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Taking Negative Spatial Autocorrelation Seriously: Deconstructing Fifty Years of Pro-Positive Bias in Spatial Statistics

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Abstract

This essay addresses the persistent understudying of negative spatial autocorrelation (SA) in spatial analysis, extending existing arguments that principal interpretations and geographic models have unduly prioritized positive dependence/correlation. It further reframes SA to explicitly recognize its negative nature as a legitimate and frequently occurring georeferenced data property. Through empirical examples, scale-sensitive analysis, and a synthesized typology of negative SA facets, this paper advances a more balanced spatial dependence/correlation understanding. It challenges the superfluous model-induced artifact view about negative SA, demonstrating its real-world empirical and substantive emergence through heretofore unexamined empirical conditional territorial reshuffling and spatial competition, using historical Texas county subdivisions as a concrete exemplar. It translates the eight established canonical positive-SA-oriented interpretations to more negative-SA-friendly multivariate geospatial contexts, highlighting competing or compensatory dynamics among attributes. This paper emphasizes that negative SA can be a localized, scale-sensitive property, one often masked by dominant positive SA in any of its geographic extents, and encourages permutation/randomization-based inference to diagnose it in negative-SA-governed geographic landscapes. In addition, this paper explores the symmetry between positive and negative SA clusters across global, regional, and local scales, and examines negative SA roles with regard to spatial outliers, territorial management, and random contrasts, such as those earmarking early diffusion processes. This paper concludes that negative SA captures inverse locational relationships, is inherently delicate and transient, and responds sensitively to changes in geographic scale, resolution, and data aggregation, meriting more candid consideration in spatial statistics/econometric modeling.



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1. Introduction

Spatial autocorrelation (SA)—the extent of neighboring attribute value (dis)similarity, describing a clustering of similar (positive) and/or juxtaposition of dissimilar (negative) values characterized by Tobler’s [1] First Law of Geography—is a half-century-old foundational concept in spatial statistics/econometrics. Although its conceptual origins trace back to the early twentieth century, with its sanctioned terminology emerging by mid-century (e.g., [2], its apparent first appearance in print), SA attained widespread methodological prominence only after, first, Cliff and Ord [3], and, second, Paelinck and Klaassen [4]

published their seminal books. Even then, a coherent and accessible SA understanding developed slowly, prompting repeated calls from applied spatial scientists for clearer explanation and interpretation. These appeals motivated the Goodchild–Griffith–Odland [5–7] monograph trilogy, and a subsequent interpretive synthesis stream aimed at clarifying its meaning, impacts, and practical relevance (e.g., [8–17]; [18], Chapter 1).

Despite their clarificatory intent, these works overwhelmingly framed SA through its positive manifestation, relegating negative SA to a peripheral or incidental role. Consequently, positive SA (PSA) has been extensively theorized, modeled, visualized, and embedded in analytical practice, whereas negative SA (NSA) remains abstractly thin, empirically underexamined, and methodologically neglected—a point noted explicitly by Lin [19] and Griffith [20], and reiterated in more recent syntheses ([18], Chapter 1). The existing literature merely states that NSA exists, essentially neglecting its mechanisms, interpretations, mathematical treatment, and analytical utility, which remain systematically underdeveloped relative to PSA. This asymmetry already was evident in Griffith’s [21] formative review, which identifies eight nuanced SA interpretations prevailing in the literature of that time: (1) self-correlation, reflecting its literal denotation; (2) map pattern, emphasizing more smoothly varying or clustered geographic distributions; (3) information content, understood as redundancy or duplicated spatial content; (4) spatial spillover effects, capturing the diffusion or covariation leakage across locations (e.g., [22]); (5) an appropriate areal unit demarcation indicator, particularly within the modifiable areal unit problem (MAUP) interpretive lens; (6) a nuisance parameter requiring estimation; (7) a surrogate for spatially structured missing variables; and (8) a diagnostic tool, especially for omitted-variables bias detection. Again, a predisposition toward PSA is discernible here. After all, these interpretations act as a conceptual silo for it, even though local NSA evidence has been accumulating in recent years.

Although nominally detached, these preceding eight interpretations were developed and operationalized almost exclusively with PSA in mind. The map-pattern interpretation presupposes synchronous spatial arrangements—gradients, trends, and/or hot/cold spots—that are emblematic of positive dependence/correlation. Information content and variance inflation implicitly assume redundancy arising from similarity more so than attribute contrast. Nevertheless, NSA still increases information content by conveying systematic local contrasts rather than homogeneous or null-hypothesis random clustering. The MAUP (also see [23]) perspective historically frames NSA as an aggregation artifact rather than an intrinsic geographic processes property. Diagnostic interpretations emphasize the global zero-SA null hypothesis, thereby diverting attention from the empirical reality that most georeferenced socioeconomic/demographic variables exhibit net PSA mixtures masking subordinate negative components. Disappointingly, none of these elucidations generated a parallel research agenda for NSA. Instead, their cumulative effect has been to normalize its absence from theory, measurement, and application. Therefore, despite routinely appearing in definitions and diagnostics, vis-à-vis PSA, NSA has not generated a commensurate body of theory/conceptualizations, models, and/or sustained empirical investigation, producing a persistent imbalance between conceptual acknowledgment and analytical development across the three SA natures (i.e., negative, zero, and positive).

Growing empirical evidence increasingly challenges this one-sided view. Geographic distributions rarely consist of purely positive dependence/correlation. Rather, they usually display scale-dependent PSA-NSA mixtures, with negative components often appearing as subordinate global, regional, and/or local contrasts that remain partially or completely hidden. In this context, zero SA acquires a refined interpretation, indicating either an absence of spatial structure or, effectively, a balance between opposing natures, not spatial

randomness per se. Several of the original eight interpretations—most notably spatial spillover, missing-variable surrogacy, and areal unit appropriateness—apply as strongly, and in some cases more directly, to NSA, particularly in settings characterized by geographic competition, exclusion, and/or partitioning. Because NSA tends to be hyper-sensitive to spatial resolution and scale—plausibly because planar adjacency geometry itself limits how strongly NSA can materialize on real maps—it provides a valuable quantification for evaluating areal unit delineations [24].

The persistence of this systemic privileging imbalance is especially striking given the semantic neutrality of the concept itself. The Greek prefix *auto* denotes self, not similarity. Following Wold's [25] time-series precedent—the first formal, normalized, discrete-time definition of the temporal autocorrelation coefficient—Besag [26] introduced this impartially toned terminology into spatial analysis. Yet early spatial auto-normal models—conventional ones systematically favor PSA because negative parameters often violate variance-covariance positive-definiteness constraints (e.g., non-negative spatial weights restrict spatial autoregressive parameters to weakly negative or positive values), making negative SA underrepresented within the confines of contemporary diagnostic tools—introduced structural asymmetries favoring PSA, and subsequent Bayesian formulations (e.g., [27]) further entrenched this tradition through exclusive reliance on conditional auto-normal priors. Furthermore, because conventional spatial weights matrices contain only non-negative entries, the Perron–Frobenius theorem guarantees a positive dominant eigenvalue while relegating negative dependence/correlation to the comparatively small negative tail of an eigenspectrum. Consequently, the smallest eigenvalue of this matrix entirely governs the admissible autoregressive parameter space for negative SA, making its feasible range graph-dependent and often substantially more restrictive than that for positive spatial dependence/correlation. This spectral constraint constitutes a fundamental technical hurdle for modeling NSA. Only recently have alternative formulations (e.g., see [18], Chapter 4) provided probability models that symmetrically treat PSA and NSA. Moreover, within the omitted-variables framework, NSA may be even more directly informative than PSA. Nonetheless, this latter empirical feature remains valuable for identifying missing covariates with smooth spatial signatures [28].

Accordingly, exploiting fresh evidence (i.e., beyond, say, [20]) and mathematical issues, this paper argues that NSA persistently has been understudied and substantively neglected—a nonscientific literature perusal suggests that 80–90% of spatial autocorrelation treatments address PSA, 10–20% focus on PSA-NSA mixtures, and <5% explicitly tackle NSA—not because it is rare, unimportant, or lacks empirical relevance (local SA indices corroborate this contention), but because dominant interpretations, modeling traditions, and analytical and computational conventions—typified in the foregoing eight canonical SA framings—have unintentionally and disproportionately prioritized positive dependence/correlation. This propensity does not mean that the research community has culpably neglected NSA. Existing exploratory tools, for example, mostly uncovered PSA, although ones currently implemented in software packages—such as GeoDa (e.g., local and global SA indices, Moran scatterplot), ArcGIS Pro (e.g., spatial statistics toolbox, which includes the Spatial Component Utilities toolset for Moran eigenvectors), R Project (e.g., *spdep*, *sf*, *spatialreg*, *adespatial* packages), PySAL (e.g., *esda*), SAS (e.g., PROC VARIOGRAM, PROC OPTMODEL/IM), MATLAB (e.g., Spatial Statistics and Machine Learning Toolbox), STATA (e.g., *spatgsa*, *spatdiag*, *spmatrix*), and GRASS GIS (e.g., SA add-ons)—enable limited NSA examination in spatial analysis workflows. Meanwhile, prevailing model-based traditions structurally disfavored NSA, fostering PSA-heavy exposition. The objective of this paper is to reframe SA in a manner that bolsters an explicit recognition of its negative nature as a legitimate, insightful, and

frequently occurring georeferenced data property beyond that done by Griffith [20] and Griffith and Li [18]; building upon an acknowledgment of the existence of NSA, it seeks to develop a broader conceptual, empirical, and methodological framework reinforcing the argument that NSA is real, measurable, and analytically useful, one advancing an understanding of its origins, manifestations, and analytical value. Through new empirical examples, scale-sensitive analysis, and a synthesized typology of NSA facets, this paper advances a more balanced and complete spatial dependence/correlation understanding. These illustrations are selected not as an exhaustive inventory of NSA occurrences, but as demonstrations of distinct mechanisms through which NSA emerges, including territorial restructuring, spatial competition, and diffusion-related processes. To help avoid cognitive overload, its space-time ideation is delayed to a later paper.

Finally, beyond empirical identification, NSA raises fundamental mathematical issues because conventional spatial models and spatial weighting structures impose constraints that disproportionately accommodate positive dependence/correlation. Consequently, understanding NSA requires not only additional empirical examples, but also examination of the algebraic and probabilistic structures underlying spatial models, such as two-parameter spatial autoregressive-moving average (SARMA) and Griffith-Moran eigenvector spatial filtering (GMESF) PSA-NSA mixture model specifications.

2. Inherent Versus Model-Based SA Nature

An ongoing ontological query in spatial analysis concerns whether SA is an inherent property or a model-induced georeferenced data artifact engendered by inappropriate entitation (i.e., the principled process of identifying, classifying, and describing discrete, homogeneous subareas within a larger territory for the purpose of mapping). This debate (e.g., [29], p. 228; [30]) bears directly on NSA, given its historical overshadowing by emphases on positive clustering. These conceptual considerations motivate the empirical examination of geographic processes that generate observable NSA patterns.

Griffith and Li [18] use a jigsaw puzzle metaphor to illustrate the three SA types: positive, (near-)zero, and negative. In this analogy, puzzle piece tabs and slots represent NSA, emerging naturally from spatial competition, such as territorial annexation or forfeiture as measured by entity area. Historical Texas (TX) county subdivisions provide a new concrete example, focusing on emergent NSA (see Appendix A); this examination differs from Griffith [31], which evaluates only these county creation timings. After independence from Mexico, the original municipios were incrementally subdivided into smaller counties. Parent counties lost area to new units, generating measurable NSA, while the newly demarcated counties exhibited compensating PSA, producing a positive–negative mixture. Formal measures—spatial weights matrices were defined utilizing the rook adjacency neighboring rule (note: no bibliometric study to date has surveyed the spatial analysis literature broadly enough to estimate its percentage usage, with it and the queen rule being the two universally accepted contiguity definition choices), an operationalization used throughout this paper—confirm these patterns. For example, Red River County’s pre-1850 subdivisions (see Figure 1; see Appendix A small sample cautionary commentary) yield Moran’s I (aka MC, for Moran coefficient) ≈ -0.35 ($z_{MC} \approx -100.50$) and Geary’s c (aka GR, for Geary ratio) ≈ 1.36 ($z_{GR} \approx -6649.91$) [32–34], a finding more statistically significant than NSA results for the Nacogdoches County segmentation [$MC \approx -0.27$ ($z_{MC} \approx -79.77$) and $GR \approx 3.95$ ($z_{GR} \approx -864.99$)]. Frontier expansions in western TX exhibit comparable negative covariation [$MC \approx -0.35$ ($z_{MC} \approx -332.31$) and $GR \approx 4.20$ ($z_{GR} \approx -19,175.57$)]. Local indices of spatial association (LISA [35]) identify these counties as outliers, demonstrating how NSA can be quantified and interpreted alongside PSA. These examples demonstrate that NSA is not merely a statistical artifact, but can arise from a recognizable geographic process that creates systematic local contrasts. Similar patterns

are observed in historical boundary sequences in Florida, Pennsylvania, and the Eire (i.e., Republic of Ireland) counties [3,36]. SA is inherent here!

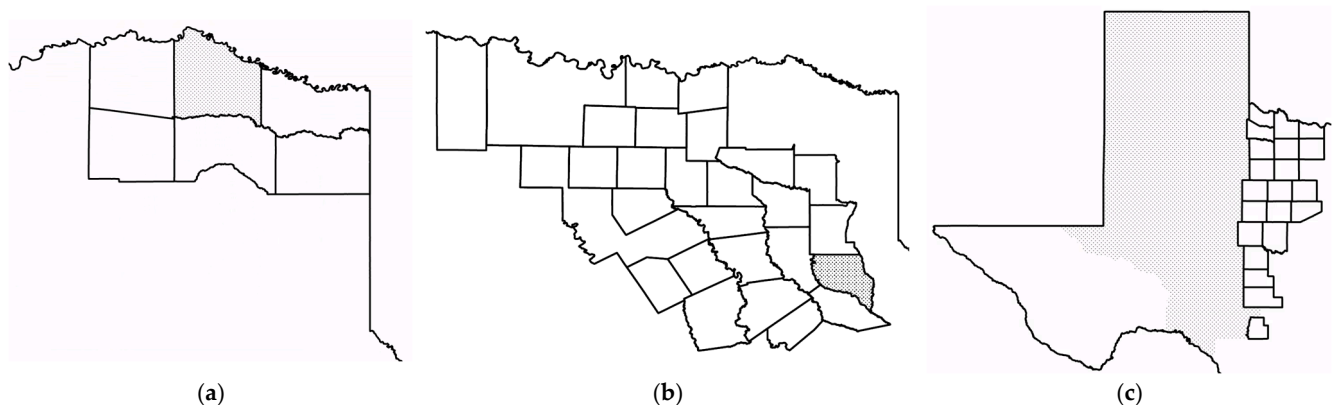


Figure 1. Empirical examples of spatial competition categories. (a) Infill—Red River County (contemporary areal unit denoted by gray dots), with 1845 subdivisions superimposed (north-east TX). (b) Infill—1832 Nacogdoches County (contemporary areal unit denoted by gray dots), with 26 1850 subdivisions superimposed (west and southwest of Red River County in (a)). (c) Frontier expansion—21 1858 counties created as an early protrusion into the wilderness territory (its remainder denoted by gray dots) along the settled fringe. Historical county boundaries follow the compilation reported by <https://www.mapofus.org/texas/> (accessed on 21 July 2026) and https://www.txgenweb.org/research/county_formation_maps.htm (accessed on 21 July 2026); cartography by the author using U.S. census TIGER/Line shapefiles.

As an aside, although spatial analysts frequently employ the queen rule, this aforementioned rook rule is preferred here because counties meeting only at a corner often have little or no meaningful geographic interaction. In addition, in many county datasets, queen contiguity adds relatively few neighbor linkages beyond those identified by the rook criterion, these additional links solely being based upon a point contact that may have limited substantive meaning. More importantly for this study, the increased connectivity associated with queen contiguity contracts the negative end of the spatial weights matrix eigenspectrum, diminishing the capacity of this matrix to represent relatively strong NSA. Therefore, restricting neighborhoods to shared boundaries of positive length provides a more conservative and substantively interpretable representation of spatial dependence/correlation while preserving a broader spectral domain for analyzing NSA. Meanwhile, because the present analysis concerns county-level areal processes and the representation of NSA, a nearest neighbor specification also seems to be a less suitable and topologically less compatible neighborhood definition.

These foregoing TX county subdivision examples illustrate that NSA can arise empirically and substantively through territorial reshuffling (i.e., the process by which the geographic arrangement of an attribute changes across an otherwise fixed geographic support, creating a different spatial pattern without altering the territorial/areal units themselves; randomization, permutations) as well as spatial competition (the topic explored in Griffith [20]), not just by merely reflecting model artifacts (most likely via residuals). Although opposing perspectives emphasize model-based sources—such as functional misspecification, omitted variables, and/or aggregation effects (e.g., MAUP)—that can induce spurious or obscure masked NSA [26,27], this historical TX case study demonstrates latent negative–positive mixtures observable in internal territorial changes. As the preceding Figure 1 Red River County discussion corroborates, these map patterns are quantifiable using formal inferential tools, including the MC, GR, and local SA statistics [35,37–39], which provide a rigorous quantifying and inferential basis for differentiating genuine

spatial dependence/correlation from artifacts. On the one hand, conditional permutation-based inference (circumventing design-based sampling infeasibility) evaluates whether these observed statistics are unlikely under a random spatial allocation null hypothesis, supporting intrinsic NSA when significance is achieved. On the other hand, NSA that fails to reach statistical significance in a model-based framework, or that is highly sensitive to model assumptions, is likely artefactual, reflecting methodological rather than substantive geographic processes that are generative and interpretative, not causal in a formal statistical/econometric sense. Moreover, this particular TX geographic landscape example naturally links to both of these inferential paradigms. Integrating this comparative perspective with the preceding eight canonical SA interpretations underscores a historical emphasis on positive clustering that reflects conceptual and methodological bias, while NSA is both real and measurable, and hence warrants systematic study and formal reporting in spatial analyses.

3. Extending the Eight Canonical SA Interpretations to a Multivariable and Multivariate Negative SA Setting

Building on the preceding intrinsic versus model-based NSA discussion, the eight canonical SA interpretations (see [21]) can be systematically extended to multivariable and multivariate geospatial contexts, ones spotlighting multiple interrelated attributes interacting across space. Historically, univariate positive geographic clustering has been the preoccupation of SA research themes, leaving NSA—particularly in multivariable/multivariate settings—insufficiently studied. Yet many real-world spatial phenomena involve competing or compensatory dynamics among attributes, territorial actions producing NSA that is both conceptually consequential and empirically observable. Recognizing this complexity is critical for acknowledging it within (e.g., [18], §2.7; [40]), as well as advancing, geospatial theory, improving quantitative geography measurement, and ensuring robust inferential spatial statistics/econometrics analysis. Revisiting the canonical univariate SA meanings suggests the following multivariable/multivariate-revised variants, with empirical examples of their multivariable contexts presented elsewhere in this paper:

1. Self-correlation: NSA may arise when different attributes exhibit inversely related covariation across neighbors—for example, in the TX county subdivisions, counties that lose area to offshoots may simultaneously recalibrate to higher population density or larger economic activity percentages, creating intrinsic negative covariation; multivariate adaptations of the MC (e.g., [41,42]) and GR [42–44] provide a formal inferential basis to quantify these dissimilarities, distinguishing them from spurious, model-induced artifacts.
2. Map pattern: NSA manifests as contrasting multivariate profile juxtapositions, often producing two-dimensional checkerboard-like or outlier patterns—in the TX illustration, the interplay between shrinking and expanding counties across multiple attributes (e.g., area size, population, and/or land use) creates locally heterogeneous clusters; LISA [35] applied to composite multivariate indices reveal the geographic distribution of these negative–positive SA mixtures, providing insights into the spatial configuration of competing geographic processes.
3. Information content: NSA in multiple variables enhances informational richness by signaling spatial heterogeneity and competitive local trade-offs across these attributes—for example, counties with decreasing area but increasing population or economic activity highlight compensatory dynamics outcomes; quantifying variance and covariation contributions across attributes ensures that NSA is recognized as substantive rather than dismissed as noise, improving the explanatory power of spatial models.

4. Spatial spillover effect: NSA emerges when a change in one attribute induces compensatory responses in neighboring units (e.g., [45–47]), such as land area reductions triggering increases in population density or economic intensity in adjacent counties—both conditional permutation-based and stochastic inferential frameworks allow geospatial researchers to determine whether these patterns reflect genuine spatial interactions, or are model specification or geographic aggregation artifacts.
5. Areal unit demarcation appropriateness (MAUP; see [48]): Multiple variables accentuate scale sensitivity—aggregating across multiple interdependent attributes can mask or artificially create NSA; examining patterns at multiple spatial resolutions, or employing multi-scale MC and GR indices, helps identify whether observed NSA is robust to unit choice, ensuring interpretability and methodological rigor.
6. Nuisance parameter: NSA can be diagnostic, not an inconvenient required estimate, indicating areas of attribute conflict, compensatory dynamics (perhaps à la comparative statics), or localized heterogeneity that are otherwise obscured in univariate analyses—acknowledging this interpretive potential transforms NSA from a methodological complication into a tool for deeper insights.
7. Missing variable surrogate: Perhaps its forte, NSA can reveal heterogeneous or contrasting unmeasured factors missed by already-accounted-for PSA, such as competing policy interventions, environmental constraints, or socioeconomic/demographic trade-offs, highlighting the utility of negative spatial dependence/correlation as a proxy for missing information (e.g., [49]) across multiple attributes.
8. Diagnostic tool: Zero SA traditionally is interpreted as a baseline, but, empirically, it actually may conceal a global PSA-NSA balance, one conditional on the adopted spatial support and neighborhood definition (i.e., subject to scale effects and the MAUP), with apparent zero SA at one spatial resolution possibly not persisting at another—thus, NSA refines diagnostic interpretation, indicating local compensatory patterns that inform model misspecification, residual analysis, and hypothesis testing.

This explicit incorporation of NSA into these eight conceptualizations, with special reference to multivariable/multivariate circumstances, enables spatial analysts to recognize that real-world spatial systems frequently comprise complex simultaneous PSA-NSA mixtures across multiple attributes.

For a truly but modest multivariate exemplification, consider the bivariate MC and GR for the two attribute variables of area (already examined) and population density for the simple Red River County case. The first United States (US) census, administered in 1850 (see https://www2.census.gov/library/publications/decennial/1850/1850a/1850-census-report-texas.pdf?utm_source=chatgpt.com (accessed on 4 July 2026)), furnishes population figures for the six counties into which the original Red River County was divided. Letting the variables X and Y respectively denote area and population density change attributable to the partitioning, $MC_{XY} \approx -0.30$ ($z_{MC_{XY}} \approx -4.79$) and $GR_{XY} \approx 1.20$ ($z_{GR_{XY}} \approx -190.81$). Switching these variate definitions yields exactly the same outcomes here because the employed binary 0–1 spatial weights matrix is symmetric.

In summary, NSA frequently is overlooked, yet it manifests clearly in real-world geographies as juxtaposed natural and/or synthetic (e.g., [18], pp. 49–50) contrasts. Historical TX county subdivisions illustrate how territorial dissection produces local contrasts in area, population, and/or economic activity, generating measurable negative-PSA cominglings. Environmental dyads, such as the orographic rainfall effect in New Zealand or desert–fertile contrasts in Chile, visually hypothesize intrinsic natural dissimilarities. Social and policy contexts—urban–rural infrastructure gaps, the Berlin Wall, food deserts, broadband disparities, and targeted Appalachian investments—further visually promote conjectures of NSA as a data-grounded phenomenon across multiple attributes. These examples provide visual

evidence that NSA can arise from genuine spatial competition, compensatory dynamics (perhaps via comparative statics), and local contrasts, rather than exclusively as a model artifact, and often coexists with broader positive clustering. Recognizing these map patterns—pure or convoluted as they may be—highlights its substantive importance for understanding spatial structure, multivariate interactions, and real-world outcomes, motivating a correction of the historical bias toward PSA exclusivity.

4. Empirical Geographic Landscapes, Scale Masking, and Inferential Negative SA Implications

As the foregoing discussion attests, this paper adopts a primarily randomization-based inferential perspective—which serves here as a null hypothesis constricted to a complete absence of any correlation, not a plausible data generative mechanism—for diagnosing NSA in fixed geographic landscapes, while treating model-based stochastic inference as complementary but potentially prejudiced when assessing spatial contrasts. Across natural, political, and socioeconomic landscapes, NSA emerges as a localized, scale-sensitive, and empirically persistent georeferenced data property, typically obscured—but not negated—by dominant positive spatial dependence/correlation. In other words, real-world geographic landscapes repeatedly demonstrate NSA as an intrinsic georeferenced data property, not merely a statistical artifact. Natural, institutional, and socioeconomic/demographic systems routinely generate localized contrasts in which adjacent areal units exhibit sharply dissimilar attribute values. These contrasts often remain hidden in aggregate summaries because they are spatially limited in extent, temporally transient, and/or embedded within broader regions dominated by PSA. Consequently, NSA is not rare; scale and mixture effects simply mask it in a preponderance of milieus.

Environmental processes offer some of the clearest empirical manifestations. The orographic rainfall effect—whose directional dependence may be modeled using structured covariance matrices (e.g., Toeplitz or certain Kronecker [26] forms) or analogous directional spatial weights matrices, although such a formal development is beyond the scope of this paper—exemplifies a physically grounded NSA mechanism: windward mountain slopes receive abundant precipitation, while immediately adjacent leeward slopes experience rain shadows. The New Zealand South Island illustrates this sharply, with the wet western coast juxtaposed against the arid Canterbury Plains. Comparable contrasts recur globally, including desert–fertile interfaces (e.g., Nile Valley–Sahara), coastal–interior gradients, and mountainous–lowland adjacencies. These settings produce stable, spatially structured dissimilarities that persist across measurement frameworks, affirming that NSA can arise independently of modeling choices. Point-source polluters furnish additional obvious exemplars.

Anthropogenic interventions reinforce this previous conclusion. Urban–rural boundaries frequently mark abrupt transitions in infrastructure provision, population density, income, and service accessibility. Historical and contemporary borders—whether de facto, such as the Berlin Wall barrier, or de jure, such as administratively imposed zoning regimes—generate immediate spatial juxtapositions of unequal economic, social, and/or environmental conditions. Similarly, policy-driven investment programs, such as the past Appalachian Regional Commission initiatives, produce geographically bounded improvements that contrast sharply with adjacent underinvested areas. These map patterns generate discernible LISA-type high–low and low–high spatial dissimilarities that are empirically detectable and substantively meaningful. In addition, and crucially, many of these contrasts are multivariate. Infrastructure access, demographic composition, economic opportunity, and environmental quality often change in concert across short distances, simultaneously reinforcing NSA across multiple variables. This multivariate embedding

further explains why univariate diagnostics may underestimate or overlook NSA when positive dependence/correlation dominates at broader scales.

Geographic scale plays a central role in this masking. Map-wide PSA frequently overwhelms localized NSA clusters, yielding net global statistics that suggest moderate positive dependence/correlation. Such positive–negative SA mixtures are typical rather than exceptional. Consequently, a near-zero global SA estimate may reflect either genuine spatial randomness or a balanced compensation between positive and negative components. Recognizing this distinction is a proper data interpretation essential, as well as a motivator of regime indicators and scale-sensitive diagnostics usage.

These empirical examples also clarify the appropriate inferential basis for diagnosing NSA. In settings where geography is fixed and the observed contrasts arise from historical, physical, and/or institutional processes, conditional permutation-based inference provides a natural foundation. Randomization tests evaluate whether or not observed local contrasts exceed what would be expected under random spatial rearrangements of attribute values, thereby distinguishing intrinsic spatial structure from chance. Stochastic model-based inference, although valuable for process modeling, can inadvertently induce or suppress NSA through specification choices, aggregation, and/or smoothing assumptions. Early spatial models that structurally favored positive dependence/correlation reinforced the misperception of NSA as anomalous or spurious. Importantly, conditional permutation-based inference provides a conservative benchmark for assessing intrinsic NSA, while model-based approaches—especially those with auto-normal priors—may under- or over-state negative components, depending upon specification and aggregation choices. This distinction underscores the need to select or enhance inferential tools that spot what is special about a negative spatial dependence/correlation, and then explicitly account for NSA-PSA mixtures and scale sensitivity.

The empirical persistence of NSA across diverse landscapes—natural and human-modified—demonstrates that it cannot be dismissed as a modeling artifact. Instead, it reflects fundamental mechanisms of spatial competition, constraints, and discontinuity. Recognizing these mechanisms clarifies why NSA often is localized, scale-dependent, and embedded within broader positive map-pattern structures. This perspective directly challenges historical PSA privileging, highlighting the need for spatial analytic frameworks that explicitly accommodate spatial contrasts alongside clustering.

In summary, empirical landscapes reveal that NSA is a genuine, recurring, and interpretable georeferenced data property. Its apparent scarcity as a subject in the literature reflects methodological conventions and scale effects far more than substantive absence. Incorporating these realities into spatial analysis enhances interpretive accuracy, providing a more complete spatial dependence/correlation understanding in complex geographic systems.

5. Symmetry in Positive and Negative SA Clusters: Global, Regional, and Local Geographic Concentrations

Conceding differential geographic clustering tendencies, Borcard et al. [50] are the first to coin the terminology classifying Moran eigenvector maps as global, regional, and local PSA portrayals. Griffith and Li ([18], p. 112) translate this nomenclature into an NSA situation, illustrating it with a Puerto Rican local SA statistics example, which has a randomization inferential basis. This section supplements the empirical reality, scale sensitivity, and inferential legitimacy of NSA established in prior sections by deciphering the conceptual mirror-imagery argument sponsoring this extension: the archetypical geographic clustering forms traditionally associated with PSA also occur under NSA, albeit reflecting systematic contrasts rather than similarities. Recognizing this symmetry reframes NSA

from a marginal, anomalous phenomenon to a central, analytically tractable georeferenced data property.

GMESF (see Appendix B (e.g., [51]) and local indices such as LISA [35] and the Getis–Ord statistics [37,38] supply operational definitions for the global, regional, and local map-pattern cluster notions, linking them to hot spots, cold spots, and intermediate gradients. Symmetrically transferring this framework to NSA identifies systematically unlike neighboring attribute value clusters. Rather than being confined to isolated outliers, NSA can manifest as spatially coherent contrasts at multiple scales: global (e.g., alternating extremes over much of a geographic landscape), regional (e.g., geographic patches of contrasting extremes), and local (e.g., high–low or low–high neighborhoods, as earmarked by LISA). Methodologically, GMESF [51] and LISA allow formal inferential assessment. Eigenvectors associated with negative Moran eigenvalues encode contrastive spatial structures, emulating the way positive eigenvalues capture clusters of synchronized values. Local indicators identify contiguous units exhibiting high–low or low–high patterns, supporting conditional permutation-based inference to distinguish intrinsic NSA from model artifacts. Crucially, these tools confirm that NSA can be systematically detected, measured, and interpreted across spatial scales, rather than being relegated to peripheral attention in empirical studies.

This symmetry perspective also clarifies the interplay between PSA and NSA within single georeferenced datasets. Many geographic attribute distributions contain mixed spatial dependence/correlation, in which globally dominant positive clustering coexists with localized negative clusters. The resulting PSA-NSA mixture is characteristic of natural, social, and institutional landscapes, explaining why standard global SA indices often underestimate negative dependence/correlation prevalence. Recognizing symmetric clustering patterns enables spatial analysts to reinterpret map patterns and diagnostic tools through a lens that accommodates both similarity and contrast.

Figure 2 provides an illustrative example, with Figure 2a,b augmenting the constructed Moran eigenvector linear combinations of the same nature and degree portrayed in Griffith and Li ([18], p. 103). It demonstrates that global, regional, and local NSA clusters reveal different scales at which neighboring counties exhibit dissimilar attribute values. Figure 2a illustrates global NSA, highlighting a broad pattern across the state of TX in which a very large region, mostly the western and southern parts of this state, displays systematic high and low value juxtapositions. With few intervening gray-colored counties, this regional map pattern has the lowest interspersal degree of such intermediate value (i.e., gray shaded) areal units; conspicuously, most of the gray-encoded counties concentrate along and near the eastern coast of the state. Figure 2b exhibits regional NSA clusters, capturing meso-scale variations where islands or near-islands of coterminous multiple-county blocks embed contrasting relative neighboring extremes, revealing spatial heterogeneity within modestly large subregions of this state. Intermediate values (i.e., gray-colored) help demarcate these moderate-sized groupings. Figure 2c emphasizes local NSA, showing fine-grained contrasts mostly between adjacent county pairs, depicting a more checkerboard-like overall map pattern, stressing the highly localized NSA nature. This geographic distribution has the highest degree of interspersed gray counties, given a rook adjacency neighbors definition. Together, these three panels demonstrate how NSA manifests across multiple spatial scales, from statewide trends to highly localized dissimilarities, providing insight into the complexity of territorial differentiation.

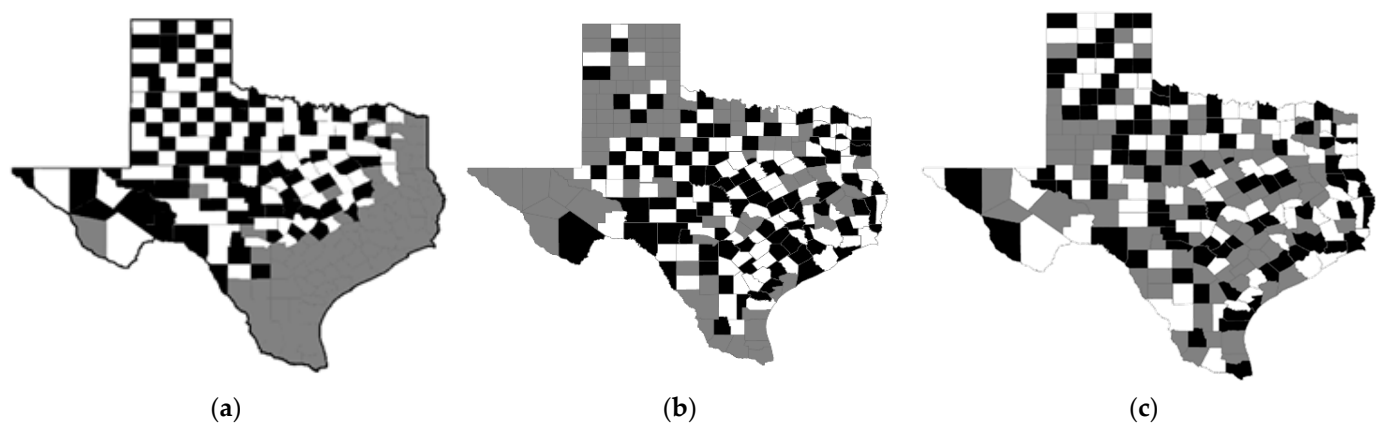


Figure 2. Specimen NSA eigenvector patterns across Texas counties ($n = 254$), portrayed as a tertile-classified choropleth map; black, gray, and white, respectively, denote relatively low, intermediate, and high numerical values. (a) A global contrast cluster (E_{254} —yields MCmin, the maximum negative (i.e., the most extreme negative) MC here (see [9]); Moran ≈ -0.63 , GR ≈ 1.51). (b) Numerous regional contrast clusters (E_{228} —yields roughly $2 \times \text{MCmin}/3$; Moran ≈ -0.48 , GR ≈ 1.62). (c) Copious local contrast clusters (E_{138} —yields roughly MCmin/4; Moran ≈ -0.16 , GR ≈ 1.08). Note: The MC-GR relationship is not necessarily strictly monotonic nor linear (e.g., see [18], Figure 3.6, p. 109); variation in neighbor cardinalities across an irregular lattice can cause local configurations to contribute differently to MC and GR calculations over a geographic landscape.

In summary, the narrative in this section demonstrates that NSA and PSA exhibit structurally analogous clustering forms across global, regional, and local scales, in a univariate context. Griffith and Li ([18], Chapter 6) comment about its multivariate contextualization, which is virtually always within a positive–negative SA mixture. This taxonomic broadening challenges the longstanding bias toward positive dependence/correlation in spatial statistics/econometrics, establishes a coherent theoretical and inferential foundation for NSA, and positions it as a critical element for understanding real-world spatial structure. By embracing symmetry in interpretation, spatial analysts can more accurately identify, measure, and model both convergent and divergent geographic phenomena, thereby enriching theoretical and applied geospatial research.

5.1. Spatial Outliers: Geographic Risk/Hazard Minimization Prompts

Spatial outliers constitute one interpretation of, particularly, local NSA clusters. In keeping with this notion, NSA often arises from both natural hazards and governmental or private interventions, manifesting as spatial outliers that form detached geographic contrasts. Its presence can be fleeting, chiefly when extreme events such as hurricanes, tornadoes, floods/tsunamis, landslides/mudslides/avalanches, or earthquakes/volcanic eruptions disrupt system inertia. These happenings transiently destroy built-up inertia in a geographic landscape, momentarily elevating SA above temporal autocorrelation importance in understanding meaningful data properties (e.g., [52,53]). As time passes and inertia restores, serial correlation regains prominence, leaving NSA as a temporary but time-specific informative statistical artifact. Real-world examples include Hurricane Katrina's impact on New Orleans (2005 [54]), the Haiti earthquake (2010+ [55]), and the Fort Worth tornado (2000+ [56]), which illustrate scattered damage zones and high-contrast clusters typical of NSA (e.g., Figure 2). An abstract counterpart occurs in the spatial optimization location-allocation problem, where a majority weight at a lone demand point creates a data spike towering over the remaining geotagged weights ([18], Chapter 8).

Government interventions often exploit or mitigate NSA. For instance, targeted public health programs identify high-risk communities for early disease detection and prevention, as in the late-1990s Clinton administration's prompted lead (Pb) initiative in Syracuse,

NY. Top-down mapping identified elevated blood lead level swaths ripe for pediatric screening, producing scattered remediated zones—an NSA manifestation. Bottom-up social processes, such as neighbors sharing health information, similarly produce localized reductions in disease prevalence. The private sector imitates these dynamics through market competition, with retail store placement or test-marketing creating sporadic spatial contrasts in locationally tagged sales figures.

Another prominent, systematic NSA source is government-mandated data suppression (see Table 1). For example, United States Department of Agriculture (USDA) farming census protocols withhold county-level data if too few farms report or if one farm dominates production. Griffith [57] developed small area frequentist imputation methods to replace restricted-access values using district- or state-level totals, maintaining statistical plausibility. Table 1 summarizes a 1997 Pennsylvania census data release analysis of five crops that discloses PSA-NSA mixtures in disseminated tabulations, with some crops displaying purely NSA, illustrating how raw SA indices can be misleading if map gaps (i.e., censored map holes) are not carefully handled. The probability model specification is for a binary 0–1 response indicating whether or not printed tables included a given crop’s data, with Moran eigenvectors as covariates to account for SA with stepwise-regression-constructed eigenvector spatial filters (ESFs). Diagnostics appearing in Table 1 confirm that only trace SA remains after constructing ESFs, and that each ESF accounts for a third or more of the geographic variance exhibited by the geographic distribution of county-anchored data absences. Conspicuous size effects are present, too, with milk—most likely the leading market value ($\hat{\rho}_{AR} \approx 0.6$)—and potato—a prominent minor specialty ($\hat{\rho}_{AR} \approx 0.4$)—commercial crop suppressions reflecting different local and regional SA complexity.

Table 1. GMESF Bernoulli logistic regression: censored government-released data values.

Crop	# of Tabulated Data Suppressions	# of Chosen Moran Eigenvectors		ESF SA		Pseudo- R^2	Residual SA	
		PSA	NSA	MC	GR		$ z_{MC} $	$ z_{GR} $
corn [†]	3	1	1	−0.05	0.97	0.51	0.80	1.56
hogs	3	0	1	−0.08	1.05	0.43	0.54	0.30
milk cows	3	1	1	−0.08	1.08	0.39	0.54	0.12
potatoes	14	5	3	−0.01	1.00	0.69	0.06	0.23
tobacco	5	0	3	−0.10	0.87	0.36	1.02	0.67

[†] The residual SA minimization criterion superseded the statistical significance criterion. Note: the multiple-comparisons complication was limited by simultaneously applying the following three stepwise selection criteria for Moran eigenvectors: statistical significance, a non-trivial residual SA reduction, and a practical prediction R^2 increase.

In sum, NSA emerges in multiple contexts that garner little overt attention as such in the literature: natural hazards, top-down or bottom-up management interventions, private sector competitions, and authoritative data suppressions. It often appears transiently, reflecting localized disruptions or interventions. In such scenarios, inattentional blindness to its presence causes a loss of critical insights into spatial processes.

5.2. Territorial Management: Zoning Regulations and Census Geography Decrees

Government interventions and policies also can induce NSA at multiple scales. Localized, selective actions—such as targeted investments, screening programs, or small-area remediation—align with the LISA [35] spatial outliers notion, producing scattered high-contrast clusters. However, more pervasive, long-term human incursions operate indirectly through structural frameworks such as land use zoning and decennial census geography delineations, which shape regional and local patterns across extended periods and multiple administrative levels.

Zoning regulates permitted activities within designated districts, spanning agriculture, commerce, industry, housing, transportation, and recreation. By prescribing allowable land uses, zoning can generate contiguous contrasts in spatial data—e.g., high-density residential areas bordering zero-population industrial corridors or parks—effectively embedding NSA into urban landscapes. Such patterns are scale-dependent: at the local level, parcels exhibit sharp contrasts; at the regional level, zoning creates similar land use clusters interspersed with contrasting areas; and, at the global/national level, zoning contributes to heterogeneous urbanization patterns readily observed spanning multiple cities. Empirical evidence underscores these effects. Griffith et al. [58] analyze 1990 US block group population densities covering 3100+ counties, finding dominant NSA in 16 of them. Salem City, Virginia (VA), exemplifies a strong local NSA case, where a few annexed parcels with extremely low or high densities generated marked contrasts relative to their surrounding urban fabric. By the 2000 census, mergers and administrative adjustments largely dissolved these contrasts (see Figure 3), illustrating the temporary nature of NSA under human-induced spatial reconfigurations. Table 2, an updated version of its 1990 counterpart ([58], p. 687), documents the positive–negative SA mixture and the portion of geographic variance accounted for across sample counties.

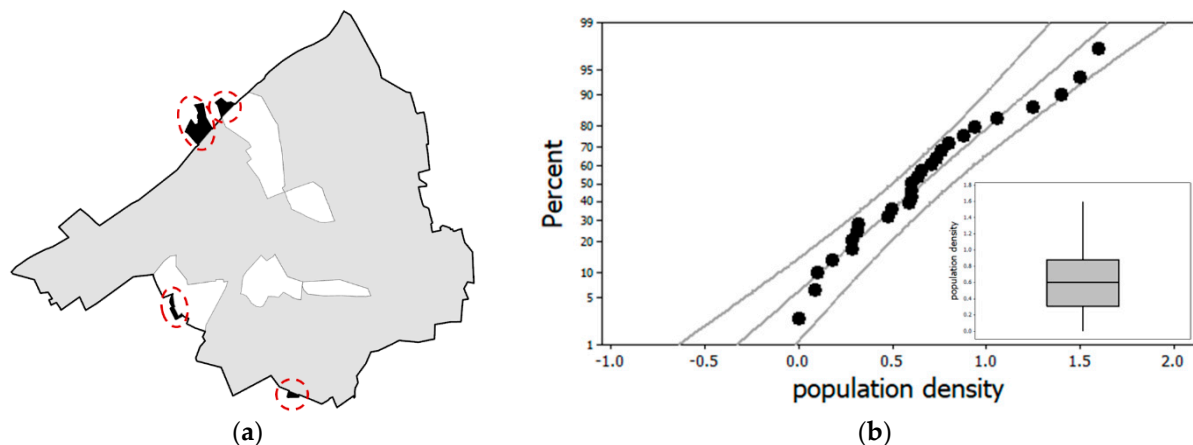


Figure 3. The geographic distribution of 1990 Salem, VA, population density by block group ($n = 27$). (a) Triad population density [i.e., people/(square mile)] choropleth map—white (>1.0 density), gray (density between 0.2 and 1.0), and black (circled in burgundy; <0.2 density). (b) A normal quantile (with a trend line and superimposed 95% confidence interval curves denoted by gray) and box plot.

Table 2. † GMESF negative binomial generalized linear modeling (GLM) regression: 1990 population density across selected US counties, by block group, displaying negative SA in their log-population density approximations (see [58]).

County	n	Raw Data		# of Chosen ESF [†] Vectors		Pseudo-R ²	Residual SA	
		MC	GR	PSA	NSA		z _{MC}	z _{GR}
Salem, VA [†]	27	−0.20	1.10	2	6	0.45	1.3	1.6
Choctaw, AL	18	−0.03	0.76	1	1	0.53	0.6	0.8
Northampton, VA	17	0.04	0.27	1	1	0.48	0.9	2.3
Cedar, NE	16	−0.10	0.82	1	2	0.70	0.0	0.9

† I am indebted to Dr. David W. Wong, Geography & Geoinformation Science Department, George Mason University, for retrieving data—1990 US Census Bureau block group counts and areas—affiliated with our original article that the National Historical Geographic Information System archives (NHGIS—<https://www.nhgis.org/>); these and our original data are not identical [e.g., the Choctaw, AL, and Northampton, VA, MCs are not $< -1/(n - 1)$ here]. In addition, densities calculated for this article employed only land area (excluding water area). Finally, our earlier paper engaged Box–Cox transformations that empowered normal curve theory approximations, whereas the analysis for this table applied GLM theory. † Supervised residual SA minimization superseded the statistical significance criterion. † Denotes an ESF (i.e., the linear combination of eigenvectors accounting for SA).

Similarly, census geography delineations—e.g., US tracts, block groups, and blocks (many of which reconfigure on a decennial basis)—impose boundaries that influence resource allocation, political representation, and public services. These artificial, state-sanctioned borders create discontinuities in spatial data, particularly at the local and regional scales, embedding NSA that may persist until boundaries are redrawn for subsequent scheduled (e.g., decennial) censuses. Collectively, zoning and census geography illustrate how, simultaneously, government policies can generate, sustain, and eliminate NSA across local, regional, and global geographic scales, modulated by temporal and administrative dynamics.

Table 3 summarizes 1990 block group counts for US counties exhibiting NSA in population density ([58], Table 7; Table 2 here). Notably, all 16 counties have relatively few areal units (n), limiting standalone SA analysis but providing instructive toy datasets. At the local scale, each resembles a LISA [35] outlier cluster, while at the regional scale, these block groups aggregate within counties, and at the global (i.e., state/national) scale, they contribute to broader spatial patterns. Changes in census geographies over time, such as block group mergers between 1990 and 2020—illustrated by the Salem City, VA, example (see Figure 3)—can mask or eliminate NSA, demonstrating a MAUP influence on detection. US counties reduced to 2–3 block groups over this period lack sufficient granularity for meaningful SA computation. Only Salem City retains enough polygons for potentially informative calculations, although caution is warranted ($n < 30$). These dynamics show that government-crafted census geographies can both generate and obscure NSA in georeferenced data, countering simplistic interpretations that NSA merely reflects inappropriate areal unit delineations (e.g., [5]). Accordingly, this example again demonstrates that NSA is not merely a statistical artifact but can arise from a recognizable geographic process that creates systematic local contrasts.

Table 3. County census block group polygon counts for investigating negative SA in 1990 population density specimens presented in Griffith et al. ([58], Table 7).

County	State [†]	Total Block Group Counts		County	State [†]	Total Block Group Counts	
		1990	2023			1990	2023
Salem City	VA	27	20	Aurora	SD	10	3
Choctaw	AL	18	15	Douglas	SD	9	3
Northampton	VA	17	14	Franklin	MS	9	10
Cedar	NE	16	9	Greene	MS	9	10
Clay	NE	13	7	Jones	NC	9	7
Todd	KY	12	9	Kemper	MS	9	9
Johnson	IL	11	11	Steele	ND	9	2
Webster	WV	11	8	Van Buren	IA	9	9

[†] Standard state abbreviations: AL—Alabama; IA—Iowa; IL—Illinois; KY—Kentucky; MS—Mississippi; NE—Nebraska; NC—North Carolina; ND—North Dakota; SD—South Dakota; VA—Virginia; WV—West Virginia.

5.3. Random Contrasts: Diffusion as an Obvious Autocorrelated Space-Time Process

A final generic NSA type this paper treats arises from true random space-time processes, distinct from fixed yet unknown heterogeneous sites like mineral reserves or petroleum fields. In these latter cases, randomness derives not from location but from discovery or adoption/infection timing. Phenomena such as innovation and disease diffusion (e.g., [59–65]) often begin with most locations exhibiting zero attribute values, fashioning stark spatial contrasts as new activity spreads, sometimes exacerbated by government quarantining. At the local scale, isolated areal units initially acquire inaugural attributes, producing NSA among early adopters and neighboring zero-value units. At the regional scale, geographic clustering of activity or adoption generates patchy contrasts across coun-

ties or metropolitan areas, whereas at the global/national scale, hierarchical structures and long-distance transmission leaps shape broad map patterns displaying spatial contrasts. The spatial configuration produced by an ongoing process of researcher relocation and associated innovation-related knowledge diffusion illustrates this mechanism. Elhorst and Zigova [66] show that a research unit's productivity is negatively affected by nearby competitors, with effects diminishing over distance. Although their analysis is cross-sectional, it captures the spatial imprint of continuing researcher relocation and associated knowledge diffusion driven by competition, illustrating NSA through competitive interactions operating across local, regional, and supra-regional scales. Using a standard normal-curve-theory-based spatial lag (SL) specification with an inverse-distance spatial weights matrix, they estimate a significant negative spatial autoregressive coefficient, indicating that research productivity is inversely related to that of nearby departments and consistent with spatial competition rather than positive spatial spillovers (PSA).

Disease diffusion also mirrors this multi-scale dynamic. Historical and contemporary outbreaks—from Finke's [67] late 18th-century world map of sicknesses to COVID-19 [28]—demonstrate how zero-inflated spatial counts at the onset create NSA opportunities and realizations. Local infection patterns emerge as scattered non-zero cases among predominantly zero-valued units, regional clusters develop as contagion permeates contiguous areas, and global/national long-range (i.e., teleportative-like) leaps generate hierarchically anchored spatial contrasts. Logistic growth curves and time-adapted GMESF model specifications capture these dynamics, documenting that NSA may arise from both spatial contiguity and hierarchical connections (e.g., [68–72]) that, for example, articulate diffusion networks. Figure 4 illustrates this last SA source for the 2017 urban hierarchy in Peru. Table 4 tabulates spatial statistical/econometric analysis output. NSA subservience necessitates combining autoregressive specifications (i.e., SL/autoregressive response (AR), and spatial error (SE)/simultaneous autoregressive (SAR))—to convert them from a one- to a two-SA-parameter model, in order for them to be capable of separately accounting for both PSA and NSA—into a solitary specification, one paralleling an SL-SE merging [73]:

$$\mathbf{Y} = \rho_{\text{SAR}} \mathbf{W} \mathbf{Y} + \mathbf{X} \boldsymbol{\beta} + \rho_{\text{MA}} \mathbf{W} \boldsymbol{\varepsilon} + \boldsymbol{\varepsilon}, \quad (1)$$

where $\mathbf{Y}$ is a n -by-1 response variable vector, $\mathbf{W}$ is an n -by- n spatial weights matrix (almost always row-standardized), $\mathbf{X}$ is a n -by- $(p + 1)$ covariate matrix (including an intercept term), $\boldsymbol{\beta}$ is a $(p + 1)$ -by-1 vector of regression coefficients, $\boldsymbol{\varepsilon}$ is an n -by-1 vector of random errors, and ρ_{SAR} and ρ_{MA} respectively are autoregressive and moving average (see [73]) spatial autocorrelation parameters. Because these last two arguments, especially in cases like the one presented here, where they share a common matrix $\mathbf{W}$, involuntarily can become confounded numerically through an increase–decrease trade-off association, their pairwise asymptotic correlation coefficient, $r_{\hat{\rho}_{\text{AR}}, \hat{\rho}_{\text{MA}}}$, almost always is an imperative diagnostic. Time-series analysis guidance (e.g., [74]) often argues for $|r_{\hat{\rho}_{\text{AR}}, \hat{\rho}_{\text{MA}}}| < 0.99$; because parameter multicollinearity affects estimation stability/inference more than forecasting accuracy, and time-series models primarily are a forecasting tool, high multicollinearity (above 0.9, but below 0.99) does not necessarily invalidate an estimated model's predictions, primarily indicating that independent estimation of the two parameters would be difficult. Meanwhile, the sign of the estimated SA parameter in MA model components appears opposite to the direction of the autocorrelation it represents due to model parameterization and interpretation (see Appendix C). Mathematically speaking, these MA term expressions are a lag polynomial in which coefficients enter with a negative sign. This arithmetic situation causes the reported parameter to reflect the opposite of the underlying dependence/correlation structure. Substantively, MA components model the propagation of direct shocks rather

than indirect persistence in a georeferenced response variable. Consequently, the apparent sign reversal does not indicate a contradiction, but rather reflects the dual representation of dependence/correlation in the posited error process versus the dependence/correlation structure of the data themselves.

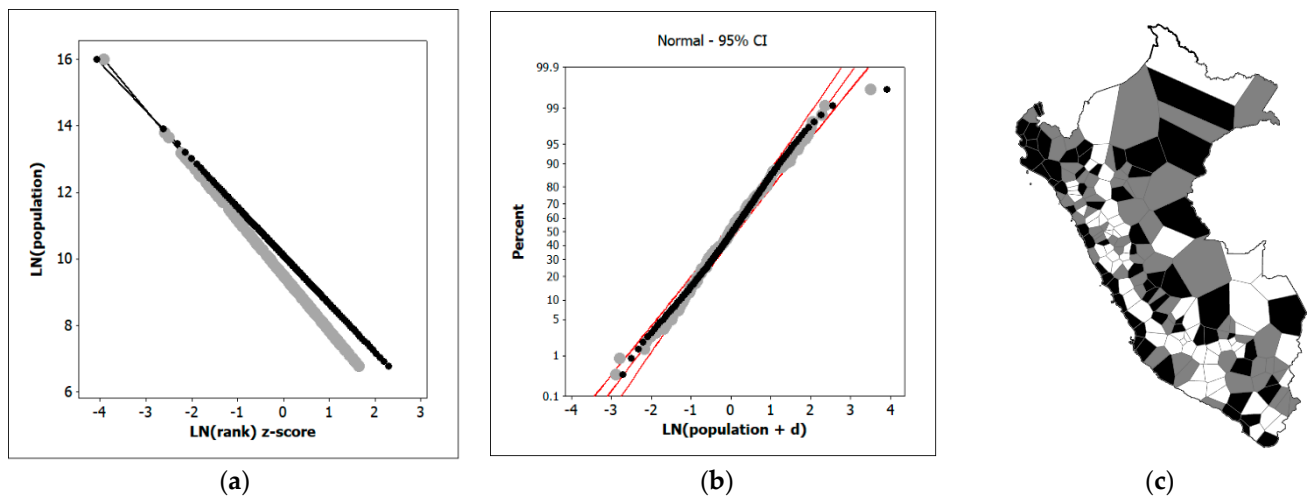


Figure 4. 2017 Peruvian urban hierarchy rank-size rule portrayals; scatterplot filled gray and black circles respectively denote raw and calibrated data. (a) Log-population versus log-rankings with trendlines (respectively gray and black). (b) Translated log-population normal quantile plots with trendline and 95% confidence intervals (red). (c) Rank-size rule calibrated LN(population + δ) tertile choropleth map; white, gray, and black, respectively, denote relatively low, intermediate, and relatively high numerical values. Source: https://www.inei.gob.pe/media/MenuRecursivo/publicaciones_digitales/Est/Lib1544/ (accessed on 21 July 2026).

Table 4. Spatial regression summary statistics for SA mixture components.

LN(Population + δ)	Auto-Normal SARMA			GLM-Normal GMESF							
	$\hat{\rho}_{AR}$	$r_{\hat{\rho}_{AR}, \hat{\rho}_{MA}}$	$-\hat{\rho}_{MA}$	ESF _p			Adjusted-Pseudo-R ²	ESF _n			Adjusted-Pseudo-R ²
				#	MC	GR		#	MC	GR	
raw	0.96	0.97	−0.93	14	0.64	0.35	0.19	24	−0.38	1.43	0.22
calibrated	0.96	0.97	−0.93	14	0.64	0.35	0.19	23	−0.38	1.45	0.22

NOTE: GLM, SAR, ESF_p, and ESF_n respectively denote generalized linear model, simultaneous autoregressive, and positive and negative eigenvector ESFs; a moving average (MA) parameter sign is the opposite of its true SA nature; and none of the $\hat{\rho}_{SAR} - \hat{\rho}_{MA}$ correlations are problematic. # denotes eigenvector selection counts (from respective candidate sets of 48 positive and 95 negative SA vectors).

As a viable and appealing alternative, GMESF impulsively enables correlation-nature-specific ESFs to account for both positive and negative SA without encountering multicollinearity complications. Without these more sophisticated tools, NSA quantification risks being lost in the shadows of PSA. Furthermore, GMESF liberates a spatial analysis from the bonds of normal curve theory. As for the Peruvian empirical example, the urban hierarchy fosters considerable NSA, substantiated here by the computed $-\hat{\rho}_{MA}$ (which are negative) and NSA ESF (ESF_n) quantities. Interestingly, both the SARMA and the GMESF models imply comparable PSA and NSA levels (à la SA parameter estimates, SA index values, and pseudo-R² magnitudes). Visual inspection of Figure 4c insinuates mostly pairwise contrasts at a local scale. As Griffith et al. [75] demonstrate, this manifestation of NSA is a typical outcome attributable to hierarchical SA: spatial dependence/correlation leaps across space in a fashion similar to non-contiguity teleporting (e.g., planes depart from one city, travel in a third dimension high over terrain, and then land in another city, i.e., hierarchical bypassing). It reflects central place theory aspects of national urban systems in which the larger cities spread out across a countrywide geographic landscape, with

intermediate and smaller settlements interspersed between their nearby larger ones. It also, once more, demonstrates that NSA is not merely a statistical artifact, but can arise from a recognizable geographic process that creates systematic local contrasts.

To conclude, random space-time processes reveal NSA as an innate geospatial data property, materializing ephemerally at local, regional, and global scales whenever new phenomena propagate across space and time, with the nucleus of this paper mostly about space.

6. Conclusions, Implications, and Remarks

This article synthesizes NSA possibilities in the literature, mostly escaping attention until now, continuing to address a chronic spatial analysis research oversight. Building upon Griffith [21], who inadvertently focuses primarily on PSA, this work emphasizes, in parallel, NSA as a meaningful, inferentially valuable univariate/multivariate georeferenced data property, one observable across local, regional, and global scales. NSA captures inverse locational relationships, is inherently delicate and transient, and responds sensitively to geographic scale, resolution, and data aggregation changes. Crucially, NSA should not be viewed as anomalous or merely a statistical artifact. Instead, it is a legitimate, interpretable spatial structure component, reflecting meaningful contrasts across geographic scales. Unlike PSA, which generally persists under spatial transformations, NSA can emerge, dissipate, or reappear amid dynamic processes—although repeatedly being mentioned in this text, addressing spatial dynamics in detail is beyond its scope, and, instead, is a theme more suitable as a standalone topic in a subsequent paper, one necessarily involving space-time panel data model specifications [76]—such as point-source spillovers, hierarchical diffusion, competitive interactions, and/or space-time randomness in innovation and disease spread. Its manifestations include local centrifugal contrasts, regional patchy clusters, and global hierarchical patterns, providing insight into spatial processes that PSA alone proves unable to fully reveal. Therefore, NSA should be understood as being other than a degenerate or aberrant outcome. It is a conditionally expected consequence of spatial processes and partitioning schemes that germinate geographically arranged local contrasts. In contrast, the literature to date generally treats NSA as uncommon (but it almost universally appears in PSA-NSA mixtures), exceptional (but the pervasiveness of spatial competition throughout the space-economy implies NSA should be encountered more frequently than the literature suggests, although competing spatial processes, aggregation effects, and/or analytical decisions often mask its signature), or primarily an artifact of methodological choices (but SARMA and GMESF specifications are beginning to supplant single-parameter spatial autoregressive defaults), with comparatively little attention devoted to it as a substantive spatial process.

At the same time, competing viewpoints, like those this paper mentions in detail, merit explicit recognition. Some argue that NSA is merely a model-based artifact, reflecting residual structures induced by misspecification, aggregation, and/or smoothing. Others suggest it is a statistical anomaly, appearing only in small or sparse datasets due to random variation (e.g., sampling error). A third perspective considers NSA to be an impossibility in non-discrete landscapes, proposing that such inverse relationships are antithetical to strictly continuous spatial surfaces. Additional perspectives highlight scale sensitivity, in which NSA is contingent upon MAUP dimensions, transitory dynamics, or mixture masking, in which its subordinate tendency almost always has it concealed by global PSA. Through engaging at least some of these alternative viewpoints, this paper clarifies that while NSA may be subjugated or obscured in certain circumstances, its repeated empirical presence across natural, social, and institutional landscapes supports its recognition as a genuine and analytically valuable geographic phenomenon. Accordingly, spatial analysts should treat NSA in meaningful ways, routinely detecting and reporting its nature and

degree, as well as modeling it. An appealing template would include: MC and GR, their z-scores based upon conditional or unconstrained permutations, local NSA diagnostics, and PSA-NSA GMESF estimation results.

From a spatial statistics/econometric perspective, an NSA presence—whether in observed geocoded attributes, model residuals, or estimated spatial parameters—should not be reflexively interpreted as anomalous, pathological, and/or misspecification evidence. Instead, when substantively supported, it may indicate competitive spatial processes, redistribution mechanisms, omitted hierarchical or other fragmented structures, or scale-dependent contrasts that are not well represented by purely contiguity-based dependence/correlation specifications. Accordingly, negative spatial autoregressive effects (i.e., SL/AR or SE/SAR), negative GMESF eigen-component formulations (i.e., ESFs [51]), and LISA [35] outlier configurations provide diagnostically valuable information about an underlying spatial data-generating process. Moreover, NSA warrants explicit consideration within spatial statistics/econometric modeling, not suppression or dismissal, because it can evolve dynamically, transitioning to PSA over time as, for example, point-source effects accumulate, hierarchical diffusion progresses, or competing spatial processes stabilize. Recognizing these temporal flips is critical for accurately interpreting NSA, particularly in longitudinal or evolving geographic datasets.

Some of the content of this paper overlaps thematically, but not substantively, with Griffith and Li [18]. Although they and Griffith [20] principally introduce and conceptually justify why NSA warrants greater attention, this paper extends that foundation by developing empirical, methodological, inferential, and application-oriented contributions. Specifically, it provides evidence-based demonstrations of how NSA manifests in geospatial data, develops methodological guidance for its measurement and incorporation into spatial analyses, and advances statistical reasoning concerning its interpretation and consequences. Furthermore, it shows how NSA can serve as a diagnostic for omitted-variable bias, zero-inflated and dynamic processes, and hierarchical spatial dependencies/correlations. In addition, this paper demonstrates how LISA [35], Moran scatterplots, and GMESF [51] can be used to identify, quantify, and interpret NSA, thereby extending analytical frameworks that have traditionally emphasized PSA. Finally, it discusses the practical implications of recognizing and correctly interpreting NSA for myriad applications, including emergency management, public health, zoning, census geography, and competitive spatial behaviors, illustrating how NSA informs the emergence, persistence, and transience of geographic contrasts. In addition, portions of this paper maintain that it is not limited to univariate phenomena. Throughout multiple interrelated attributes, it arises from compensatory dynamics, spatial competition, and hierarchical and/or diffusive processes. Multivariable/multivariate manifestations can reveal complex spatial trade-offs, uncover latent heterogeneity, and improve the diagnostic power of spatial models. Recognizing these multiple-variable patterns ensures that NSA is interpreted as a substantive georeferenced data component, not a statistical artifact, providing richer insight into spatial structure than univariate analyses alone. Meanwhile, the most obvious implications and final remarks pertain to those integrated into the translation of the eight canonical SA meanings originally compiled by Griffith [21]. NSA matters! As such, it is a meaningful, scale-sensitive georeferenced data property, capturing inverse spatial relationships spanning its inherent geographic scales. It is transient, influenced by temporal dynamics, administrative delineations, and data aggregation, and requires more sophisticated detection tools than PSA.

Future research should explore, among other NSA themes, the following: (1) formal methods for detecting and quantifying it under varying scales and resolutions, (2) integrating it into dynamic, hierarchical, and network-based spatial models, (3) its interplay in mixed spatial processes, and (4) temporal evolution and policy intervention implications for

its emergence, persistence, and mitigation. Collectively, such endeavors promise to elevate NSA from a frequently overlooked artifact to a robust tool for understanding complex spatial phenomena and informing spatially sensitive decision-making across local, regional, and global contexts.

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Abbreviations

The following abbreviations are used in this manuscript:

ESF	Eigenvector spatial filter
GLM	Generalized linear model
GR	Geary ratio
GMESF	Griffith-Moran eigenvector spatial filtering
LISA	Local indices of spatial association
MC	Moran coefficient
MAUP	Modifiable areal unit problem
NSA	Negative spatial autocorrelation
NY	New York
Pb	Lead
PSA	Positive spatial autocorrelation
SA	Spatial autocorrelation
SARMA	Spatial autoregressive-moving average
TX	Texas
USDA	United States Department of Agriculture
VA	Virginia

Appendix A

On the Making of Areal Units in Geographic Landscapes

Areal unit formation, such as counties, is an endogenous geographic process governed by territorial competition, so that neighboring units are deliberately differentiated, embedding negative SA by design. Consequently, an initial surface partitioning is neither exchangeable nor consistent with partition-invariance assumptions, and hence conditional (like for local SA indices) rather than standard permutation-based or random-partition null hypotheses are suitable for constructing an inferential basis for SA indices here—especially frontier expansions (e.g., Figure 1c), in which the source territories must be fixed, demonstrate why the unconstrained approach is unsuitable. Although random partitioning provides a conceptual baseline for understanding how variation in support influences spatial statistics, it assumes all tessellations are equally likely, whereas competitive boundary formation produces a single, structured realization. The observed boundaries themselves are substantively meaningful and historically stable, providing fixed support for inference, with accompanying SA indices primarily describing structural negative SA rather than stochastic deviation. To assess whether or not these observed statistics are unusually high or low relative to what might occur under the same generative mechanism demands non-causal process-based territorial competition simulations: each simulated partition must represent a plausible alternative outcome of the competitive process in question, creating a reference SA index sampling distribution that

is consistent with the structural forces shaping county boundaries. This is because the SA literature suggests that, under a null hypothesis of random attributes on a fixed support, SA index sampling distributions may be approximated by a normal distribution. But this should be with caution, recognizing that proper simulation experiments furnish a more robust context. Otherwise, only SA description is possible. Nevertheless, this proposed sampling-assisted approach effectively renders random values for SA indices in a principled, process-informed way, allowing inference to distinguish between autocorrelation inherent in the surface partition under study and additional spatial patterns in the attributes of interest.

Appendix B

Griffith-Moran Eigenvector Spatial Filtering (GMESF): A Concise Statement

GMESF is a technique used in spatial statistics to describe and account for spatial autocorrelation (SA) appearing in linear or generalized linear model (GLM) regression residuals by incorporating specially constructed SA surrogates as additional covariates for entry into a posited model. As such, in its simplest version, it filters SA from regression residuals, compresses it into a single synthetic variate, and transfers it to the model specification mean, converting that constant into a variable.

At its core, GMESF starts with a spatial weights matrix that encodes an articulation of the two-dimensional geographic neighborhood configuration for n areal units. From a doubly centered version of this n -by- n construct, it derives a set of orthogonal eigenvectors (often called Moran eigenvectors because the aforementioned matrix modification derives from the numerator of the Moran coefficient (MC)) that represent distinct map patterns (i.e., natures and degrees of SA) of spatial variation—ranging from broad, smooth gradients, through regional groupings, to fine-scale local clusterings. Each eigenvector has an associated MC value, indicating the strength and type (positive, zero, or negative) of SA it captures.

Judicious selection of a subset of the n eigenvectors removes spatially structured noise from the response variable map—almost always a mixture of positive and negative SA ones mostly dominated by moderate-to-strong PSA vectors—by including these synthetic map patterns as additional predictors in a regression equation. The partial regression results constituting their computed linear combination create the relevant eigenvector spatial filter (ESF). In this way, the revised regression model explicitly accounts for spatial dependence/correlation that otherwise would violate the independence assumption accompanying standard regression residuals. The outcome is a model with reduced (ideally eliminated) residual SA, bolstering statistical inference and improving interpretability.

In other words, to succinctly encapsulate, GMESF represents SA through orthogonal map patterns in a way that incorporates key ones into a regression model to effectively absorb—thereby simultaneously removing it from residuals while transferring it to the global mean term—spatial dependence/correlation. Its greatest technical advantage is that it enables the full use of conventional statistical methods, especially those capable of being executed in a regression format, in the presence of SA without modification or adjustment.

Appendix C

The SMA Spatial Autocorrelation Parameter Sign

A SARMA specification has two parameters, a spatial autoregressive and a spatial moving average one, such as, respectively, ρ_{SAR} and ρ_{MA} in Equation (1). Letting $\mathbf{X}\beta = 0$ for simplicity, and then rewriting that equation as

$$\mathbf{Y} = (\mathbf{I} - \rho_{\text{SAR}}\mathbf{W})^{-1}(\mathbf{I} - \rho_{\text{MA}}\mathbf{W})\epsilon,$$

implies that the covariance matrix for this data conceptualization is

$$\Sigma = \sigma^2(\mathbf{I} - \rho_{\text{SAR}}\mathbf{W})^{-1}(\mathbf{I} - \rho_{\text{MA}}\mathbf{W})(\mathbf{I} - \rho_{\text{MA}}\mathbf{W}^T)(\mathbf{I} - \rho_{\text{SAR}}\mathbf{W}^T)^{-1},$$

where superscript T denotes the matrix transpose operation. The matrix inverse infinite series expansion is

$$(\mathbf{I} - \rho_{\text{SAR}}\mathbf{W})^{-1} = \mathbf{I} + \rho_{\text{SAR}}\mathbf{W} + \rho_{\text{SAR}}^2\mathbf{W}^2 + \dots$$

This first-order expansion is a clear and transparent way to justify the opposite sign convention. It demonstrates mathematically that the SAR and SMA parameters enter this foregoing covariance structure with opposite signs. The approximation it furnishes is

$$\Sigma = \sigma^2[\mathbf{I} + 2(\rho_{\text{SAR}} - \rho_{\text{MA}})\mathbf{W}],$$

which makes the opposing effects of the SAR and MA parameters immediately apparent while avoiding an unnecessarily lengthy derivation.

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