

Article

Enhanced Optical Triangulation Method for Piezoelectric Stack

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Abstract

The precise control of piezoelectric actuators is limited by inherent hysteresis, creep, and nonlinear behavior, which necessitate high-resolution displacement sensing for effective closed-loop operation. Although optical interferometers can achieve nanometer and sub-nanometer resolution, their practical implementation is often constrained by complex optical alignment, sensitivity to environmental disturbances, and limited robustness in high-speed measurements. Optical triangulation sensors offer a more robust and straightforward alternative; however, their resolution is typically insufficient for nanometer-scale displacement measurements. In this study, a novel optical triangulation sensor based on a two-stage geometric optical amplification scheme is proposed for measuring the expansion of piezoelectric stacks. The method relies purely on geometric optical amplification and does not require interferometric techniques or complex signal processing. Using off-the-shelf optical components and an industrial imaging sensor, the proposed system achieves a displacement resolution of 109.6 nm, a repeatability of 74.62 nm, and an accuracy of 98.81% with a maximum error of 207.14 nm under hysteresis measurements. The achieved resolution is primarily limited by the spatial resolution of the camera sensor, indicating that further improvements are possible through optimization of the optical configuration or the use of higher-resolution imaging devices. Owing to its simplicity and robustness, the proposed sensor is well suited for real-time closed-loop control of piezoelectric actuators.

Keywords: nanometer resolution; piezoelectric actuator; laser triangulation

1. Introduction

Piezoelectric actuators are widely used in applications, including precision positioning and robotics [1–4], despite their nonlinear features such as hysteresis and creep [5,6]. The positioning precision depends on the displacement resolution of the piezoelectric stack. Various non-contact laser displacement-sensing methods exist to measure the expansion of piezoelectric actuators [7]. Among these, interferometric and triangulation sensors have been employed to measure the expansion of piezoelectric stacks with nanometer resolution [8–12].

Interferometers are capable of measuring displacement with nanometer resolution [13]. However, these approaches require complex modulation processes to achieve accuracy and suppress harmonic noise components [14,15], and their utilization in commercial applications is also limited by high cost and space constraints. Corner cube reflectors can help reduce space requirements [13]. Self-mixing laser diode (SM-LD) interferometry offers a lower-cost alternative for moderate-resolution applications [16]. The implementation of SM-LD interferometry requires advanced signal-processing algorithms and specialized optical components for cost-effective, real-time operation [16]. Due to this restriction of SM-LD interferometry, this setup is unsuitable for real-time applications [17].



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Optical beam deflection (OBD) is another non-contact optical method for nanometer-precision displacement sensing that utilizes a position-sensitive detector (PSD) or quadrant photodiode. While OBD systems offer resolution comparable to that of interferometers [18], they are designed to measure the motion of objects with a fixed initial side and a moving end (cantilever). This configuration usually requires knowledge of the sample length and the distance between the sample and the detector to calculate linear displacement. Due to this feature, they are not well suited for measuring the free expansion of piezoelectric stacks.

Laser triangulation sensors are frequently used in commercial applications for displacement measurement due to their simplicity, low cost, and ability to operate in real time without specialized signal-processing algorithms. They also provide faster measurements than confocal sensors. However, the resolution of standard triangulation sensors is generally limited to several micrometers [19]. Although position-sensitive detectors (PSDs) offer higher resolution and superior real-time performance compared to charge-coupled device (CCD) and quadrant sensors [20], CCD sensors are more available and are predominantly used in commercial systems.

Additionally, charge-based control techniques in piezoelectric actuators enable precise nanometer-level measurement and control. Dedicated circuits have been developed to measure charge using charge-based methods [21]. To control the piezoelectric actuator, class-D amplifier circuits are commonly used for their simplicity and for pulse-width modulation (PWM) [22].

This article presents a novel non-contact optical displacement measurement technique with nanometer resolution using an enhanced laser triangulation method. The approach preserves the simplicity and cost-effectiveness of standard laser triangulation sensors and does not require complex signal-processing algorithms or modulation schemes, unlike interferometric methods. The technique employs a commercially available CCD or CMOS image sensor to measure laser spot displacement resulting from piezoelectric expansion with nanometer precision.

2. Materials and Methods

In an optical triangulation measurement system, the sensor relies on light reflecting from the sample. Typically, the image sensor, mostly a CCD/CMOS camera, is positioned vertically over the sample, and the light source is placed at a specific angle relative to the sample to optimize resolution, which determines the angle of reflection and the sensor's resolution. When the sample moves along the camera axis, due to the angle of light reflected light is higher than the actual displacement.

In conventional optical triangulation, the camera observes the light spot directly on the sample. However, in the proposed method, the camera does not directly observe the light on the sample; instead, it observes the reflected light from the sample projected onto a semi-transparent screen.

The reflection angle α and the screen angle relative to the reflection surface θ are treated as sensor variables. To observe the reflected laser's projection, a semi-transparent screen is used. The camera is positioned to observe the screen perpendicularly, with its focal point aligned to the screen surface. This configuration allows the camera and screen module to be sealed as a single unit, simplifying the sensor system and enabling easy angle adjustment.

To observe the projection of the reflected laser at a specific angle, a screen is used, and the camera is positioned perpendicularly to the screen. A semi-transparent screen is used for the camera placement to observe both sides from the screen. This method simplifies the measurement system, enabling easy screen angle adjustment by rotating the camera and screen module as a single unit.

Since the piezoelectric stack's surface lacks sufficient reflectivity to project a clear laser profile onto the screen, an external reflection surface is required. However, standard mirrors are unsuitable due to the diffraction caused by secondary reflections from the glass substrate in front of the reflective surface, which would distort the laser profile and degrade measurement accuracy. To eliminate these optical artifacts, a first-surface mirror was attached to the piezoelectric stack's surface, aligned with the axis of movement. Furthermore, the mirror must have a low flatness tolerance to guarantee that the reflected laser beams at the initial and displaced positions remain strictly parallel. Due to the flatness of the reflection surface, the line $\vec{BE}$ and the line $\vec{AF}$ must be parallel to each other.

The geometric principles of the proposed two-stage amplification system are modeled using the vector definitions illustrated in Figure 1. In this model, Point A represents the initial position of the reflection surface prior to actuation. Upon the application of voltage, the piezoelectric stack expands vertically, shifting the reflection surface to a new position denoted by Point C. Consequently, the displacement vector $\vec{AC}$ represents the physical expansion of the piezoelectric material, and its magnitude corresponds to the variable h . This vertical displacement induces a shift in the reflection path, creating an intermediate optical path difference represented by the vector $\vec{DF}$, which is obtained by geometrically translating the initial reflection vector $\vec{AB}$, forming the virtual triangle DEF, in the upper triangle, with a magnitude denoted by k . Finally, the reflected laser beam is projected onto the semi-transparent screen, where the resulting displacement is defined by the vector $\vec{DE}$, corresponding to the measured output x . The trigonometric relationships between these vectors form the basis of the system's transfer functions.

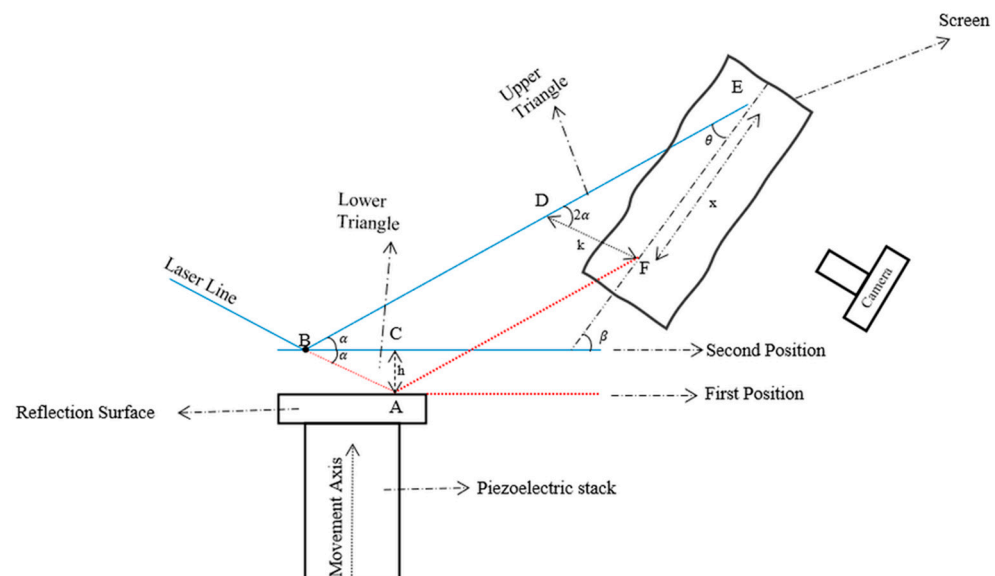


Figure 1. Working principle of the two-stage measurement system. The red dotted lines represent the path of the laser beam when the reflection surface is at its initial position (First Position), and the blue solid lines show the laser path after the surface has moved to the second position.

Between the positions, the reflectance surface movement is caused by the expansion of the piezoelectric stack. The method relies on the displacement of the reflectance surface being equal to the expansion of the piezoelectric stack. Therefore, the reflectance surface must be precisely aligned with the predefined movement axis of the piezoelectric stacks.

With these conditions, the difference between the reflected light points from the reflectance surface on the screen at the first and second positions has a magnification ratio that is proportional to the expansion of the piezoelectric stack, which can be calculated

using the trigonometric function at each triangle, specifically the lower triangle (ABC) and upper triangle (DEF).

The proposed measurement system is modeled as a two-stage geometric amplifier, with optical gain determined solely by the angular configuration. In this framework, the reflection angle α and the screen inclination angle θ function as the primary state variables governing the system's response. Consequently, the amplification at each stage is characterized by specific transfer functions, denoted as $G1(\alpha)$ and $G2(\alpha, \theta)$, which represent the gain contributions of the lower (ABC) and upper (DEF) triangles, respectively.

The first magnification function is calculated from the lower triangle, using the sine theorem. Because of that, the difference between the reflected laser at the first position and the second position can be described as a function of the expansion of the piezoelectric stack. For the geometric derivation, the expansion of the piezoelectric stack is constrained strictly to the vertical axis, aligned with the vector $\vec{AC}$. Consequently, the angle $B\hat{C}A$ is geometrically defined as 90° . Under this orthogonality condition, the first magnification function is shown in (1).

$$G1(\alpha) = \frac{k}{h} = \frac{1}{\sin \alpha} \quad (1)$$

The second magnification function is derived from the geometric relations of the upper triangle using the sine theorem. The difference between the reflected lasers on the screen can be described as a function of the difference between the reflected lasers. The parallelism of the reflected laser, the angle of $E\hat{D}F$, is equal to the angle of $E\hat{B}A$. Therefore, using the sine theorem, the magnification function of the upper triangle is calculated in (2).

$$G2(\alpha, \theta) = \frac{x}{k} = \frac{\sin 2\alpha}{\sin \theta} \quad (2)$$

The expansion of the piezoelectric stack between the first and second positions can be described as a function of the difference in the reflected laser intensity on the screen between these two positions, yielding a total magnification function for the proposed method. The total magnification function is given at (3), and the modified form of the total magnification ratio is given at (4)

$$G_{total}(\alpha, \theta) = G1(\alpha) \times G2(\alpha, \theta) = \left(\frac{1}{\sin \alpha} \right) \times \left(\frac{\sin 2\alpha}{\sin \theta} \right) \quad (3)$$

$$G_{total}(\alpha, \theta) = \frac{x}{h} = \left(\frac{2 \cos \alpha}{\sin \theta} \right) \quad (4)$$

The conditions are given at (5) and (6) to achieve the maximum value of the total magnification function. Therefore, the angle of reflection α should be minimized (approaching 0), and the difference between the reflection angle and the screen angle from the reflection surface should be minimized (approaching 0), as indicated by the limits:

$$\lim_{\alpha \rightarrow 0} \cos \alpha = 1 \quad (5)$$

$$\lim_{\theta \rightarrow 0} \sin \theta = 0 \quad (6)$$

The sensitivity analysis of the total magnification factor, $G_{total}(\alpha, \theta)$, reveals distinct behaviors for the two angular state variables, as illustrated in Figure 2. The total magnification ratio is maximized when the angles α and θ approach zero. When the angle θ is held constant, the angle α has a linear response to the total magnification. When the angle " α " is held constant, the angle θ has a nonlinear, parabolic response to the total magnification. Therefore, the angle α has a minor impact on the total magnification function rather than the angle θ .

Consequently, by fine-tuning the angle of θ parameter, extremely high amplification ratios can be achieved within a compact geometric configuration. This characteristic allows the sensor to be designed with a maximum magnification ratio in the minimum required space.

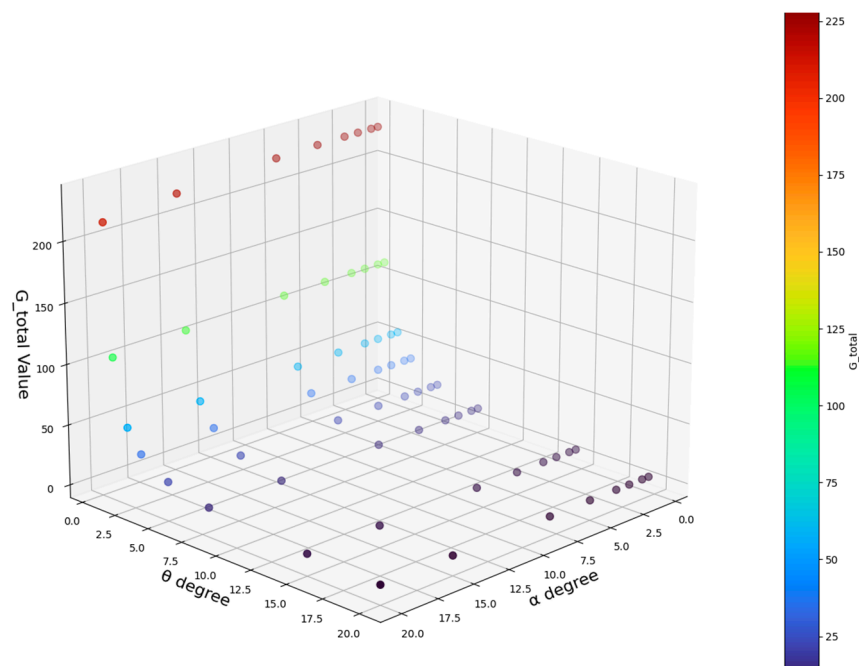


Figure 2. Relation between the $G_{total}(\alpha, \theta)$ value and angle of θ and α .

To measure the laser positions on the screen, a CCD/CMOS camera is used as an alternative to the PSD quadrant sensor when a high magnification ratio is applied. To calculate the resolution of the sensor, the magnification ratio is considered with the spatial resolution.

A 5W red line laser was used as the laser source. However, high geometric magnification inevitably expanded the laser profile on the screen, resulting in reduced intensity and profile broadening on the screen. Therefore, a 100 mm focal-length focusing lens is placed in front of the laser source to focus the laser. The system with a high-geometric amplification design is shown in Figure 3.

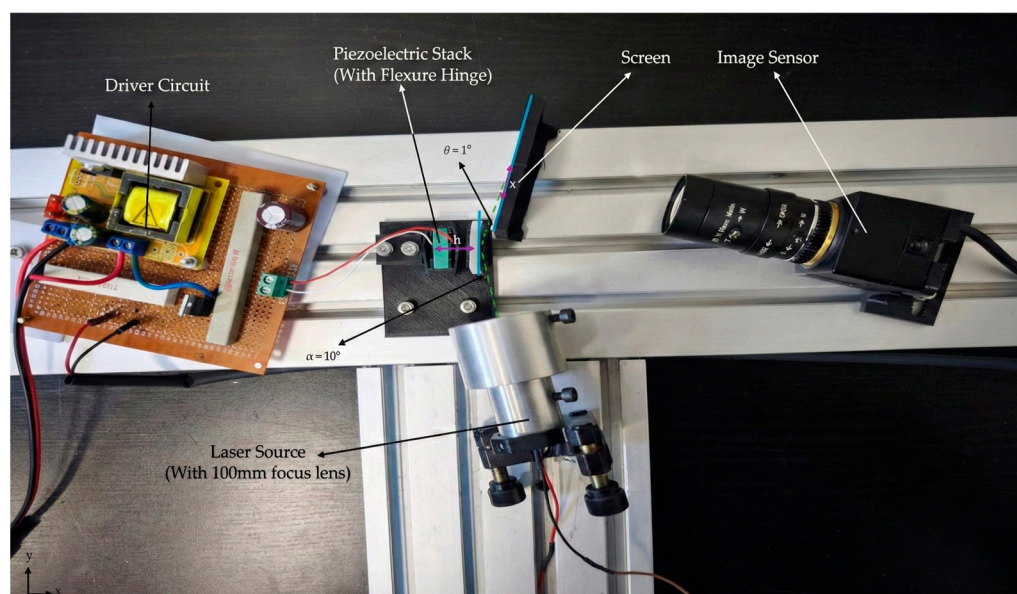


Figure 3. Proposed measurement system, highlighting the optical components, movement axes, and the geometric parameters.

The screen roughness value causes salt-and-pepper noise in the image captured by the camera. To reduce the noise level, lower roughness value materials should be chosen.

The imaging system consisted of a 2 MP “ELP USB Camera module” configured with an exposure time of 15.6 ms, and the camera lens aperture was manually controlled to achieve a dark image while retaining the entire laser profile. The camera’s spatial resolution was set to 11.45 μm along the measurement axis.

An “AE0505D16F” piezoelectric stack was employed, driven by a custom-designed, voltage-controlled driver circuit. The control signal was modulated using a 490 Hz PWM technique at 100 V amplitude, and an 8 Hz RC low-pass filter was used to filter the modulated signal output.

Figure 4a shows the raw image acquired under red-laser illumination. The red channel is extracted to generate the grayscale image, as shown in Figure 4b. A median filter is then applied to suppress salt-and-pepper noise caused by the screen surface roughness, resulting in the filtered image shown in Figure 4c. The Otsu thresholding method is used for the segmentation, as shown in Figure 5b. To reduce the error caused by the black spot within the laser profile on the segmented image, morphological methods were employed.

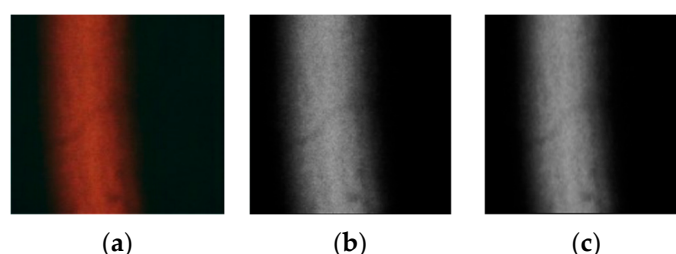


Figure 4. (a) The raw image. (b) The Red plane. (c) After the median filter on the red plane.

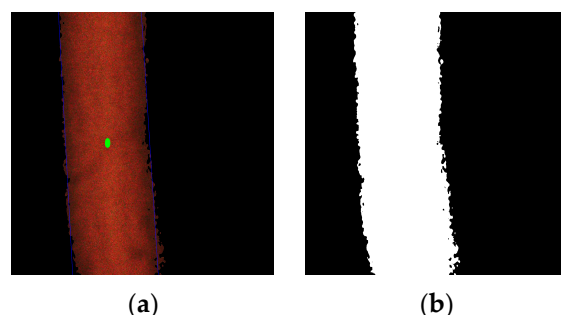


Figure 5. (a) Fitted lines and tracking point. (b) Segmented image.

Edge detection is performed by computing the first derivative of the segmented image. The geometric center of the edges is identified as the tracking point, enabling determination of the piezoelectric stack’s displacement.

The fitted lines of the two edges and the tracking point are shown in Figure 5a. The tracking points are determined for different voltage levels applied to the piezoelectric stack. Changes in voltage affect the position of the image and the tracking point. The image shifts to the right as the applied voltage increases. The shifting distance is used to measure the expansion of the piezoelectric stack. The difference between the first and last tracking points indicates the pixel shift in the laser profile. The expansion of the piezoelectric stack is calculated using (7).

$$d = (p \times s) / m \quad (7)$$

where

d: amount of expansion

p: number of pixels between two tracking points
s: spatial resolution of the camera
m: Magnification ratio

To provide a ground truth of the sensor, an optical microscope (CX40, Olympus Corporation, Tokyo, Japan). equipped with a 40× objective lens was used to observe the piezoelectric stack's expansion at different voltages. The microscope's spatial resolution was calibrated with a Thorlabs 50-mm divide ruler, which measured 70 nm. Piecewise regression is used to display the measurement points.

The sensor measurements were conducted under the identical operating conditions and the same actuation sequence as defined in the calibration methodology. The maximum total expansion of the piezoelectric stack was measured at 6192.67 nm under 60 V. These reference values served as a calibration standard for the proposed sensor, which determined the pixel-to-nanometer ratio. All calibration data is shown in Table 1 as the microscope mean and the microscope standard deviation.

Table 1. Comparison between the microscope and sensor measurements at high gain.

Voltage (V)	Microscope Mean (nm)	Microscope Std (nm)	Sensor Mean (nm)	Sensor Std (nm)	Mean Difference (nm)
10	0	71.70	0	260.88	0
20	1110.66	0	1187.57	87.15	76.90
30	2375.33	17.46	2482.74	141.26	107.40
40	3467.33	33.00	3616.17	52.02	148.84
50	4666.67	28.00	4740.59	60.77	73.92
60	6192.67	34.29	6192.67	75.73	0

Equation (7) provides the analytical framework, using the empirical calibration value instead of the theoretical s/m ratio (Equation (7)) ensures statistical accuracy. This method bypasses the practical difficulty of measuring the angle θ in high-magnification design, as the total magnification ratio exhibits a parabolic dependence on θ . The calibration factor is derived by dividing the physical displacement (measured by the microscope) by the corresponding pixel shift (recorded by the proposed sensor) under identical voltage conditions.

3. Results

3.1. Measurement

Measurements were performed using both the proposed sensor and the microscope over an applied voltage range from 10 V to 60 V in increments of 10 V. This measurement sequence was repeated ten times to obtain the calibration dataset. The mean values and standard deviations of the measured displacements are presented in Figure 6, where the horizontal axis denotes the applied voltage and the vertical axis denotes the measured displacement. The corresponding numerical results are summarized in Table 1.

It should be noted that the 10 V measurements were recorded immediately after applying 60 V in the measurement sequence. As a result, this data point is influenced by transient hysteresis and creep effects of the piezoelectric actuator rather than steady-state behavior.

A thorlabs silver-coated mirror with $\lambda/10$ at 633 nm is used for the reflectance surface. The screen is made from a 1.5 mm-thick substrate without any dataset given for the flatness value.

The roughness of the screen causes measurement deviations, affecting system precision. A median filter was applied to reduce noise; however, this slightly shifted the laser profile location. As shown in Table 1, this deviation is less than 3 pixels. Figure 6 compares the proposed sensor with the reference microscope.

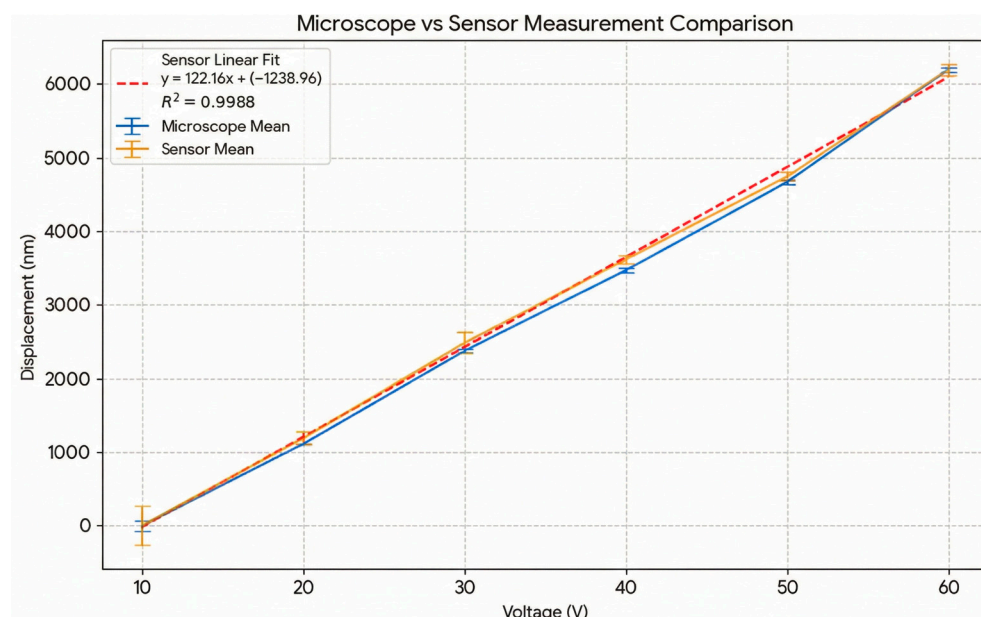


Figure 6. Comparison between microscope and system data at high gain.

3.2. Comparative Assessment of Intrinsic Precision and Accuracy Across Gain Configurations

Table 1 compares the microscope and sensor measurements under high-gain conditions. The angle α was set to 10° , and the angle θ was set to 1° . Under this configuration, the sensor calibration factor was measured as 109.6 nm. All values in Table 1 are reported in nanometers; corresponding pixel values can be obtained by dividing the calibration factor by the pixel size. The measurements correspond to the high-gain configuration, where the input voltage was varied from 10 V to 60 V in 10 V increments.

It should be noted that the 10 V measurements were recorded immediately after applying 60 V in the measurement sequence. Consequently, this data point is influenced by transient piezoelectric hysteresis and creep effects rather than steady-state actuator behavior. The observed max–min variation in this region (~ 850 nm) is comparable to the actuator hysteresis amplitude (~ 805 nm), indicating that the dominant contribution arises from actuator memory effects rather than sensor noise. Therefore, the 10 V data were excluded from the precision analysis.

To evaluate the precision and accuracy of the proposed sensor, the microscope measurements were used as the reference, since their standard deviation remained below the spatial resolution of the microscope, making them suitable for comparison with the sensor results.

The precision or repeatability of the proposed sensor was measured using the standard deviation of the sensor. The average sensor precision is calculated as 83.39 nm, and with this value, the precision of the high magnification system is described as 1.35% (Full Scale Error). The worst-case repeatability was observed at the 30 V operating point, where the standard deviation reached 141.26 nm, which is near 1.3 pixels.

The accuracy of the proposed sensor was defined with the RMSE value of the bias, which was calculated as 86.67 nanometers (1.4% Full Scale Error) with a maximum bias limited to 148.84 nm (2.40% Full Scale Error) even under worst-case conditions. Thus, the global accuracy of the high magnification system was calculated as 98.60%.

Table 2 presents the measurements obtained using the same camera sensor region and the same angle α as in the first configuration, but with an angle θ greater than 1° . Under this condition, a calibration factor of 516 nm/pixel was obtained. When the same voltage inputs were applied to the piezoelectric stack, similar pixel-level deviations were observed.

Table 2. Comparison between the microscope and sensor measurements at low gain.

Voltage (V)	Microscope Mean (nm)	Microscope Std (nm)	Sensor Mean (nm)	Sensor Std (nm)	Mean Difference (nm)
10	0	71.70	0	1021.2	0
20	1110.66	0	1113.77	345.48	3.10
30	2375.33	17.46	2613.73	311.02	237.90
40	3467.33	33.00	4010.23	581.82	542.90
50	4666.67	28.00	4822.17	315.45	155.50
60	6192.67	34.29	6192.67	306.42	0

The measurements in Tables 1 and 2 were taken from different regions of the same camera sensor plane. In Table 1, the largest deviations occurred at an input voltage of 30 V, whereas in Table 2, they occurred at 40 V. This measurement indicates that the roughness of the camera surface area affects the system's precision at the pixel level.

The global accuracy was calculated as 95.96% (RMSE: 250.25 nm, 4.04% FSE), with a maximum bias of 542.90 nm (8.77% FSE) occurring at points where surface irregularities coincided with quantization steps. Furthermore, the precision (repeatability) across 15 cycles was maintained at 284.94 nm (4.60% FSE).

The accuracy of the proposed sensor is influenced by the flatness of both the reflective surface and the camera sensor plane. Variations in surface flatness along the laser travel path cause small changes in the effective angle θ , which in turn alter the calculated optical gain. This effect becomes more pronounced at high gain, where even a minor variation in θ leads to a noticeable change in the gain value.

In Table 1, the maximum mean difference corresponds to approximately 1.35 pixels, whereas in Table 2 it is about 1.05 pixels. When evaluated in terms of pixel displacement, the low-gain configuration appears more accurate. However, when converted to physical displacement units, the high-gain configuration provides significantly higher measurement accuracy due to the larger optical amplification factor.

Despite this variation in pixel-level deviation, the intrinsic performance metrics show a striking similarity, confirming the system's robustness. Specifically, the intrinsic accuracy was found to be 0.79 pixels for the high-gain configuration and 0.48 pixels for the low-gain configuration. Similarly, the intrinsic precision (repeatability) remained stable at sub-pixel levels (0.76 px vs. 0.55 px). This similarity indicates that the measurement performance is not strictly limited by the local surface flatness, rather, the system effectively compensates for these physical imperfections.

In the high-gain configuration, each region of the camera sensor corresponds to a slightly different effective gain and calibration factor due to surface flatness variations. In the present study, a single calibration factor was used for simplicity. To further reduce flatness-related errors, a spatially varying calibration approach can be implemented, in which multiple calibration values are assigned to different sensor regions using a piecewise calibration function. This approach can significantly reduce errors associated with surface flatness variations.

3.3. Hysteresis and Rate-Dependent Measurements

To evaluate the proposed sensor's response to the piezoelectric nonlinear effects, the hysteresis behavior of the piezoelectric actuator was experimentally characterized using both a microscope and the proposed sensor. The applied voltage was increased from 0 V to 60 V in increments of 10 V and subsequently decreased back to 0 V using the same voltage steps. This voltage cycling procedure was repeated ten times to ensure repeatability. After each voltage step, a settling time of 1 s was allowed before recording the

displacement measurement. By introducing this waiting period, rate-dependent nonlinear effects were minimized, enabling the observation of the quasi-static hysteresis behavior of the piezoelectric material. The corresponding measurement results are shown in Figure 7.

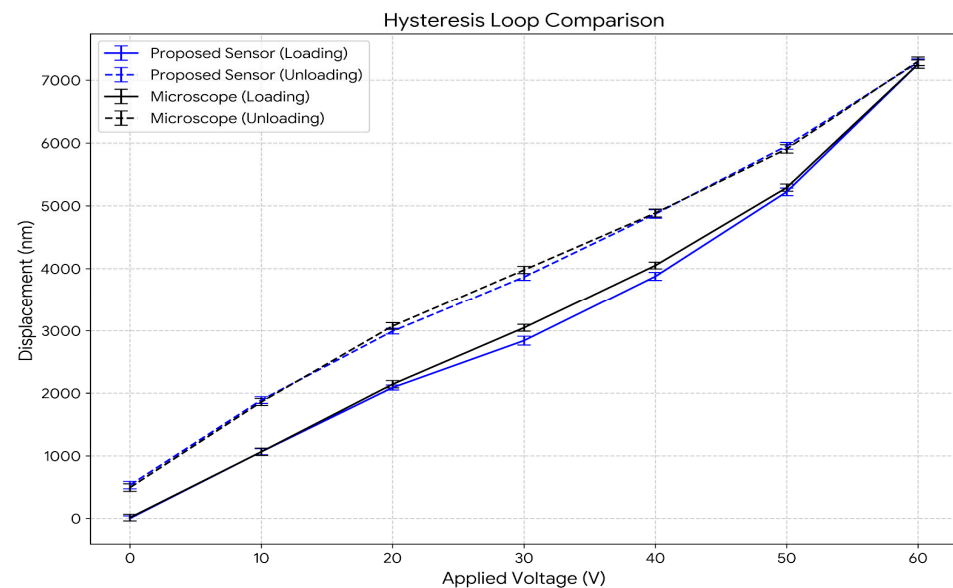


Figure 7. Hysteresis behavior of the proposed sensor.

The measurements indicate that the full-scale expansion of the piezoelectric stack is 7300.62 nm. Under hysteresis conditions, the sensor exhibits an accuracy of 98.81%, corresponding to an RMSE of 86.83 nm. The precision (repeatability) of the sensor is evaluated as 1.02%, with a standard deviation of 74.62 nm. The maximum measured bias is 207.14 nm, which corresponds to a full-scale error of 2.84%. The detailed measurement results are summarized in Table 3.

Table 3. Comparison between the microscope and sensor measurements in hysteresis.

Voltage (V)	Sensor Loading (nm)	Microscope Loading (nm)	Sensor Unloading (nm)	Microscope Unloading (nm)
0	0.00 ± 40.04	11.42 ± 52.14	531.83 ± 60.06	492.57 ± 60.70
10	1063.66 ± 49.04	1063.66 ± 57.86	1889.89 ± 53.91	1859.58 ± 55.28
20	2089.33 ± 40.04	2142.87 ± 57.86	2991.53 ± 45.87	3073.70 ± 55.28
30	2839.58 ± 70.07	3046.72 ± 55.28	3865.25 ± 53.91	3977.54 ± 57.86
40	3874.75 ± 63.31	4049.49 ± 52.99	4871.93 ± 67.15	4885.88 ± 61.62
50	5223.31 ± 60.06	5290.58 ± 57.86	5954.58 ± 53.91	5906.64 ± 66.36
60	7255.66 ± 66.40	7255.66 ± 66.36	7284.15 ± 53.91	7300.62 ± 66.36

In addition to hysteresis characterization, the rate-dependent behavior of the piezoelectric material was also experimentally investigated, as it represents another significant source of nonlinearity. In this analysis, measurements were performed at different actuation rates to evaluate the capability of the proposed sensor to capture dynamic nonlinear effects. A direct comparison with microscope-based measurements was not conducted for this experiment, as the hysteresis results already confirmed that the proposed sensor is capable of accurately measuring the displacement of the piezoelectric actuator. Therefore, the rate-dependent measurements were used to assess relative changes in sensor response rather than absolute displacement accuracy. The corresponding experimental results are presented in Figure 8.

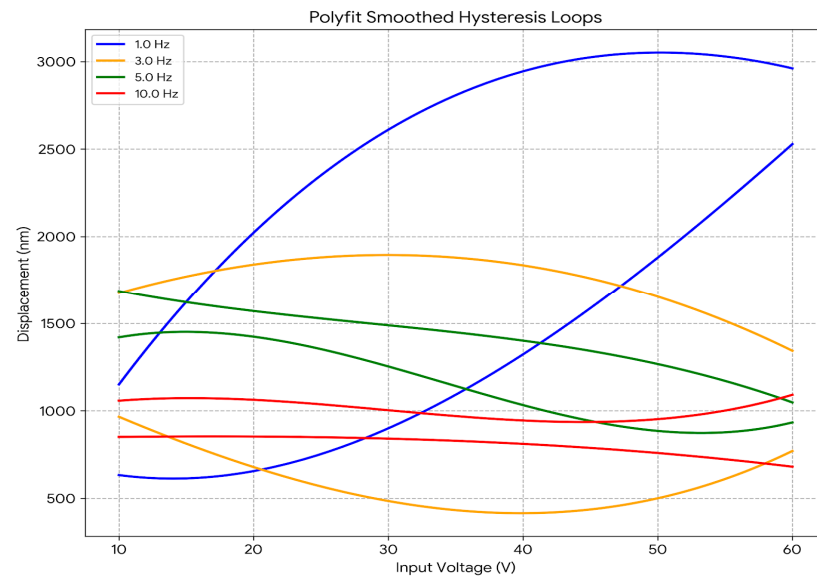


Figure 8. Experimental rate-dependent hysteresis loops of the proposed sensor system measured at excitation frequencies of 1, 3, 5, and 10 Hz. The data are processed using cycle-averaging and Savitzky–Golay smoothing to reduce high-frequency noise while preserving the hysteresis behavior.

The quantitative results extracted from the smoothed hysteresis loops are summarized in Table 4. A clear trade-off between the operating frequency and the achievable output displacement is observed. At low excitation frequencies (e.g., 1.0 Hz), the sensor operates effectively within the quasi-static regime, exhibiting a displacement amplitude of 2710 nm and a phase delay of 38.0° .

Table 4. Frequency-dependent hysteresis and phase lag characteristics.

Frequency (Hz)	Max Displacement (nm)	Hysteresis Width (nm)	Relative Hysteresis Error (%)	Phase Lag (Approx. Degrees)
1.0	2710	2036.66	75.27	38.0
3.0	1600	1471.22	92.19	134.3
5.0	1070	432.91	40.63	129.3
10.0	730	502.51	68.47	111.9

As the excitation frequency increases, the sensor performance changes markedly. For frequencies of 3.0 Hz and above, the phase lag increases substantially, reaching 134.3° , while the displacement amplitude decreases to 730 nm at 10 Hz. It should be noted that the current optical readout system operates at a nominal frame rate of approximately 30 frames per second. According to fundamental control and signal processing principles, reliable signal reconstruction typically requires a sampling rate at least ten times higher than the excitation frequency. Consequently, the effective operating range of the proposed sensor is practically limited to excitation frequencies up to approximately 3 Hz, providing about ten samples per actuation cycle.

Nevertheless, experiments at 5 Hz and 10 Hz were intentionally performed to investigate the sensor response under undersampled conditions. As anticipated, the results at these higher frequencies exhibit pronounced phase lag and significant attenuation of the displacement amplitude.

4. Conclusions

The experimental results demonstrate that nanometer-scale displacement measurement can be achieved using a two-stage optical triangulation architecture constructed from

readily available optical components and an industrial imaging sensor. A displacement resolution of 109.6 nm and an accuracy of 98.81%, with a precision (repeatability) of 74.62 nm, were achieved without the use of interferometric techniques, vibration isolation, or complex signal-processing algorithms, even under hysteresis-loop measurements, highlighting the robustness and practical applicability of the proposed approach. Under the hardware limitations of the 30 FPS image sensor, the effective bandwidth of the proposed method was demonstrated to be up to 3 Hz.

It is important to emphasize that the achieved resolution is primarily limited by the spatial resolution of the imaging sensor employed in the experimental setup. Since the proposed sensing principle relies purely on geometric optical amplification, the displacement resolution scales directly with the effective pixel size and the imaging geometry. Therefore, the reported resolution does not represent a fundamental limitation of the sensor architecture. Further improvements in measurement resolution can be achieved by increasing the optical magnification, extending the projection distance, or employing imaging sensors with higher spatial resolution, while preserving the simplicity of the system.

Compared with interferometric displacement measurement techniques, which routinely achieve sub-nanometer resolution at the expense of increased system complexity and sensitivity to environmental disturbances, the proposed method offers a complementary solution that prioritizes simplicity, robustness, and ease of implementation. For piezoelectric actuator applications involving displacement ranges on the order of tens of micrometers—such as the 30 μm range demonstrated in this study—the proposed sensor provides sufficient resolution for effective closed-loop control while significantly reducing system complexity and overall cost.

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