



Article

# Black–Scholes 50 Years Later: Has the Outperformance of Passive Option Strategies Finally Faded?

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**Abstract:** Slightly over fifty years ago, the Black–Scholes option pricing model revolutionized investing by enabling a shift from linear to non-linear payoff structures. Myron Scholes later published two papers documenting the performance of passive option strategies that outperformed the underlying index on a risk–return basis. The options market has evolved considerably over the last fifty years from an open outcry trading structure with options being single-listed to a high-frequency computer-based market. This paper re-evaluates the trilogy of foundational studies to determine whether passive-option-enhanced portfolios still produce superior performance in the current high-frequency options market environment.

**Keywords:** equity options; option-enhanced indices/portfolios; option strategies; reward-to-risk portfolio performance



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## 1. Introduction

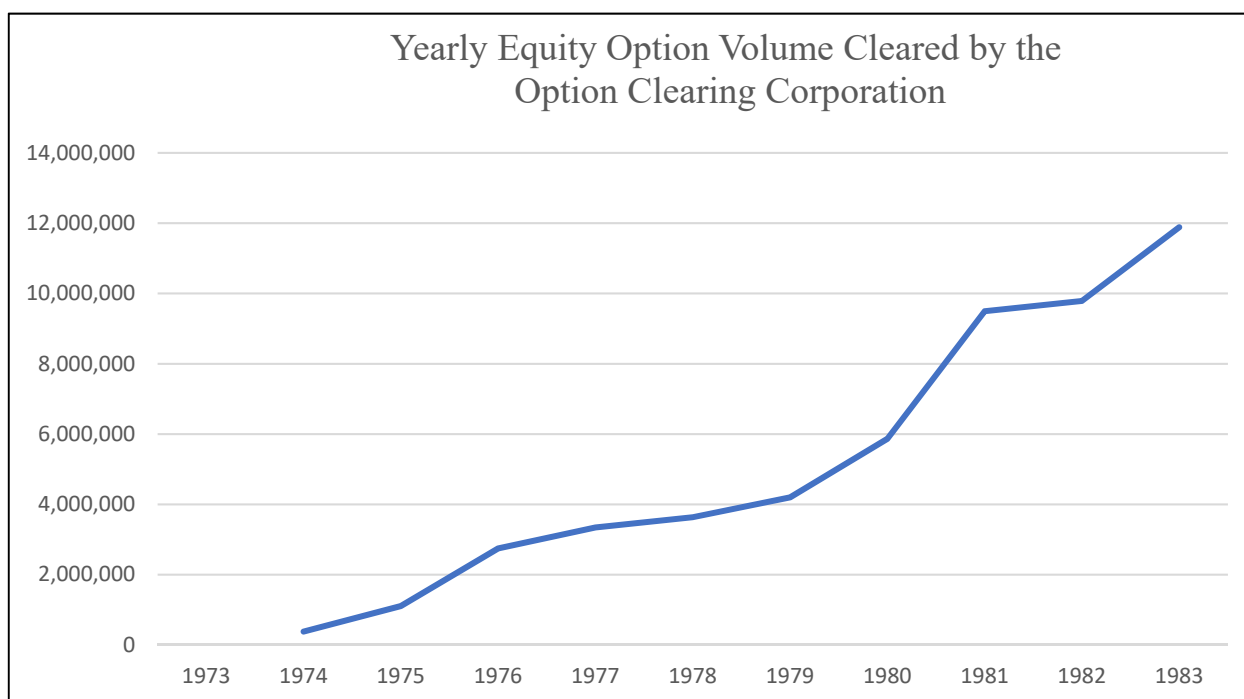
A little over fifty years ago in 1973, the Black–Scholes model was published. The Black–Scholes model was the first publicly available model to value European-style options (Black and Scholes 1973). The formula enables investors to estimate the fair value of options using only one unknown variable, volatility. It also provides partial derivatives called Greeks, which predict changes in the option value based on the underlying asset price, volatility, time to expiration, and interest rate inputs. The Greeks allow asset managers to tailor portfolio risk through the use of option strategies, which may be one of the most valuable contributions of the Black–Scholes model to investing. This paper does not focus on the mathematical derivation of the Black–Scholes option pricing model. Instead, it emphasizes the practical applications of option strategies as an investment tool, as outlined in the work of (Merton et al. 1978). Adding options to portfolios lets asset managers tailor desired risk/reward criteria (Fong et al. 2005). For this research, we use listed index settlement prices provided and managed by the Chicago Board of Options Exchange (CBOE).

Before Black–Scholes and the CBOE, options trading was limited. It operated as a phone market primarily limited to institutional investors. Black–Scholes allowed for options to be exchange-listed (Houser et al. 2023). The timing was perfect since the Chicago Board of Trade (CBOT) had been petitioning the SEC for permission to list equity options. Black–Scholes played a pivotal role in the SEC granting permission to publicly list equity options.

“I think the SEC very quickly thought of options as a useful mechanism in the securities markets and it’s probably—that’s my judgment—the effect of Black–Scholes. I never heard the word “gambling” again in relation to options” (MacKenzie and Millo 2003).

Although the CBOE was granted permission to list and trade options, it is often overlooked that options order flow was limited in the early years. This is shown by the slow growth in options volume in the early years as shown in Figure 1. CBOE management at first struggled to encourage members to make markets in equity options. Despite the

SEC approval, the investment community remained reluctant to incorporate options into traditional portfolios (MacKenzie and Millo 2003). In 1977, the CBOE began listing puts. This further increased the usefulness of options to investment managers. Later, in 1978, Myron Scholes co-published a research paper documenting that option strategies could outperform a diversified portfolio of stocks (Merton et al. 1978). Then, in 1982, Merton et al. published a second paper detailing the benefits of put strategies (Merton et al. 1982). These seminal papers by Merton et al., along with the original Black–Scholes paper, are central to this study, as together they laid the groundwork for the development of listed options. The main goal of this paper is to reevaluate the widely held belief that including options in a portfolio improves its risk/return ratio. As shown in Figure 1, option volumes increased after the publication of the Merton et al.’s papers (Merton et al. 1978, 1982). While it cannot be known if the increase in option volume was related to the Merton et al. paper or a result of better marketing by the CBOE, we are sympathetic to the paper as a true cause.



**Figure 1.** Growth of option volume cleared in early years.

This brings us to the central hypothesis of this article. We posit that structural changes in the markets over the past fifty years have morphed the original results. There are several questions we pose. The first question is whether the structural changes in the option market have eliminated the increase in the return-to-risk ratio from including option overlays in a portfolio as described by Merton et al. (1978, 1982). Second, we would like to understand if this change happens because of the methodology of Merton et al., or if it is something market-driven. Our discussion first surveys the literature, explaining the modeling choices that were made in the first paper. We then review the literature that has been written in the field of passive option strategies. Finally, we replicate the study using exchange data. The exposition of this paper avoids any intra-expiry trading. Moreover, it is the case that most index options (which figure prominently in this paper and the literature) are European in expiry style. This alleviates any need to deal with early exercise or assignment. Discussions of alternative models of option pricing are only used as evidence to show proliferation of the knowledge and techniques as time progressed. We aim to answer whether the results of Scholes still hold or if they have changed. If the results have changed, we are interested in attributing those changes to an epoch or portfolio style.

## 2. Historical Review

In 1952, Harry Markowitz proposed portfolio selection and soon after mean–variance optimization (Blay 2024). These papers became the foundation of modern portfolio management. It was a foundation built on the tradeoff between risk and return. Markowitz showed that portfolio diversification effectively manages the tradeoff between expected return and risk. This efficiency arises because while stocks tend to be positively correlated, their correlations are not perfect, allowing risk to be reduced through diversification. The efficiency was because though equities are positively correlated to each other, the correlation is less than 100%. Combining relatively uncorrelated assets meant that the volatility of the portfolio could be lower than its constituents separately. Markowitz's mean–variance optimization framework forced asset managers to shift their focus from maximizing returns to creating a portfolio that maximizes expected returns given a target portfolio variance (risk) (Rubinstein 2002). Markowitz is universally credited with bringing systematic approaches to risk and return.

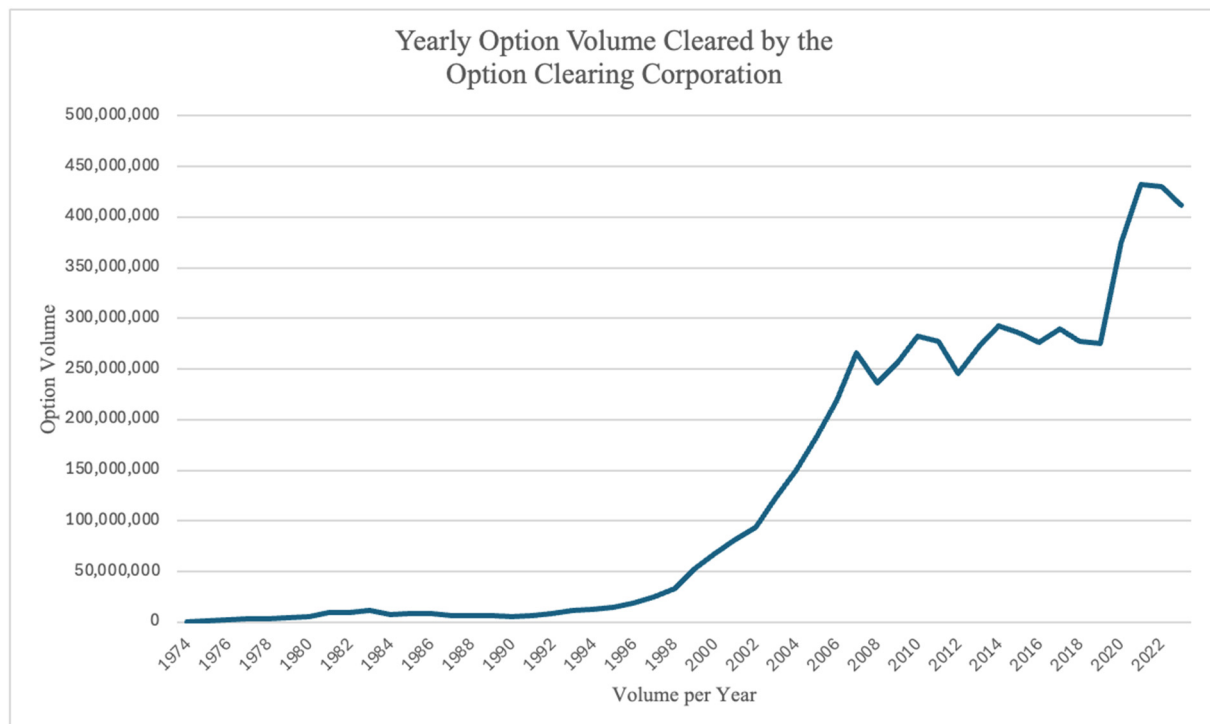
In Edward Thorp's groundbreaking book "Beat the Market: A Scientific Stock Market System", applying nascent computer algorithms to generate a fair option price was discussed (Thorp and Kassouf 1967). Through his understanding of finance and gaming, he likely developed an option pricing algorithm before Black–Scholes (Haug and Taleb 2007). Thorp scrutinized Markowitz's portfolio theory by highlighting key limitations, particularly its reliance on assumptions about the normal distribution of returns and static correlations (Thorp 1975). More importantly, these two publications described how options could create portfolios that would outperform the market. The illiquid telephone market for equity options at the time hindered practical implementation of these strategies. The world would have to wait a few more years.

The seminal Black–Scholes paper was the catalyst for the creation of the CBOE. It coaxed regulators into shifting their views from options as a form of gambling to a risk management tool for asset managers (Black and Scholes 1973). This was documented in an early interview with Burton R. Rissman about the creation of the CBOE:

"Black-Scholes was really what enabled the options to thrive since options were now listed on an SEC regulated exchange. It gave a lot of legitimacy to the whole notion of hedging and efficient pricing, whereas we were faced, in the late 60s–early 70s with the issue of gambling. That issue fell away, and I think Black-Scholes made it fall away. It wasn't speculation or gambling; it was efficient pricing. I think the SEC very quickly thought of options as a useful mechanism in the securities markets and it's probably—that's my judgment—the effects of Black-Scholes. I never heard the word "gambling" again in relation to options" (MacKenzie and Millo 2003).

The Black–Scholes model was the first widely adopted framework for pricing options and calculating the Greeks, fundamentally changing the landscape of financial derivatives. Before its publication, option valuation relied largely on Bachelier's early-twentieth-century framework (Bachelier 1900; Murphy 1990). Over the past fifty years, the use of equity/fixed income, foreign exchange, and index options has surged, driven by the Black–Scholes framework and later models, such as Cox–Ross–Rubinstein (Cox et al. 1979), Garman–Kohlhagen (Garman and Kohlhagen 1983), and the common stochastic volatility models (Heston 1993; Mehrdoust and Saber 2014). The continuous evolution in option modeling has enabled the pricing of increasingly complex derivatives, including pricing discounted perpetual game call options (Zaevski 2020). This overview of the continuous evolution in option pricing models is not exhaustive, as this paper focuses on option strategies and calculates returns using settlement prices of listed securities.

The growth of option trading underscores their expanding role in equity portfolios, underscoring their expanding role in modern finance. As shown in Figure 1, the number of contracts traded in 2022 was expected to exceed 400 million as shown in Figure 2 (Historical Volume Statistics n.d.)



**Figure 2.** Historical growth of equity and index open option contracts.

Even with the SEC's approval, the question remained whether options were gambling instruments masquerading as risk management tools. To shift the early perception that option trading was gambling, and not an investment tool, Merton et al. published two studies of how options might be incorporated into a simple portfolio, the first of which was published in 1978 (Merton et al. 1978, 1982). This research sought to educate investors about the benefits of including options in their investment portfolios. Because of the recent listing of options, a back test of basic option strategies was essential to inform retail investors about their potential benefits. Merton et al. conducted this research by simulating the impact of options on two portfolios: a broad market proxy of 136 equities and the Dow Jones Thirty index. Using a twelve-year period, the back test incorporated historical volatility and applied the Black–Scholes model to price the options. This paper examined two options investment strategies tailored for asset managers. The original research focused only on call strategies because the CBOE originally listed only calls (MacKenzie and Millo 2003). Merton et al. compared these option strategies to the traditional approach of building a diversified portfolio to manage risk. Their research objective was to assess whether these option strategies could effectively reduce risk and improve returns.

The option strategies and returns from the original research are below:

**Covered Call:** This involves a long position in equity while also selling a call option on the same equity. The premium received from selling the call provides income and partially offsets potential losses in the stock position. The result of this research is shown below in Table 1.

**Long Call + Commercial Paper:** The authors introduced a portfolio comprising 90% commercial paper and 10% Long Call options. Using different strike prices (10% of in-the-money and at-the-money, 10% out-of-the-money, and 20% out-of-the-money) allowed them to create a spectrum of risk profiles catering to a range of investor preferences as shown in Table 2.

**Table 1.** Covered call returns from (Merton et al. 1978) research.

|                                 | Exercise  | Exercise  | Exercise  | Exercise  |         |
|---------------------------------|-----------|-----------|-----------|-----------|---------|
|                                 | Price=    | Price=    | Price=    | Price=    |         |
|                                 | 0.9 Stock | 1.0 Stock | 1.1 Stock | 1.2 Stock | Stock   |
|                                 | Price     | Price     | Price     | Price     |         |
| 136 Stock-Semiannual Statistics |           |           |           |           |         |
| Covered Call                    | 3.30%     | 3.70%     | 4.50%     | 5.30%     | 7.90%   |
| Standard Deviation              | 4.90%     | 7.10%     | 9.30%     | 11.20%    | 16.60%  |
| Return to Risk                  | 0.67      | 0.52      | 0.48      | 0.47      | 0.48    |
| Highest Return                  | 14.60%    | 19.30%    | 24.70%    | 30.40%    | 54.60%  |
| Lowest Return                   | −9.90%    | −14.40%   | −17.40%   | −19.20%   | −21.00% |
| Upside Capture                  | 0.27      | 0.35      | 0.45      | 0.56      |         |
| Downside Capture                | 0.47      | 0.69      | 0.83      | 0.91      |         |
| Dow Jones-Semiannual Statistics |           |           |           |           |         |
| Covered Call                    | 2.90%     | 2.90%     | 3.20%     | 3.50%     | 4.10%   |
| Standard Deviation              | 3.70%     | 6.20%     | 8.60%     | 10.40%    | 13.70%  |
| Return to Risk                  | 0.78      | 0.47      | 0.37      | 0.34      | 0.30    |
| Highest Return                  | 12.30%    | 16.90%    | 22.90%    | 29.50%    | 49.10%  |
| Lowest Return                   | −5.40%    | −9.20%    | −11.90%   | −13.80%   | −16.40% |
| Upside Capture                  | 0.25      | 0.34      | 0.47      | 0.60      |         |
| Downside Capture                | 0.33      | 0.56      | 0.73      | 0.84      |         |

**Table 2.** Long Call + Commercial Paper returns from (Merton et al. 1978) research.

|                                 | Exercise  | Exercise  | Exercise  | Exercise  |         |
|---------------------------------|-----------|-----------|-----------|-----------|---------|
|                                 | Price=    | Price=    | Price=    | Price=    |         |
|                                 | 0.9 Stock | 1.0 Stock | 1.1 Stock | 1.2 Stock | Stock   |
|                                 | Price     | Price     | Price     | Price     |         |
| 136 Stock-Semiannual Statistics |           |           |           |           |         |
| Long Calls + Commercial Paper   | 6.30%     | 8.20%     | 11.10%    | 16.20%    | 7.90%   |
| Standard Deviation              | 7.80%     | 10.60%    | 15.70%    | 27.20%    | 16.60%  |
| Return to Risk                  | 0.81      | 0.77      | 0.71      | 0.60      | 0.48    |
| Highest Return                  | 25.70%    | 34.70%    | 59.90%    | 121%      | 54.60%  |
| Lowest Return                   | −4.70%    | −5.20%    | −5.70%    | −6.10%    | −21.00% |
| Upside Capture                  | 0.47      | 0.64      | 1.10      | 2.22      |         |
| Downside Capture                | 0.22      | 0.25      | 0.27      | 0.29      |         |
| Dow Jones-Semiannual Statistics |           |           |           |           |         |
| Long Calls + Commercial Paper   | 4.20%     | 5.10%     | 7.20%     | 10.60%    | 4.10%   |
| Standard Deviation              | 7.30%     | 10.10%    | 14.60%    | 25.70%    | 13.70%  |
| Return to Risk                  | 0.58      | 0.50      | 0.49      | 0.41      | 0.30    |
| Highest Return                  | 27.10%    | 34.40%    | 42.60%    | 88.10%    | 49.10%  |
| Lowest Return                   | −4.60%    | −5.70%    | −7.50%    | −7.90%    | −16.40% |
| Upside Capture                  | 0.55      | 0.70      | 0.87      | 1.79      |         |
| Downside Capture                | 0.28      | 0.35      | 0.46      | 0.48      |         |

In 1978, the CBOE finally listed put options. As the option order flow increased, Merton et al. updated their results using the same time period and stocks (Merton et al. 1982). The second paper effectively completes their effort to document the benefits of equity options. Taken together, the two papers cover the four traditional option investment strategies that are commonly used by asset managers. By considering these four results in a consistent framework, an asset manager can determine which option strategy is most beneficial to his or her investment style.

**Protective Put:** This strategy involves holding a long position in a stock while also buying a put option on the same stock. The put option serves as insurance against potential losses in the stock position as shown in Table 3.

**Table 3.** Protective Put results from (Merton et al. 1982).

|                                 | Exercise                     | Exercise                     | Exercise                     |         |
|---------------------------------|------------------------------|------------------------------|------------------------------|---------|
|                                 | Price=<br>0.9 Stock<br>Price | Price=<br>1.0 Stock<br>Price | Price=<br>1.1 Stock<br>Price | Stock   |
| 136 Stock-Semiannual Statistics |                              |                              |                              |         |
| Protective Put                  | 7.30%                        | 6.70%                        | 5.90%                        | 7.70%   |
| Standard Deviation              | 12.00%                       | 9.50%                        | 7.10%                        | 16.10%  |
| Return to Risk                  | 0.61                         | 0.71                         | 0.83                         | 0.48    |
| Highest Return                  | 43.60%                       | 37.90%                       | 31.20%                       | 54.60%  |
| Lowest Return                   | −8.20%                       | −4.00%                       | −1.50%                       | −21.00% |
| Upside Capture                  | 0.80                         | 0.69                         | 0.57                         |         |
| Downside Capture                | 0.39                         | 0.19                         | 0.07                         |         |
| Dow Jones-Semiannual Statistics |                              |                              |                              |         |
| Protective Put                  | 4.70%                        | 4.50%                        | 4.00%                        | 4.60%   |
| Standard Deviation              | 10.40%                       | 7.90%                        | 5.60%                        | 13.70%  |
| Return to Risk                  | 0.45                         | 0.57                         | 0.71                         | 0.34    |
| Highest Return                  | 40.80%                       | 35.10%                       | 28.10%                       | 49.10%  |
| Lowest Return                   | −7.00%                       | −1.80%                       | 0.30%                        | −16.40% |
| Upside Capture                  | 0.83                         | 0.71                         | 0.57                         |         |
| Downside Capture                | 0.43                         | 0.11                         | −0.02                        |         |

Cash-Secured Put: This strategy involves holding cash (commercial paper) while also selling a put option. If the put option is exercised, the investor is obligated to buy the underlying asset at the specified price, using the cash held to cover the purchase. If the option remains out-of-the-money, the investor earns the put premium as shown in Table 4.

**Table 4.** Cash-Secured Put results from (Merton et al. 1982).

|                                 | Exercise                     | Exercise                     | Exercise                     |         |
|---------------------------------|------------------------------|------------------------------|------------------------------|---------|
|                                 | Price=<br>0.9 Stock<br>Price | Price=<br>1.0 Stock<br>Price | Price=<br>1.1 Stock<br>Price | Stock   |
| 136 Stock-Semiannual Statistics |                              |                              |                              |         |
| Cash-Secured Put                | 3.50%                        | 4.10%                        | 5.00%                        | 7.70%   |
| Standard Deviation              | 3.90%                        | 5.60%                        | 7.20%                        | 16.10%  |
| Return to Risk                  | 0.90                         | 0.73                         | 0.69                         | 0.48    |
| Highest Return                  | 14.40%                       | 18.80%                       | 23.60%                       | 54.60%  |
| Lowest Return                   | −6.00%                       | −6.40%                       | −5.60%                       | −21%    |
| Upside Capture                  | 0.26                         | 0.34                         | 0.43                         |         |
| Downside Capture                | 0.29                         | 0.30                         | 0.27                         |         |
| Dow Jones-Semiannual Statistics |                              |                              |                              |         |
| Cash Secured Put                | 3.30%                        | 4.00%                        | 5.10%                        | 4.60%   |
| Standard Deviation              | 3.10%                        | 4.80%                        | 6.20%                        | 13.70%  |
| Return to Risk                  | 1.06                         | 0.83                         | 0.82                         | 0.34    |
| Highest Return                  | 12.10%                       | 16.90%                       | 23.10%                       | 49.10%  |
| Lowest Return                   | −2.20%                       | −4.00%                       | −2.80%                       | −16.40% |
| Upside Capture                  | 0.25                         | 0.34                         | 0.47                         |         |
| Downside Capture                | 0.13                         | 0.24                         | 0.17                         |         |

The methodological limitations of this research must be acknowledged. Merton et al.'s findings are based on simulated returns rather than live market data. While simulations have advantages, they also fail to capture the nuances of real market behavior including an implied volatility skew that is not flat and whose shape changes. This study also ignores the fact that bid/ask spreads were wide during this period. The calculation of profit and loss for each strategy should include the addition of the spread.

Despite the methodological limitations of the original papers, they provided valuable insights into the tradeoffs associated with using options strategies for portfolio management. Here, we summarize the key insights:

**Drawdown Mitigation:** The results confirm that the analyzed option strategies had lower maximum drawdowns compared to equity benchmarks. This means that the options strategies were effective in protecting against large losses, which is important when trying to manage risk. But this downside protection often meant these strategies generated lower returns than investing directly in the underlying stocks. The Long Call + Commercial Paper strategy proved to be an exception, as it generated higher returns with a maximum drawdown in line with the other option strategies.

**Superior Return to Risk:** The option strategies achieved a return-to-risk ratio equal to or exceeding the equity benchmarks. The success of the option strategies can be attributed to their ability to reduce volatility more than they reduce total return. Reducing volatility can make an investment more attractive to risk-averse investors. It means that the value of the investment is less likely to fluctuate.

The Long Call + Commercial Paper strategy is an exception to the rule that adding options to a portfolio normally reduces the portfolio variance. This strategy generated higher returns than the equity benchmark with a higher standard deviation. The higher generated returns more than compensated for the added volatility, resulting in a superior return/risk ratio, making it an attractive alternative for those willing to accept more risk.

**Upside/Downside Capture:** Long volatility strategies, such as Protective Put and Long Call, outperform short volatility strategies (covered call and Cash-Secured Put) in terms of their upside/downside capture. This outcome is not unexpected, as short volatility strategies inherently limit upside potential. That said, short volatility strategies had lower standard deviations of returns, underscoring the tradeoff that investors often face when balancing return and risk.

The research conducted by Merton et al. expanded the toolkit available to investors for constructing portfolios with specific risk–reward profiles. By incorporating call and put options into a portfolio, investors could now receive tailored risk management and/or income generation. A successful risk mitigation strategy not only reduces risk but also offers a good value proposition by being cost-effective in relation to the benefits. This perspective encourages decision-makers to critically assess the tradeoffs between risk reduction and their associated costs. These studies demonstrated the power of options to provide flexible investment solutions beyond the constraints of traditional equity and bond allocations.

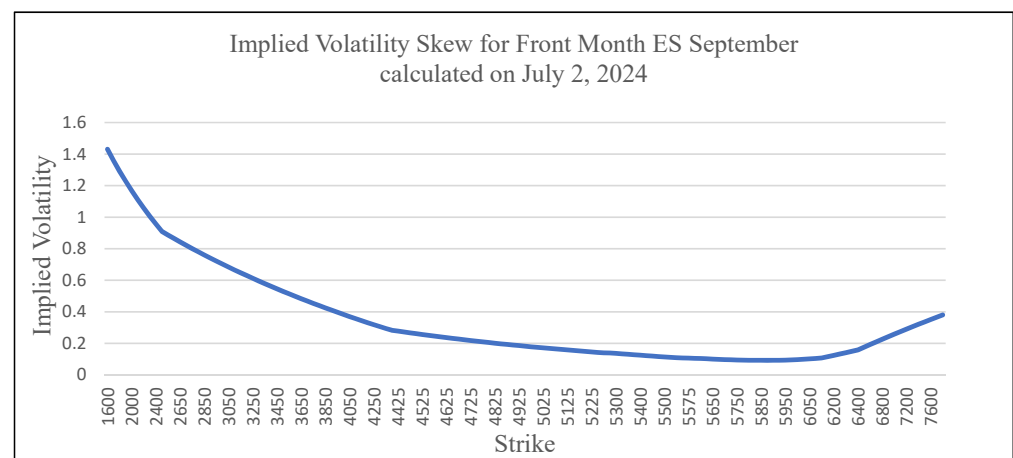
These studies had serious issues from an investor viewpoint. The studies relied on the concept of mark to model, which included the assumption of a single implied volatility value per equity or index. In mark to model, an investor selects an option pricing model and what they believe are the correct inputs to the model. The investor then calculates the value of the option. The original mark-to-model approach was logical in the Merton et al. paper since option markets lacked historical settlement prices and were not liquid. The next major issue with the original papers was the rapid changes in the microstructure of the options market that occurred as the option markets matured into a liquid electronic market. By the end of the 1990s, the results of the previous articles no longer provided a realistic assessment of the performance of these strategies. Therefore, it became clear that the original study using mark to model needed to be replicated using market data versus simulated data.

### 3. Option Market Microstructure Change

The option market grew steadily after options were exchange-listed in 1973. The market structure of option trading changed in 1987 when the CBOE introduced the Retail Automated Execution Platform System (RAES). The RAES was the first auto-execution platform that let retail customers route orders to the CBOE to be filled at the quoted price. The RAES shifted the option market from where the screen price was an indication of price

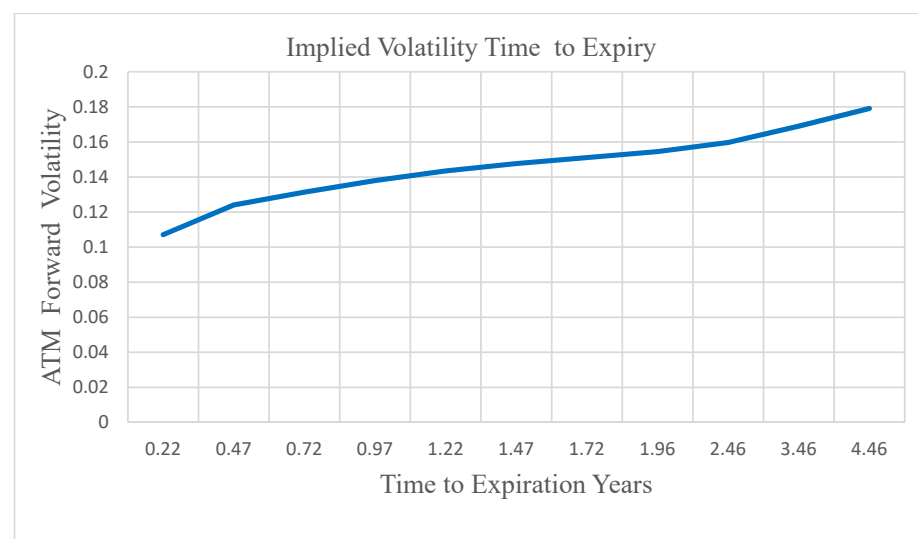
but not executable, to a market where retail customers were guaranteed execution. Market makers could no longer change or widen the bid/ask spread when a customer tried to buy an option ([An Unlikely Rebel at Helm of the Cboe—Chicago Tribune n.d.](#)).

The 1987 stock market crash, known as “Black Monday,” transformed financial markets and led to the discovery of the implied volatility skew. Before the crash, the Black–Scholes model’s assumptions led to the belief that implied volatility was constant across strike prices. However, after the crash, it became evident that implied volatility varied across different strikes. Figure 3 shows the front month skew for the Emini futures contract. The skew (smirk) is shown with the lower strikes trading at a significant premium to the higher-priced strikes. The presence of a skew forced trading behavior to change. Simple delta hedging was necessary, but not a sufficient condition for risk management. A market participant would have to worry that lower strikes could become significantly more expensive.



**Figure 3.** Emini S&P500 front month option skew on 2 July 2024.

Originally, it was assumed that a stock’s volatility (for option pricing purposes) was not affected by expiration date. The Black–Scholes model treats implied volatility as a fixed constant. The implied volatility should be the same whether the option expires in 30 days, 60 days, or any arbitrary time period. However, as shown in Figure 4 again using Emini futures options as provided by CommodityVol.com, implied volatility also exhibits a term structure.



**Figure 4.** Term structure example from Emini futures option on 2 July 2024.

The deviations from the original Black–Scholes assumptions meant that the results of the original series of papers could not be relied on. In 2004, Feldman and Roy of Ibbotson Associates reviewed the returns of options-based investment strategies, focusing specifically on the CBOE S&P 500 Buy-Write Index (BXM) strategy (Feldman and Roy 2004). The BXM strategy is a long position in the SPX Index while simultaneously selling an at-the-money call option on the same SPX Index. This new research focused on the period after the implementation of RAES-like systems on all equity option exchanges, together with option chains that contained both implied volatility skew and a term structure. Feldman and Roy showed that BXM not only outperformed the S&P 500, but also had a significantly lower standard deviation, as shown in Table 5.

**Table 5.** Covered call research from Feldman and Roy 2004.

|                    | BXM                                 |         |
|--------------------|-------------------------------------|---------|
|                    | Exercise=<br>BXM Exercise = 1.0 SPX | S&P 500 |
| 6 Month Return     | 6.03%                               | 5.90%   |
| Standard Deviation | 6.93%                               | 10.33%  |
| Return to Risk     | 0.869                               | 0.571   |
| Highest Return     | 2.95%                               | 4.77%   |
| Lowest Return      | −2.54%                              | −4.86%  |
| Upside Capture     | 0.62                                |         |
| Downside Capture   | 0.52                                |         |

The observation period of the Feldman and Roy’s research, spanning from 1 June 1988 to 31 March 2004, is significant because it encompasses a period of considerable market volatility and major market events. This study also used approximately twelve years of data, which is the same length as the Merton et al. paper. This period included the rise and fall of the technology bubble of the late 1990s and early 2000s. The time period also contains a major shift in pricing options as the concept of skew and kurtosis in returns is also filtering into option methodology (Corrado and Su 1997; Das and Sundaram 1997). Feldman and Roy elected to use mid-market quotes for the options. They could have used the appropriate bids and asks for the option legs. They also calculate their results for each month end versus running the position to the option expiration date. Thus, the settlement prices could contain both future and option manipulations by market makers that are trading the month-end roll order flow (Mouakhar and Roberge 2010; Taylor 2016). This is the main drawback of their work.

Feldman and Roy’s finding that BXM outperformed the S&P 500 with lower volatility may have resulted from unique market circumstances. The overwhelming optimism of investors during the technology bubble and the unique market structure of that period is well documented. Their results might suggest that optimistic investors overpay for upside participation.

The unique market structure during this period may have resulted from the single listing of stock options, rather than the multiple listings common in today’s market. Stock options were single-listed because the SEC chose the Big Board listing model for the option markets. That is, an equity option had a listing exchange much like some equities were listed on the NYSE while others were listed at the AMEX. Equity options were finally cross-listed in 1999, which changed the dynamics of option trading as exchanges competed for customer order flow. The cross-listing of options, along with electronic trading, set in motion two major changes in the equity option market. The first was the narrowing of the bid/ask spread (Cheng et al. 2005). Second was the start of the speed war stemming from cross exchange arbitrage (Barboza 1999). We would expect that this might improve markets for the investor class as market makers gave up a little bit of their edge.

The Feldman and Roy study provides empirical evidence that the covered call strategy (BXM) outperformed the index’s return with lower volatility. This conclusion by Feldman

and Roy differs from the results of the Merton et al. paper. They proposed that the implied volatility of options consistently exceeds realized volatility. Merton et al., on the other hand, calculated their simulated option prices using historical (statistical) volatility; therefore, using the mid-price of listed options would be more accurate. Feldman and Roy hypothesized that this elevated implied volatility was due to higher demand for call options relative to their supply, leading to BXM's outperformance (Feldman and Roy 2004).

#### 4. Current Review of Strategies

The growth in options trading has been remarkable since equity options were cross-listed in 1999, with the single-day notional value of options traded regularly exceeding stock trading volume. An estimated USD 1.1 quadrillion in notional value has been traded through seventy-one billion contracts (Basar 2023). Option trading has achieved an immense scale.

The most dramatic shift in pricing options was the Unlisted Trading Privileges Act of 1994, which led to high-frequency trading. The transition from individual pit traders to electronic high-frequency trading (HFT) firms shifted the option market pricing structure used in the previous studies (Houser et al. 2023). The primary effect, from approximately 1998 to 2008, was that equity bid/ask spreads fell by over 80% and liquidity increased by more than nine-fold (Reg NMS for Dummies | Nasdaq n.d.). Regulation SHO further altered equity options pricing dynamics.

CBOE tradable strategy indices have been used to further examine the performance of options portfolios. Data were collected on expiration dates. This should eliminate any pricing anomalies for at least the final value of the strategy. Option value at expiration is the difference between the strike price and the closing price of the underlying. Therefore, we use expiration dates as opposed to month ends to remove any modeling or trading bias for the final price of the strategy. Our research is similar in style and approach to Feldman and Roy's except for these changes (Feldman and Roy 2004). We discuss the data and results in the following sections.

##### 4.1. Data

Unlike the previously discussed research studies, our analysis uses settlement prices from CBOE strategy indices. These CBOE tradable indices have over fifteen years of settlement prices covering the original strategies listed in the Merton paper. These settlement prices are publicly available and transparent, allowing researchers to analyze these strategies without calculating the price of the embedded options. These settlement data also avoid many of the known issues of pricing index future options (Zaevski 2024). We used the settlement prices that were collected on each expiration date to avoid any interim price anomalies from the firms that traded the futures rolls (Taylor 2016).

Many investors have complex models to determine a theoretical value of the option/strategy at the time of purchase that differs from the actual settlement. Most quantitative analysts also have alpha models for the implied volatility. In finance, investors try to gain an edge by using advanced prediction models to buy undervalued securities and sell overvalued securities. Yet, both Merton et al.'s original research and Feldman and Roy's research focused on passively buying the options for the strategy. Thus, advanced option models and alpha strategies are not part of this research.

##### 4.2. Design

The goal of the original research by Merton et al. was to determine whether including options in portfolios benefited investors. It was decided to follow the original research methods of using monthly index values. To create monthly values from the published CBOE daily indices, we had a choice: we could follow the (Merton et al. 1978) expiration-to-expiration methodology, or we could use month-end data. We chose expiration-to-expiration return series using option expiration dates to accurately reflect strategy returns and remain consistent with the methodology of Merton et al. (Merton et al. 1978, 1982).

The choice of the expiration as the end date for the strategy was to eliminate all known market maker manipulation that normally occurs at month end as investment firms roll their futures and options positions (Taylor 2016). During the roll, market makers can and do adjust the theoretical price of the options to maximize their profit. At expiration, the price of the option is the difference between the strike and the closing price of the index, so the market makers cannot manipulate the price of the option.

Finally, twelve years of data were used to maintain consistency with previous research studies. The time period from 3 January 2012 to 20 June 2023 was selected for this research. This period length was selected to make it consistent with previous studies. To make sure the microstructure of the market contained no known technological or structural jumps, we chose 2012 as the starting point, thereby avoiding the period of the housing crisis from 2008 to 2010.

#### 4.2.1. Call-Write Strategies

**BXM**—The BXM strategy is a long position in the SPX Index while also selling an at-the-money call option on the same SPX Index. This is the same option strategy reviewed by Feldman and Roy.

**BXY**—The BXY strategy is a modification of the BXM strategy that uses out-of-the-money strikes. BXY is a long position in the SPX Index while also selling the call to option with the strike closest being two percent out-of-the-money.

**BXMD**—The BXMD strategy is a modification of the BXM strategy that uses out-of-the-money strikes. BXMD is a long position in the SPX Index while also selling a call option whose strike is the closest to a thirty delta.

**BXMH**—BXMH is a Half Buy-Write strategy. This strategy is a modification of the BXM strategy in which the number of options sold has been reduced by 50 percent.

**BXMC**—Conditional buy-write strategy. The BXMC strategy is a modification of the BXM strategy. The BXMC strategy includes logic based on the VIX Index. When the VIX Index level is greater than twenty, the strategy follows BXM. When VIX is less than twenty, then the strategy follows BXMH.

#### 4.2.2. Put-Write Strategies

**PUT**—The PUT strategy sells an at-the-money put on the underlying security. It invests the balance of the account in one- and three-month treasury bills.

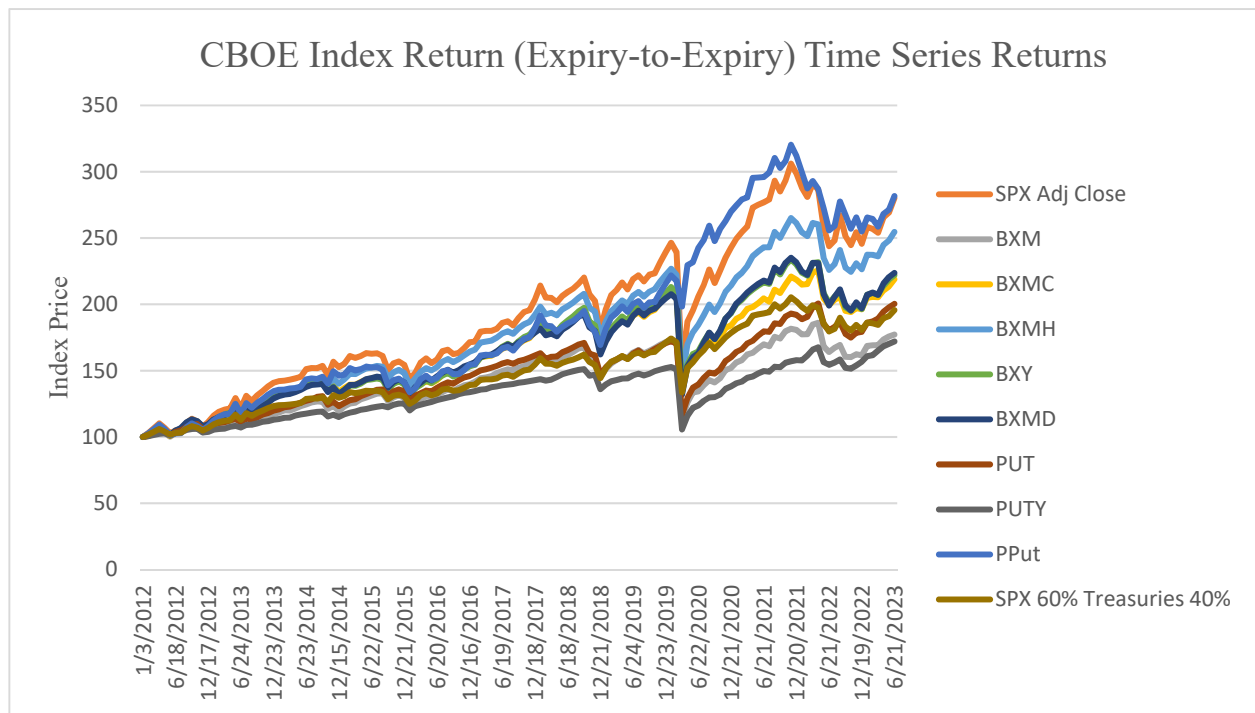
**PUTY**—The PUTY strategy sells a 2% out-of-the-money put on the underlying. It invests the balance of the account in one- and three-month treasury bills.

#### 4.2.3. Protective Put

**PPUT**: The PPUT strategy is a long position in the SPX Index while also buying a put option whose strike is closest to being 5% out-of-the-money.

We collected the index value on the expiration date to construct the expiration-to-expiration time series. The data were then converted into a time series of natural log returns. The indices were rebased to a level of 100. This is purely cosmetic, but aids in quick comparisons. The expiry-to-expiry returns used in this research are shown in Figure 5 below. Table 6 documents the correlation structure of the data. The correlations are uniformly high but are not perfectly correlated.

The returns are highly correlated since all the indices share the S&P 500 Index return as the driving process. Also, as demonstrated in (Merton et al. 1978, 1982), the S&P 500 index outperforms all other strategies on a pure return basis. Consistent with the previous research, we calculate the return-to-risk ratio. In the previous studies, it was shown that on a return-to-risk basis some option strategies outperformed the index. We find that some strategies outperform the index, while others do not, see Table 7.



**Figure 5.** CBOE indices return (expiry-to-expiry) time series return.

**Table 6.** Expiry-to-expiry correlation between strategies.

|                        | SPX Adj. Close | BXM   | BXMC  | BXMH  | BXY   | BXMD  | PUT   | PUTY  | PPut  | SPX 60% Treasuries 40% |
|------------------------|----------------|-------|-------|-------|-------|-------|-------|-------|-------|------------------------|
| SPX Adj. Close         | 1.000          |       |       |       |       |       |       |       |       |                        |
| BXM                    | 0.952          | 1.000 |       |       |       |       |       |       |       |                        |
| BXMC                   | 0.977          | 0.991 | 1.000 |       |       |       |       |       |       |                        |
| BXMH                   | 0.996          | 0.974 | 0.991 | 1.000 |       |       |       |       |       |                        |
| BXY                    | 0.987          | 0.985 | 0.997 | 0.995 | 1.000 |       |       |       |       |                        |
| BXMD                   | 0.991          | 0.982 | 0.994 | 0.997 | 0.999 | 1.000 |       |       |       |                        |
| PUT                    | 0.966          | 0.986 | 0.986 | 0.983 | 0.981 | 0.984 | 1.000 |       |       |                        |
| PUTY                   | 0.929          | 0.984 | 0.973 | 0.957 | 0.961 | 0.961 | 0.991 | 1.000 |       |                        |
| PPut                   | 0.968          | 0.851 | 0.901 | 0.947 | 0.922 | 0.931 | 0.885 | 0.824 | 1.000 |                        |
| SPX 60% Treasuries 40% | 0.998          | 0.942 | 0.971 | 0.993 | 0.980 | 0.986 | 0.961 | 0.922 | 0.976 | 1.000                  |

**Table 7.** Return to risk for strategies, 3 January 2012 to 21 June 2023.

| Strategy                     | Annual Return | Annualized Risk | Return to Risk |
|------------------------------|---------------|-----------------|----------------|
| Buy-Write (BXM)              | 5.97%         | 12.89%          | 0.4629         |
| Conditional Buy-Write (BXMC) | 7.88%         | 13.41%          | 0.5875         |
| Half Buy-Write (BXMH)        | 9.43%         | 14.88%          | 0.6333         |
| 2% OTM Buy-Write (BXY)       | 8.12%         | 14.19%          | 0.5721         |
| 30-Delta Buy-Write (BXMD)    | 8.31%         | 14.89%          | 0.5577         |
| Put-Write (PUT)              | 6.95%         | 12.37%          | 0.5619         |
| 2% OTM Put-Write (PUTY)      | 5.41%         | 10.79%          | 0.5015         |
| 5% Put Protection (PPUT)     | 9.70%         | 11.59%          | 0.837          |
| Logic Put Protection         | 11.42%        | 12.66%          | 0.9017         |
| 60-40 Corporate Bonds        | 7.53%         | 10.16%          | 0.7414         |
| SPX 100%                     | 10.69%        | 17.44%          | 0.6128         |

Evaluating these strategies provides insights for investors seeking to optimize their portfolios and improve risk-adjusted returns. It shows how important it is to align strategy choices with investment goals, risk tolerance, and market expectations, while considering

the dynamic nature of financial markets. As shown in Table 7, the Buy-Write Index significantly underperforms the S&P 500 Index on both an absolute and risk-adjusted basis. The days of easy profits from premium writing seem to have disappeared. The mostly bull market of the sample period seems not to have exhibited the optimistic call buying of the previous study (Feldman and Roy 2004).

In fact, none of the simple options strategies have outperformed the S&P 500. However, some of the nuanced strategies have outperformed the S&P 500 Index. The call strategies, BXMC and BXMH, both use simple logic to deviate from traditional option overlay strategies. BXMH sells only half the S&P 500 position. The BXHM strategy seems to confirm that in a rising market, a call seller is not compensated enough to offset rising asset prices. This may be because the VIX average and median value for this recent study are significantly lower than the period used by Feldman and Roy.

VIX is negatively correlated with the index value (Fleming et al. 1995). The BXMC strategy seems to exploit the short-term negative correlation between VIX and the index price. As shown in Figure 2, VIX usually has long periods of low volatility and short period of high volatility since equity options were multiple listed. The periods of high volatility are usually around major market events such as the internet bubble, the 2008 housing bubble implosion, and COVID-19.

The BXMC index has selection logic that passively trades between two strategies. When the VIX Index is above 20, it becomes BXM, portfolio-selling call options. When the VIX is below 20, the BXMH strategy is implemented, selling only half as many options. As shown in Table 8, the average value of VIX from 1 January 1990 to 21 June 2023 is 19.58. The average VIX Index value for the research period of (3 January 2012–20 June 2023) is 17.78.

**Table 8.** VIX Index descriptive statistics.

| VIX Index Values | 1990 to 2023 | 1994 to 2004 | 2012 to 2022 |
|------------------|--------------|--------------|--------------|
| Max              | 69.36        | 42.07        | 68.33        |
| 75th Percentile  | 22.96        | 23.99        | 24.32        |
| Median           | 17.75        | 19.58        | 19.23        |
| Average          | 19.59        | 20.21        | 20.57        |
| 25th Percentile  | 14.04        | 15.38        | 14.69        |
| Min              | 9.63         | 10.23        | 10.23        |

## 5. The Black Swan of COVID-19

The call-based and short put strategies experienced substantial losses at the onset of the COVID-19 pandemic, serving as a stark reminder of the tail risks within these strategies. This hints that while the mean risk/return ratios were better, the outperformance exposes the investor to the negative results of Black Swans (Taleb 2007). Additional studies may want to include measures of convexity or tail risk in their results.

One of the benefits of using options identified in previous studies was the ability of options to improve portfolio risk/return ratios. However, during COVID, the overall US economy was highly constrained. As a result, this period of time provides an opportunity to ask a unique question. During a period of extreme tail risk due to unique economic conditions, should an investor remain fully invested? We calculated the strategy's returns excluding the April, May, June, and July maturities for 2020, which effectively includes the period when the economy was constrained during the COVID-19 pandemic.

To counterbalance the logic of exiting the market, it has been shown that portfolios that remain fully invested beat portfolios that trade in and out of the market. The recognition that a significant part of overall investment returns can be attributed to a relatively small number of trading days underscores the importance of continuously implementing the same strategy over a long period. As shown in the table produced by JP Morgan Asset Management on 9 March 2022, exiting a market after a major event is normally harmful to an investor. In 2020 during COVID, the largest up days were 24 March at 9.4%, March 16th at 9.3%, and 6 April at 7.0% (Konish 2022).

Here are the results after removing the COVID periods (Table 9).

**Table 9.** Return to risk for strategies from 3 January 2012 to 21 June 2023 excluding COVID.

| Option Strategies for Time Period Excluding COVID | Annual Returns | Annual Realized Volatility | Return to Risk |
|---------------------------------------------------|----------------|----------------------------|----------------|
| BXM                                               | 5.10%          | 13.38%                     | 38.10%         |
| PUTY                                              | 4.62%          | 11.23%                     | 41.12%         |
| PUT                                               | 6.04%          | 12.83%                     | 47.04%         |
| BXY                                               | 7.27%          | 14.80%                     | 49.09%         |
| BXMC                                              | 7.18%          | 13.98%                     | 51.35%         |
| SPX Adj. Close                                    | 9.64%          | 18.21%                     | 52.95%         |
| BXMH                                              | 8.51%          | 15.53%                     | 54.78%         |
| BXMD                                              | 8.57%          | 15.52%                     | 55.26%         |
| PPut                                              | 8.59%          | 11.19%                     | 76.31%         |

The results in Table 9 underscore the dynamic nature of options-based strategies and their performance in different market environments, particularly during periods of high volatility such as spring 2020. Under typical market conditions, our results suggest that option strategies that incorporate some logic can significantly outperform the S&P 500 (SPX) on a risk-adjusted basis. After removing spring 2020, which included an extreme period for the VIX Index, only BXMH, BXMD, and PPUT delivered favorable risk-adjusted results. These results seem to show that an investor should only write calls against a portfolio's entire long position when implied volatility is at an extreme since BXMC slightly underperformed when the COVID period was removed.

Somewhat surprisingly, BXMH also performed well in both periods. From the results, it appears that investors that implement a passive option overlay are not being properly compensated since BXM has the worst return-to-risk ratio.

The results also confirm previous research that Black Swans seem to not be properly priced into the option markets (Welch 2016). It is refreshing that this research confirms that PPUT is a beneficial strategy for investors. The PPUT strategy is the only strategy whose reward-to-risk ratio did not significantly change in value when the COVID period was removed.

As a final robustness check, we decided to create a unique strategy based on the facts above. The strategy is an active one based on simple option market making logic, which is to sell implied volatility right after a large movement down. This logic is partially due to the fact implied volatility is elevated in these periods as shown by Figure 6 and exhibits a quick reversion. The strategy is to continuously purchase the PPUT index except after a one-standard-deviation drop in the previous month for SPX. For the following two months' purchase, SPX is used instead of PPUT. As shown in Table 10 and Figure 7, this strategy outperformed the index on both a return basis and on a return-to-risk basis. This strategy also outperformed the traditional mix of stocks and treasury bills along with stock and corporate bonds as shown in Figure 7.

**Table 10.** Comparison of efficient frontier and logic-driven put strategy.

| Efficient Frontier       | Return | Risk   | Return to Risk |
|--------------------------|--------|--------|----------------|
| 70% SPX and 30% T-Bill   | 7.74%  | 12.21% | 0.6341         |
| 80% SPX and 20% T-Bill   | 8.72%  | 13.95% | 0.6252         |
| 90% SPX and 10% T-Bill   | 9.71%  | 15.70% | 0.6183         |
| SPX 100%                 | 10.69% | 17.44% | 0.6128         |
| 60-40 Corporate Bonds    | 7.53%  | 10.16% | 0.7414         |
| 5% Put Protection (PPUT) | 9.70%  | 11.59% | 0.837          |
| Logic Put Protection     | 11.42% | 12.66% | 0.9017         |

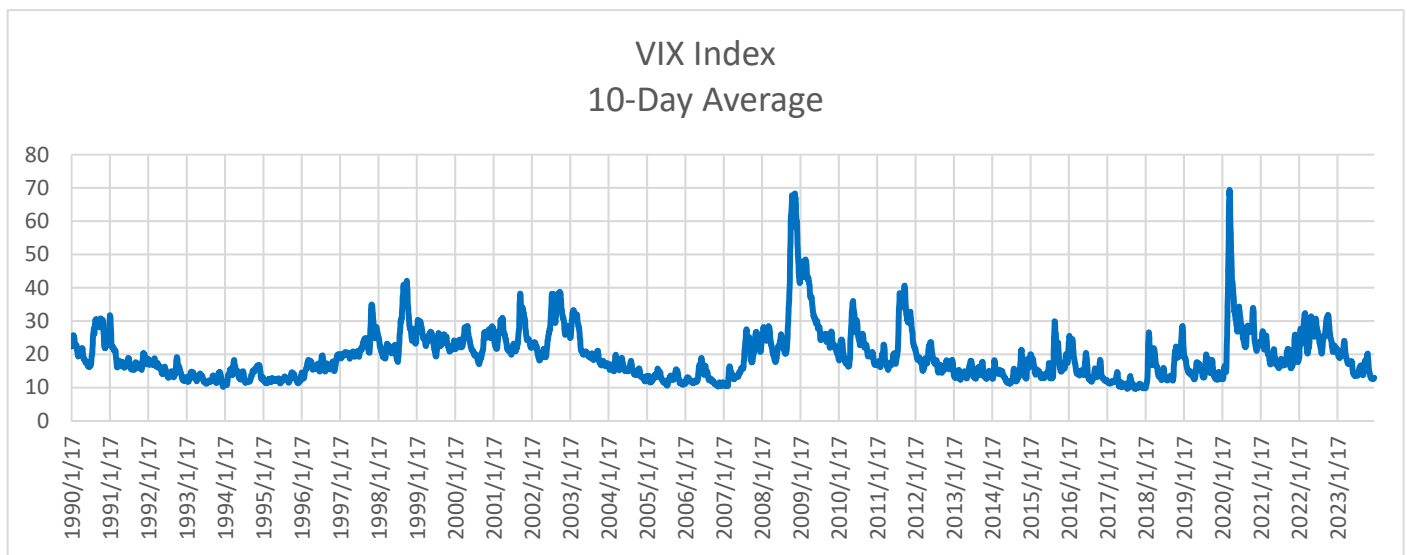


Figure 6. VIX Index smoothed using a ten-day moving average.

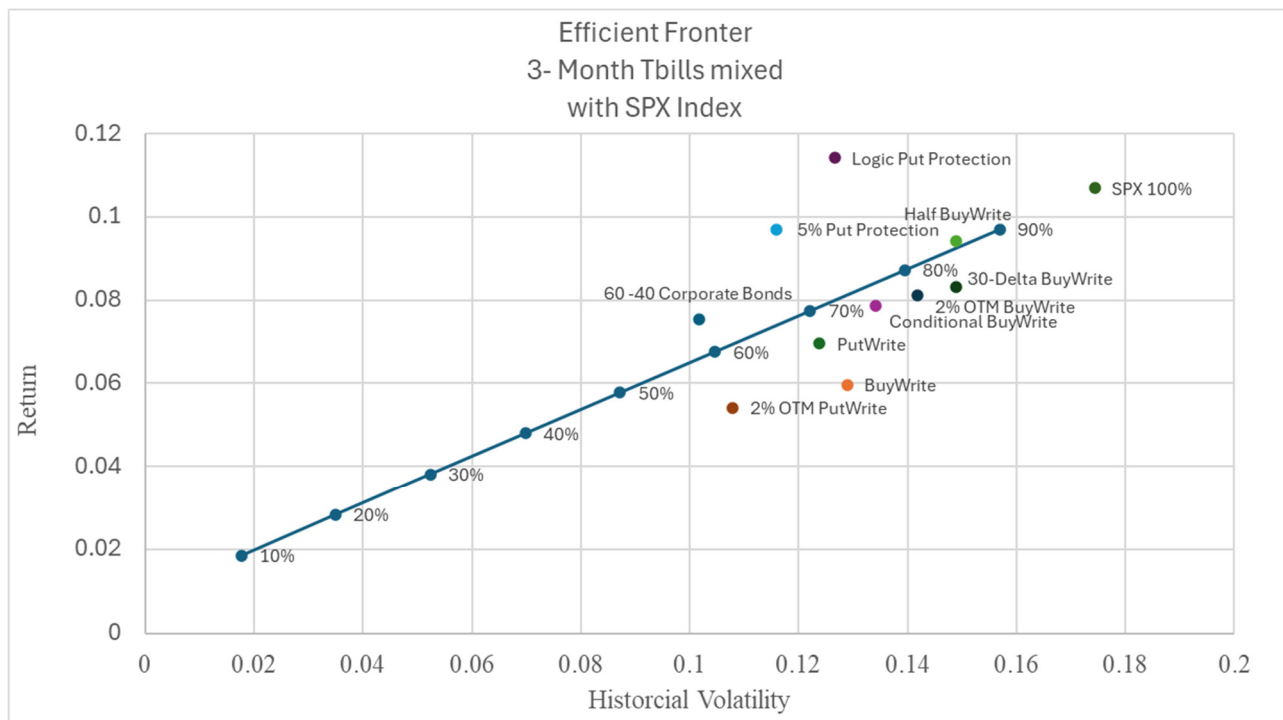


Figure 7. Efficient frontier.

## 6. Conclusions

The contributions of the Black–Scholes model over the last fifty years to the field of investing are monumental. The simplicity of the Black–Scholes model and its application has withstood the test of time. The foundational principles remain effective tools for practitioners seeking to harness the power of non-linear payoff structures.

The recent review shows that the original option strategies recommended by Merton et al. no longer provide a favorable return-to-risk ratio. It is likely that these returns were illusory, driven by their assumptions. After all, they were based on a simulation.

Recent data demonstrate that simple options strategies no longer add value to a portfolio or an index. However, our research shows that three well-known and somewhat dynamic option strategies have outperformed the S&P 500 Index on a return-to-risk basis.

Furthermore, we find that the favorable performance observed in previous studies can be revitalized by incorporating simple signals of the market regime in their construction.

An interesting finding of this study is that PPUT consistently outperforms the S&P 500 Index on a return-to-risk basis. Even more astonishing is that by adding the simple logic of avoiding puts after a one-standard-deviation draw down, it outperforms the index on a return basis with significantly lower risk. A key reason for the PPUT index and the logic-based PPUT strategy outperformance may be that as more firms implement covered call strategies, they inadvertently reduce implied volatility levels, underpricing the risk in the tails of the distribution. This hypothesis needs to be tested with a larger set of indices.

Investors who can overcome the emotional bias of needing their portfolio to outperform its benchmark annually can capitalize on the favorable risk–reward outcomes of these strategies (Pompian and CFA 2015). Our findings are also consistent with Taleb’s finding in *Fooled by Randomness*:

“I try to make money infrequently, as infrequently as possible, simply because I believe that rare events are not fairly valued, and that the rarer the event, the more undervalued it will be in price.”. (Taleb 2007)

The dramatic surge in option trading volume underscores the growing importance of these derivative instruments as tools for the modern portfolio manager. We have shown care needs to be applied in employing these instruments. The original findings seem not to hold as well as they used to. We attribute these results to more impartial data, a more sophisticated set of market participants, technological change, and shifts in the market institutions driven by changes in regulation and market practice. The simple edge from options is gone, and participants can still profit from options, but they must be cleverer.

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