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Winner Strategies in a Simulated Stock Market

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Abstract: In this study, we explore the dynamics of the stock market using an agent-based simulation platform. Our approach involves creating a multi-strategy market where each agent considers both fundamental and technical factors when determining their strategy. The agents vary in their approach to these factors and the time interval they use for technical analysis. Our findings indicate that investing heavily in reducing the value–price gap was a successful strategy, even in markets where there were no trading forces to reduce this gap. Furthermore, our results remain consistent across various modifications to the simulation's structure.

Keywords: agent-based simulation; stock market; fundamental traders; technical traders

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1. Introduction

John Maynard Keynes (1936) once compared the stock market to a "beauty contest", in which participants select the most aesthetically pleasing pictures, but the prize is awarded to those who align with the most popular choices. Keynes argued that the outcome of such a contest was not the most attractive picture, but rather the one which people guess should be the dominant choice. However, there is a crucial difference between a stock market and a "beauty contest": in a stock market, companies generate profits and distribute a portion of them as dividends. Earnings and dividends change the zero-sum game of a stock market into a positive-sum game. At least, since the publication of *Security Analysis* by Graham and Dodd (1934)[1], investors have tried to determine the fundamental values of stocks by projecting future dividends and comparing those to market prices. In the authors' words, "although in the short run, the stock market acts such as a voting machine, in the long run, it plays the role of a weighing machine [measuring values, not opinions]".

In this paper, we model the stock market's dynamics as a mixed game, considering both the "beauty contest" aspect of the market and the fundamental values of the stocks. Our goal is to identify the strategies that lead to the highest possible returns for investors in an interactive environment where the prices themselves are the result of the strategies selected. We demonstrate that, even if participants, on average, do not give much consideration to the fundamental values of stocks, over time, the market behaves like a weighing machine and the fundamental strategies ultimately emerge victorious.

The remainder of this paper is structured as follows: Section 2 presents a comprehensive literature review. Section 3 provides a detailed description of the model. Section 4 discusses the model's robustness and primary findings. Section 5 offers concluding remarks, while the appendices contain supplementary discussions and calculations.

2. Literature Review

The market comprising risky assets, investors, and capital flows, presents a compelling platform for study from various perspectives. Investors endeavor to adopt strategies that yield higher returns while also withstanding unfavorable market conditions. To identify suitable stocks, investors track fundamental values, price trends, or a combination of both. Fundamental values of stocks are typically estimated by predicting future dividends and discounting them by appropriate discount rates. However, the dividends themselves and the required rates of return, serving as discount rates, are intertwined with the investors' predisposition towards different stocks through the strategies they choose. This interdependency creates a loop that links asset prices to investors' strategies.

In an attempt to break out of this loop, classical finance aims to view the market from a macro-level, seeking common factors that can explain the overall dynamics. Fama and French (1993, 2015), and Ross (1976) are the very first researchers among many who employ this common factor view. In regards to the efficacy of investment strategies, a multitude of approaches have been proposed and evaluated in comparison to one another, e.g., Jegadeesh and Titman (1993), Shleifer and Vishny (1997). In the financial literature, the works of Kai-Ineman and Tversky (1979) should be noted, as they developed behavioral finance and have made valuable contributions, especially in explaining market anomalies.

Another avenue for comprehensively examining the market, its assets, investors, and capital flows is through the employment of an agent-based model (ABM). This model considers the market as an arena for agents with distinct investment strategies to interact, thus determining asset prices endogenously. Through the manipulation of parameters in these models, controlled experiments can be designed and executed to test hypotheses.

A noteworthy early ABM was proposed by Kim and Markowitz (1989) to simulate the 1987 market crash. This model features two trader types: negative feedback traders, who construct their portfolios based on a return-variance framework, and positive feedback traders, who seek to limit their losses by emulating a put option. The study aims to investigate the conditions under which a market becomes unstable.

Another prominent ABM is the Levy et al. (1994) model, which incorporates a risky asset that pays dividends. Agents in this model maximize their portfolios based on a logarithmic utility function and calculate expected returns by comparing the dividends with the prevailing price levels. The Levy model is widely utilized to demonstrate the occurrence of market booms and busts.

During the 1990s, the Santa Fe Institute for the study of complex systems developed an ABM specifically designed for financial markets, commonly referred to as the Santa Fe Institute Artificial Stock Market (SFI-ASM). Palmer et al. (1994) published an early iteration of the model, with significant findings presented by Arthur et al. (1996) and LeBaron et al. (1999). LeBaron subsequently investigated several variations of the SFI-ASM in 2001 and 2002 (LeBaron (2001, 2002)). Within the SFI-ASM, agents respond to fundamental and technical signals and modify their strategies utilizing a genetic algorithm (GA). Each agent is provided with a set of rules from which it selects via a classifier. The complex system shares certain characteristics with an actual stock market, particularly in a fast-updating mode. For a comprehensive explanation of the model, we refer the reader to Ehrentreich (2008).

Evstigneev et al. (2009) developed an artificial stock market wherein stock dividends are randomly paid out following a uniform distribution, with investors able to select from a limited number of established strategies. The authors demonstrate that, irrespective of the scenario utilized, the sole surviving strategy that dominates the market is one that evaluates stocks based on their anticipated dividends. In this model, all agents maintain their strategies unchanged, resulting in a passive evolution, as defined by LeBaron (2011).

Each of the three ABMs described above assumes rule-based agents, which adhere to the rules specified by the model. However, an advanced type of agent, referred to as goal-based, strives to achieve objectives without adhering to any predetermined rules. The Lux (1998) and Lux and Marchesi (2000) (LM) model presents these agents, wherein two types of traders participate: fundamental and technical (optimists or pessimists). One distinguishing characteristic of the LM model is the direct interaction between agents. In most ABMs, agents primarily interact through the endogenous pricing mechanism, whereby their trading activities impact prices, providing insights into the behaviors of other agents. However, in the LM model, agents actively adjust their strategies by engaging in direct communication with their counterparts and comparing returns, deviating from the conventional interaction methods of most ABMs. The LM model can replicate unique features of financial markets, such as heavy tails for returns and high volatility. In our model, we also incorporate goal-based agents.

A recent trend in ABMs is to incorporate machine learning techniques, similar to the approach adopted in actual financial markets (Meng and Khushi (2019) provides a comprehensive review on the topic). Natranj and Leidner (2019) and Maeda et al. (2020) are two papers that equip agents with deep-learning capabilities to enhance their modeling and analytical capacities.

Numerous researchers, including El Oubani and Lekhal (2022) and Westerhoff (2008), employ ABMs to investigate specific regulations on financial markets. Initially, they construct a stock market that mimics real-world characteristics, then introduce a specific rule and examine its impact on the market. In this type of research, little attention is paid to the performance of individual market participants, particularly over a prolonged period.

In a series of papers, Evstigneev et al. (2006, 2009, 2016), Palczewski et al. (2016), and Hens and Schenk-Hoppé (2020) develop evolutionary finance models that examine a market involving fundamental traders, who compare stock prices with perceived fundamental "values", and technical traders, who trade by observing price trends. These papers investigate the performance of fundamental investors and focus on identifying the dominant strategy over an extended duration.

Among the papers referenced, the work of Hens and Schenk-Hoppé (2020) bears the closest resemblance to our own design. The authors construct an artificial market featuring three types of funds— fundamental, trend-chasing, and noise trading—and two assets—risky and riskless. Each agent is assigned a patience parameter, representing their endurance in the fund they initially select. By manipulating the patience parameter, the authors demonstrate that the greater the patience of fundamental investors, the larger their market share and the greater proportion of risky assets they attract to their portfolio.

Similarly, we investigate a market containing a risky and a riskless asset. However, rather than dividing investors into three distinct groups with varying degrees of patience, we permit investors to form a spectrum, encompassing pure fundamentalists to pure trend-chasers, along with a diverse range in between.

An examination of over fifty years of contemporary finance literature reveals that scholars have predominantly concentrated on broader market characteristics, such as market efficiency and the risk–reward relationship. Nevertheless, in recent years there has been a discernible shift towards the methods of portfolio management and stock selection, as well as the attributes a stock must possess to surpass its peers. Noteworthy works in this vein include those by Frazzini et al. (2018), Asness et al. (2019), Kozak et al. (2020), Hou et al. (2022), and Gai (2022). Our study aims to address the same question, but from an agent-based standpoint.

3. Model

This section presents the introduction of our proposed model. Our model assumes that investors possess knowledge of the underlying fundamental value of the stock market and hold the belief that price patterns exist. These investors aim to enhance their returns by forecasting these patterns. To develop our model, we follow the framework proposed by LeBaron (2001) and describe each component of our model design in detail.

3.1. Agents

Agent-based models place a significant emphasis on the role of agents, as highlighted by LeBaron 2001. The sophistication of agents is a crucial factor in these models, with various levels of complexity being possible. For example, simple rule-based agents have been utilized in studies by Evstigneev et al. (2009), while other studies have employed learning agents that maximize utility, such as the well-known Santa Fe Institute's Artificial Stock Market (SFI-ASM).

The present study establishes the agents' objectives as the maximization of wealth at the conclusion of the simulation. Given the underlying assumption of a friction-less market, this objective is akin to maximizing the growth rate during each period. This approach, commonly referred to as the Kelly strategy (Kelly 1956), shares similarities with the maximization of logarithmic utility as described by (Hakansson 1971). While the appropriateness of the Kelly strategy as a utility function remains a topic of discussion (Rubinstein 1976; Samuelson and Merton 1974), it appears to be compatible with the research objectives of this study. It should be noted that agents are not granted autonomy in determining their consumption levels, a matter that will be further elaborated upon later. By employing the Kelly strategy, evaluating the success of agents becomes relatively straightforward, with the agent accumulating the highest wealth being declared the winner. Additionally, agents are motivated to avoid bankruptcy at all costs, as bankruptcy results in negative infinity utility and eliminates any possibility of accruing future wealth.

We assume that cash dividends are paid during each period, and agents consume these dividends in the same period. If agents were to retain these extra resources, the cumulative demand would increase, leading to a consistent inflation of prices. To ensure that all agents trade under equal conditions, the ratio of consumption to wealth must be the same across all agents. Consequently, in each period, some agents must divest themselves of the risky asset to obtain sufficient cash, while others possess excess cash that can be invested.

At the onset of the simulation, all agents are endowed with an equal amount of money. While agents share certain characteristics, they are assigned distinct parameters to ensure the uniqueness of their strategies. These agents estimate the anticipated return of holding the stock for a single period, utilizing the following formula:

$$R = \frac{D}{P} + \frac{1}{P}\frac{V-P}{cat} + tfp \times TREND, \tag{1}$$

where D is the expected dividend for the next time period and P is the current price, so that the first term stands for the expected dividend yield. The variable V represents the fundamental value of stock. The other parameters in Formula (1) are described in the following paragraph.

The agents in this study are in search of trends in the price trajectory. Each agent is assigned a fixed time period, denoted as *dur*. At each time step t, an agent examines the market returns in two consecutive periods, specifically $[t-2(dur), t-dur]$ and $[t-dur, t]$. If the returns exhibit the same sign in both periods, agents infer the presence of a trend and assign to the *TREND* in Formula (1): -1 for consecutive negative signs and $+1$ for consecutive positive signs. If no trend is detected, agents set the value of *TREND* equal to zero. Given this design and the stochastic nature of price movements, approximately half of the agents will observe a trend in a given stock price, while the other half will not. The agents possess identical estimations of the fundamental value of the risky asset, but they differ in their estimates of the time required for price convergence to this value. To account for this variability, each agent is assigned a parameter, denoted as "catalyst" (*cat*). Agents with a low *cat* anticipate a rapid convergence, with the fundamental component of the expected return (the second term in Formula (1)) becoming exceedingly large in absolute terms. The values of *cat* and *dur* are fixed for each agent and are sampled from an exponential distribution with a mean of 100, approximately two years. Similarly, the "trend following preference" parameter, denoted as *tfp*, represents the degree of trend chasing the agents have; it is fixed and randomly sampled from a uniform distribution ranging from 0 to 1.2.

This study intentionally deviates from the design of a realistic market, a decision that requires clarification. In actual stock markets, trader beliefs frequently initiate a self-fulfilling phenomenon. Any influencing factor, regardless of its relevance to the underlying business's realities, can prompt movement in the stock price if traders act upon it. In this dynamic, the first movers often secure significant profits. To determine whether a value investor can genuinely achieve superior performance independent of this effect, we develop a market that, unlike the actual market, does not inherently favor value investing on average. We refer to this as a “fundamentally neutral” market. As we highlight in our conclusion, our primary findings become even more pronounced when simulating with more realistic assumptions.

To construct a fundamentally neutral market, the direction of the categorical parameters is reversed for fifty percent of the agents. As a result, half of the agents partake in the acquisition of stocks when deemed overvalued, while the other half behave in an opposing manner. Although this assumption may appear unrealistic, we chose it to neutralize the effect of fundamental investors demand and are therefore assured that the advantages displayed by the fundamental strategies are primarily attributed to dividends rather than being solely driven by demand.

In the present configuration, the agents possess three distinct degrees of freedom, which are denoted by the parameters *cat*, *dur*, and *tfp*. In order to generate interactions among as many strategies as possible, these three parameters are randomly assigned for each agent. Consequently, the number of strategies is equivalent to the number of agents. It is noteworthy to mention that the reference made to the number of agents or strategies pertains to a singular value in accordance with the aforementioned explanation.

As previously emphasized, the agents’ logarithmic utility implies the avoidance of bankruptcy by utmost effort, because any possibility of negative wealth corresponds to a negative infinity utility. However, in discrete-time simulations, the potential for a sudden price movement always exists, thereby introducing the risk of bankruptcy for any amount of leverage or short-selling of the risky asset. So, it is never advantageous for agents to incur even a minimal amount of liability. Therefore, we exclude the borrowing of money and short selling of stock from our simulation.

3.2. Assets

The market comprises two assets, namely a risk-free asset that exhibits a zero rate of return and possesses an infinite supply (cash) and a risky asset (stock) that is characterized by a fixed, limited supply and pays dividends in each period. The dividends generated by the stock follow a discrete-time stochastic process that is mean-reverting in nature, where the mean-reverting coefficient is relatively small and is employed to stabilize the dividend level at a steady state. Owing to their lack of awareness regarding the mean-reverting dividends, agents rely on the most recent dividends to estimate the fundamental value. To this end, the dynamics of dividends from the perspective of agents can be expressed mathematically as follows:[2]

$$\frac{D_{t+1}}{D_t} = 1 + \sigma_D Z_t, \quad (2)$$

where D_t is the dividend at time t and σ_D is the variance of dividend growth. The stock is subject to two distinct sources of risk, namely the volatility of its price and the uncertainty associated with the stochastic dividend process, which jointly contribute to the movement of returns. At the commencement of each simulation step, referred to as a “tick,” the dividend payout is disclosed, leaving price movements as the sole source of risky returns.

We assume that the agents consider a two-state probability distribution for the returns, which conforms to the expectation and variance of the actual returns. This probabilistic framework enables the agents to apply the well-known Kelly criteria and determine the optimal investment in risky asset, which can be expressed as follows (see Appendix B for more details):

$$\alpha^* = \frac{1}{\sigma^2}\mu, \quad (3)$$

where α^* is the optimal fraction of wealth allocated to the risky asset, μ is the expected return of investment, and σ^2 is the variance rate of the risky asset (variance of its price percentage change). If all agents allocate this same fraction to the stock, α^* would be $V/(V+C)$, where V signifies the fundamental value of the stock and C represents the total amount of cash available in the market.

The fundamental value is characterized as the equilibrium price that would result in the market clearing if every agent were to hold the risky asset exclusively for its dividend yield. In such a scenario, the expected return for each agent would be given by $\mu = D/V$. Moreover, the fundamental values exhibit a linear relationship with dividends, at least for minor price fluctuations. Consequently, the percentage changes in dividends and fundamental values are subject to the same stochastic process, with equal variances $\sigma^2 = \sigma_D^2$. Altogether, Formula (3) transforms to:

$$\frac{V}{V+C} = \frac{1}{\sigma_D^2}\frac{D}{V}. \tag{4}$$

This quadratic equation has the following positive solution:

$$V = \frac{1}{2\sigma_D^2}(D + \sqrt{D^2 + 4CD\sigma_D^2}), \tag{5}$$

which represents a shared reference point that all agents take into account when appraising the stock price.

3.3. Market

In the context of our discrete-time market simulation, each period encompasses approximately one week, with the aim of approximating the variance of dividends. In this study, we contend that the total demand of the risky asset in each period is a decreasing function of the price, and subsequently we can identify the price that clears the market. Our analysis focuses on Formula (1), which represents the return of the risky asset in each period. This return is composed of three components, namely the dividend yield ($\frac{D}{P}$), the fundamental yield ($\frac{1}{P}\frac{V-P}{cat}$), and the technical premium ($tfp \times TREND$). The dividend yield is established as a decreasing function of price, and therefore our attention is directed to the other components of return. For the second term, the fundamental yield, we observe that its positivity or negativity is contingent on the state of the price (P) relative to the fundamental value (V) and the sign of the *cat*, which we deliberately set as +1 for half of the agents and –1 for the other half. Thus, the fundamental yield can be regarded as the neutral component of return with respect to the price. Finally, for the technical premium of return, as previously stated in Section 3.1, the agents rely on the two preceding time intervals, $[t-2(dur), t-dur]$ and $[t-dur, t]$, which eliminates the potential technical preference for price increases[3]. In summary, in each period, the dividend yield component of return is the foremost decreasing part concerning the price, and thus the demand is also a decreasing function of price. Therefore, calculating the clearing price can be accomplished through a simple numerical resolution process that involves solving the optimization problem, where the total demand is equated to the fixed supply.

3.4. Simulation

To calculate the technical component of returns, our agents rely on historical data, which is not accessible to them at the start of the simulation. To address this issue, we train the market using agents' decisions and clearing prices for several hundred periods, without including actual trades in the model. During the training period, agents' wealth and portfolios remain unchanged. Once sufficient historical data is generated, actual trades commence. Throughout each period, agents consume all dividends, and we assume a passive evolution, as described in LeBaron (2011), where agents do not adjust their investment strategy. However, over time, some agents become wealthier, while others become relatively poorer, resulting in an increasing number of successful trades and gradual evolution of the market.

Each agent has a unique set of parameters, resulting in multiple unique strategy simulations running simultaneously. As demonstrated in the subsequent section, ample evidence suggests that only a few hundred agents are required to run the model effectively.

At the beginning of the simulation, each agent is provided with equal proportions of the stock and an equal amount of additional cash. As previously stated, the initial few periods correspond to the “simulation” mode, where supply, demand, and pricing mechanisms are implemented, but no actual trades are conducted. Therefore, at the onset of the “real” mode, each agent possesses adequate price history to determine its technical return. The agents do not engage in any direct interactions with one another, and pricing serves as the sole mechanism of interaction.

4. Results

4.1. Classification of Results

In an ABM, four distinct states emerge based on the known or unknown status of inputs and outputs. This demarcation, however, is somewhat idealistic, as the categorization of an input as known or unknown is not always unambiguous. For instance, while a general comprehension of investors’ decision-making frameworks may be available, numerous investors in any given market possess investment strategies that remain incompletely understood, even by the investors themselves. In the present analysis, we seek to differentiate between anticipated results and those revealed upon the simulation’s completion. In this section, we initially present the findings, demonstrate their significance, and verify that they do not stem from random occurrences. Subsequently, we delve into the insights these results provide concerning the attributes of successful and unsuccessful strategies.

4.2. Identifying Winners by Scores

In simulations such as ours, the availability of data is not a concern, and the primary issues relate to computational power and storage capacity. However, the key challenge arises during the data interpretation phase. Standard statistical tests and p-values, which are commonly used for data interpretation, may not be the most suitable choice in such scenarios, as elaborated in Appendix A. In this section, we present the results alongside the rationale for the tests employed to verify their robustness.

Due to the stochastic nature of simulations, it is essential to conduct multiple simulations and aggregate their outcomes. While summing the wealth of individual agents at the end of each simulation may seem straightforward, the challenge arises when the collective wealth of all agents is significantly elevated due to the occurrence of a randomly high valuation of the risky asset. As a result, the impact of individual simulations may not be uniformly distributed. Normalizing the wealth of all agents does not address this issue, as the primary competition is often concentrated among a few agents, and the normalized wealth of the remaining agents is effectively zero.

To mitigate this issue and obtain more robust results, we employ a scoring scheme similar to those used in sporting events. In each iteration of the simulation, the wealthiest agent is awarded 10 points, the next in rank receives 8 points, the third receives 6 points, and so on. Our comprehensive scoring system is designed as $10, 8, 6, 5, 4, 3, 2, 1, 0, 0, \ldots$, where agents beyond the top eight receive no points. This scoring system enables the simplified and efficient analysis of hundreds of simulation rounds with numerous agents, as presented in Tables 1–4.

Another aspect that we seek to examine is *the role of the random price seed in the simulation outcomes*. If there exist winning strategies, altering the random price seed[4] should not result in significant changes to the results. To test this hypothesis, we conducted 100 simulations of the market using the same 200 agents and assessed the stability of the winning strategies. As presented in Table 1, several winning strategies retained their positions across different random price seeds. For instance, the top-performing strategy for agent 68 garnered approximately 50% of the possible maximum 1000 points, a position that can be substantiated by standard statistical tests (see Appendix A).

In order to demonstrate the statistical significance of our findings, we conducted a series of simulations a total of 100 times. Each simulation was run 100 times, resulting in a total of 10,000 simulations. The $(5\%, 95\%)$ confidence interval for each value is shown in Table 1. During the course of these simulations, we observed that the outcomes were sensitive to the randomness present in the data. Even after 2000 ticks, we found that for each random seed, the rankings varied, indicating that no single strategy was absolutely dominant. We chose a duration of 2000 ticks for our simulations, which roughly corresponds to a few decades. This period is short enough to be considered a life-long investment, yet long enough to allow for unambiguous observation of the results.

Table 1. Agents' scores, positions, and the corresponding confidence intervals, in 10,000 simulations

		Number of Gained Positions					
Agent	**Score**	**1st**	**2nd**	**3rd**	**Rank:4th... 6th**	**Rank:7th... 9th**	**Other Ranks**
68	**474** (397, 547)	**26** (18, 33)	**12** (8, 17)	**9** (5, 14)	**13** (8, 19)	**5** (2, 9)	**35**
194	**420** (353, 506)	**29** (22, 38)	**10** (5, 15)	**4** (1, 7)	**7** (4, 11)	**5** (2, 9)	**46**
184	**332** (276, 384)	**2** (0, 4)	**16** (10, 23)	**14** (9, 20)	**23** (16, 30)	**8** (4, 12)	**38**
16	**287** (239, 340)	**0** (0, 2)	**6** (2, 10)	**17** (11, 24)	**29** (20, 37)	**9** (5, 15)	**38**
118	**268** (224, 314)	**5** (2, 8)	**12** (7, 17)	**9** (5, 14)	**14** (8, 19)	**12** (7, 17)	**48**
138	**216** (164, 274)	**6** (3, 10)	**9** (3, 14)	**6** (3, 10)	**10** (5, 14)	**6** (3, 10)	**63**
110	**154** (118, 187)	**0** (0, 0)	**0** (0, 1)	**0** (0, 2)	**32** (23, 39)	**18** (13, 24)	**49**
172	**142** (113, 174)	**0** (0, 0)	**0** (0, 1)	**0** (0, 1)	**29** (22, 38)	**26** (20, 34)	**45**
144	**105** (67, 149)	**2** (0, 4)	**3** (0, 7)	**3** (1, 7)	**9** (4, 14)	**5** (2, 9)	**78**
1	**83** (37, 140)	**4** (1, 8)	**3** (1, 6)	**2** (0, 4)	**3** (0, 6)	**1** (0, 3)	**88**

Note: Ten best agents according to their average scores. In each row, the scores and top positions gained are presented (in **bold** font) along with the confidence interval (in parentheses) for 100 simulations. Some agents have significant advantages over others. This observation rejects the randomness of agents' returns.

To assess the *robustness of our findings with respect to the number of agents* involved in the simulations, we initiated the simulations with 10 agents and added additional agents in each iteration. We then recorded the results and normalized the scores of the agents based on the percentage of simulations in which they participated. The scores are presented in Table 2. Our analysis indicates that there is no significant difference between the normalized and real scores, and the best agents remained the same with only minor changes in their ranking. This finding suggests that changing the number of agents involved in the simulations would not have a dramatic impact on the results.

Table 2. Agents' original and normalized scores in 100 simulations.

Agent	Original Score	Reduced Score	Normalized Score
68	453	467	667
194	402	38	475
184	317	33	275
16	284	545	568
118	245	234	509
138	177	46	128
110	155	144	288
172	147	29	161
1	112	23	23
144	83	3	9

Note: The scores of the top 10 agents in Table 1 (original score) are compared with the scores they received in the new simulations with a fewer number of agents (reduced score). To achieve a fair comparison, we increased the agents' scores based on the percentage of simulations in which they participated (normalized score).

In prior research, such as that conducted by Evstigneev et al. (2009), simulations have been used to approximate real-world market conditions. These studies have shown that fundamental investors often require a significant amount of time to capture a meaningful market share. In our current study, we have set the dividend and price variance levels to be roughly equivalent to actual weekly levels. *To observe the stages of change during the simulations*, we captured eight snapshots at various time intervals (ticks 10, 20, 50, 100, 200, 500, 1000, and 1500) before reaching the final tick value of 2000. We report the scores associated with these snapshots in Table 3, including only those time stamps in which at least one agent achieves a top ten ranking.

Table 3. Agents' scores over time.

	Time								
Agent	**10**	**20**	**50**	**100**	**200**	**500**	**1000**	**1500**	**2000**
1	0	8	154	216	252	243	153	136	112
8	0	0	92	100	66	13	9	12	5
16	418	449	201	88	57	90	156	213	284
22	356	357	148	37	20	31	34	20	39
32	512	489	200	57	34	16	47	25	28
35	91	20	38	37	49	51	41	39	32
68	782	781	370	147	86	151	277	349	453
69	6	26	58	86	96	100	88	78	50
74	0	0	240	224	149	74	59	20	32
89	0	7	107	143	161	160	107	101	67
97	0	7	72	88	97	100	72	63	49
110	293	279	113	43	30	57	100	125	155
112	119	78	27	11	1	6	12	15	18
118	29	79	73	55	93	157	198	274	245
119	0	9	138	147	162	141	83	83	70
125	0	0	87	117	137	133	97	49	38
129	0	43	115	54	62	33	20	7	6
138	0	27	229	483	312	322	328	262	177
144	0	0	0	15	28	20	57	58	83
172	235	195	78	20	17	31	64	104	147
173	0	26	130	22	22	10	3	0	0
179	73	22	22	31	38	38	35	29	21
181	0	0	42	80	95	83	66	64	55
184	656	658	294	128	71	109	175	255	317
194	0	28	72	169	206	286	354	400	402
sum_t	3535	3408	2104	1874	1667	1802	1945	2219	2375
sum_T	2413	2504	1584	1364	1152	1466	1862	2176	2375

Note: Agents' scores at different time stamps before the final time. The sums of the scores for the top-ten agents are presented in the snapshots shown at the top of each column. The last row represents the sums of the scores of the final top-ten agents at each snapshot. From this table, we infer that changing the tick number beyond 2000 would not affect our results significantly.

Our findings suggest that, typically, the winning agents exhibit a clear advantage at an early stage of the simulations, although this advantage may not be apparent in the first few ticks. As the simulations progress, the relative wealth of the top-performing agents increases, with their rankings reflecting their continued edge.

Table 3 reveals a range of behaviors among the simulated strategies. Some of the top-performing agents demonstrate their advantages from the beginning of the simulation, while others gradually increase their scores over time. Conversely, certain strategies that initially appear promising may lose their edge, either gradually or abruptly. These latter strategies appear to be more technical in nature, as they tend to lose their advantages in a market dominated by fundamental factors.

The last two rows of the table provide additional insights into the distribution of scores over time. The row labeled sum_t shows the sum of scores for the top ten agents at each time interval, from 10 to 2000. At the beginning of the simulation, there is a high concentration of scores among the top-performing agents. However, over time, their advantages tend to diminish and the scores become more dispersed. Toward the end of the simulation, other groups of agents appear to gain more relative strength in the market. The final row of the table, labeled sum_T, is almost monotonically increasing, as we track the final top ten agents and expect them to improve their performance through each trial until the end.

In our next test, we conduct simulations in which we concurrently adjust the agents' parameters, rather than making random adjustments. Specifically, we multiply the parameters *cat* and *dur* by a value between 1/3 and 3 and examine the resulting outcomes. To compare these results with those from our initial simulations, we sum the scores of the agents and present them in Table 4. Our analysis indicates that there is no significant difference between the results in Table 4 and those from the earlier simulations. This finding is both essential and trivial, as it demonstrates that the intensity with which agents incorporate value–price gaps or trend patterns in their calculations does not significantly impact their performance. Rather, it is the relative intensity with which they react to market conditions, as compared to their peers, that determines their success.

Thus far, our findings have demonstrated that our results are not random and are robust when subjected to certain modifications. In the next section, we will identify winning strategies based on their parameters.

Table 4. Changing agents' parameters concurrently.

Agent	Original Score	Scaled Score
68	453	469
194	402	75
184	317	369
16	284	280
118	245	73
138	177	48
110	155	217
172	147	169
1	112	55
144	83	0

Note: In 100 simulations, we scale the agents' *cat* and *dur* parameters from approximately one-third to up to three times the original ones and compare the total scores.

4.3. Parameter Distribution of Winners

Up to this point, we have used fixed agents, meaning that agent #9, for example, was always the same agent. In this section, we will randomly assign parameters to the agents. This approach will serve two purposes. Firstly, *it will enable us to verify the robustness of our results*. Secondly, *it will allow us to obtain the parameter distribution of the winning strategies*. As previously described, each agent has three specific parameters: catalyst (*cat*), duration (*dur*), and trend following preference (*tfp*). For our verification test, we conduct 100 simulations, each with different agents. To analyze the results, we examine the *cat*, *dur*, and *tfp* parameters of the winning agents and compare them to the parameter distribution of the entire population. The results of this analysis are presented in the three parts of Table 5.

Table 5 shows that the most effective method for maximizing the likelihood of winning is to choose a low positive *cat* number. Agents within the *cat* range of 1 to 5 have more than ten times the chance of reaching the top positions than their percentage of the population would suggest. For instance, these agents occupy the first position 26 times, despite the likelihood of having this *cat* number being only 2% in the population. The next best strategy corresponds to a low negative *cat* number. This observation may seem paradoxical, as it suggests that two opposite strategies could yield similarly good results. However, it is worth noting that a low absolute value of *cat* (either positive or negative) creates an extreme strategy with positions that change dramatically and quickly. This type of trading sometimes creates a trend that other agents adopt, thereby incurring advantageous benefits for the original creators of the trend, even if the strategy has no other original edge. Hence, agents with a low negative *cat*, i.e., powerful counter value investment strategies, have a significant chance of winning. Nevertheless, we also observe in the table that agents with a low positive *cat* have far more chances of winning (see the first part of Table 5, rows $-1 \ldots -5$ and $1 \ldots 5$). Furthermore, the agents with negative *cat* generally perform worse than those with a positive *cat*. Additionally, in the mid-range *cat* (from 20 to 200), it can be observed that fundamental strategies (positive *cat*s) lead to the best overall performances.

Another observation that can be drawn from Table 5 is the potential correlation between low *dur* parameters and the likelihood of achieving good results. However, this correlation may be due to excessive trading and potential trend-creating consequences, which could have a high chance of backfiring (as discussed in the next section).

Table 5. Parameter distribution of winners.

			cat			
	Number of Gained Positions					
cat Range	1st	2nd	3rd	Rank:4th... 6th	Rank:7th... 9th	All (PDF)
−900 · · · − 500	0	0	1	0	0	<1%
−500 · · · − 300	0	0	0	0	0	2%
−300 · · · − 200	3	0	0	0	0	4%
−200 · · · − 100	5	0	1	0	2	12%
−100 · · · − 50	2	5	4	4	6	12%
−50 · · · − 20	3	3	2	13	12	11%
−20 · · · − 10	2	7	3	11	24	4%
−10 · · · − 5	4	3	5	26	25	2%
−5 · · · − 1	9	8	12	21	10	2%
1 · · · 5	26	34	32	70	23	2%
5 · · · 10	4	17	16	72	62	2%
10 · · · 20	3	3	4	29	79	4%
20 · · · 50	13	6	5	11	18	11%
50 · · · 100	6	3	3	6	11	12%
100 · · · 200	12	1	5	11	9	12%
200 · · · 300	1	3	1	14	9	4%
300 · · · 500	6	6	4	9	10	2%
500 · · · 900	1	1	2	3	0	<1%
			dur			
	Number of Gained Positions					
dur Range	1st	2nd	3rd	Rank:4th... 6th	Rank:7th... 9th	All (PDF)
1 · · · 5	41	8	11	18	16	5%
5 · · · 10	5	5	2	13	11	5%
10 · · · 20	7	9	9	18	25	9%
20 · · · 50	17	24	23	65	58	21%
50 · · · 100	13	16	26	69	71	24%
100 · · · 200	12	28	15	76	78	23%
200 · · · 300	5	5	9	25	25	9%
300 · · · 500	0	4	5	15	15	4%
500 · · · 700	0	1	0	1	1	1%
700 · · · 1000	0	0	0	0	0	<1%
			tfp			
	Number of Gained Positions					
tfp Range	1st	2nd	3rd	Rank:4th... 6th	Rank:7th... 9th	All (PDF)
0.0 · · · 0.1	9	5	6	36	47	10%
0.1 · · · 0.2	10	9	19	35	37	10%
0.2 · · · 0.4	8	16	9	32	33	10%
0.4 · · · 0.5	13	12	10	36	31	10%
0.5 · · · 0.6	8	7	13	33	27	10%
0.6 · · · 0.7	6	15	10	23	24	10%
0.7 · · · 0.8	8	8	15	31	20	10%
0.8 · · · 1	12	14	7	22	33	10%
1.0 · · · 1.1	16	7	4	28	19	10%
1.1 · · · 1.2	9	7	7	23	28	10%

Note: The number of top-ranking agents for different ranges of *cat*, *dur*, and *tfp*. We present the distribution of specified parameters for winning agents in each of the three parts of the table, comparing them to the probability distribution of the same parameter (last column). Each row represents the range of the parameter being studied, and each column shows the observed ranks. It is important to note that in the fourth and fifth columns, we add the number of agents across three ranks, resulting in numbers three times the usual (the sum of each column rounded to 300 instead of 100).

To further investigate the relationships between the agents' parameters and their chances of winning, we study these parameters pairwise in a two-part table. The first part of Table 6 is designed to show the number of top-ranking agents for each pair of (*cat*–*dur*) in the specified ranges. The number in each cell represents the total number of top-ten agents with the associated *cat–dur* parameters of the cell. The numbers in circles indicate the total number of first-ranked agents. Although having a small *cat* or *dur* parameter appears to confer a considerable advantage (as observed in Table 5), we do not observe any clear superior *cat–dur* strategy.

Table 6. The *cat–dur*, and *dur–tfp* distributions of winners.

cat* vs. *dur										
	Number of Gained Positions for Each Time Stamp									
cat Range	**5**	**10**	**20**	**50**	**100**	**200**	**300**	**500**	**700**	**1000**
−900 · · · − 500	1	0	0	0	0	0	0	0	0	0
−500 · · · − 300	0	0	0	0	0	0	0	0	0	0
−300 · · · − 200	1 ①	0	0	1 ①	1 ①	0	0	0	0	0
−200 · · · − 100	4 ①	0	1 ①	2 ②	1 ①	0	0	0	0	0
−100 · · · − 50	3	1	1	4	4	7 ②	2	0	0	0
−50 · · · − 20	3 ①	2 ①	2	9	9	9 ①	3	1	0	0
−20 · · · − 10	0	3 ①	2	13 ①	13	16	7	2	0	0
−10 · · · − 5	0	6 ①	4	18 ①	19 ①	16	4 ①	2	0	0
−5 · · · − 1	4 ①	2	5 ①	19 ③	12	16 ②	5 ②	0	1	0
1 · · · 5	4 ②	6	19 ③	49 ⑤	42 ⑨	48 ⑥	14 ①	13	0	0
5 · · · 10	5 ①	2	13	37 ②	51	53 ①	15	10	0	0
10 · · · 20	5 ②	7	8	15	45 ①	41	16	13	1	0
20 · · · 50	24 ⑪	5 ①	9 ①	9	5	5	2	2	0	0
50 · · · 100	15 ⑥	1	5	8	3	1	0	0	0	0
100 · · · 200	16 ⑧	3 ①	4 ①	10 ②	4	2	0	1	0	0
200 · · · 300	5 ①	0	1	2	5	12	2	1	0	0
300 · · · 500	7 ⑤	1	0	6	8	5	6 ①	2	1	0
500 · · · 900	1 ①	0	1	1	0	5	0	0	1	0
dur* vs. *tfp										
	Number of Gained Positions for each *tfp*									
dur Range	**0.12**	**0.24**	**0.36**	**0.48**	**0.60**	**0.72**	**0.84**	**0.96**	**1.08**	**1.20**
1 · · · 5	5 ④	9 ①	10 ④	16 ⑤	8 ④	11 ②	9 ④	11 ⑤	11 ⑦	7 ④
5 · · · 10	10	7 ②	2	5 ①	4	5	2	3 ①	0	1 ①
10 · · · 20	10	13	6 ①	8 ①	6	6 ①	6 ①	9 ①	3 ①	7 ①
20 · · · 50	20 ①	26 ②	17 ②	32 ④	25 ②	10	23 ③	15 ①	18 ②	17
50 · · · 100	33 ③	28 ①	17	19	22 ②	15 ①	16	28	23 ④	18 ②
100 · · · 200	28	22 ②	39 ①	28 ②	18	19 ①	23	24 ④	16 ②	19
200 · · · 300	7 ①	10 ②	9	7	7	12 ①	8	5	5	6 ①
300 · · · 500	4	3	7	2	6	6	6	5	3	5
500 · · · 700	0	0	1	0	0	0	0	0	3	0
700 · · · 1000	0	0	0	0	0	0	0	0	0	0

Note: The simultaneous effects of two parameters on winning strategies. The number in each cell represents the total number of top-ten agents with the associated parameters. The numbers in circles indicate the total number of first-ranked agents. Our analysis suggests that each parameter class has its effect individually, as observed, for instance, for low *dur* values coupled with low *tfp* values.

The second part of Table 6 examines the effects of *dur* and *tfp* parameters from the perspective of winning strategies. Interestingly, our analysis shows that low *dur*, i.e., short-period trading, does not necessarily need to be coupled with high *tfp*, i.e., forceful trading on trends, to achieve good results. In fact, high *tfp* can lead to strategies with excessive risks and damage outcomes. However, low *tfp* does not result in successful strategies either.

4.4. Parameter Distribution of Losers

To ensure that the results reported in Table 5 are not simply the outcome of taking extreme risks, we now investigate the characteristics of the worst-performing agents. We begin by examining the parameter intervals of the losing agents. In Table 7, we present the results in five tiers, ranging from the worst 1% to the worst 50%. Our earlier observations indicated that a low *cat* value, whether positive or negative, creates a risky strategy with a significant chance of poor performance, but a low negative *cat* value has a higher chance of failure (as observed in Table 5). Here, in the mid-range *cat* values in Table 7, we observe that a positive *cat* value has a lower chance of being at the bottom. Additionally, we observe that low *dur* values increase the probability of poor performance. This latter observation suggests that the good performance of low *dur* may be solely due to the chance associated with an extremely risky strategy. Another noteworthy observation pertains to the effect of *tfp* on bad performances. In previous sections, we were unable to identify a winning strategy based on *tfp*. However, in Table 7, we observe that increases in *tfp* are uniformly associated with a higher chance of being among the bottom tiers.

Table 7. The parameter distribution of losers.

			cat			
			Number of Gained Positions			
cat Range	worst 1%	worst 5%	worst 10%	worst 25%	worst 50%	all
−900 ··· −500	0	0	0	0	0	<1%
−500 ··· −300	8	10	9	6	4	2%
−300 ··· −200	9	10	9	8	6	4%
−200 ··· −100	13	14	16	15	16	12%
−100 ··· −50	5	7	8	10	15	12%
−50 ··· −20	2	5	5	11	14	11%
−20 ··· −10	1	2	3	7	6	4%
−10 ··· −5	1	2	4	4	3	2%
−5 ··· −1	2	3	5	4	3	2%
1 ··· 5	2	2	2	2	1	2%
5 ··· 10	0	2	3	2	1	2%
10 ··· 20	0	3	5	4	3	4%
20 ··· 50	10	6	7	10	7	11%
50 ··· 100	12	11	9	8	8	12%
100 ··· 200	22	15	10	6	9	12%
200 ··· 300	8	4	3	2	3	4%
300 ··· 500	4	3	2	1	1	2%
500 ··· 900	0	0	0	0	0	<1%
			dur			
			Number of Gained Positions			
dur Range	worst 1%	worst 5%	worst 10%	worst 25%	worst 50%	all
1 ··· 5	10	7	5	4	3	5%
5 ··· 10	33	23	15	9	6	5%
10 ··· 20	31	26	22	15	11	9%
20 ··· 50	18	25	29	28	25	21%
50 ··· 100	4	10	15	21	23	24%
100 ··· 200	2	6	10	15	21	23%
200 ··· 300	0	2	4	5	7	9%
300 ··· 500	0	0	1	2	4	4%
500 ··· 700	0	0	0	0	0	1%
700 ··· 1000	0	0	0	0	0	<1%
			tfp			
			Number of Gained Positions			
tfp Range	worst 1%	worst 5%	worst 10%	worst 25%	worst 50%	all (PDF)
0.0 ··· 0.1	1	2	3	5	8	10%
0.1 ··· 0.2	2	5	5	7	9	10%
0.2 ··· 0.4	8	5	6	8	9	10%
0.4 ··· 0.5	8	7	8	9	10	10%
0.5 ··· 0.6	11	11	10	11	10	10%
0.6 ··· 0.7	11	11	11	11	10	10%
0.7 ··· 0.8	13	12	13	11	11	10%
0.8 ··· 1	15	13	13	11	11	10%
1.0 ··· 1.1	14	17	15	13	11	10%
1.1 ··· 1.2	18	17	16	13	11	10%

Note: The worst performing agents for different ranges of *cat*, *dur*, and *tfp*. The structure of this table is similar to Table 5 but instead of winning agents, each column represents a tier of losing agents, from the worst 1% to the worst 50% (bottom half).

5. Conclusions

In this study, we constructed a simulated stock market in which every investor selects an optimal portfolio based on both fundamental and technical considerations. Specifically, investors invest in one risky asset (i.e., a stock) that pays dividends and one riskless asset. The investors in our model agree on the stock's fundamental value. However, the differences in agents' investing strategies arise from their predictions of the value–price gaps and price trends. Unlike many models in the literature (e.g., (Levy et al. 1994), EHS, SFI-ASM), our agents do not simply review the price-dividend ratio of a stock. Instead, they analytically calculate the stock's fundamental value and use it as a benchmark for the stock price. Our primary goal is to study the performance of different strategies, rather than the market environment as a whole. To the best of our knowledge, beyond the work of Lo et al. (2018), which studies the relative performance of agents from a similar perspective, little research has been presented from this perspective in the literature.

In this study, we develop an unconventional market simulation in which the intrinsic value of the risky asset is disregarded on average. This experimental design enables us to examine three strategies that yield superior returns: (1) betting on the convergence of the value–price gap, (2) betting on the divergence of the value–price gap, and (3) betting on the continuation of the price trend.

We observe that the performance of the second strategy is an artifact of our simulation design, as this approach would be untenable in the real world. The entire market would act against such an irrational strategy, leaving it with no chance of success. In contrast, although the third strategy exhibits some effectiveness, its return is not as favorable as that of the first strategy, and its associated risk is substantially higher.

It is essential to note that our setup is heavily biased in favor of momentum traders, as it lacks counter-momentum traders to counterbalance their influence. Furthermore, our design disadvantages value investors due to the presence of unrealistic counter-value investors. We propose that value investing, unlike other strategies, does not partake in a zero-sum game. Consequently, its performance is not solely reliant on forecasting the actions of other investors and outperforming them, although this may be beneficial.

Our simulations were conducted under certain assumptions that deviate from the real world, such as the presence of counter-value investors and the absence of counter-momentum traders. However, simulations conducted under more realistic assumptions, more closely mirroring the real world where most investors act upon the convergence of the value–price gap rather than its divergence, and where some investors deploy counter-momentum strategies effectively wagering on the reversal of price trends, demonstrate that, under these conditions, value investors can achieve even superior returns with reduced risk, while the performance of momentum traders tends to deteriorate. We have chosen to omit these results from the present discussion as the results already presented sufficiently illustrate our primary findings.

Through running the strategies in a biased environment, in contrast to value investors, and subjecting them to several robustness checks, we can confidently conclude that the fundamental strategies, which heavily bet on value–price gaps, exhibit superior performance.

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Appendix A. Statistical Significance

In simulation studies, abundant data is typically available; however, researchers must develop suitable methods for filtering out relevant information from the data set. Given that the amount of data generated can be as much as required, the key factors to be considered are time and computing power in order to achieve a desired level of statistical significance. In this paper, we delve further into this issue and propose an ad hoc scoring scheme, analogous to those used in sporting events, to compare agents and present the outcomes of 100 simulations for multiple agents in a single table. This approach is exemplified by the application of conventional statistical methods to evaluate the significance of the results.

Consider the best agent among the competing 200 agents, who received 474 scores and ranked first 26 times out of 100. By the binomial distribution, the probability of this record is:

$$P = 200 \times \sum_{k=26}^{100} \begin{pmatrix} 100 \\ k \end{pmatrix}^{k} (199/200)^{100-k},$$

although the calculation of the expression is relatively straightforward, using an approximation would better illustrate our point. We replace the last term with one and the initial term with its maximum value to obtain the following upper bound:

$$p < 200 \times 60 \times \begin{pmatrix} 100 \\ 26 \end{pmatrix} (1/200)^{k},$$

which is lower than 10^{-10}.

We expand the proposed method to address the evaluation of agents that may not be prominent. To illustrate this, we consider the case of agent 16, who did not rank first in any of the 100 simulations, but was ranked lower than 38 only 13 times. Utilizing a binomial test, we demonstrate that the likelihood of achieving a performance better than the 10th position is lower than 10^{-10}. These examples highlight the ease with which statistical significance can be established through appropriate testing, provided that the pertinent questions are framed and significant relationships are identified. Indeed, it is essential to pose the correct questions and pinpoint the crucial connections to discern the statistical relevance of the results. A trained observer can readily recognize when a question of statistical significance is moot.

Appendix B. Kelly in Investment

We consider an investor who is confronted with a risky asset that exhibits a two-state outcome: a gain of g with a probability of p and a loss of l with a probability of $q = 1 - p$. To maximize the expected growth rate, the investor must determine the optimal fraction (α) of their wealth to invest in this asset. Thus, the problem can be formulated as follows:

$$\max r(\alpha) = (1 + \alpha g)^{p} \cdot (1 - \alpha l)^{q},$$

or taking logarithm,

$$\max \ln r(\alpha) = p \ln(1 + \alpha g) + q \ln(1 - \alpha l).$$

For the latter the first-order condition reads, as follows:

$$\left.\frac{dr}{d\alpha}\right|_{\alpha=\alpha^*} = \frac{pg}{1 + \alpha^* g} + \frac{-ql}{1 - \alpha^* l} = 0,$$

leading to

$$\alpha^* = \frac{p}{l} - \frac{q}{g}.$$

The expected return and variance of this risky asset are, respectively, given by the following relations

$$\mu = pg - ql,$$
$$\sigma^2 = pq(g+l)^2,$$

where $p = q = 0.5$; for g and l at the same order of magnitude, one obtains the following:

$$\sigma^2 = pq(g+l)^2 \simeq 0.5 \cdot 0.5 \cdot 4gl \simeq gl.$$

Thereby,

$$\alpha^* = \frac{p}{l} - \frac{q}{g} = \frac{pg - ql}{gl} \simeq \frac{\mu}{\sigma^2}.$$

Notes

1. the oldest source we could find in this regard was Fetter (1904), which was cited in Herbener and Holcombe (1999).
2. In the simulation, a slightly modified formula is utilized to ensure that the dividends remain positive and do not deviate significantly from the initial value
$$d_{t+1} = \bar{d} + \rho(d_t - \bar{d}) + \sigma_d Z_t$$
where d_{t+1} is the logarithm of the dividend at time $t+1$, $\bar{d}$ is the long-run average of the logarithm of the dividend, and ρ is the mean-reversion coefficient.
3. A higher price increases the probability of agents calculating a positive trend, subsequently leading to a positive technical premium. This is why the technical component of the return is computed using the previous periods' stock prices.
4. In a pseudo-random number generator such as the one we used, a seed is needed to initiate the generator. Starting with different seeds results in different series of numbers.

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