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Abstract: The public transport system is responsible for the displacement of a large part of the population, particularly in developing countries. This fact makes it relevant to evaluate the performance of public transport to provide an efficient and effective service. The purpose of this study is to conduct a performance evaluation of the public transport operation in the Metropolitan Region of Fortaleza (MRF), in the State of Ceará, Brazil. The analysis is based on DEA and the Malmquist index, based on three inputs (total operating time, fleet age, and the mileage traveled) and two outputs (fare revenue and number of passengers). Data were obtained through automated fare collection systems (AFCs) that were implemented in the MRF. Although there were no major fluctuations in performance during the analyzed period, the results indicate that the system’s performance declined in certain years. In addition, the analysis enables a better understanding of route performance, considering the operating company or the area of operation, which helps to diagnose and comprehend the operation more effectively. By analyzing the operational performance over time, the proposed approach provides an additional contribution by offering a comprehensive overview to the involved stakeholders, fostering decision-making processes based on evidence.

Keywords: Malmquist index; data envelopment analysis; benchmarking; performance evaluation; public transport; performance indicators

1. Introduction

Mass transportation systems are crucial in large cities and metropolitan areas globally, not only because they enable the movement of large numbers of people but also because they play a significant role in addressing environmental concerns and promoting the well-being of citizens, according to Belwal and Belwal [1], Booysen et al. [2], Javid et al. [3], Wang et al. [4]. As supported by Belwal and Belwal [1], public transport services play a crucial role in addressing exclusion and poverty among the lower-income segments of the population in the region served. Debnath [5] states that efficient transport plays a significant role in the development of any region. Furthermore, Alonso et al. [6] highlight that a pressing challenge faced by developing countries is to promote the adoption of sustainable transportation options for daily commuting in order to mitigate pollution.

Urban centers face significant challenges in transporting people, including inadequate infrastructures, increased private vehicle ownership leading to congestion, and an
inefficient public transport system [7]. Public transport, often managed and regulated by government institutions, aims to provide high-quality transportation options that address the needs of the population, as affirmed by OECD [8], which emphasizes the role of transport authorities in ensuring efficient and quality service from public or private operators. The services are crucial in facilitating travel for various activities such as work, education, leisure, and healthcare, by offering coverage in terms of space and time.

According to Sun et al. [9], evaluating the performance of bus routes can be challenging when limited indicators are available. Therefore, regular performance evaluation is essential to ensure the efficient use of available resources and the effective transportation of people. Evaluation processes can also support managers in making informed decisions by providing operational indicators that specify the details of service provision, including location, timing, and operational features.

Wei et al. [10] reinforce the importance of evaluating the public transport system by stating that it is an essential and challenging issue for researchers and transport authorities, in view of limited funding and growing public needs. Evaluating the performance can assist transportation agencies in identifying underperforming services, planning potential investments, justifying past investments, and communicating accomplishments and challenges [11,12].

De Carvalho and Marques [13] suggest that using rewards and penalties based on indicators can be an effective way to motivate and maintain a high level of performance. The authors also emphasize the importance of using simple and intuitive indicators that can be easily monitored.

Assessing the quality of services can be achieved through qualitative measures (such as user opinion surveys) or quantitative measures (such as utilizing historical service data). However, conducting qualitative assessments involves interviewing service users, which can be challenging to carry out regularly due to its subjective nature and high costs. Conversely, a quantitative evaluation relies on historical data or is obtained through field research, which is also costly and infrequently conducted, as mentioned by Zhou et al. [14].

Due to the development of new technologies, the utilization of intelligent transportation systems (ITSs) has become increasingly common, as affirmed by Qureshi and Abdullah [15]. Such systems include automated transit management systems (ATMSs), which are composed of other systems, such as automated fare collection systems (AFCs) and automated vehicle location (AVL). Gkiotsalitis et al. [16] note that public transport systems have increased the amount of equipment and sensors that allow a passive collection of large amounts of data. The utilization of massive amounts of data allows for the analysis and evaluation of the service, which is crucial in assessing the performance of public transport routes with accuracy and monitoring the quality of service provided.

Jarboui et al. [17] note that research in this field is limited to certain regions, namely Europe, North America, and Asia. The authors highlight that the countries with the most research on this topic are the United States, Norway, Germany, and India. However, it should be noted that there is already some research on this topic in other regions, albeit in small numbers. This could be due to a lack of data or a shortage of qualified professionals.

According to Litman [18], the impacts of public transport services can be diverse, including both indirect and external effects (i.e., those that affect people who do not use the service). The author also outlines four categories that can be targeted for improvements in public transport: service expansion, service enhancement, demand management, and transit-oriented development (TOD).

Kurauchi and Schmöcker [19] mention that electronic ticketing systems provide spatial and temporal information on users, vehicles, and routes. These data can be used by authorities for various purposes, such as:
• To obtain the number of passengers by fare type, which enables the measurement of the network load by the type of user and the assessment of the economic viability of the route;
• To share revenues between operators;
• To consult stop-wise distribution of passengers boarding and alighting, by fare type;
• To compute origin–destination matrices for each route, subset of routes, or the entire network.

Additionally, electronic ticketing systems offer the potential for a wide range of disaggregated or aggregated analyses per trip, depending on the intended purpose. As noted by Arbex and da Cunha [20], electronic ticketing data, as well as other data passively generated by public transport systems, allow for a much more precise and continuous technical evaluation and monitoring of the implementation of changes. Furthermore, Liu et al. [21] developed a replication methodology that includes cleaning, filtering, processing, and converting collected data into information, such as bus travel paths and subway system schedules.

According to Tsolas [22], there are only a few studies that have analyzed the efficiency of the urban transport system based on a data envelopment analysis (DEA), especially regarding production and sales processes. The authors used a two-stage DEA to evaluate the performance of a group of electric bus routes in Greece. They also note that, with the series of data available, a DEA can provide a dynamic assessment of public transport performance.

Georgiadis et al. [23], Karim and Fouad [24] highlight that there are limited studies that have utilized DEAs and considered routes as decision-making units (DMUs) in the context of public transport. The authors note that only a small percentage (1.6%) of the studies have measured the performance of routes, with the majority focusing on operators (76%) and systems (21%). Furthermore, it is worth noting that the majority of route performance analysis studies were published after 2010.

Thus, this work aims to evaluate the operational efficiency of regular public transport routes in the Metropolitan Region of Fortaleza (MRF) through a DEA and the Malmquist index. The approach fills the research gap in the use of massive data to assess the performance of public transport operations over time, providing operators and managers with information to support decision-making. Furthermore, this work proposes a performance analysis based on the routes, which is a perspective made possible by the detailed data available from the ticketing system. This approach allows for a more detailed analysis of the operational performance than those studies where DMUs are the operators or the transportation systems.

The following section provides a literature review of the DEA model and Malmquist index, along with a description of the variable selection process. Section 3 outlines the methodology and dataset employed to evaluate the operational efficiency of the service. Section 4 presents the results of the application of this methodology to the case study, followed by a detailed discussion of the results in Section 5. Lastly, the article concludes with implications and final considerations.

2. Literature Review

The literature review of the DEA model, the Malmquist index, and the selection of variables.

2.1. DEA

The DEA is a method based on linear programming that measures the relative efficiency of entities called decision-making units (DMUs) based on input and output information. Sun et al. [9] used DEA to categorize and evaluate existing routes, noting that few studies used new technologies to assess the quality of the service. Wei et al. [10] developed a new method to evaluate the overall performance of public transport services, using a combination of mathematical programming methods, GIS-based analysis and multi-objective
spatial optimization techniques. Lee et al. [25] mentioned that the DEA model has four assumptions: return to scale, convexity, free disposability, and input or output orientation. The mathematical formulation of the model is presented in Equation (1), as per Seiford and Zhu [26].

$$\max \sum_{r=1}^{s} u_r y_{ro} + u_o$$  \quad (1)$$

subject to:

$$\sum_{i=1}^{m} v_i x_{io} = 1$$
$$\sum_{r=1}^{s} u_r y_{ro} - \sum_{i=1}^{m} v_i x_{ij} + u_o \leq 0 \quad j = 1, \ldots, n;$$
$$u_r, v_i \geq 0$$ \quad and \quad $$u_o$$ \quad is \quad free$$

wherein,

$m$: total number of inputs;
$s$: total number of outputs;
$n$: total number of DMUs;
$x_{io}$: $i$th input for DMU$_o$;
$y_{ro}$: $r$th output for DMU$_o$;
$j = 1, 2, \ldots, n$;
$v_i$: weight vector of input $x$;
u_r$: weight vector of output $y$.

The constraints are orientation, efficiency, and non-negativity. The constraints of orientation (input-oriented DEA variable returns to scale—VRS model) lean toward minimizing the inputs, which means that all DMUs seek to minimize the number of inputs required to produce their outputs. The efficiency constraint requires that all DMUs be efficient or operate at a point of maximum efficiency, i.e., producing the maximum amount of outputs with the minimum amount of inputs. Moreover, the non-negativity constraint prevents the weights of inputs and outputs from being negative.

The DEA models proposed by Charnes (constant returns to scale—CRS) and Banker (VRS) are called standard, conventional, or basic models, according to Lovell and Rouse [27]. Additionally, there are other models adapted from these, such as the super-efficient model (Sun et al. [9], Lovell and Rouse [27]), Malmquist index (Färe et al. [28]), and network DEA (Färe et al. [29], Daraio et al. [30]). Applications and literature reviews of other models can be seen in work by Kuah et al. [31], Cavaignac et al. [32].

2.2. Malmquist Index

The Malmquist index was introduced by Caves et al. [33]; it was first used in the DEA literature by Berger and Humphrey [34] and further developed by Färe et al. [28]. This is a commonly used method to measure productivity in different periods ($t$ and $t + 1$). It is based on the concept of a distance function, which measures the relative efficiency of DMUs in transforming inputs into outputs. The Malmquist index calculates the productivity change over a period of time by comparing the efficiency of a DMU at two different points in time. Färe et al. [28] propose, through the geometric mean of two indices, the expression inside the square brackets in Equation (2), for periods $t$ and $t + 1$. This equation represents the Malmquist index, where the first part of the equation is related to the change in technical efficiency and the second part (in the square root) is related to technological change.

$$M_{t,t+1} = \frac{E^{t+1}(x^{t+1}, y^{t+1})}{E^t(x^t, y^t)} \sqrt{\frac{E^t(x^t, y^t)}{E^{t+1}(x^t, y^t)}} \sqrt{\frac{E^{t+1}(x^{t+1}, y^{t+1})}{E^{t+1}(x^{t+1}, y^{t+1})}}$$

(2)$$

wherein,

$M_{t,t+1}$: Malmquist index between period $t$ and $t + 1$;
x$^t, y^t$: Inputs and outputs for period $t$;
x$^{t+1}, y^{t+1}$: Inputs and outputs for period $t + 1$;
Efficiency for period $t$; 
Efficiency for period $t + 1$.

The decomposition of the Malmquist index into technical efficiency and technological change allows us to identify whether the increase is attributed to technological advancements, improvements in technical efficiency, or both. The assessment of technical efficiency measures how a production unit’s efficiency compares to other units within the same period. It determines whether the unit is more efficient or less efficient in terms of production compared to its counterparts. On the other hand, technological change evaluates whether the technology utilized by the production unit has improved or deteriorated over time. This is determined by comparing the production efficiency in one period to that in another period.

According to Oliveira et al. [35], the index can be greater than, equal to, or less than 1, where greater than 1 indicates performance growth, while a value less than 1 reflects a reduction for periods $t$ and $t + 1$.

### 2.3. Selection of Variables

The selection of variables for the application of the DEA or Malmquist index model is a topic extensively discussed in the literature, according to the number of variables and how to define the choice; Georgiadis et al. [23], De Borger et al. [36], Holmgren [37] note that there is no consensus for defining the variables. The authors state that the selection of variables is often determined by practical issues, such as data availability or the method used.

The selection of variables was based on those used in previous studies, which represent aspects related to the operation of the service, in order to analyze the following categories: availability, time, service offered, maintenance, and economics, according to European (CEN/TC320 [38]) and American (Kittelson et al. [39]) technical regulations. In addition, a correlation analysis was performed between the available variables, although Dyson et al. [40] state that omitting variables purely because they have a high correlation should be avoided.

The study chose to use only variables with absolute values, avoiding the use of ratios or rates, as recommended by Dyson et al. [40] and Emrouznejad and Amin [41]. The authors suggest an alternative to using rates or indices as inputs and outputs, which is to include the numerator as an input and the denominator as an output if the rate is used as an input. Conversely, if the rate is an output, the numerator should be used as an output and the denominator as an input. In addition to these issues, Dyson et al. [40] pointed out some pitfalls that must be avoided or be carefully handled in the application of a DEA, namely non-homogeneous units or environments, non-verification of economies of scale, a high number of inputs and outputs, use of qualitative data, undesirable inputs and outputs, and external and constrained factors. To ensure accurate results, the study aimed to follow the protocols outlined by the authors and avoid these pitfalls.

The total operating time was used in several studies, such as in work by Sheth et al. [42], Güner and Coşkun [43], Tran et al. [44]. This variable represents the actual period of service provision during the year and is related to the costs of the operation (vehicle and labor costs).

In the study by Hanumappa et al. [7], other inputs, namely fleet age and mileage traveled, were used. Fleet age is an indicator of maintenance costs, as older vehicles are expected to require more maintenance (see Taboada and Han [45]) compared to other variables, such as service quality and type of service. On the other hand, mileage traveled, such as the total operating time, relates to fleet and labor costs, but it also varies due to the traffic conditions on the route.

The number of passenger variables is the most frequently used output in the analyzed publications (see Karim and Fouad [24], Güner and Coşkun [43], Azambuja et al. [46], Sampaio et al. [47], Araújo et al. [48], Sun et al. [49], Fitzová et al. [50], Fitzová and Matulová [51]). It is a crucial variable as it represents a primary focus of the public trans-
port service provision, meeting the travel needs of passengers while providing a high
quality service.

The other output variable used is fare revenue, which is utilized in work by Fitzová
and Matulová [51], Zhu et al. [52], Zhang et al. [53]. Fare revenue is useful in assessing the
economic viability of the service, and, similar to the number of passengers, it represents an
important outcome of the public transport supply.

In addition to the studies presented, Daraio et al. [30] conducted a review of studies
with the main measures used for efficiency analysis, among them, are the operating time
(8.9%), fleet age (14.5%), mileage traveled (53.2%), number of passengers (16.9%), and fare
revenue (11.3%). The percentages of papers reported for the variables are in parentheses.

The use of ticketing system data may underestimate passenger demand due to
fare evasion (passengers with no card, passengers who use the transport several times with
the same card or passengers who do not validate the card), as noted by
Barabino et al. [54], National Academies of Sciences et al. [55], Lee [56].

3. Materials and Methods

Monitoring and analyzing the operation of public transport networks from various
perspectives is essential due to the significance of public transport in urban and metropoli-
tan regions, and the challenges involved in meeting the population’s needs with a safe,
reliable, efficient, effective, and reasonably priced service. These perspectives may include,
but are not limited to, availability, accessibility, time, security, information, services offered,
and economic, social, and environmental factors. Figure 1 presents the steps of the research.

![Flowchart of research steps.](image)

The first step corresponds to the definition of the research objective, which is to
evaluate the operational efficiency of regular public transport routes. The second step
involves the collection of data, considering factors such as importance, availability, and
cost. Step 3 entails the selection of input and output variables for the model, as described in
Section 2.3. In Step 4, the approach (DEA method and Malmquist index) is chosen. Step 5
involves applying the models to the case study, followed by steps 6 and 7, which include
analyzing the results, discussing the findings, and drawing conclusions, respectively.
3.1. Data Envelopment Analysis and the Malmquist Index

The initial approach employed in the analysis was the conventional DEA model. As stated before, in this model, DMUs that lie on the frontier receive an efficiency index of 1, while those with an index less than 1 are deemed inefficient. The model can be CRS (constant returns to scale) or VRS (variable returns to scale) and can be either input- or output-oriented.

This approach enables the evaluation and comparison of different routes, allowing for the assessment of their performance and identification of areas that require improvement to enhance efficiency. By computing the efficiency index of each route, it is possible to scrutinize them and diagnose required adjustments to provide a better quality of service. The performance evaluation of each route is based on comparisons with other routes within the same public transport system, serving the same area and population.

After the initial analysis, the next step is to conduct a performance analysis of the routes over time to determine the variation in their efficiency. This is achieved using the Malmquist Total Productivity Index.

3.2. Data Set—Case Study

The case study focused on 15 out of 19 municipalities in the Metropolitan Region of Fortaleza (MRF) that have a regular metropolitan public transport service. These 15 municipalities are served by 7 different companies, as shown in Figure 2. It is important to note that the remaining four municipalities were not included in the study as they did not have a regular metropolitan public transport service during the specified period. The provision of this service plays a crucial role in integrating the municipalities within the MRF, which directly impacts economic activities in the region and its surroundings. The management and regulation of the service are the responsibility of the Government of the State of Ceará, with the task delegated to the Regulatory Agency for Delegated Public Services of the State of Ceará (ARCE).

![Figure 2. Map of the Public Transport System of the Metropolitan Region of Fortaleza—Ceará.](image-url)
In 2022, the metropolitan public transport system recorded over 800,000 bus trips and approximately 17 million passenger trips. The service operates using urban buses with two doors, where passengers enter through the front door and validate their transport card or pay cash to the collector. After that, all users must pass through the turnstile (ticket barrier), ensuring low fare evasion and accurate passenger counting. Additionally, multi-fare routes have a collector, and all vehicles are equipped with a surveillance camera system. The system only registers boarding data and does not record alighting information. The pricing system is based on fare zones, also known as fare rings, which are structured according to the distance covered, resulting in varying fare ranges.

The construction of the database refers to the period from 2016 to 2022. The database contains 246.1 million records (70.3 Gb). In addition to the data validation, the database also includes information about the routes, itineraries, schedules, fare values, and mileage. Each validation record contains 29 fields. For this study, the following were used:

- OperatorID: Code of the company responsible for the vehicle;
- VehicleID: Vehicle identification;
- RouteID: Route identification;
- TripDepTime: Departure date and time of the trip;
- TripFinishTime: Finish date and time of the trip;
- Fee: Fee paid by the user.

The analysis was conducted from the perspective of managers and operators, as other perspectives would have required additional data beyond what was available through the ticketing or monitoring system. From the viewpoint of managers and operators, performance metrics were derived from the desired level of service quality, which was established by defining service goals, and the actual service quality, which was measured by the level of quality achieved in daily service operations. By comparing the planned and actual service operations, managers and operators could identify inefficiencies, weaknesses, and potential improvement guidelines. While there are several other aspects of the public transport service that could be analyzed, such as cost/financial feasibility, investments, attractiveness, externalities, or flexibility to adapt to demand, this research focuses specifically on the performance of public transport operations.

Prior to conducting the data analysis, it was necessary to pre-process the raw data. Raw data often contain incorrect, illogical, or missing records, making it necessary to prepare the data before analysis. Trépanier et al. [59] discussed the pre-processing step, which involved identifying and correcting errors before proceeding with the analysis.

In the exploratory analysis, extreme values of operating time were found, either too low or too high, which were not reasonable for the operation and were excluded from the data set. Therefore, records with travel times shorter than what could be achieved with the speed of the road (60 km/h) were removed. In addition, an analysis of outliers was performed. According to Pearson [60], outlier detection can be performed as an automatic mathematical procedure, either by the three-sigma rule, the Hampel identifier, or the boxplot rule. However, the interpretation of the results required expert knowledge and experience. This study used the boxplot rule to remove outliers for the operating time variable. Working days were selected for the analysis, while weekends and holidays were excluded due to their different behaviors. Additionally, routes that did not belong to the metropolitan service or did not operate consistently throughout the year were removed from the analysis as the focus was on regular routes.

Table 1 shows the number of trips selected after each stage of data processing.

After constructing the database containing only data from regular metropolitan routes, the analysis proceeded to the next stage, which involved examining the selected variables.
Table 1. Processing steps for public transport ticketing data.

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</tr>
</thead>
<tbody>
<tr>
<td>Raw data</td>
<td>1,307,043</td>
<td>1,309,493</td>
<td>1,220,498</td>
<td>1,176,309</td>
<td>683,971</td>
<td>757,634</td>
<td>811,309</td>
</tr>
<tr>
<td>Exclusion of outliers</td>
<td>1,079,537</td>
<td>1,079,941</td>
<td>1,028,157</td>
<td>965,857</td>
<td>614,075</td>
<td>677,839</td>
<td>685,941</td>
</tr>
<tr>
<td>Exclusion of holidays and weekends</td>
<td>810,479</td>
<td>814,921</td>
<td>814,892</td>
<td>730,367</td>
<td>475,067</td>
<td>515,687</td>
<td>526,602</td>
</tr>
<tr>
<td>Exclusion of non-regular routes</td>
<td>806,565</td>
<td>805,283</td>
<td>753,370</td>
<td>720,836</td>
<td>429,743</td>
<td>501,355</td>
<td>516,207</td>
</tr>
</tbody>
</table>

The variables were selected based on the following aspects: importance in the operation of public transport, application in other studies (see Section 2), and availability of data. Based on these aspects, the following variables were calculated:

- Total operating time: Sum of the operating time of each trip per route for the entire year. The operating time was obtained by subtracting the arrival time and the departure time, provided by the ticketing system. This variable allows for analyzing temporal aspects of each route operation;
- Fleet age: Sum of the age of the vehicle used per trip and per route throughout the year. The age of the vehicle was obtained through the vehicle register provided by ARCE and the vehicle used in each trip was included in the ticketing system. This variable is related to the maintenance and comfort of the vehicles;
- Mileage traveled: Sum of the distance traveled for each trip per route for the whole year. The length of the route was obtained through the registration of the routes provided by ARCE and the number of trips was obtained by the ticketing system. This variable allows for analyzing the availability and the service offered;
- Fare revenue: Total fare revenue per trip for each route for the entire year. The fare revenue is the sum of the amount paid by the user and is provided by the ticketing system. This variable reflects the economic and financial viability of the route and service provided;
- Number of passengers: Total number of passengers per trip for each route for the whole year. The number of passengers is the sum of the number of records in the ticketing system. The variable allows for analyzing the demand of each route.

The public transport supply is represented by the input variables (total operating time, fleet age, and mileage traveled) while the demand is represented by the output variables (fare revenue and number of passengers). Table 2 provides the descriptive statistics of these variables for the year 2022.

Table 2. Descriptive statistics of the variables used—2022.

<table>
<thead>
<tr>
<th>Descriptive Statistics</th>
<th>Total Operating Time—In Hours</th>
<th>Fleet Age—In Years</th>
<th>Mileage Traveled—In km</th>
<th>Fare Revenue—In BRL</th>
<th>Number of Passengers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average</td>
<td>10,024.35</td>
<td>40,598.88</td>
<td>214,912.55</td>
<td>1,332,475.09</td>
<td>298,168</td>
</tr>
<tr>
<td>Median</td>
<td>4012.46</td>
<td>26,428</td>
<td>126,000</td>
<td>671,289.61</td>
<td>99,822</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>9785.06</td>
<td>37,844.34</td>
<td>203,021.42</td>
<td>1,353,761.75</td>
<td>339,442</td>
</tr>
<tr>
<td>Minimum</td>
<td>372.11</td>
<td>1401</td>
<td>4647.15</td>
<td>13,242.15</td>
<td>7155</td>
</tr>
<tr>
<td>Maximum</td>
<td>37,165.04</td>
<td>131,257</td>
<td>800,403.70</td>
<td>5,445,283.96</td>
<td>1,389,964</td>
</tr>
<tr>
<td>Sum</td>
<td>591,436.38</td>
<td>2,395,334</td>
<td>12,679,840.39</td>
<td>78,616,030.49</td>
<td>17,591,929</td>
</tr>
</tbody>
</table>

4. Results

To analyze the operational efficiency of the routes, an input-oriented DEA with variable returns to scale was applied to each route per year from 2016 to 2022. Table 3 provides a yearly summary, including the number of routes analyzed, the efficient routes, and descriptive statistics of the performance index obtained.
During the study period, there were changes in the transport network, with fluctuations of up to 20 routes between 2019 and 2020, indicating adjustments made to the network. However, despite these changes, the average performance index only showed a slight fluctuation of 0.071, suggesting that the network adjustments did not have a significant impact on performance. Additionally, there has been an increase in the average operating performance index since 2018.

In a DEA approach, the difference between the input/output levels of the inefficient DMUs and the corresponding input/output levels of the efficient DMUs provide a measure of the relative inefficiency or potential for improvement of the inefficient ones. In this work, the efficient routes are used as benchmarks for the inefficient routes in terms of input–output ratios, representing the optimal utilization of inputs (operating time, fleet age, and mileage traveled) to produce outputs (number of passengers and fare revenue). For example, in 2022, route 67 operated by company 5, which connects the capital city of Fortaleza to the municipalities of Eusébio, Itaitinga, and Pacatuba, was an efficient route and served as a benchmark for 32 less efficient routes. Routes 9 and 83, operated by companies 2 and 6, respectively, serving the municipalities of Maranguape and Maracanaú, were efficient in all years and served as references for up to 40 inefficient routes in 2019. Additionally, an additional 40 routes were efficient in at least one of the seven years.

Table 3. Measuring the operational performances of the bus routes.

<table>
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<tbody>
<tr>
<td>Number of routes</td>
<td>68</td>
<td>69</td>
<td>68</td>
<td>73</td>
<td>53</td>
<td>62</td>
<td>59</td>
</tr>
<tr>
<td>Efficient routes</td>
<td>15</td>
<td>16</td>
<td>12</td>
<td>14</td>
<td>14</td>
<td>17</td>
<td>13</td>
</tr>
<tr>
<td>Average</td>
<td>0.746</td>
<td>0.757</td>
<td>0.744</td>
<td>0.750</td>
<td>0.783</td>
<td>0.813</td>
<td>0.814</td>
</tr>
<tr>
<td>Median</td>
<td>0.762</td>
<td>0.793</td>
<td>0.784</td>
<td>0.758</td>
<td>0.810</td>
<td>0.842</td>
<td>0.852</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.223</td>
<td>0.212</td>
<td>0.216</td>
<td>0.190</td>
<td>0.204</td>
<td>0.181</td>
<td>0.175</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.216</td>
<td>0.290</td>
<td>0.287</td>
<td>0.331</td>
<td>0.129</td>
<td>0.256</td>
<td>0.337</td>
</tr>
</tbody>
</table>

The other analysis performed was to assess the operational performance over time through the Malmquist index. The same variables of the classic DEA model were considered. Table 4 presents the main results of this analysis. Only DMUs that operated in both periods \( t \) and \( t + 1 \) were included in the analysis. Any routes that did not operate in both periods were excluded.

Table 4. Measuring the operational performance of public transport over time—Malmquist index.

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of routes</td>
<td>66</td>
<td>64</td>
<td>63</td>
<td>49</td>
<td>50</td>
<td>57</td>
<td>45</td>
</tr>
<tr>
<td>Routes with increasing productivity</td>
<td>24</td>
<td>34</td>
<td>24</td>
<td>7</td>
<td>27</td>
<td>32</td>
<td>11</td>
</tr>
<tr>
<td>Average</td>
<td>1.046</td>
<td>0.993</td>
<td>1.100</td>
<td>0.826</td>
<td>1.026</td>
<td>1.269</td>
<td>0.896</td>
</tr>
<tr>
<td>Median</td>
<td>0.912</td>
<td>1.013</td>
<td>0.956</td>
<td>0.805</td>
<td>1.027</td>
<td>1.036</td>
<td>0.751</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.621</td>
<td>0.166</td>
<td>0.988</td>
<td>0.187</td>
<td>0.246</td>
<td>1.027</td>
<td>0.502</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.229</td>
<td>0.349</td>
<td>0.317</td>
<td>0.443</td>
<td>0.547</td>
<td>0.146</td>
<td>0.147</td>
</tr>
<tr>
<td>Maximum</td>
<td>3.442</td>
<td>1.485</td>
<td>8.389</td>
<td>1.385</td>
<td>1.893</td>
<td>6.879</td>
<td>2.811</td>
</tr>
</tbody>
</table>
It is possible to see that the number of DMUs with productivity growth varied over time, with 34 routes in 2018 compared to 2017, and only 7 routes in 2020 compared to 2019. It is worth mentioning that the results for 2020 may have been affected by external factors, particularly the COVID-19 pandemic, which had its first wave between March and June 2020. The mean and median index values were consistently close to 1, except for the 2- and 3-year comparisons, where the mean and median were below 1, respectively.

When comparing only the initial (2016) and final (2022) years, only 11 routes showed productivity growth out of a total of 45 routes that remained operating during the period. The mean and median values of the index were less than 1, indicating a decrease in the system’s productivity between 2016 and 2022.

5. Discussion

The analysis and results obtained, both by route and over time, allow for the identification of critical areas that require attention and intervention by managers and service operators. One approach to interpreting the results is to evaluate them by company, considering that they have similar resources allocated to all routes. This can help determine measures that can be taken to minimize inefficiencies. Another aspect to consider in terms of operational performance is the spatial dimension. Routes that have similar origin–destination patterns and itineraries may serve a particular region with common demand. Therefore, it is essential to analyze what factors contribute to one route being inefficient while another similar route is efficient.

Apart from the aforementioned methods, analyzing the system’s performance over time enables the establishment of correlations with external factors that may influence its operations. The outbreak of the COVID-19 pandemic in 2020, for instance, had a significant impact on public transportation. In the region under analysis, a decrease in the number of trips and routes was observed. Furthermore, the pandemic had an adverse effect on the system’s productivity, which is evident in its lowest performance index (0.129) throughout the entire period studied, as well as in the mean and median values of the Malmquist index for consecutive years.

The operational performance index of three routes for company 1, obtained for each year, is presented in Figure 3. This visualization allows for a comparison of performance between the routes and reveals that routes 1 and 3 had high indices over the period, being efficient in some years. In contrast, route 2 consistently had lower values compared to the other two routes of the analyzed operator.

![Figure 3. Analysis of three routes of company 1.](image-url)
Figures 4 and 5 present graphs related to the performance of Route 2, which was operated by three different companies. Both the performance index (Figure 4) and the Malmquist index (Figure 5) showed a decline, indicating a decrease in operational performance over time.

Beginning in 2020, when route 2 was operated exclusively by company 1, an improvement in its operational performance could be observed. This suggests that there may have been an oversupply of resources for low demand, coupled with inadequate fare revenue, which negatively impacted the route’s performance in the past.

Despite the stability of the operating performance during the analyzed period, as indicated by the average operational performance in Table 3 and the average Malmquist index in Table 4, it is important to highlight that the public transport network experienced a reduction in supply (number of trips and routes) and demand (number of passengers), as mentioned earlier. This reduction may have resulted in a decrease in the quality of service provided and left certain regions without access to metropolitan public transport.
Therefore, when conducting the performance analysis, it is crucial to consider additional data and information to avoid drawing premature conclusions.

Therefore, by readjusting the number of trips or changing the schedule, it would be possible to enhance operational efficiency. This analysis can also be extended to other routes, identifying the critical areas that require improvement.

6. Conclusions

The objective of this study was to propose a methodology for evaluating the operational performance of public transport and to apply it to the public metropolitan bus transport system of MRF. The results obtained by the DEA method can provide valuable information for managers and transport operators to make better decisions and improve the quality of service.

This research provides valuable insight for managers and operators in the specific area, as well as proposes a novel approach for analyzing operational performance that can be replicated by managers and operators in other areas or transportation systems. Additionally, it offers valuable input for researchers in the field of public transport who aim to compare methodologies and performances across different public transport systems.

By comparing the results obtained from the classic DEA and the Malmquist index, the analysis was enhanced by evaluating the performance for a specific year and its progression over time. The pre-processing analysis revealed issues in the data recording stage, emphasizing the need for the managing body and operators to improve the ticketing system process. This includes recording the start and end of each trip and the direction of the route. This improvement would help reduce the number of outliers, thereby increasing the reliability of the records.

When attempting to improve route efficiency by reducing inputs without altering output values, the model yielded significant reduction values. To minimize inefficiencies, it is suggested to reduce travel time and/or the number of trips, reorganize the network through route grouping, and review the distribution of bus stops, among other potential modifications. These modifications should consider time slots and the spatial structure of the network to avoid compromising service quality.

The study presents the limitation of not incorporating cost variables (such as operational costs, waiting time costs, etc.) due to the unavailability of disaggregated data during trip and route levels.

One potential continuation of this research involves incorporating external factors into the analysis and evaluating their impacts on public transport performance. Another possibility is to conduct studies that focus on specific companies or sets of routes that share similar characteristics. This approach could provide more nuanced insights into the factors that influence operational efficiency and help guide decision-making at a more granular level.

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**Conflicts of Interest:** The authors declare no conflict of interest.

**Abbreviations**

The following abbreviations are used in this manuscript:

- AFC: automated fare collection
- ATMS: automated transit management system
- CRS: constant returns to scale
- DEA: data envelopment analysis
- DMU: decision-making units
- GIS: geographic information system
- MRF: Metropolitan Region of Fortaleza
- TOD: transit-oriented development
- VRS: variable returns to scale

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