
- Bauwens, Luc, Arie Preminger, and Jeroen V. K. Rombouts. 2010. Theory and inference for a markov switching garch model. *The Econometrics Journal* 13: 218–44. [CrossRef]
- Bollerslev, Tim. 1986. Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics* 31: 307–27. [CrossRef]
- Bollerslev, Tim. 1990. Modelling the coherence in short-run nominal exchange rates: A multivariate generalized arch model. *The Review of Economics and Statistics* 72: 498–505. [CrossRef]
- Bouoiyour, Jamal, and Refk Selmi. 2016. Bitcoin: A beginning of a new phase. *Economics Bulletin* 36: 1430–40.

- Briere, Marie, Kim Oosterlinck, and Ariane Szafarz. 2015. Virtual currency, tangible return: Portfolio diversification with bitcoin. *Journal of Asset Management* 16: 365–73. [CrossRef]
- Brockwell, Peter J., and Richard A Davis. 1991. Stationary time series. In *Time Series: Theory and Methods*. New York: Springer, pp. 1–41.
- Carbone, Anna, Giuliano Castelli, and H Eugene Stanley. 2004. Time-dependent hurst exponent in financial time series. *Physica A: Statistical Mechanics and Its Applications* 344: 267–71. [CrossRef]
- Charles, Amélie, and Olivier Darné. 2019. Volatility estimation for bitcoin: Replication and robustness. *International Economics* 157: 23–32. [CrossRef]
- Cheah, Eng-Tuck, and John Fry. 2015. Speculative bubbles in bitcoin markets? an empirical investigation into the fundamental value of bitcoin. *Economics Letters* 130: 32–36. [CrossRef]
- Choi, In. 2002. *Econometrics: By fumio hayashi*, princeton university press, 2000. *Econometric Theory* 18: 1000–6. [CrossRef]
- Chou, Ray Yeutien, Chun-Chou Wu, and Nathan Liu. 2009. Forecasting time-varying covariance with a range-based dynamic conditional correlation model. *Review of Quantitative Finance and Accounting* 33: 327–45. [CrossRef]
- Chu, Jeffrey, Stephen Chan, Saralees Nadarajah, and Joerg Osterrieder. 2017. Garch modelling of cryptocurrencies. *Journal of Risk and Financial Management* 10: 17. [CrossRef]
- Corbet, Shaen, Brian Lucey, Andrew Urquhart, and Larisa Yarovaya. 2019. Cryptocurrencies as a financial asset: A systematic analysis. *International Review of Financial Analysis* 62: 182–99. [CrossRef]
- Dickey, David A., and Wayne A. Fuller. 1979. Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American Statistical Association* 74: 427–31.
- Diebold, Francis X., and Roberto S. Mariano. 1995. Comparing predictive accuracy. *Journal of Business and Economic Statistics* 13: 253–63.
- Dyhrberg, Anne Haubo. 2016. Bitcoin, gold and the dollar—A garch volatility analysis. *Finance Research Letters* 16: 85–92. [CrossRef]
- Engle, Robert. 2002. Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business & Economic Statistics* 20: 339–50.
- Engle, Robert F. 1982. Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. *Econometrica: Journal of the Econometric Society* 50: 987–1007. [CrossRef]
- Fiszeder, Piotr, Marcin Faldziński, and Peter Molnár. 2019. Range-based dcc models for covariance and value-at-risk forecasting. *Journal of Empirical Finance* 54: 58–76. [CrossRef]
- Fry, John, and Eng-Tuck Cheah. 2016. Negative bubbles and shocks in cryptocurrency markets. *International Review of Financial Analysis* 47: 343–52. [CrossRef]
- Ghalanos, Alexios. 2015. *The Rmgarch Models: Background and Properties*. Version 1.3-0. Available online: https://cran.r-project.org/web/packages/rmgarch/vignettes/The_rmgarch_models.pdf (accessed on 20 September 2021).
- Ghalanos, Alexios. 2018. *Introduction to the Rugarch Package*. R Vignette. pp. 1–3. Available online: https://cran.r-project.org/web/packages/rugarch/vignettes/Introduction_to_the_rugarch_package.pdf (accessed on 20 September 2021).
- Gilli, Manfred, Dietmar Maringer, and Enrico Schumann. 2019. *Numerical Methods and Optimization in Finance*. Cambridge: Academic Press.
- Glosten, Lawrence R., Ravi Jagannathan, and David E. Runkle. 1993. On the relation between the expected value and the volatility of the nominal excess return on stocks. *The Journal of Finance* 48: 1779–801. [CrossRef]
- Granger, Clive W. J., and Roselyne Joyeux. 1980. An introduction to long-memory time series models and fractional differencing. *Journal of time Series Analysis* 1: 15–29. [CrossRef]
- Hannan, Edward J., and Barry G. Quinn. 1979. The determination of the order of an autoregression. *Journal of the Royal Statistical Society: Series B (Methodological)* 41: 190–95. [CrossRef]
- Hansen, Peter R., and Asger Lunde. 2005. A forecast comparison of volatility models: Does anything beat a garch (1, 1)? *Journal of Applied Econometrics* 20: 873–89. [CrossRef]
- Harvey, David, Stephen Leybourne, and Paul Newbold. 1997. Testing the equality of prediction mean squared errors. *International Journal of Forecasting* 13: 281–91. [CrossRef]
- Hyun, Steve, Jimin Lee, Jong-Min Kim, and Chulhee Jun. 2019. What coins lead in the cryptocurrency market: Using copula and neural networks models. *Journal of Risk and Financial Management* 12: 132. [CrossRef]
- Jarque, Carlos M., and Anil K. Bera. 1987. A test for normality of observations and regression residuals. *International Statistical Review/Revue Internationale de Statistique* 55: 163–72. [CrossRef]
- Katsiampa, Paraskevi. 2017. Volatility estimation for bitcoin: A comparison of garch models. *Economics Letters* 158: 3–6. [CrossRef]
- Katsiampa, Paraskevi, Shaen Corbet, and Brian Lucey. 2019. High frequency volatility co-movements in cryptocurrency markets. *Journal of International Financial Markets, Institutions and Money* 62: 35–52. [CrossRef]
- Kim, Jong-Min, Chulhee Jun, and Junyoun Lee. 2021. Forecasting the volatility of the cryptocurrency market by garch and stochastic volatility. *Mathematics* 9: 1614. [CrossRef]
- Kristoufek, Ladislav. 2013. Bitcoin meets google trends and wikipedia: Quantifying the relationship between phenomena of the internet era. *Scientific Reports* 3: 1–7. [CrossRef] [PubMed]
- Lahmiri, Salim, Stelios Bekiros, and Antonio Salvi. 2018. Long-range memory, distributional variation and randomness of bitcoin volatility. *Chaos, Solitons & Fractals* 107: 43–48.
- Liu, Jinan, and Apostolos Serletis. 2019. Volatility in the cryptocurrency market. *Open Economies Review* 30: 779–811. [CrossRef]
- Liu, Ruiping, Zhichao Shao, Guodong Wei, and Wei Wang. 2017. Garch model with fat-tailed distributions and bitcoin exchange rate returns. *Journal of Accounting, Business and Finance Research* 1: 71–75. [CrossRef]

- McDonald, Oonagh. 2021. The Bank of England Should Beware the Lure of Digital Currency. Available online: <https://www.ft.com/content/96b14a1d-c9ad-4c55-8494-741c9adf8220> (accessed on 12 September 2021).
- Nelson, Daniel B. 1991. Conditional heteroskedasticity in asset returns: A new approach. *Econometrica: Journal of the Econometric Society* 59: 347–70. [[CrossRef](#)]
- Parker, Richard I., Kimberly J. Vannest, and Leanne Brown. 2009. The improvement rate difference for single-case research. *Exceptional Children* 75: 135–50. [[CrossRef](#)]
- Phillips, Peter C. B., and Pierre Perron. 1988. Testing for a unit root in time series regression. *Biometrika* 75: 335–46. [[CrossRef](#)]
- Pindyck, Robert S., and Daniel L. Rubinfeld. 1998. *Econometric Models and Economic Forecasts*. Boston: Irwin/McGraw-Hill, Volume 4.
- Rabemananjara, Roger, and Jean-Michel Zakoian. 1993. Threshold arch models and asymmetries in volatility. *Journal of Applied Econometrics* 8: 31–49. [[CrossRef](#)]
- Revoll, John, and Tom Wilson. 2022. Switzerland Tests Digital Currency Payments with Top Investment Banks. Available online: <https://www.reuters.com/technology/switzerland-tests-digital-currency-payments-with-top-investment-banks-2022-01-13/> (accessed on 14 January 2022).
- Tiwari, Aviral Kumar, Ibrahim Dolapo Raheem, and Sang Hoon Kang. 2019. Time-varying dynamic conditional correlation between stock and cryptocurrency markets using the copula-adcc-egarch model. *Physica A: Statistical Mechanics and Its Applications* 535: 122295. [[CrossRef](#)]
- Triacca, Umberto. 2016. Lesson19: Comparing Predictive Accuracy of Two Forecasts: The Diebold-Mariano Test. Available online: <https://www.phdeconomics.sssup.it/documents/Lesson19.pdf> (accessed on 10 January 2022).
- Trucíos, Carlos. 2019. Forecasting bitcoin risk measures: A robust approach. *International Journal of Forecasting* 35: 836–47. [[CrossRef](#)]
- Urquhart, Andrew, and Hanxiong Zhang. 2019. Is bitcoin a hedge or safe haven for currencies? an intraday analysis. *International Review of Financial Analysis* 63: 49–57. [[CrossRef](#)]
- Wang, Jinghua, and Geoffrey M. Ngene. 2020. Does bitcoin still own the dominant power? an intraday analysis. *International Review of Financial Analysis* 71: 101551. [[CrossRef](#)]
- Zakoian, Jean-Michel. 1994. Threshold heteroskedastic models. *Journal of Economic Dynamics and Control* 18: 931–55. [[CrossRef](#)]