

Article

Micro Breakage-Induced Contact Network Evolution and Performance Degradation of Graded Aggregates Under Cyclic Loading

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Abstract

Graded aggregates used in high-speed railway subgrades often undergo micro breakage (i.e., corner and edge spalling) under cyclic loading, whereas the mechanism linking local particle spalling to contact-network evolution and permanent deformation remains unclear. To address the lack of a physically based micro breakage criterion and a continuous local shape-updating scheme in existing DEM approaches, this study proposes an energy-driven micro breakage method. The method uses the relationship between contact elastic energy and fracture energy for newly created free surfaces as the breakage criterion. It also employs a radial function representation to simulate local particle shape evolution. The proposed method is subsequently validated through multilevel tests and shows reasonable agreement with the experimental results for particle micro breakage and the associated mechanical responses. Furthermore, dynamic triaxial simulation results show that micro breakage exhibits a distinct cumulative characteristic and increases with loading frequency and amplitude. Meanwhile, micro breakage drives particle rearrangement and new contact formation, leading to skeleton densification and weakening of the strong force-chain network, which in turn accelerates plastic deformation accumulation. These coupled processes constitute an important mechanism governing the progressive degradation of graded aggregates under cyclic loading. This study clarifies the associated particle-scale mechanism and provides a reference for performance optimization.

Keywords: high-speed railway subgrade; graded aggregates; micro breakage; cyclic loading; contact network; performance degradation; discrete element method

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1. Introduction

Graded aggregates are widely used in load-bearing structures subjected to cyclic loading, such as high-speed railway subgrades and road bases [1–3]. The long-term service condition of these materials is directly related to the stability and durability of engineering structures. However, under long-term cyclic loading, graded aggregates often undergo micro breakage, mainly in the form of corner and edge spalling [4–7]. In contrast to overall breakage, micro breakage is difficult to detect and develops cumulatively, with its effects becoming progressively apparent during service. Existing research has shown that

micro breakage can induce particle rounding and weaken interparticle interlocking, thereby altering the aggregate skeleton and affecting material behavior [8–10]. Hence, it is necessary to systematically reveal the evolution of micro breakage in graded aggregates under cyclic loading and the underlying mechanisms, so as to provide a basis for evaluating long-term service behavior of high-speed railway subgrades.

Particle breakage significantly affects the performance of granular materials and remains a central topic in this field. It is generally classified into overall breakage and micro breakage [11–13]. In overall breakage, particle size and assembly gradation are significantly affected, and the resulting changes can usually be directly identified by sieve analysis [14–16]. In contrast, micro breakage mainly involves local shape change and fine generation, and therefore often requires more refined observation techniques [17,18]. For example, Wang et al. [19], Xiao et al. [20], and Xie et al. [21] quantified particle shape at different loading or abrasion stages using X-ray CT scanning and three-dimensional reconstruction. The results show that particle roundness gradually increases with loading time, whereas the aspect ratio and particle size change insignificantly. Nevertheless, particle shape can usually only be obtained at discrete stages through costly methods in such research. It is difficult to simultaneously obtain other mesoscopic indicators during the micro breakage. By comparison, the discrete element method (DEM) shows significant advantages in characterizing contact states, force chain structures, and anisotropic evolution, and is therefore widely used to investigate mesoscopic mechanisms in granular materials [22–24]. Aela et al. [25] systematically reviewed DEM applications to railway ballast, including contact mechanics, particle-shape representation, particle breakage and abrasion, and coupled numerical approaches. Ngo et al. [26] combined impact tests with a coupled discrete–continuum model to investigate ballast deformation and breakage, contact-force distributions, coordination number, and energy evolution under impact loading. These studies demonstrate the capability of DEM to reveal particle-scale mechanisms underlying ballast performance. Nevertheless, there are still clear limitations in existing numerical methods for micro breakage. As a result, previous studies have relied on comparative simulations using particle shapes obtained from experiments, and the effect of micro breakage on material response is only analyzed indirectly [27–29]. This poses a challenge for investigating the continuous micro breakage of graded aggregates and the induced mesoscopic evolution. Hence, it is necessary to propose a numerical method that can accurately characterize micro breakage.

It is critical for particle breakage simulation to establish an appropriate breakage criterion and the corresponding breakage-path representation [30,31]. Existing particle-breakage simulation methods mainly include the particle bonding method (PBM) [32,33], the fragment replacement method (FRM) [34,35], and empirical methods [36,37]. Among them, PBM and FRM are widely used in overall breakage simulations, with breakage criteria supported by a relatively clear physical basis. Nevertheless, there are still limitations in local-shape resolution of PBM and FRM for micro breakage simulation. By contrast, empirical methods are computationally efficient in representing micro breakage, whereas the corresponding breakage criteria lack a clear physical basis. It is necessary to develop a DEM-based method for simulating micro breakage, with emphasis on a physically meaningful breakage criterion and efficient shape representation of the breakage path. In terms of the breakage criterion, the energy criterion can establish a link between energy evolution and cumulative characteristics under cyclic loading [38,39]. In terms of shape representation, the method based on Ball elements combined with rolling-resistant contact is computationally efficient, while the ability to represent local shape details remains limited [40–43]. Moreover, the spherical harmonic DEM method provides a new way for high-accuracy representation of irregular particle geometry [44–46]. Hence, the combination of the energy criterion, Ball elements, contact correction, and spherical harmonic rep-

resentation provides a feasible method for continuous simulation of micro breakage in graded aggregates.

To address the above issues, an energy-driven micro breakage method is proposed. In this method, the relationship between contact elastic energy and the fracture energy associated with newly created free surfaces is used as the breakage criterion. Meanwhile, Ball elements with contact correction capability are introduced into the spherical harmonic DEM framework. The Ball elements are used to carry particle kinematics, while a surface radial function is adopted to represent particle shape. The accuracy of the proposed method is validated at multiple levels through single-particle tests, Los Angeles abrasion tests, and cyclic loading tests. Furthermore, dynamic triaxial simulations are further conducted to investigate the mesoscopic evolution and performance degradation induced by micro breakage in graded aggregates. This study contributes to establishing a physically based and computationally efficient framework for micro breakage simulation and provides a reference for understanding and mitigating the progressive degradation of graded aggregates under cyclic loading.

2. Energy-Driven Micro Breakage Method

2.1. Energy-Based Criterion

As shown in Figure 1a, graded aggregates are susceptible to micro breakage under long-term cyclic loading at low stress. This process is mainly characterized by local spalling at high-curvature regions (i.e., corners and edges), together with the generation of fine particles [4,8–10]. It should be noted that the performance of graded aggregates is progressively degraded by continuous micro breakage. Nevertheless, there is still a lack of a physically meaningful criterion for micro breakage in the existing discrete element method (DEM). It is difficult to investigate the mesoscopic evolution and performance degradation of graded aggregates induced by micro breakage under cyclic loading.

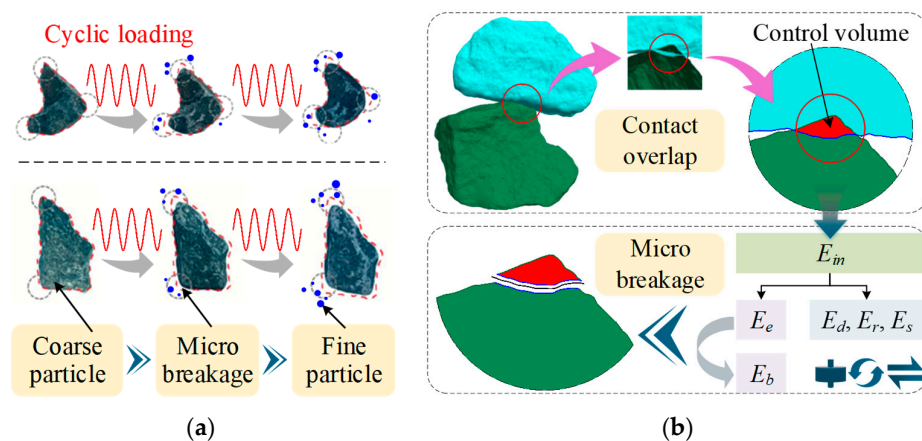


Figure 1. Schematic illustration of the energy-based criterion for micro breakage in DEM: (a) Micro breakage definition and (b) Energy transformation in contact.

To address this issue, an energy criterion for micro breakage is established based on the first law of continuum mechanics [47]. Firstly, under the assumptions of small deformation and negligible heat exchange, the energy balance of an arbitrary control volume $V \subset \Omega$ (boundary ∂V , outward unit normal $\mathbf{n}$) can be expressed by Equation (1).

$$E_{\text{ext}} = E_{\text{int}} + E_{\text{kin}} + E_{\text{d}} \quad (1)$$

where E_{ext} is the work done by external forces, E_{int} is the change in internal energy, E_{kin} is the change in kinetic energy, and E_d is the irreversible energy dissipation (e.g., sliding, rolling, damping, and breakage).

Secondly, the basic units in DEM are assumed to be non-deformable rigid bodies, and particle deformation is equivalently represented by contact overlaps. As shown in Figure 1b, the overlap volume corresponding to the contact overlap region is defined as the control volume V_i . It should be noted that this control volume is an equivalent representation of the local high-energy region near the contact, rather than the actual fragment volume generated by particle micro breakage. This assumption is consistent with laboratory observations showing that micro breakage of graded aggregates under repeated loading mainly occurs as local spalling and abrasion at particle corners and edges [20,48]. Then, V_i is treated as a continuum, where ∂V_i is the intersection boundary of the contacting particles and $\mathbf{n}$ is the contact direction. Based on this, the energy balance of V_i in conventional DEM is expressed by Equation (2), in which E_{ext} is mainly converted into E_e and E_d , and E_d includes sliding E_s , rolling E_r , and damping dissipation E_{damp} . It should be noted that an explicit breakage energy term is absent in conventional DEM. Meanwhile, E_s , E_r , and E_d are mainly dissipated continuously, whereas E_e continues to accumulate with increasing contact overlap. Hence, the energy available for micro breakage is mainly stored in the contact region as E_e .

$$\begin{cases} E_{\text{ext}} = E_e + E_d & , \text{Conventional DEM} \\ E_{\text{ext}} = E_e + E_d + E_b & , \text{This study} \end{cases} \quad (2)$$

where E_e is the elastic energy, uniquely determined by the elastic state variables at the contact, E_d is the irreversible dissipated energy, and E_b is the micro breakage energy.

Thirdly, it is crucial for the characterization of micro breakage to separate E_b from E_e in a physically meaningful manner. The Griffith energy criterion in linear elastic fracture mechanics states that crack propagation and the formation of newly created free surfaces require the corresponding fracture energy [49]. In DEM, micro breakage is equivalently represented as local spalling associated with the contact overlap region. Accordingly, the newly created free surface A_i is approximated by the geometric boundary of the overlap region. Based on this, E_b is expressed by Equation (3).

$$E_b = G_c A_i \quad (3)$$

where G_c is the breakage energy per unit area (critical energy release rate).

Finally, the micro breakage criterion is defined by comparing the relative magnitudes of E_e and E_b . It should be noted that the comparison between E_e and E_b at a given instant is essentially an instantaneous criterion. Micro breakage occurs only when the elastic energy associated with a single contact event exceeds the corresponding breakage energy. Nevertheless, micro breakage in graded aggregates under long-term cyclic loading exhibits a cumulative character. When the instantaneous breakage conditions remain unmet, local damage can accumulate progressively during repeated loading-unloading cycles and eventually lead to breakage. Furthermore, a cumulative breakage factor ℓ is introduced to characterize the cumulative evolution of micro breakage under long-term cyclic loading, as expressed in Equation (4). Accordingly, two breakage modes are considered, namely the cumulative mode (model I) and the instantaneous mode (model II). The micro breakage is described using Equation (5). After each micro breakage event, ℓ is updated according to the corresponding breakage mode, thereby ensuring the continuity and progressive nature of micro breakage [Equation (6)].

$$\ell_{\Delta t+1} = \ell_{\Delta t} + \frac{\max(0, E_{e,\Delta t+1} - E_{e,\Delta t})}{G_c A_i} \quad (4)$$

$$\text{Breaking} = \begin{cases} \text{True} & , \quad l_{n+1} > 1 \quad (\text{model I}) \\ \text{True} & , \quad E_e > E_b \quad (\text{model II}) \\ \text{False} & , \quad \text{otherwise} \end{cases} \quad (5)$$

$$l = \begin{cases} 1 - l & , \quad \text{model I} \\ 0 & , \quad \text{model II} \\ 1 & , \quad \text{otherwise} \end{cases} \quad (6)$$

where $\Delta t + 1$ and Δt are the current and previous time steps, respectively.

2.2. Shape Representation and Update Strategy

It is critical to accurately calculate E_e and A_i in the contact region for characterizing micro breakage within the proposed energy criterion. Nevertheless, geometric approximations such as bonded sub-particle representations or discretized boundaries are mostly adopted in existing approaches for simulating irregular particles [50–52]. As a result, it is difficult to accurately represent high-curvature regions such as corners and edges, which poses a challenge to the accurate determination of A_i . Hence, a high-resolution representation method for irregular particles is proposed to ensure compatibility with the energy criterion.

As shown in Figure 2, irregular particles in the assembly are represented by the Ball kinematic carrier, the shape radial function $r(\theta, \varphi)$ [Equation (7)], and the rolling-resistant Hertz (RRHertz) contact model. Notably, the motion of the Ball is updated according to the Newton-Euler equations. The RRHertz contact model is developed from the Hertz contact model by incorporating a rolling-resistance component, and the detailed formulation is given in Refs. [41,53–55]. When particles come into contact, the contact curvature κ is obtained from $r(\theta, \varphi)$ and then used to correct the stiffness coefficient k in the RRHertz model [Equations (8)–(10)]. This further affects the contact force $\mathbf{F}$ and moment $\mathbf{M}$ between particles, and consequently the translational velocity $\mathbf{v}$ and angular velocity $\boldsymbol{\omega}$ of each particle. Meanwhile, the contact-point direction in $r(\theta, \varphi)$ is determined by the Ball orientation through the rotation matrix $\mathbf{R}$.

$$r(\theta, \varphi) = \sum_{n=0}^{N_s} \sum_{m=-n}^n a_n^m Y_n^m(\theta, \varphi), \quad \theta \in [0, \pi] \text{ and } \varphi \in [0, 2\pi] \quad (7)$$

$$k_n = \frac{2\pi}{\alpha} \sqrt{\frac{\mathbb{E}}{(\kappa_m + \kappa_n) \mathbb{K}^3}} G|\delta|^{0.5} \quad (8)$$

$$k_s = \frac{\pi b \left(2\eta_1 \mathbb{K} - 2\eta_2 \frac{\mathbb{K} - \mathbb{E}}{e^2} \right)^{-1} + \pi b \left[2\eta_1 \mathbb{K} + 2\eta_2 \frac{(1 - e^2) \mathbb{K} - \mathbb{E}}{e^2} \right]^{-1}}{2} \quad (9)$$

$$k_r = k_s \left(\frac{1}{r_A} + \frac{1}{r_B} \right)^{-2} \quad (10)$$

where a_n^m is the spherical harmonic coefficient, N_s is the truncation order. $Y_n^m(\theta, \varphi)$ is the spherical harmonic basis function, the detailed formulation can be found in Ref. [46]. G is the equivalent shear modulus, α is the aspect ratio of the contact ellipse, δ is the normal overlap, κ_m and κ_n are the relative curvatures at the contact, $\mathbb{K}$ and $\mathbb{E}$ are the complete elliptic integrals of the first and second kinds, respectively, as functions of the contact el-

lipse eccentricity e . η_1 and η_2 are the combined compliance coefficients associated with the shear moduli and Poisson's ratios of the two contacting particles, corresponding to the isotropic and directional terms in Mindlin's full-stick solution, respectively. r_A and r_B are the radii of the two particles at the contact ends, respectively.

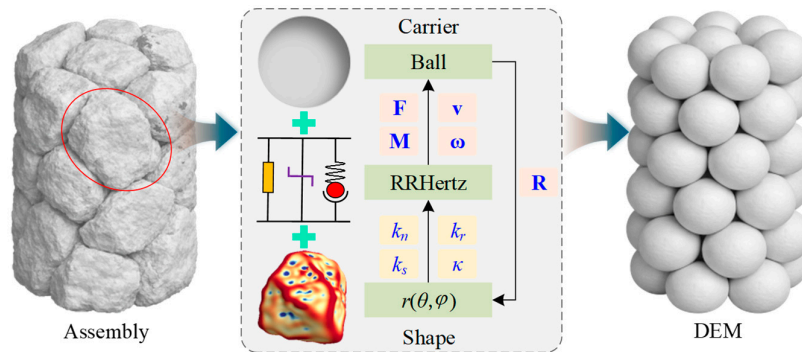


Figure 2. Schematic illustration of the equivalent calculation scheme based on a ball carrier and radial-function-based contact stiffness correction.

Additionally, the identification of corner and edge regions on particles is a prerequisite for the applicability of the energy-driven micro breakage criterion. It can be observed that corners and edges generally correspond to relatively large local curvature. Hence, the curvature is used to determine whether the contact point is in a corner or edge region. Specifically, a quadratic surface is fitted to the local surface in the neighborhood of the contact point [Equation (11)], and the maximum $\kappa_{\max}$ and minimum $\kappa_{\min}$ principal curvatures are calculated from the second derivatives of the fitted surface [56,57], as shown in Equations (12) and (13).

$$z = f(x, y) \approx a + b(x - x_0) + c(y - y_0) + d(x - x_0)(y - y_0) + e(x - x_0)^2 + h(y - y_0)^2 \quad (11)$$

$$\mathbf{H}(x_0, y_0) = \begin{bmatrix} f_{xx}(x_0, y_0) & f_{xy}(x_0, y_0) \\ f_{yx}(x_0, y_0) & f_{yy}(x_0, y_0) \end{bmatrix} \quad (12)$$

$$\kappa_{\max}, \kappa_{\min} = \text{eig}(\mathbf{H}) \quad (13)$$

where a, b, c, d, e and h are the coefficients obtained by least-squares fitting, $\text{eig}(\cdot)$ is the operator that computes the eigenvalues of a matrix.

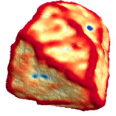
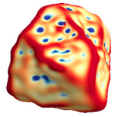
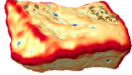
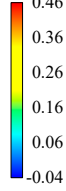
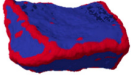
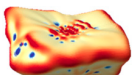
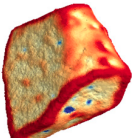
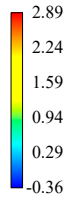
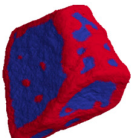
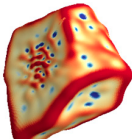
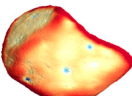
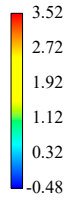
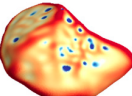
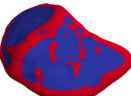
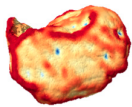
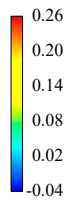
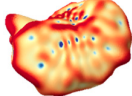
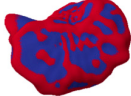
Existing research shows that $\kappa_{\max}$ exhibits higher sensitivity than $\kappa_{\min}$ to geometric discontinuities such as corners and edges [58]. The curvature function $g(\kappa)$ is further defined in Equation (14). When $g(\kappa) = 1$, the contact point is identified as being in a corner or edge region.

Table 1 presents the $\kappa_{\max}$ distribution and the identified corner and edge regions on both the real shape and $r(\theta, \varphi)$. A close agreement is observed between the two results, indicating that the adopted method accurately captures the corner and edge features of the particle.

$$g(\kappa) = \begin{cases} 1, & |\kappa_{\max}|^{-1} < r_{\text{ins}} \\ 0, & |\kappa_{\max}|^{-1} \geq r_{\text{ins}} \end{cases} \quad (14)$$

where r_{ins} is the maximum inscribed sphere radius of the particle.

Table 1. Comparison of $\kappa_{\max}$ distribution and identified corner-edge regions between the real particle shape and $r(\theta, \varphi)$.

No.	Indicators			
	Shape	$\kappa_{\max}$		$g(\kappa)$
1	Real			
	$r(\theta, \varphi)$			
2	Real			
	$r(\theta, \varphi)$			
3	Real			
	$r(\theta, \varphi)$			
4	Real			
	$r(\theta, \varphi)$			
5	Real			
	$r(\theta, \varphi)$			

Accurate updating of local shape changes induced by micro breakage is essential for continuous simulation. The $r(\theta, \varphi)$ effectively captures the micro shape evolution induced by micro breakage through updates of the spherical harmonic coefficients. The specific updating procedure is described as follows.

- Calculation of overlap-region parameters

According to the basic assumptions of Hertzian contact theory, the contact plane of irregular particles is equivalently treated as an ellipse, and A_i is calculated from the curvature at the contact point, as given in Equation (15). The overlap volume V_i of particle

contact is then obtained, which corresponds to the control volume in micro breakage, as given in Equation (16).

$$A_i = 2\pi\delta\sqrt{\frac{1}{\kappa_{\max}\kappa_{\min}}} \quad (15)$$

$$V_i = \frac{1}{2}A_i\delta = \pi\delta^2\frac{1}{\sqrt{\kappa_{\max}\kappa_{\min}}} \quad (16)$$

- Calculation of a_0^0 associated with the Ball radius

After micro breakage, the updated Ball radius r_i^* can be obtained from V_i , as given in Equation (17). Then, according to the relation between the constant term in $r(\theta, \varphi)$ and the equivalent radius, a_0^{0*} is obtained from r_i^* , as given in Equation (18).

$$r_i^* = \left(r_i^3 - \frac{3V_i}{4\pi}\right)^{\frac{1}{3}} \quad (17)$$

$$a_0^{0*} = r_i^*\sqrt{4\pi} \quad (18)$$

- Calculation of a_n^m ($n \geq 1$) associated with local shape

As shown in Figure 3, the contact normal $\mathbf{n}$ is first transformed into the particle coordinate system through the orientation matrix $\mathbf{R}$. The resulting unit vector $\mathbf{u}_i$ defines the action direction of micro breakage on the particle surface [Equation (19)].

$$\mathbf{u}_i = \mathbf{R}^T \mathbf{n} \quad (19)$$

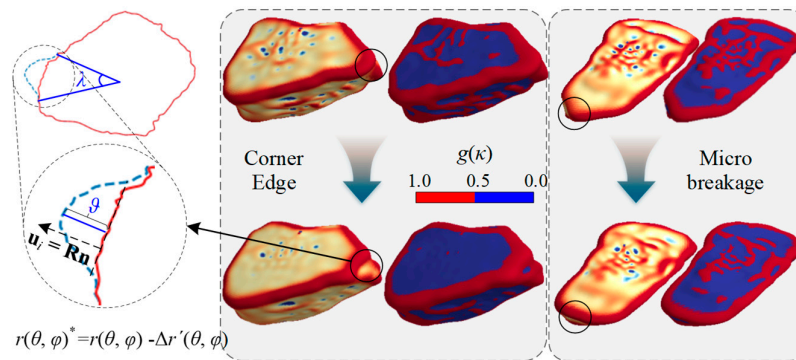


Figure 3. Geometric representation of micro breakage through radial function updating.

To localize the breakage effect on the spherical domain, an angular distance $\gamma(\theta, \varphi)$ is introduced between an arbitrary spherical direction and $\mathbf{u}_i$. This quantity is defined from the spherical unit vector $\mathbf{s}(\theta, \varphi)$, as given in Equations (20) and (21).

$$\mathbf{s}(\theta, \varphi) = (\sin\theta\cos\varphi, \sin\theta\sin\varphi, \cos\theta)^T \quad (20)$$

$$\gamma(\theta, \varphi) = \arccos[\mathbf{u}_i \cdot \mathbf{s}(\theta, \varphi)] \quad (21)$$

The spatial extent of breakage is then described by a Gaussian kernel $K(\gamma)$ [Equation (22)]. With this kernel, the local radial loss $\Delta r(\theta, \varphi)$ is constructed in Equation (23), where λ controls the angular spread and ϑ represents the breakage depth. A smooth kernel is adopted to avoid discontinuous truncation of the surface update and to ensure numerical stability during shape evolution.

$$K(\gamma(\theta, \varphi)) = \exp\left(-\frac{\gamma(\theta, \varphi)^2}{2\lambda^2}\right) \quad (22)$$

$$\Delta r(\theta, \varphi) = \vartheta K(\gamma(\theta, \varphi)) \quad (23)$$

where λ is the angular width parameter of $K(\gamma)$, ϑ is the breakage depth.

To avoid the global scale bias caused by the nonzero spherical mean of $\Delta r(\theta, \varphi)$, a mean-removal operation is introduced. The spherical average $\langle \Delta r \rangle$ is calculated [Equation (24)], and the zero-mean correction $\Delta r'(\theta, \varphi)$ is then obtained by Equation (25).

$$\langle \Delta r \rangle = \frac{1}{4\pi} \int_0^{2\pi} \int_0^\pi \Delta r(\theta, \varphi) \sin \theta d\theta d\varphi \quad (24)$$

$$\Delta r'(\theta, \varphi) = \Delta r(\theta, \varphi) - \langle \Delta r \rangle \quad (25)$$

After that, $\Delta r'(\theta, \varphi)$ is expanded onto the spherical harmonic basis $Y_n^m(\theta, \varphi)$, yielding the coefficient increment Δa_n^m ($n \geq 1$) [Equation (26)]. The post-breakage coefficients a_n^{m*} ($n \geq 1$) are subsequently obtained through Equation (27). The projection in Equation (26) is evaluated numerically by discrete quadrature, and the corresponding implementation is provided in Appendix A.

$$\Delta a_n^m = \int_0^{2\pi} \int_0^\pi \Delta r'(\theta, \varphi) Y_n^m(\theta, \varphi) \sin \theta d\theta d\varphi, \quad n \geq 1 \quad (26)$$

$$a_n^{m*} = a_n^m - \Delta a_n^m, \quad n \geq 1 \quad (27)$$

With the updated coefficients, the post-breakage radial function $r(\theta, \varphi)^*$ is finally reconstructed by Equation (28).

$$r(\theta, \varphi)^* = r(\theta, \varphi) - \Delta r'(\theta, \varphi) \quad (28)$$

3. Numerical Model

3.1. Dynamic Triaxial Setup

To investigate the micro breakage-induced mesoscopic evolution and mechanical degradation of graded aggregates under cyclic loading, a series of dynamic triaxial simulations is conducted. The main modeling procedure is summarized below, and additional implementation details are provided in Refs. [3,59,60]. As shown in Figure a, the established model has a size of $300 \times 300 \times 300$ mm, and the particle size ranges from 5 to 45 mm. The minimum particle size of breakable particles is 7.1 mm. Moreover, the model boundary conditions are determined according to the loading characteristics of the surface layer of high-speed railway subgrade beds. Existing research shows that the confining pressure of railway subgrade fillers in triaxial tests is generally within 20–40 kPa [2,3,61], and therefore, the confining pressure σ_3 is set to 30 kPa. As shown in Figure b, the loading frequencies are taken as 10, 15, 20, 25, and 30 Hz, which are generally consistent with the frequency characteristics of cyclic loading induced by high-speed trains. Furthermore, to analyze the evolution of micro breakage in graded aggregates under different loading levels, the dynamic stress amplitudes σ_1 are set to 80, 100, 120, 140, and 160 kPa. Considering the high computational cost associated with repeated contact calculations, micro breakage detection, and particle-shape updating in DEM, the total number of loading cycles is set to 1000.

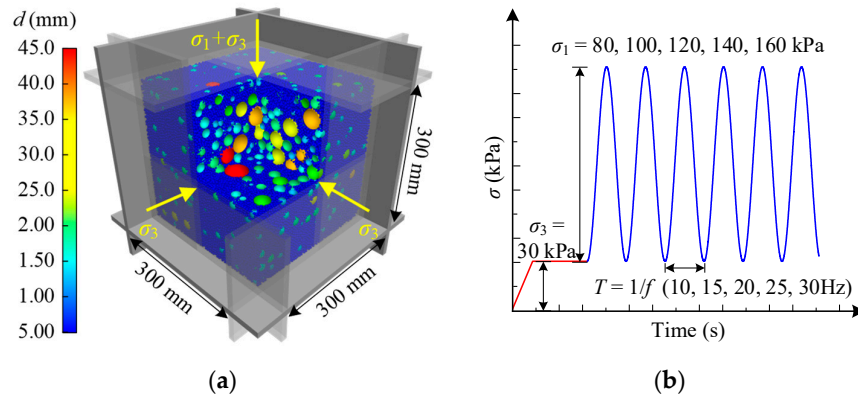


Figure 4. Numerical model: (a) Model setup and (b) Loading.

Additionally, interparticle interactions in DEM are governed by the contact parameters, and the macroscopic response of the specimen is strongly affected by these parameters. Hence, appropriate contact-model selection and parameter calibration are essential for ensuring the reliability of the numerical simulations. As described in Section 2.2, particle-particle interactions are represented by the RRHertz contact model, whereas particle-wall interactions are represented by the Linear contact model. A hierarchical calibration strategy is adopted in this study. The shear modulus and Poisson's ratio in the RRHertz contact model, together with the normal and tangential stiffnesses in the Linear contact model, are constrained by single-particle tests and static triaxial tests. The interparticle friction coefficient and rolling resistance coefficient are calibrated using the angle of repose test, while the particle-wall friction coefficient is calibrated using the inclined plane test. The damping coefficients for particle-particle and particle-wall contacts are determined from drop tests. Detailed calibration procedures, target responses, and data analyses have been reported in previous studies [10,21,62]. The final parameter set adopted for the dynamic triaxial simulation of graded aggregates is listed in Table 1.

Table 1. Contact parameters used in the dynamic triaxial simulation of graded aggregates.

Type	Parameter	Symbol	Value	Unit	Source
Global	Particle Density	ρ	2750	kg/m ³	Material test
	Numerical Damp	D_a	0.5	—	Numerical calibration
	setting Time step	Δt	1×10^{-6}	s	Stability analysis
Contact	Shear modulus	G	5×10^8	Pa	Single-particle and triaxial tests
	Particle to Poisson's ratio	ν	0.25	—	Material property
	particle Friction coefficient	f	0.5	—	Repose angle tests
	(RRHertz) Rolling resistance coefficient	f_r	0.2	—	
	Normal damping ratio	$d_{p,n}$	0.27	—	Ball drop test
	Shear damping ratio	$d_{p,s}$	0.27	—	
	Normal stiffness	k_n	1×10^8	Pa	Boundary material property
	Particle to Tangential stiffness	k_s	0.85×10^8	Pa	
	boundary Friction coefficient	f_w	0.21	—	
	(Linear) Normal damping ratio	β_{nw}	0.62	—	
Micro breakage	Tangential damping ratio	β_{sw}	0.62	—	Ball drop test
	SH truncation order	N_s	20	—	Reconstruction accuracy
	Discrete quadrature points	N_q	3200	—	Convergence analysis
	SH coefficients	a_n^m	—	—	Reconstructed geometries

Critical breakage energy per unit area	G_c	1.0×10^4	J/m ²	Single-particle test
Minimum breakable radius	$r_{b,min}$	2	mm	
Initial cumulative breakage factor	ℓ_0	0	—	Initial condition

3.2. Verification of the Breakage Method

To evaluate the performance of the proposed energy-driven micro breakage method, three independent validation tests, namely single-particle compression, Los Angeles abrasion, and cyclic loading, are conducted using separately prepared specimens, covering responses from the particle scale to the assembly scale [10,20,48]. The test material is graded aggregate used as the surface layer filler of the high-speed railway subgrade bed, with limestone as the parent rock. As shown in Figure a, a multifunctional testing system with a maximum loading capacity of 100 kN and a force resolution of 1 N is used to conduct loading tests on a single graded aggregate particle. The loading rate is set to 0.5 mm/min, and the force and deformation responses of the particle are recorded in real time during the loading. It is worth noting that G_c is first calibrated based on the single-particle compression tests, and the calibrated value $G_c = 1.0 \times 10^4$ J/m² is adopted as the model input parameter in the subsequent simulations.

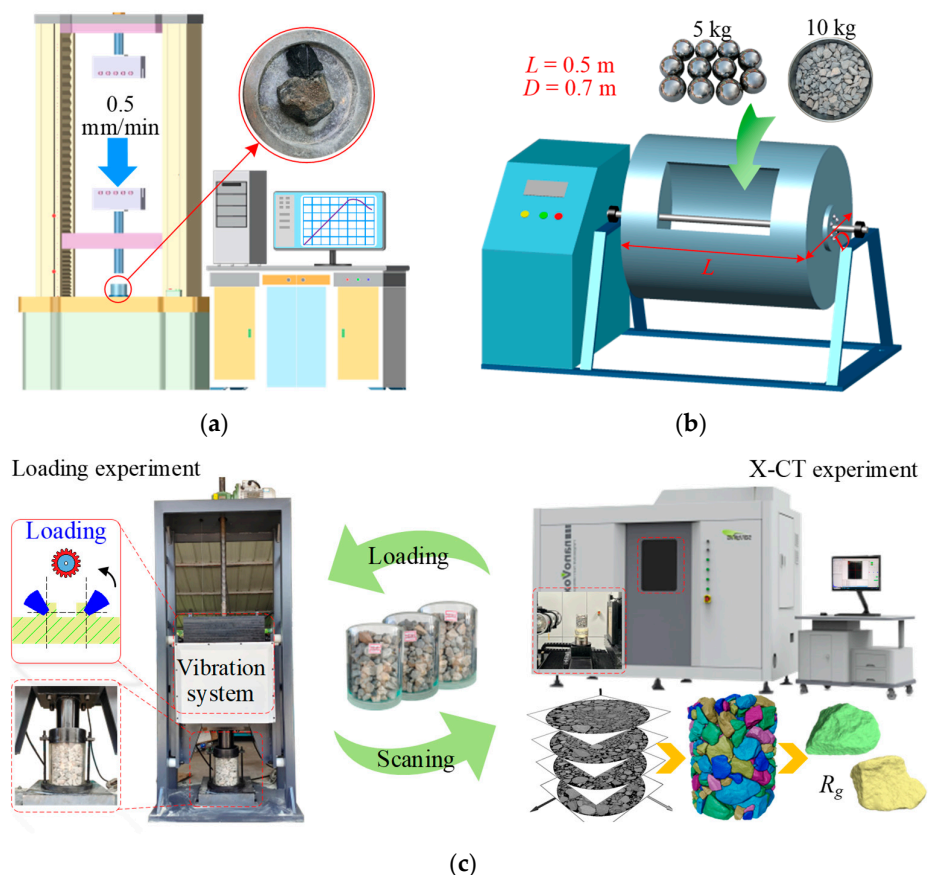


Figure 5. Verification tests: (a) Single-particle test, (b) Los Angeles abrasion test, and (c) Cyclic loading test.

As shown in Figure b, a Los Angeles abrasion test is conducted on graded aggregates with particle sizes ranging from 20 to 45 mm. The mass of the graded aggregate is 10 kg, together with 12 steel balls with a total mass of 5 kg. The inner length and inner diameter

of the test drum are 0.5 m and 0.7 m, respectively. The abrasion ratio LAA of the graded aggregates is recorded after 200, 500, 1000, 1500, and 2000 revolutions, and LAA is calculated using Equation (29).

$$\text{LAA} = \frac{m_0 - m_1}{m_0} \times 100\% \quad (29)$$

where m_0 and m_1 are the initial aggregate mass and the post-test mass, respectively.

Furthermore, as shown in Figure c, a cyclic loading test is conducted on a cylindrical specimen composed of limestone graded aggregates with particle sizes ranging from 10 to 45 mm. The mass fractions of the particle-size groups of 10–15, 15–20, 20–25, 25–30, 30–40, and 40–45 mm are 20%, 20%, 20%, 20%, 10%, and 10%, respectively. The specimen has a diameter of 150 mm and an initial height of 150 mm. The aggregates are thoroughly mixed according to the prescribed gradation and placed into the cylindrical container in layers. The specimen is prepared in four layers to an initial dry density of 1760 kg/m³, corresponding to an initial porosity of 0.36. The aggregates are tested under dry conditions. After sample preparation, the specimen surface is levelled, and the loading plate is installed on the top of the specimen. The excitation force is generated by the rotation of an eccentric block. The vibration frequency is set to 32 Hz, the static load is 600 kg, the eccentric mass is 2.4 kg, and the eccentricity is 3.32 mm. The specimen is continuously loaded for 350 s, and the loading-plate displacement is recorded throughout the test. Further details of the apparatus and data-processing procedure are provided in Ref. [10].

The total loading duration is 350 s. To capture the local shape evolution induced by micro breakage during loading, X-CT scans are conducted on the sample after 50 s and 350 s of loading, respectively. After the first loading stage (i.e., 50 s), the specimen is unloaded and transferred to the X-ray CT system for scanning and is then reinstalled in the loading apparatus for continued loading. During unloading, transfer, and scanning, the specimen remains inside the original rigid cylindrical container. The specimen orientation and loading position are maintained during reinstallation. Based on the scanning results, STL models of single particle shapes are obtained through three-dimensional reconstruction, and the particle roughness R_g is further calculated. R_g is determined from $\kappa_{\max}$, as given in Equation (30). Further details of the X-CT test parameters and slice image processing procedure can be found in Ref. [20].

$$R_g = \frac{\sum g(\kappa) |\kappa_{\max}|^{-1}}{N \cdot r_{\text{ins}}} \quad (30)$$

where N is the total number of identified corners and edges.

As shown in Figure 6a, the proposed method reasonably reproduces the nonlinear force-displacement response, multiple local force drops, and progressive micro breakage behavior observed in the single-particle compression tests. The representative micro breakage triggering points P1, P2, and P3 are identified from the experimental and DEM curves. Figure 6b,c further compare the displacement and force at these triggering points. The relative errors in displacement range from 3% to 20%, while those in force range from 0% to 15%. These results indicate that the proposed method can quantitatively capture the staged variation in particle load-bearing capacity associated with local micro breakage.

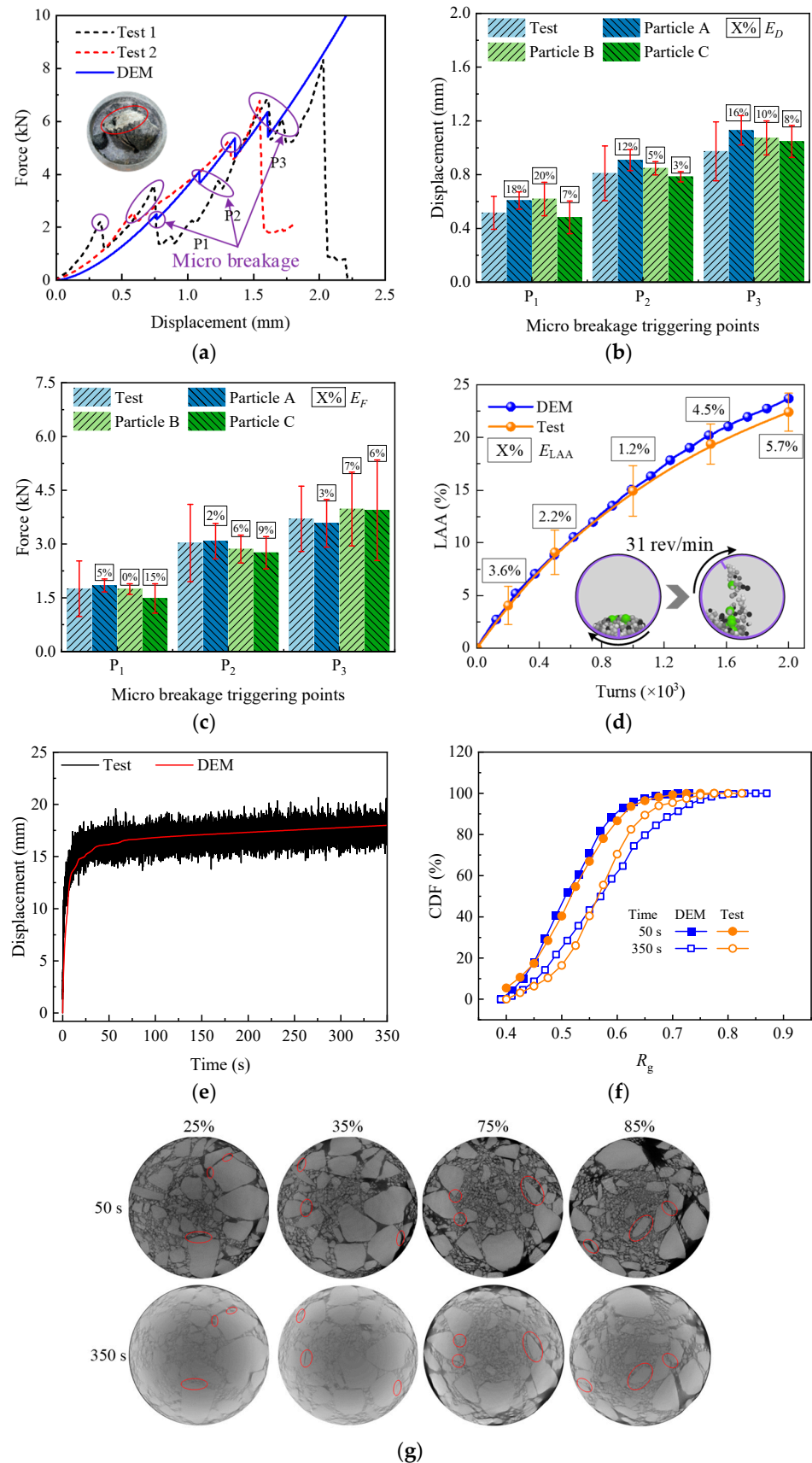


Figure 6. Quantitative validation results: (a) Force-displacement responses in single-particle compression tests, (b) Displacement errors at representative micro breakage triggering points, (c) Force

errors at representative micro breakage triggering points, (d) LAA evolution in the Los Angeles abrasion test, (e) Displacement evolution during cyclic loading, (f) CDFs of R_g during cyclic loading; and (g) Representative X-ray CT slices at different normalized specimen heights after 50 s and 350 s of cyclic loading.

As shown in Figure 6d, the LAA values obtained from both the experiment and DEM increase continuously with the number of drum turns. The relative errors at the five representative abrasion stages are 3.6%, 2.2%, 1.2%, 4.5%, and 5.7%, respectively, all of which are below 6%. The close agreement demonstrates that the proposed method can reasonably reproduce the cumulative mass loss caused by repeated impact and abrasion.

The macroscopic deformation and particle-shape evolution under cyclic loading are compared in Figure 6e–g. As shown in Figure 6e, the specimen displacement increases rapidly during the initial loading stage and then gradually approaches a relatively stable evolution stage. The DEM result follows the experimental displacement evolution reasonably well. Figure 6f shows that the cumulative distributions of R_g obtained from the DEM simulation and experiment are consistent at both 50 s and 350 s. With increasing loading time, the distributions shift toward larger R_g values, indicating the progressive weakening of particle angularity and particle rounding.

Figure 6g presents representative X-ray CT slices at different normalized specimen heights after 50 s and 350 s of cyclic loading. The red ellipses indicate representative regions exhibiting local particle-morphology changes. The comparison shows changes in particle contours, particle arrangement, and local packing structure during cyclic loading, providing direct visual support for the particle-scale morphology evolution reflected by the R_g distributions. Overall, the comparisons of force-displacement response, triggering-point errors, LAA evolution, specimen displacement, R_g distribution, and CT images demonstrate that the proposed method can reasonably characterize micro breakage-induced mass loss, macroscopic deformation, and particle-shape evolution from the particle scale to the assembly-response scale.

It should be noted that fragments smaller than 2 mm are not explicitly generated as independent particles in the present DEM model. Their formation is represented through equivalent local material loss; therefore, the independent mechanical contribution of these fine fragments is not quantified in the present simulations.

To evaluate the effect of key parameters on the model response, a parameter sensitivity analysis is conducted based on the single-particle test. The shear modulus G , G_c , particle size d , spherical-harmonic truncation order N_s , discrete quadrature point number N_q , and time step Δt are selected as the sensitivity parameters.

As shown in Figure 7a, with increasing G , the force-displacement curves shift upward as a whole, and both the curve slope and contact force level increase markedly. The number of micro breakage events N_b also increases gradually. This indicates that a larger G accelerates the accumulation of contact energy. As shown in Figure 7b, increasing G_c raises the micro breakage threshold, reduces the number of force drops in the force-displacement curve, and generally decreases N_b . This indicates that G_c is a key parameter controlling the micro breakage frequency. In Figure 7c, with increasing d , the bearing capacity of the single particle increases significantly, whereas N_b tends to become stable for larger particle sizes. This suggests that particle size mainly affects the bearing capacity, while its influence on the number of micro breakage events is relatively limited.

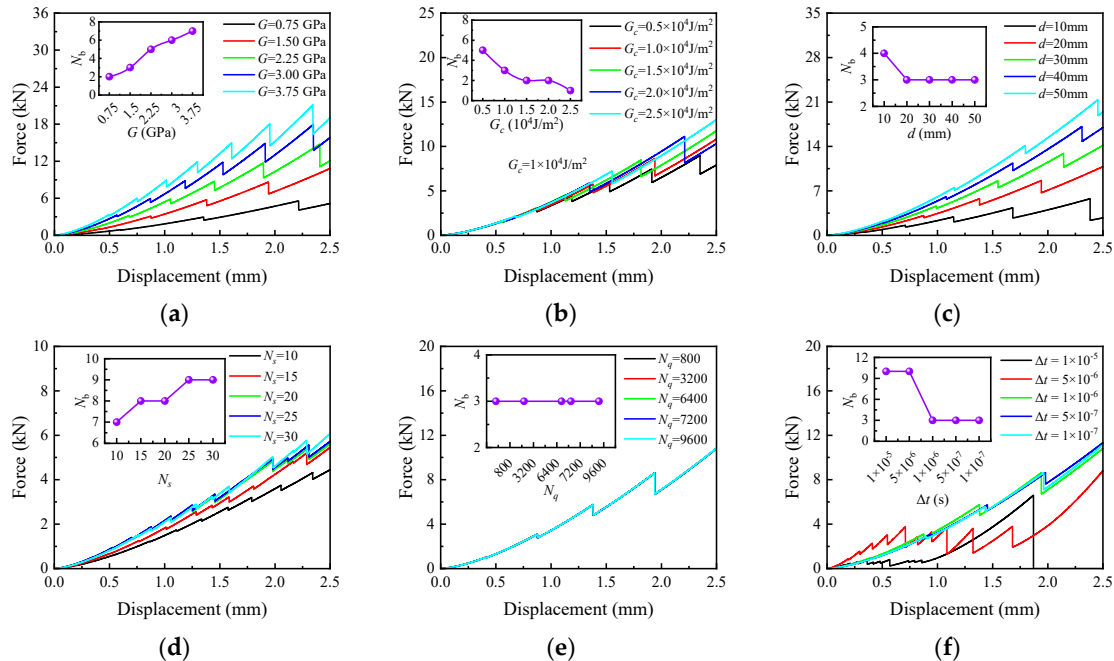


Figure 7. Sensitivity analysis of key parameters in the single-particle test: (a) G , (b) G_c , (c) d , (d) N_s , (e) N_q , and (f) Δt .

For the shape and numerical parameters, in Figure 7d, as N_s increases, more local shape details are represented, and both the force-displacement response and N_b change to some extent. This indicates that the shape-representation resolution affects the local contact state and micro breakage triggering. In Figure 7e, the force-displacement curves and N_b are almost consistent under different N_q values, indicating that the number of spherical quadrature points has a limited effect on the calculation results and that the adopted quadrature resolution is sufficient. Figure 7f shows that a relatively large time step leads to noticeable deviations in both the force-displacement response and N_b . When Δt is reduced to 1×10^{-6} s or below, the force-displacement curves and N_b become nearly stable.

In summary, the validation results show reasonable agreement between the numerical and experimental responses under the considered single-particle, abrasion, and cyclic-loading conditions. The proposed method can therefore be used to investigate micro breakage-induced mesoscopic evolution within the validated range.

4. Results and Analysis

This section first quantifies the effects of cyclic loading frequency and amplitude on LAA, mechanical coordination number CN , average contact force f_c , and axial plastic strain ε_p of graded aggregates. Based on these results, the mesoscopic evolution and performance degradation induced by micro breakage are further investigated.

4.1. Micro Breakage Characteristics

As shown in Figure , the LAA increases with the number of loading cycles N under different loading conditions, and its increase rate becomes higher as the frequency and amplitude increase. This indicates that micro breakage of graded aggregates under cyclic loading exhibits a clear cumulative characteristic. In Figure 8a, when $N = 10^3$, the LAA increases with frequency, with a more significant increase observed when the frequency exceeds 20 Hz. Notably, existing research indicates that granular materials such as railway ballast exhibit more significant dynamic effects when the frequency exceeds 20 Hz [63,64].

This suggests that the dynamic response of the particle system is significantly enhanced above 20 Hz, thereby promoting the accumulation of micro breakage.

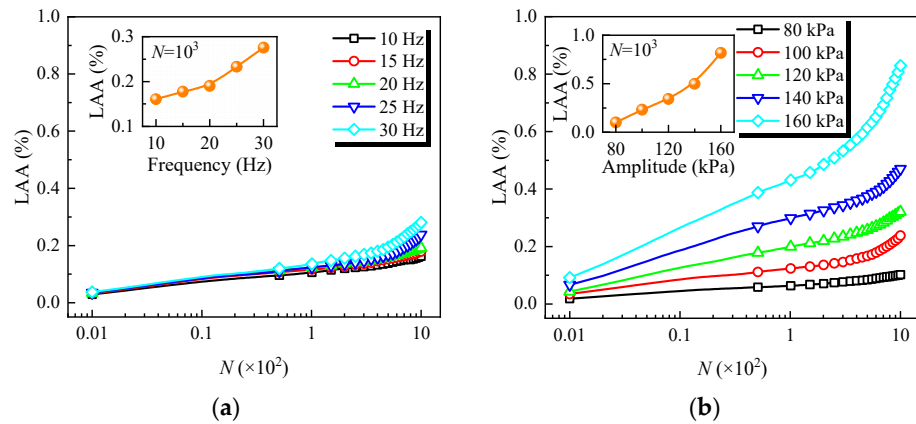


Figure 8. Effects of loading parameters on LAA: (a) Frequency and (b) Amplitude.

Additionally, as shown in Figure b, the LAA increases with loading amplitude, and its sensitivity to amplitude is significantly greater than that to frequency. This indicates that stress level is the main factor controlling micro breakage at small N . As N increases, the cumulative characteristic associated with frequency gradually becomes more significant, and its effect on micro breakage correspondingly increases. This is attributed to the fact that a larger amplitude directly increases the elastic energy stored at contacts, making high-curvature regions more likely to reach breakage conditions. In contrast, a higher frequency mainly enhances the cumulative effect (i.e., cumulative breakage factor ℓ) during cyclic loading and has a relatively limited influence on the contact energy level in a single cycle. The increase in LAA also implies more corner and edge spalling, progressive particle rounding, and fine generation. These changes facilitate particle rearrangement, thereby promoting volumetric contraction of the sample.

Furthermore, as shown in Figure , the curvature-sensitive particle roughness R_g under different loading conditions is quantified. R_g gradually increases with N , and the increase is more significant at higher frequency and amplitude, which is generally consistent with the variation of LAA in Figure . This indicates that micro breakage under cyclic loading gradually weakens the corner and edge features of particles, leading to a more rounded overall shape. Moreover, as N increases from 0 to 1000, the variation in R_g remains relatively small, and particle shape evolution mainly involves local adjustments, reflecting the cumulative characteristic of micro breakage.

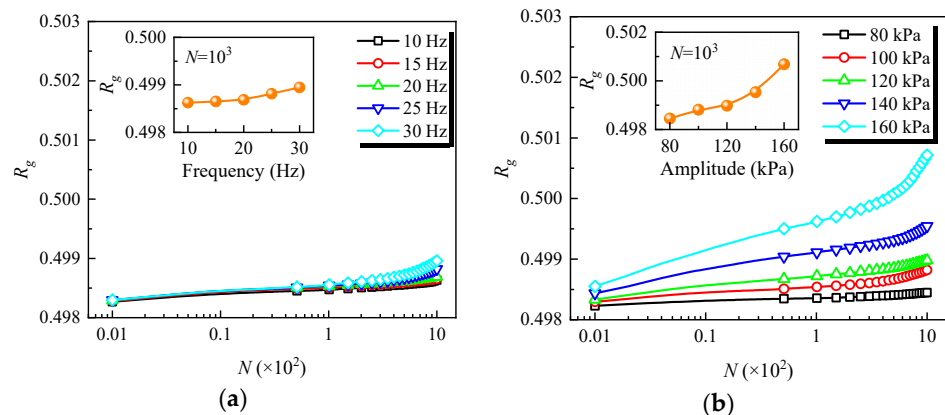


Figure 9. Effects of loading parameters on R_g : (a) Frequency and (b) Amplitude.

In summary, the increase in micro breakage with N is more significant under high frequency and large amplitude, reflecting a significant cumulative characteristic. Hence, it is necessary to further investigate the mesoscopic evolution and performance degradation of graded aggregates induced by micro breakage. This is important for revealing the stage-dependent degradation mechanism within the simulated cyclic-loading range and for supporting service performance evaluation.

4.2. Mechanical Coordination Number

The coordination number CN is an important mesoscopic parameter characterizing the compactness of the particle system and is calculated by Equation (31). As shown in Figure a,b, the mechanical coordination number CN increases with N under different loading conditions, and its sensitivity to amplitude is significantly greater than that to frequency. This indicates that the particle contact network is continuously rearranged under cyclic loading. As a result, the graded aggregate skeleton gradually approaches a denser state, with amplitude showing a more significant influence. Furthermore, to investigate the effect of micro breakage on the contact structure, CN is compared between the breakage and non-breakage conditions.

$$CN = \frac{2C - N_1}{N_a - N_1 - N_0} \quad (31)$$

where C is the number of inter-particle contacts, N_1 is the number of particles with only one contact, N_0 is the number of particles without contact, and N_a is the total number of particles.

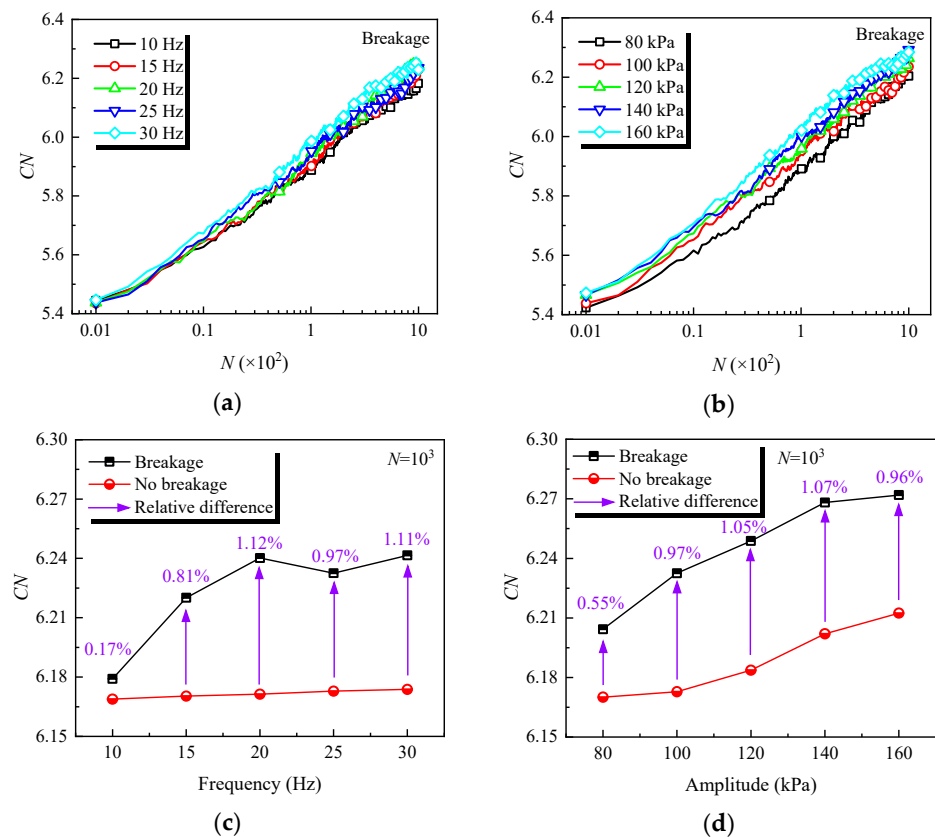


Figure 10. Effects of loading parameters on CN : (a) Frequency under breakage conditions, (b) Amplitude under breakage conditions, (c) Relationship between frequency and CN , and (d) Relationship between amplitude and CN .

As shown in Figure c, CN increases with frequency under both breakage and non-breakage conditions, with a more significant increase under breakage conditions, indicating that micro breakage enhances the compactness of graded aggregates. When the frequency exceeds 20 Hz, CN under breakage conditions remains nearly stable. Relative to the non-breakage case, CN under breakage conditions increases by 0.17% at 10 Hz and by 1.12% at 20 Hz, after which the difference remains approximately 1% between 25 and 30 Hz. This indicates that 20 Hz is a threshold value, which is consistent with the variation of LAA in Figure . When the frequency exceeds 20 Hz, micro breakage continues to accumulate, whereas its contribution to CN growth becomes limited.

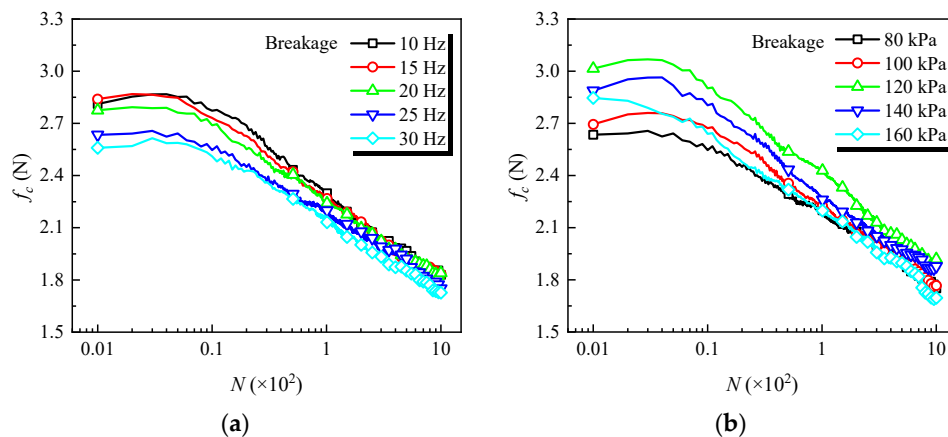
Similarly, as shown in Figure d, CN increases with amplitude under both breakage and non-breakage conditions, with a more significant increase under breakage conditions. Compared with the non-breakage case, the corresponding relative difference under breakage conditions increases from 0.55% at 80 kPa to 1.07% at 140 kPa, and then decreases to 0.96% at 160 kPa. This indicates that a larger loading amplitude significantly promotes the particle rearrangement induced by micro breakage. In summary, micro breakage promotes particle rearrangement and drives the graded aggregate skeleton toward a denser state. As the skeleton gradually becomes stable and dense, the promoting effect of micro breakage on CN growth gradually weakens.

4.3. Contact Force

The average contact force f_c characterizes the stress level within the force-chain network of the particle assembly and is calculated using Equation (32). As shown in Figure a,b, f_c gradually decreases with increasing N under different loading conditions. The time-history evolution of CN indicates that the number of contacts within the sample continuously increases as N increases, with the external load being distributed among more contacts. As a result, the contact force concentrated in strong force chains is progressively dispersed, leading to a gradual decrease in f_c .

$$f_c = \frac{\sum_i^C F_i}{C} \quad (32)$$

where F_i is the magnitude of contact i .



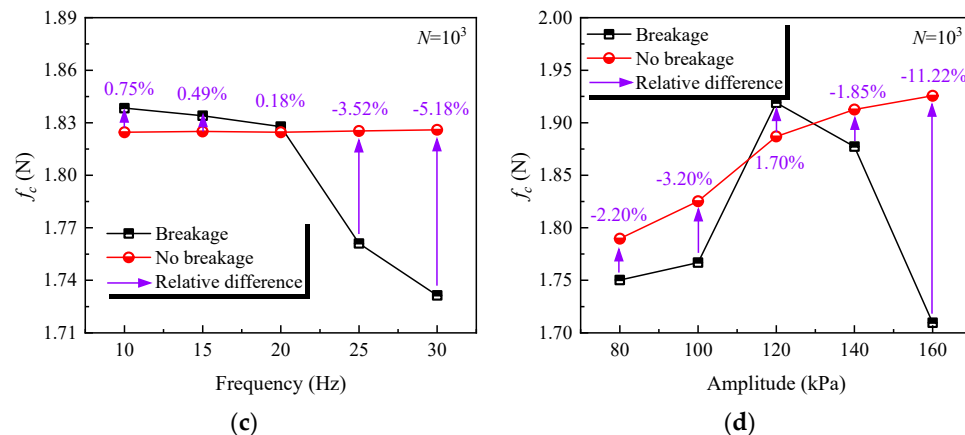


Figure 11. Effects of loading parameters on f_c : (a) Frequency under breakage conditions, (b) Amplitude under breakage conditions, (c) Relationship between frequency and f_c , and (d) Relationship between amplitude and f_c .

As shown in Figure c, under non-breakage conditions, the effect of frequency on f_c is insignificant. In contrast, under breakage conditions, f_c decreases progressively with increasing frequency, and the reduction becomes more significant when the frequency exceeds 20 Hz. When the frequency exceeds 20 Hz, micro breakage increases significantly and promotes the formation of more contacts within the sample. Moreover, compared with the non-breakage case, the relative difference of f_c under breakage conditions is about 0.5% at 10–20 Hz, whereas it becomes −3.52% and −5.18% at 25 Hz and 30 Hz, respectively. This indicates that micro breakage has a more significant effect on f_c than loading frequency alone.

Additionally, as shown in Figure d, f_c gradually increases with amplitude under non-breakage conditions, indicating that increasing amplitude enhances the stress level within the sample. It is worth noting that a higher amplitude promotes micro breakage, whereas an increase in micro breakage leads to a reduction in f_c . In other words, micro breakage and increasing amplitude affect f_c in opposite ways. This leads to a trend in which f_c initially increases and then decreases with increasing amplitude under breakage conditions. When the amplitude exceeds 120 kPa, corresponding to an LAA greater than 0.35%, the evolution of f_c gradually shifts from being amplitude-dominated to micro breakage-dominated. In summary, micro breakage promotes densification-induced skeleton rearrangement and weakens the existing strong force-chain network. With the continuous accumulation of micro breakage during cyclic loading, this evolution from rearrangement with increased contacts to strong force-chain weakening further reduces the strength of the skeleton structure.

4.4. Macroscopic Response

The plastic strain ε_p represents the accumulated irrecoverable deformation of the sample under cyclic loading and can be used to evaluate the performance of graded aggregates. As shown in Figure a,b, ε_p increases with N under different loading conditions, indicating the continuous accumulation of plastic deformation in the sample under cyclic loading. Compared with frequency, the effect of amplitude on ε_p is more significant, which is consistent with the mesoscopic evolution characteristics. Similarly, mesoscopic evolution indicates that micro breakage promotes particle rearrangement and new contact formation, while weakening the strong force-chain network. As a result, the skeleton structure progressively degrades during the simulated cyclic-loading stage and is reflected in the increase in ε_p .

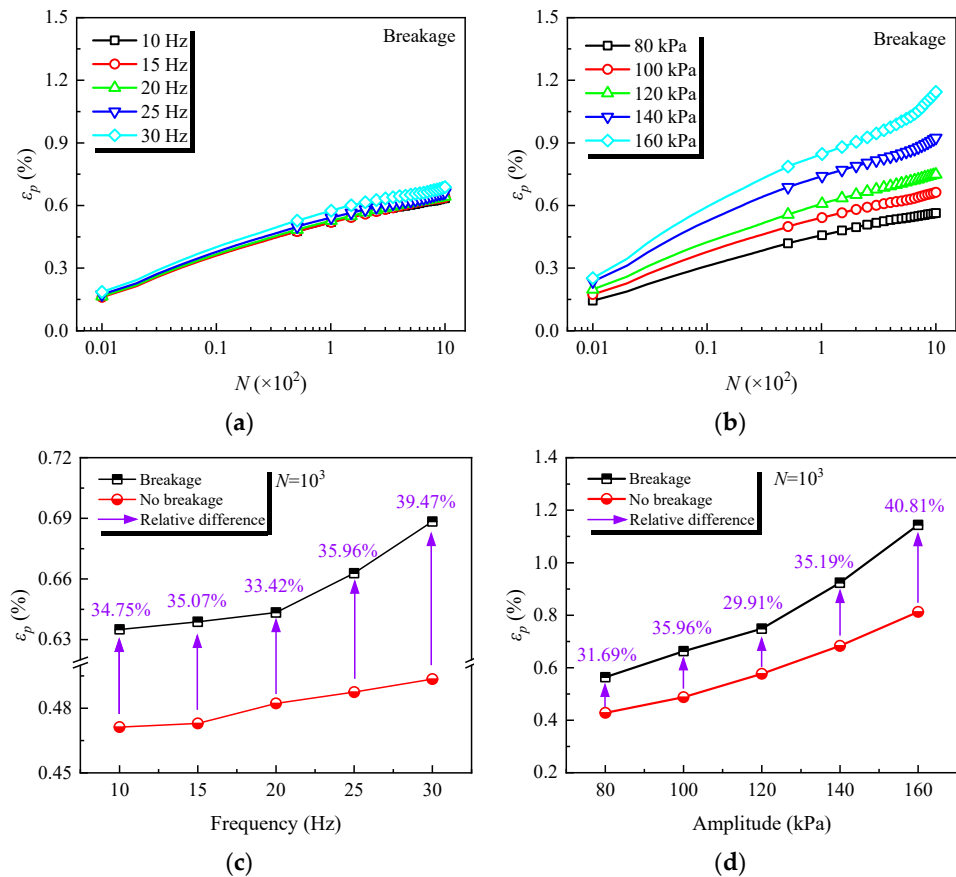


Figure 12. Effects of loading parameters on ε_p : (a) Frequency under breakage conditions, (b) Amplitude under breakage conditions, (c) Relationship between frequency and ε_p , and (d) Relationship between amplitude and ε_p .

As shown in Figure c,d, ε_p under both breakage and non-breakage conditions gradually increases with increasing frequency and amplitude. Compared with the non-breakage cases, ε_p is approximately 30–40% higher under breakage conditions. The relative difference increases with further increases in frequency and amplitude. This indicates that higher frequency and amplitude further enhance the promoting effect of micro breakage on skeleton rearrangement and the irrecoverable deformation accumulation. Moreover, the increase in ε_p indicates that the accumulated plastic deformation of graded aggregates continuously increases and the structural stability gradually decreases. This indicates that micro breakage accelerates the performance degradation of the graded aggregate skeleton, and this degradation becomes more significant at higher frequency and amplitude.

5. Discussion

To further clarify the intrinsic relationships between the micromechanical indicators and macroscopic deformation discussed in Section 4, Figure 13 illustrates the mechanism by which micro breakage induces the performance degradation of graded aggregates.

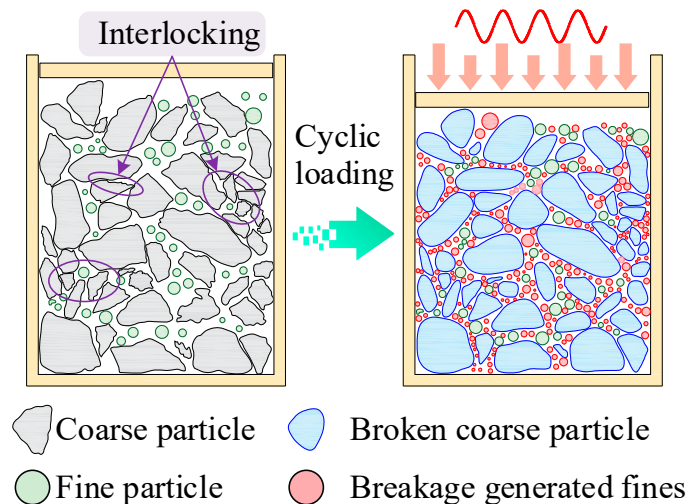


Figure 13. Schematic illustration of the micro breakage-induced degradation mechanism of graded aggregates under cyclic loading.

In the initial state, irregular coarse particles form a stable load-bearing skeleton through angular interlocking. Under cyclic loading, contact energy tends to concentrate at high-curvature corners and edges, gradually triggering local micro breakage. Micro breakage causes local particle-volume reduction, corner blunting, and particle rounding, thereby weakening the geometric constraints between particles and promoting particle sliding, rotation, and rearrangement. As the skeleton continues to adjust, the specimen becomes denser and develops more effective contacts, resulting in an increase in CN , while the distribution of the external load among a larger number of contacts reduces f_c and promotes the weakening and redistribution of the original strong force chains. The resulting contact-network reconstruction and skeleton densification are irreversible, and their continued accumulation ultimately leads to an increase in plastic strain. Therefore, micro breakage first alters the local particle volume and morphology, then weakens particle interlocking and induces skeleton rearrangement, and finally promotes cyclic deformation accumulation and material degradation.

It should be noted that the energy-driven micro breakage method can describe particle corner spalling, particle rounding, and the resulting evolution of the contact network under cyclic loading. The current formulation still involves several assumptions and limitations that should be considered when interpreting the results. First, the contact overlap region is treated as an equivalent control volume representing the local high-energy zone, and its geometric scale does not correspond exactly to the actual spalled volume. Therefore, the method cannot reproduce actual crack propagation paths or irregular fracture surfaces. Second, the particle-shape updating scheme is mainly applicable to corner abrasion and local micro breakage, and its ability to represent severe fragmentation modes, such as through-going cracking, particle splitting, and large-fragment detachment, remains limited. In addition, breakage-generated fines are represented by equivalent local material loss, without explicitly considering their contacts, migration, or void-filling effects. Seepage, drainage, and pore-pressure evolution are also not included. Future work should incorporate explicit fragment generation, crack propagation, and fine-particle migration to extend the applicability of the method to severe fragmentation and hydro-mechanical coupling conditions.

6. Conclusions

This study develops an energy-driven DEM-based micro breakage method to investigate micro breakage of graded aggregates under cyclic loading and the resulting mesoscopic evolution and performance degradation. The proposed method enables continuous simulation of micro breakage and local particle shape evolution. Then, a series of dynamic triaxial simulations is conducted to investigate the micro breakage-induced mesoscopic evolution and macroscopic deformation of graded aggregates under different loading conditions. Based on the results obtained in this study, the main conclusions are summarized as follows.

- (1) An energy-driven micro breakage method applicable to irregular particles is proposed based on the energy balance and radial function $r(\theta, \varphi)$. The results of the single-particle test, Los Angeles abrasion test, and cyclic loading test indicate that the proposed method can reasonably reproduce the force–displacement response, representative micro breakage triggering behavior, LAA evolution, R_s evolution, and sample deformation.
- (2) The dynamic triaxial results indicate that micro breakage in graded aggregates exhibits a cumulative characteristic, with both LAA and R_s increasing with loading cycles N . Both higher loading frequency and amplitude promote micro breakage, while amplitude exerts a stronger effect. A larger amplitude mainly increases the instantaneous elastic energy in the contact, whereas a higher frequency mainly accelerates the accumulation of the cumulative breakage factor ℓ .
- (3) Micro breakage promotes particle rearrangement and new contact formation, while inducing skeleton densification and force-chain weakening in graded aggregates. This is reflected by the increase in coordination number CN and the decrease in average contact force f_c . This mesoscopic evolution further increases plastic deformation accumulation and reduces the structural stability of graded aggregates during the simulated cyclic-loading stage.
- (4) Under the conditions considered in this study, 20 Hz is a critical frequency at which micro breakage in graded aggregates becomes significantly enhanced. When the frequency exceeds 20 Hz, the dynamic response of the sample increases significantly, resulting in greater plastic deformation. Accordingly, as train operating speeds increase, micro breakage may exert a greater influence on the cyclic degradation of graded aggregates.

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Data Availability Statement: Data are contained within the article.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A. Numerical Implementation of Coefficient Update (Discrete Quadrature and Projection)

A fixed quadrature point set $\mathbf{O}$ is selected on the unit sphere, as given in Equation (A1).

$$\mathbf{O} = \{(\theta_k, \varphi_k), w_k\}_{k=1}^{N_q} \quad (\text{A1})$$

where (θ_k, φ_k) are the spherical coordinates of the k th quadrature point, N_q is the number of discrete quadrature points, w_k is the corresponding surface-integration weight, satisfying Equation (A2).

$$\sum_{k=1}^{N_q} w_k \approx 4\pi \quad (\text{A2})$$

Hence, $\langle \Delta r \rangle$ for an arbitrary spherical function can be approximated by discrete quadrature, as shown in Equation (A3).

$$\langle \Delta r \rangle \approx \frac{1}{\sum_{k=1}^{N_q} w_k} \sum_{k=1}^{N_q} w_k \Delta r(\theta_k, \varphi_k) \quad (\text{A3})$$

Then, for the coefficient increment Δa_n^m of a_n^{m*} ($n \geq 1$), the continuous projection can be discretized using the approximation in Equation (A4).

$$\Delta a_n^m \approx \sum_{k=1}^{N_q} w_k \Delta r'(\theta_k, \varphi_k) Y_n^m(\theta_k, \varphi_k), \quad n \geq 1 \quad (\text{A4})$$

To evaluate Equation (A4) efficiently as a single matrix-vector product, all coefficients with $n \geq 1$ are first re-indexed by j ($j = 1, 2, \dots, N_{>0}$), where $N_{>0} = (N_s + 1)^2 - 1$. The spherical harmonic value matrix $\mathbf{Y}_{>0}$ and the diagonal weight matrix $\mathbf{W}$ are then established as shown in Equations (A5) and (A6).

$$\mathbf{Y}_{>0} \in \mathbb{R}^{N_q \times N_{>0}}, \quad [\mathbf{Y}_{>0}]_{k,j} = Y_j(\theta_k, \varphi_k) \quad (\text{A5})$$

$$\mathbf{W} = \text{diag}(w_1, w_2, \dots, w_{N_q}) \quad (\text{A6})$$

Accordingly, the coefficient increments vector $\Delta \mathbf{a}_{>0}$ and the radial correction vector $\Delta \mathbf{r}(\theta, \varphi)$ are defined by Equations (A7) and (A8), respectively.

$$\Delta \mathbf{a}_{>0} = [\Delta a_1, \Delta a_2, \dots, \Delta a_{N_{>0}}]^T \quad (\text{A7})$$

$$\Delta \mathbf{r}'(\theta, \varphi) = [\Delta r'(\theta_1, \varphi_1), \Delta r'(\theta_2, \varphi_2), \dots, \Delta r'(\theta_{N_q}, \varphi_{N_q})]^T \quad (\text{A8})$$

As a result, Equation (A4) can then be written as Equation (A9).

$$\Delta \mathbf{a}_{>0} = \mathbf{Y}_{>0}^T \mathbf{W} \Delta \mathbf{r}'(\theta, \varphi) \quad (\text{A9})$$

It is worth noting that $\mathbf{Y}_{>0}$ and $\mathbf{W}$ remain fixed throughout the simulation, and the matrix $\mathbf{B}$ can be computed before the calculation starts, as given in Equation (A10).

$$\mathbf{B} = \mathbf{Y}_{>0}^T \mathbf{W} \in \mathbb{R}^{N_{>0} \times N_q} \quad (\text{A10})$$

The $\Delta \mathbf{a}_{>0}$ for each micro breakage event requires only a single matrix-vector multiplication, as given in Equation (A11).

$$\Delta \mathbf{a}_{>0} = \mathbf{B} \Delta \mathbf{r}'(\theta, \varphi) \quad (\text{A11})$$

Finally, the updated coefficient vector $\mathbf{a}_{>0}$ is obtained by Equation (A12).

$$\mathbf{a}_{>0} = \mathbf{a}_{>0} - \Delta \mathbf{a} \quad (\text{A12})$$

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