



Article

# A Pore Pressure Generation Model for Strain-Controlled Cyclic Tests on Sandy Soils

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## Abstract

This paper presents a modified version of the Booker et al. cycle-based pore pressure generation model for use with strain-controlled cyclic loading tests on sandy soils. The original Booker et al. model has been widely used in geotechnical engineering because of its simplicity and computational efficiency; however, it was developed using stress-controlled cyclic test data and does not accurately capture the pore pressure response observed in strain-controlled cyclic loading. In strain-controlled tests, excess pore pressure typically develops rapidly during the early stages of loading and more gradually as liquefaction is approached, resulting in a pore pressure–cycle ratio relationship fundamentally different from that produced by the original model. To address this limitation, a Modified Booker Model was developed through curve fitting of laboratory test data obtained from strain-controlled cyclic triaxial and cyclic direct simple shear tests. The model was evaluated using the results of 51 cyclic triaxial tests and 101 cyclic direct simple shear tests performed on clean sands and sand–silt mixtures with fines contents both above and below the threshold fines content (TFC). The model produced excellent agreement with measured pore pressure responses, yielding average coefficients of determination ( $R^2$ ) ranging from 0.921 to 0.989 depending on the test type and soil conditions. The results indicate that the Modified Booker Model provides a practical, accurate, and computationally efficient method for predicting pore pressure generation during strain-controlled cyclic loading of sands and silty sands.

**Keywords:** cyclic loading; pore pressure generation; liquefaction; strain-controlled testing; cyclic triaxial tests; cyclic direct simple shear tests



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## 1. Introduction

Pore pressure generation models are widely used in geotechnical engineering to estimate the development of excess pore water pressures under applied loading conditions. These predictions facilitate the evaluation of changes in effective stress and, consequently, improve assessments of soil behavior, including stress distribution, shear strength, and deformation response. Existing models span a broad range of complexity, from relatively simple empirical relationships to advanced numerical formulations implemented using finite element or finite difference methods.

Although advanced numerical models can achieve a high level of predictive accuracy, their application is often associated with substantial costs related to specialized software, computational resources, and technical expertise. For many engineering projects, particularly those constrained by limited budgets or resources, the use of such sophisticated

tools may not be practical or economically justified. In these cases, simplified pore pressure generation models provide a practical alternative.

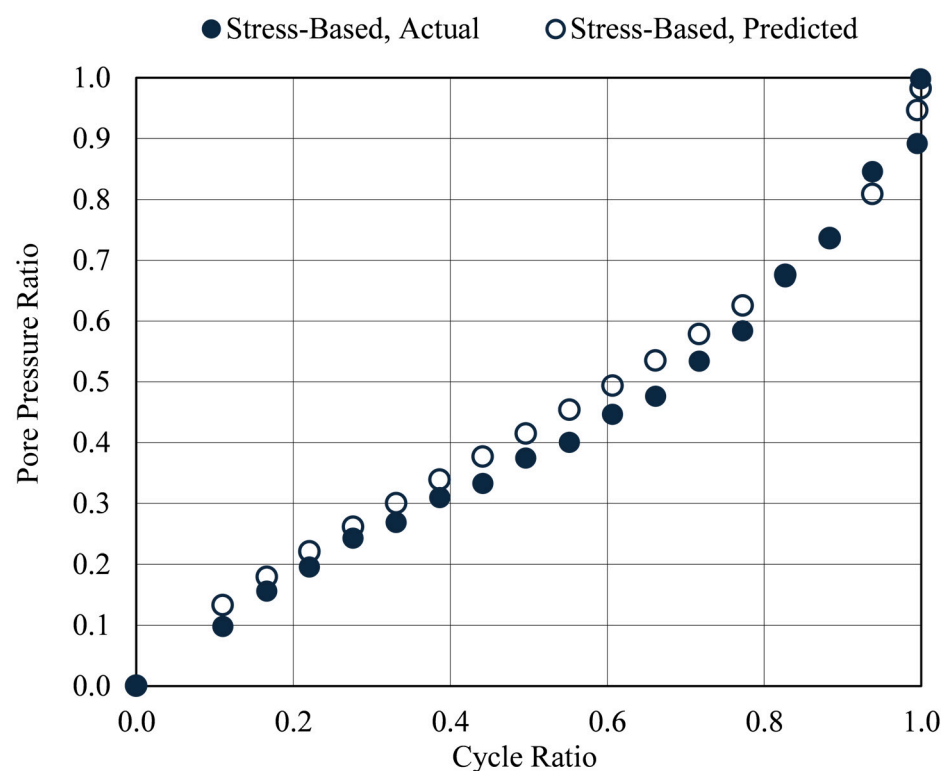
Despite their simplicity, these models can produce sufficiently accurate results for many routine engineering applications, especially under relatively uncomplicated loading conditions. In addition to their acceptable predictive capability, simplified models offer important advantages in terms of cost efficiency and accessibility. They can be implemented using commonly available programming environments or spreadsheet-based tools, thereby avoiding the need for expensive proprietary software. Furthermore, their lower computational requirements enable efficient execution on standard desktop computers.

In the mid-1970s, researchers at the University of California, Berkeley, developed a simple model for predicting pore pressure generation in soils subjected to cyclic loadings [1,2]. This model uses the cycle ratio and one calibration parameter. While the model is easy to implement, it has the drawback that its application is limited to stress-controlled tests.

This paper will begin by providing background information on the Booker et al. cycle-based pore pressure generation model, stress- and strain-controlled cyclic tests, and threshold fines content (TFC). Next, a new model will be introduced and evaluated.

### 1.1. Statement of the Problem

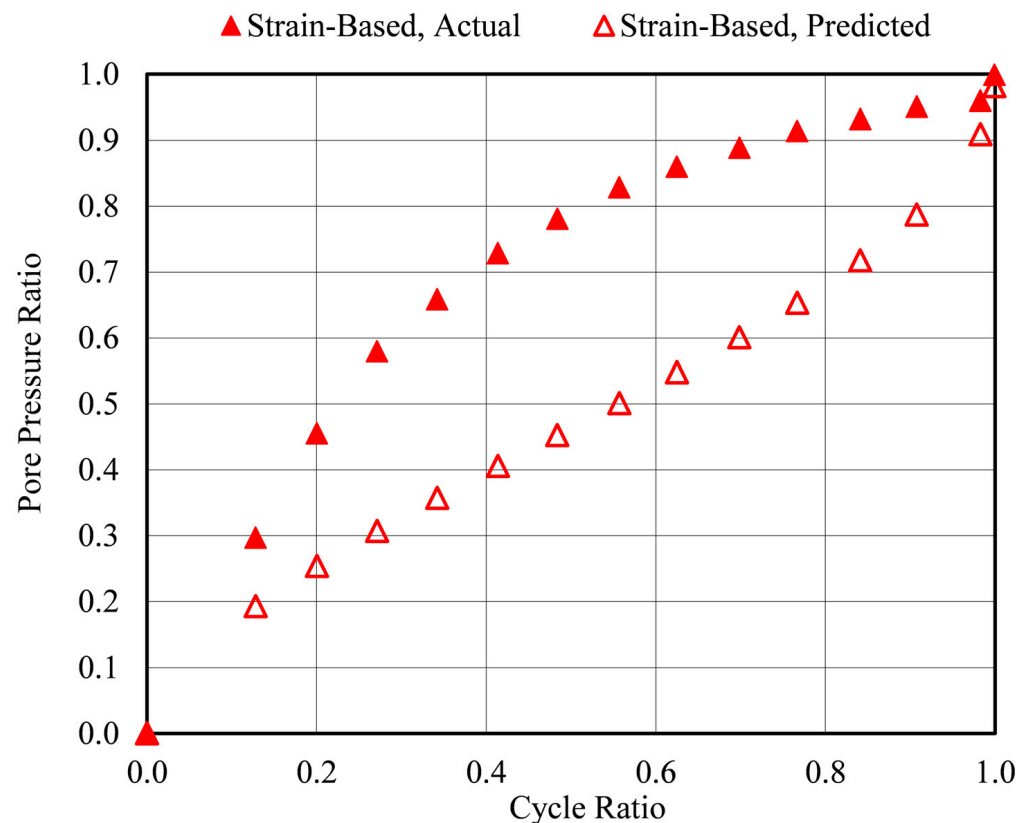
The Booker et al. model [2] was developed to predict pore pressure generation in sands subjected to cyclic loading. It was based on the results of a series of stress-based cyclic triaxial tests. When the model is used to predict the pore pressure ratio using the cycle ratio from a stress-controlled cyclic triaxial test, it generally works quite well. Figure 1 presents the measured pore pressures (plotted in terms of the pore-pressure ratio) versus the cycle ratio for a stress-controlled cyclic triaxial test performed on Ottawa C-109 sand prepared to a relative density of 40% and loaded at a cyclic stress ratio 0.20. The specimen reached initial liquefaction in 9.1 cycles of loading.



**Figure 1.** Booker et al. model applied to a stress-controlled cyclic triaxial test.

Also plotted in the figure are the pore pressure ratios predicted by the Booker et al. model using a calibration factor,  $\Theta$ , of 0.7 as recommended by Lee and Albaisa [3]. The figure shows that the model does an excellent job of accurately predicting the measured pore pressure ratios, capturing both the shape and the magnitude of the data.

The model can just as easily be applied to the data from a strain-controlled test; however, when this is done, the results are quite different. Figure 2 presents the measured pore pressure ratio versus the cycle ratio for a strain-controlled cyclic triaxial test performed on Ottawa C-109 sand prepared to a relative density of 40% and loaded at an axial strain of  $\pm 0.20\%$ . The specimen reached initial liquefaction in 7.0 cycles of loading.

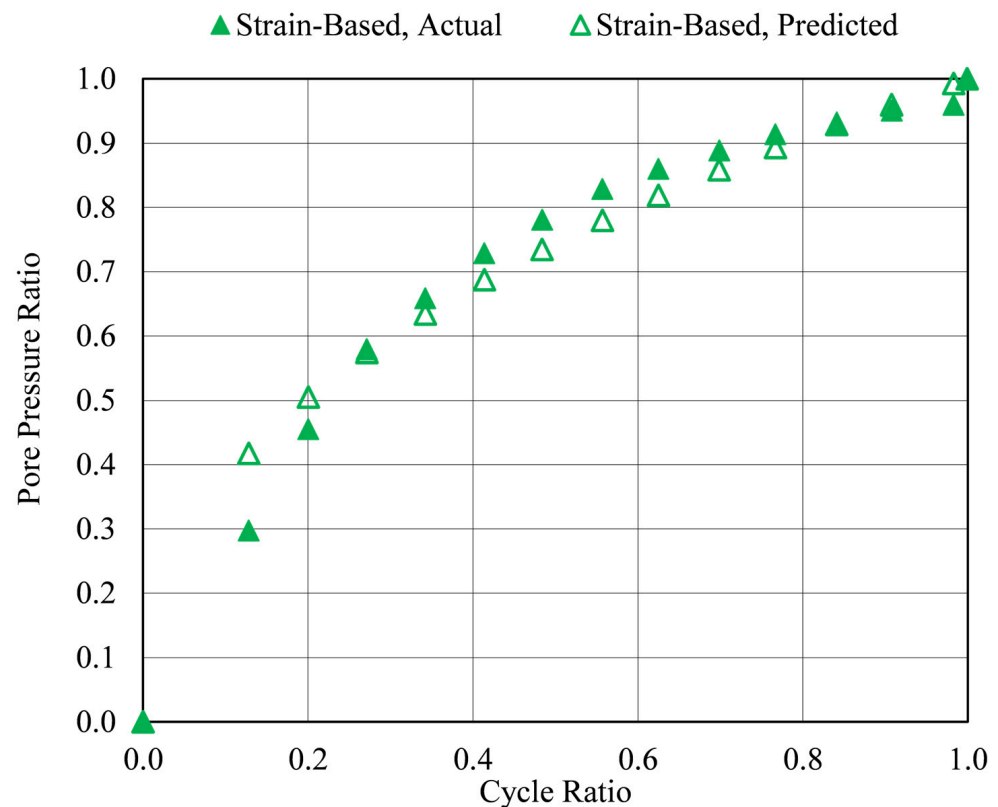


**Figure 2.** Booker et al. model applied to a strain-controlled cyclic triaxial test.

Also plotted in the figure are the pore pressure ratios predicted by the Booker et al. model using a calibration factor,  $\Theta$ , of 0.7. In the figure, it can be seen that the model does a very poor job of predicting the measured pore pressure ratios, capturing neither the shape nor the magnitude of the measured behavior.

The model's inability to accurately predict pore pressures in strain-controlled tests results from the inherent differences in a soil's response to stress- and strain-controlled cyclic loadings. Because the Booker et al. model cannot replicate the shape of the pore pressure–cycle ratio curve, a new model, named the “Modified Booker Model”, was developed for predicting pore-pressure development under strain-controlled cyclic loadings.

Figure 3 presents the measured pore pressure response originally presented in Figure 2. It also plots the pore pressure ratios predicted using the Modified Booker Model with a calibration factor,  $\beta$ , of 0.85. As seen in the figure, the Modified Booker Model does an excellent job of accurately predicting the measured pore pressure ratios, capturing both the shape and the magnitude of the data.



**Figure 3.** Modified Booker Model applied to the strain-controlled cyclic triaxial test previously shown in Figure 2.

### 1.2. Literature Review

Excess pore pressure generation models play a fundamental role in geotechnical earthquake engineering because they provide a means of predicting the progressive reduction in effective stress that may ultimately lead to liquefaction. Early empirical studies by Seed and co-workers established the relationship between excess pore pressure generation and the normalized number of loading cycles, forming the basis of stress-based liquefaction analyses [1,2]. Building on this work, Booker, Rahman, and Seed [4] developed a simplified pore pressure generation model that has become one of the most widely used models in geotechnical engineering. Owing to its simple mathematical formulation, computational efficiency, and ability to represent the progressive accumulation of pore pressure, the Booker model has been widely implemented in effective stress analyses, finite element simulations, spreadsheet-based calculations, and preliminary engineering design. The model has been successfully applied to problems involving earthquake loading, traffic-induced vibrations, machine foundations, and offshore wave loading. Subsequent investigations by Lee and Albaisa [3] and Berrill and Davis [5] further demonstrated the applicability of simplified pore pressure generation models for evaluating liquefaction susceptibility under cyclic loading.

Recognizing the limitations of stress-based approaches under irregular loading histories, researchers subsequently developed strain-based and energy-based pore pressure generation models. Martin, Finn, and Seed [6] introduced a strain-based framework that relates cyclic shear strain to volumetric contraction, while Byrne [7] proposed a simplified formulation suitable for engineering practice. Energy-based approaches, beginning with the work of Nemat-Nasser and Shokooh [8], relate excess pore pressure development to the cumulative energy dissipated within the soil and more accurately represent variable-amplitude loading. Additional constitutive formulations by Finn et al. [9] and Iai et al. [10] further advanced the prediction of pore pressure generation during seismic loading. Collec-

tively, these developments have established stress-based, strain-based, and energy-based models as practical tools for evaluating cyclic soil behavior and liquefaction potential under a wide range of laboratory and field loading conditions.

A primary application of the Booker et al. model is liquefaction assessment, where it is used to estimate excess pore pressure buildup and the associated reduction in effective stress in loose saturated sands. The model has also been incorporated into finite element and finite difference analyses when a simplified constitutive representation is appropriate. Its modest computational requirements make it well suited for routine engineering applications and projects where advanced constitutive modeling or extensive laboratory testing is impractical.

Despite these advantages, the Booker et al. model has several important limitations. The largest limitation of the Booker et al. model is that it is only capable of producing accurate pore pressure generation predictions for stress-controlled tests. This is because both are stress-controlled cyclic tests, and both exhibit pore pressure ratios that increase slowly at low cycle ratios and then increase more rapidly as the cycle ratio approaches unity (as shown in Figures 1 and 2). This is in contrast to strain-controlled cyclic tests in which the pore pressure ratio increases rapidly at low cycle ratios and then increases more slowly as the cycle ratio approaches unity (as shown in Figures 2 and 3).

Another limitation of the model is that it is fundamentally empirical or semi-empirical in nature and therefore relies heavily on calibration against experimental data. As a result, its predictive accuracy may decrease when applied to soils or loading conditions that differ significantly from those used during model development. In particular, the model may not adequately capture complex soil behaviors such as anisotropy, strain localization, cyclic mobility, or stress-path dependency.

Another limitation is that the model generally assumes simplified loading conditions and idealized soil behavior. Consequently, its applicability may be restricted in problems involving highly irregular loading histories, multidirectional stress states, or strongly heterogeneous soil deposits. In addition, because the model focuses primarily on pore pressure generation, it may provide limited insight into other aspects of soil response, such as post-liquefaction deformation or progressive failure mechanisms.

Therefore, while the Booker et al. model remains a useful and practical tool for preliminary analyses and routine engineering applications, caution should be exercised when applying it to complex geotechnical problems where more advanced constitutive models may be required.

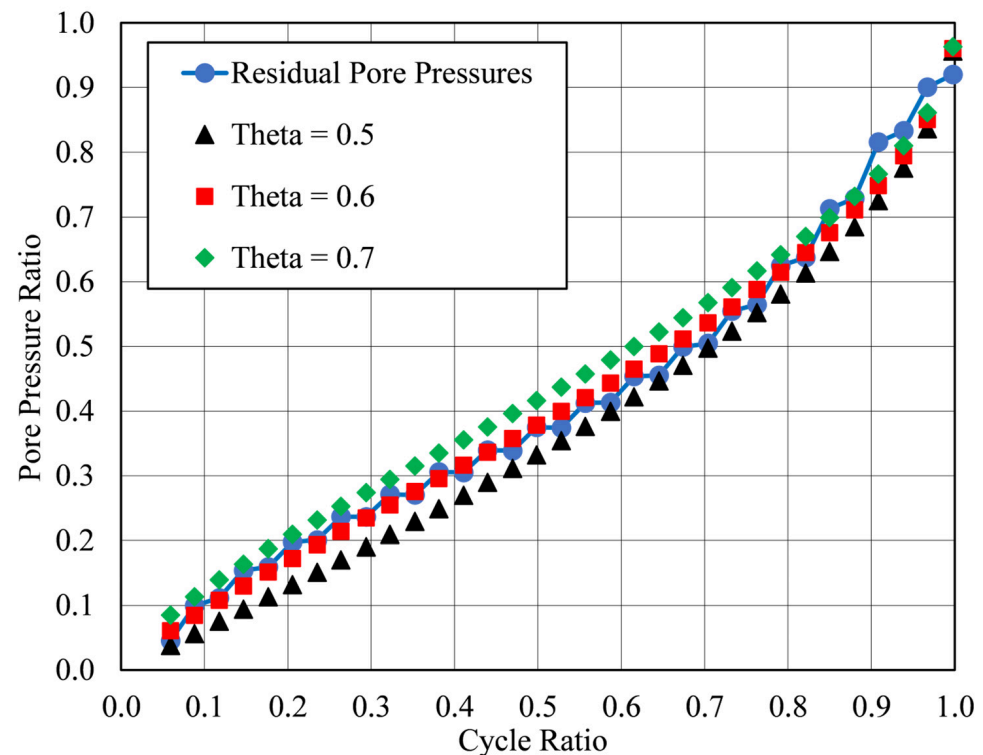
The model requires two input parameters: the cycle ratio and an empirical calibration constant,  $\theta$ . The relationship is expressed as Equation (1):

$$r_u = \frac{\pi}{2} \arcsin \left[ \left( \frac{N}{N_L} \right)^{\frac{1}{2\theta}} \right] \quad (1)$$

In this equation,  $r_u$  represents the pore pressure ratio,  $N$  is the number of applied load cycle of interest, and  $N_L$  is the number of cycles required to initiate liquefaction. The calibration constant,  $\theta$ , characterizes the rate of pore pressure build-up and is typically determined through laboratory-based stress-controlled cyclic loading tests. Lee and Albaisa [3] found that a value of  $\theta = 0.7$  provided a good fit to experimental data across a range of sand types and relative densities under stress-controlled conditions.

Figure 4 plots the residual pore pressures measured during a stress-controlled cyclic triaxial test performed on a specimen of New Martin Sand at a relative density of 40%. It was loaded at a cyclic stress ratio of 0.23 and reached initial liquefaction in 17.1 cycles. Also included in the figure are the residual pore pressures predicted by the Booker et al. model

with three values of  $\Theta$ . In the figure, it can be seen that the model does an excellent job of predicting the residual pore pressures for this stress-controlled test for all three values of  $\Theta$ .



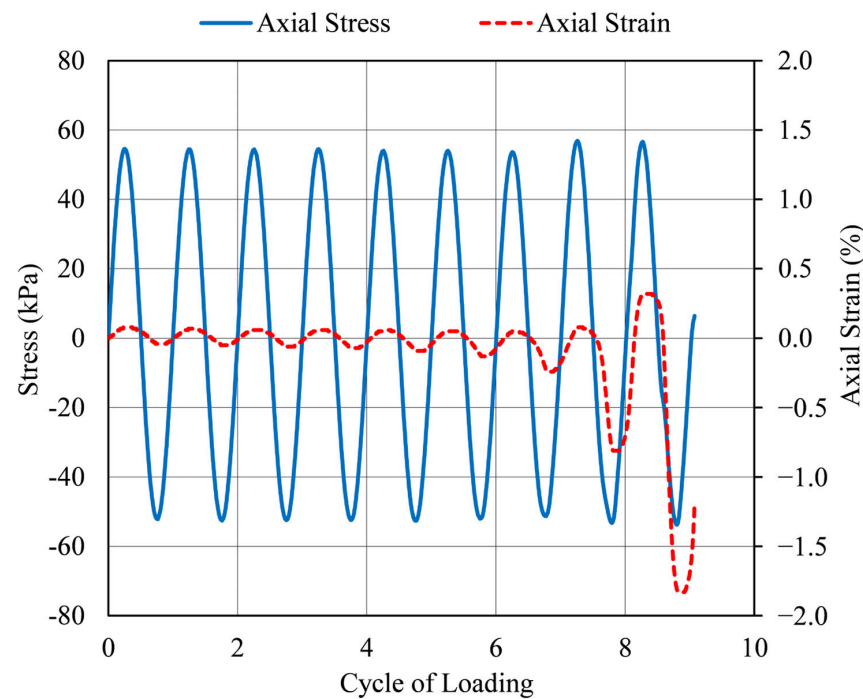
**Figure 4.** Application of the Booker et al. model for a stress-controlled cyclic triaxial test using three values of  $\Theta$ .

### 1.3. Stress- and Strain-Controlled Cyclic Tests

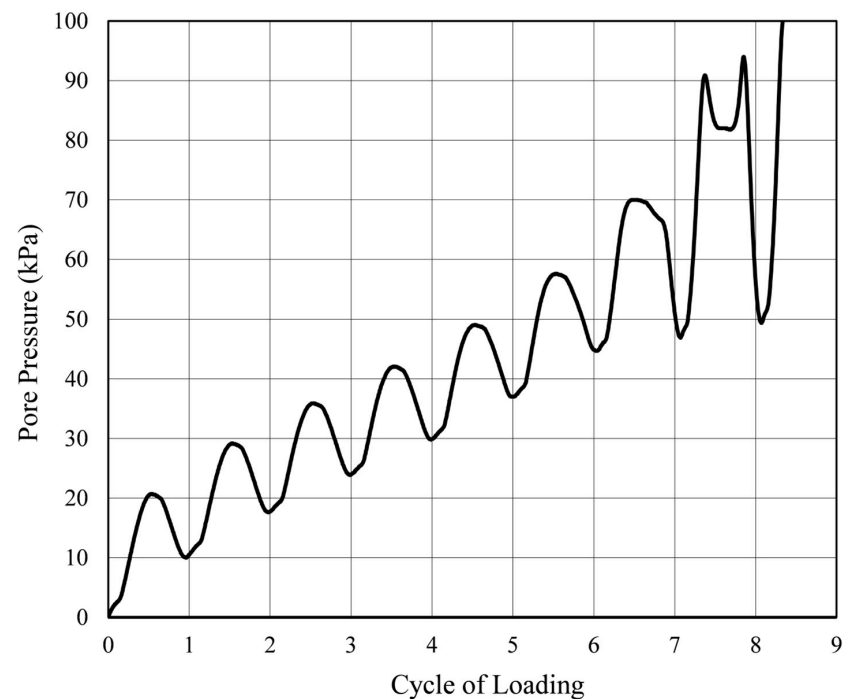
The patterns of stresses and strains that similarly prepared specimens experience are distinctly different for the cases of stress-controlled and strain-controlled loadings. Because of these differences, the patterns of excess pore water generation in similarly prepared specimens are also different in stress-controlled and strain-controlled cyclic triaxial and cyclic direct simple shear tests.

In stress-controlled cyclic triaxial and cyclic direct simple shear tests, the maximum and minimum amplitudes of the applied stress remain constant throughout the test, as shown in Figure 5, which presents the stresses applied and the resulting strains in a stress-controlled cyclic triaxial test. The applied stresses and resulting strains in a cyclic direct simple shear test follow the same patterns; however, the axial stress is replaced with shear stress, and the axial strains are replaced with shear strains.

As a result of the constant maximum and minimum deviator stresses acting on the specimen, it is the amplitude of the applied strain induced in the specimen that controls the pattern of pore pressure generation. As pore pressures are generated in response to the applied stress, the effective stress acting on the specimen and its ability to resist deformation decrease. This leads to increasing shear strains within the specimen even though the maximum stress amplitude applied to the specimen remains at a constant level. As a result, little pore pressure is generated during the early part of the test due to the small levels of straining; however, in the later stages of the test, larger amounts of pore pressures are generated in the soil due to the increased shear strains. This behavior may be seen in Figure 6, which shows that the pore pressure increases slowly over the first few cycles of loading when the axial strains are small, and then increases rapidly as the specimen nears liquefaction and undergoes large strains.



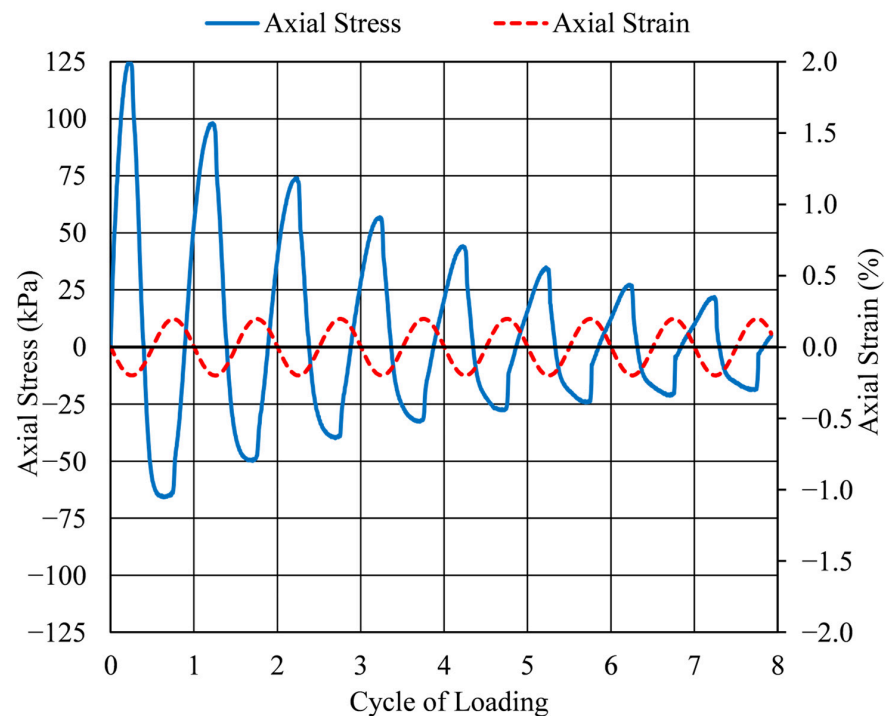
**Figure 5.** Stresses and strains in a stress-controlled cyclic triaxial test.



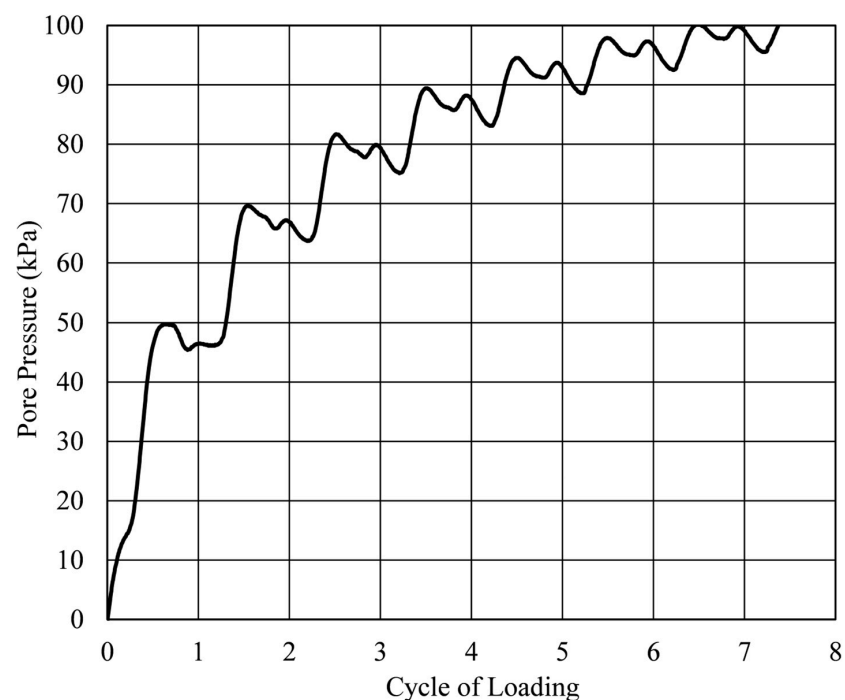
**Figure 6.** Pore pressure development in a stress-controlled cyclic triaxial test.

Conversely, in strain-controlled cyclic triaxial and cyclic direct simple shear tests, the maximum and minimum amplitudes of the applied strain remain constant throughout the test, as shown in Figure 7. As a result, it is the resistance to deformation produced by the soil (i.e., the stress) that controls the excess pore pressure generation. Again, as excess pore pressures are generated in response to the cyclic loading, both the effective stress acting on the specimen and thus its ability to resist deformation decrease. This leads to a decreasing amount of resistance being required to maintain the level of straining as the loading continues to be applied at a constant level. As a result, there is a relatively large amount of pore pressure generated during the early part of the test when the resistance

to the applied displacements is high. In the later stages of the test, when pore pressures are high and effective stresses are low, smaller amounts of pore pressures are generated in the soil due to the lower levels of resistance that the soil can develop to the applied deformations. This behavior may be seen in Figure 8, which shows that the pore pressure increases rapidly over the first few cycles of loading when the soil's resistance is small, and then increases more slowly as the specimen nears liquefaction and offers little resistance to the applied strain.



**Figure 7.** Stresses and strains in a strain-controlled cyclic triaxial test.



**Figure 8.** Pore pressure development in a strain-controlled cyclic triaxial test.

#### 1.4. Threshold Fines Content

The concept of threshold fines content was introduced to describe the fines content at which sand–silt mixtures transition from sand-like behavior, characterized by a relatively high shear strength and low compressibility, to silt-like behavior with a lower shear strength and higher compressibility [11]. Similar terminology has been used in the literature, including “limiting silt content” [12], “limiting fines content” [13], “limiting fines value” [14], “critical fines content” [15], and “transitional fines content” [16].

Previous studies have shown that the static shear strength and liquefaction resistance of sand–silt mixtures are strongly dependent on the fines content relative to the threshold fines content. Mixtures with fines contents exceeding this threshold generally exhibit a lower shear strength and cyclic resistance than mixtures below the threshold [11].

In cohesionless soils containing non-plastic fines, the threshold fines content corresponds to a transition in soil fabric from a sand-dominated matrix, where fines occupy voids between sand grains, to a fines-dominated matrix in which sand grains become isolated within the silt matrix. Below the threshold, behavior is governed primarily by sand-to-sand contacts, whereas above the threshold, silt-to-silt interactions control the mechanical response.

## 2. Materials and Methods

This section will first present the proposed Modified Booker Model and then discuss the soils tested and the test methodologies used. The proposed model was named the “Modified Booker Model” as a homage to the original model on which it is largely based. This was done even though the original model is based on a sinusoidal function and the modified model is based on a simple power function.

#### 2.1. The Proposed Model

As previously noted, the primary drawback to the Booker et al. model is that it only produces accurate results for stress-controlled loadings. It does not accurately predict pore pressure development in strain-controlled tests.

In order to better predict pore pressures in strain-controlled cyclic tests, the Booker et al. model was modified. The new form of the model was developed through curve fitting of test data from strain-controlled cyclic triaxial and cyclic direct simple shear tests. The modified form is referred to as the “Modified Booker Model” and is presented as Equation (2).

$$r_u = \left[ \frac{N}{N_L} \right]^\beta \quad (2)$$

where  $N$  is the cycle of loading at which the pore pressure ratio,  $r_u$ , is calculated;  $N_L$  is the number of cycles required to induce liquefaction; and  $\beta$  is the calibration constant.

The new model follows a simple power relationship in which the pore pressure ratio generated in the specimen during cyclic loading is equal to the cycle ratio (the ratio of the current cycle of loading to the number of cycles of loading required to trigger liquefaction) raised to the power  $\beta$ . In the model,  $\beta$  is an empirical curve-fitting constant determined through results of laboratory cyclic loading tests performed on specimens of the soil being investigated. The modified functional form better captures strain-controlled behavior because it reflects the fundamentally different manner in which pore pressures develop under constant-strain-loading conditions. In stress-controlled loading, pore pressure generation accelerates as the soil weakens and strains increase. Conversely, in strain-controlled loading, the imposed strain amplitude remains constant, causing large stresses and rapid pore pressure generation during the early stages of loading when the soil stiffness is highest. As excess pore pressures accumulate and effective stress decreases, progressively smaller

stresses are required to maintain the imposed strains, resulting in slower rates of pore pressure generation as liquefaction is approached.

The original Booker et al. model was formulated to reproduce the concave-upward pore pressure response characteristic of stress-controlled cyclic loading. However, strain-controlled cyclic tests exhibit the opposite trend, where pore pressure ratios increase rapidly during the initial loading cycles and then gradually level off as the cycle ratio approaches unity. The modified functional form reverses the curvature of the pore pressure–cycle ratio relationship, allowing it to more accurately represent the observed experimental behavior. As a result, the Modified Booker Model captures both the magnitude and shape of the measured pore pressure response significantly better than the original formulation.

Despite its strong predictive capability, the Modified Booker Model has several limitations. The model is empirical in nature and was calibrated using laboratory tests performed primarily on clean sands and non-plastic sand–silt mixtures under controlled cyclic loading conditions. Consequently, its applicability to soils containing plastic fines, cementation, aging effects, or significant anisotropy has not yet been established. In addition, the model assumes relatively simple, uniform loading conditions and does not explicitly account for multidirectional loading, irregular earthquake loading histories, stress-path dependency, or post-liquefaction deformations. The laboratory tests were also conducted at relatively low loading frequencies and under controlled boundary conditions that may not fully represent complex field behavior. Therefore, additional validation is needed before the model can be confidently applied to all field loading conditions.

## 2.2. Soil Tested and Testing Methodology

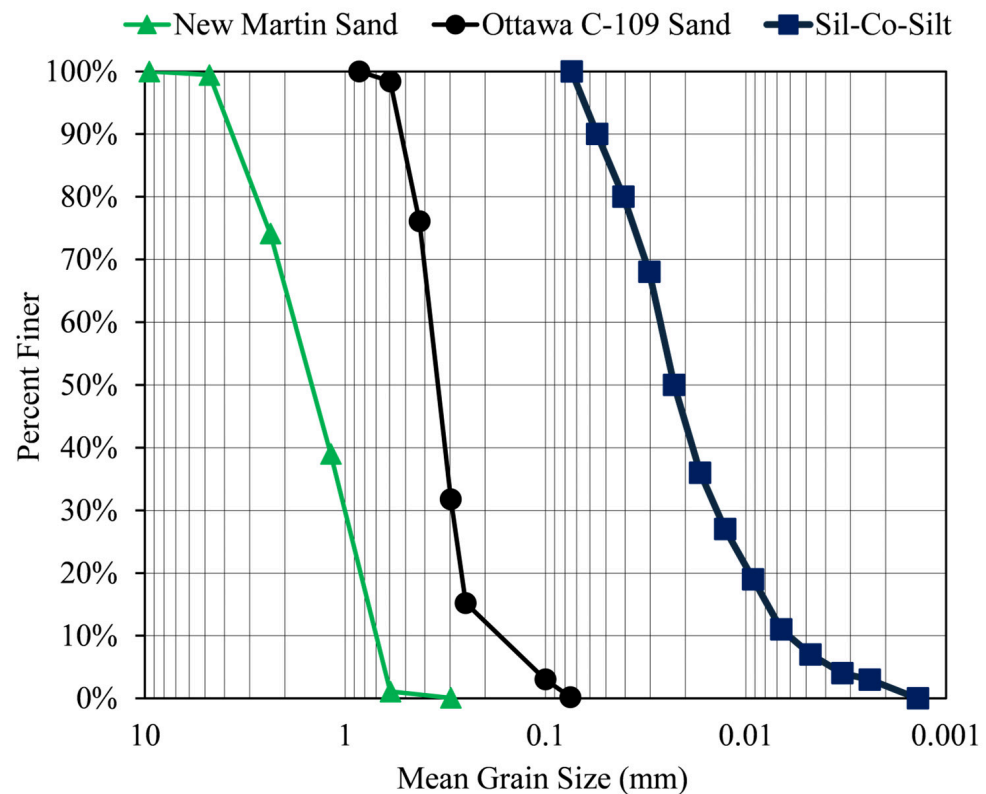
In order to test the validity of the new model, the model was evaluated using the results of 51 strain-controlled cyclic triaxial tests and 101 strain-controlled cyclic direct simple shear tests. The cyclic triaxial tests were performed on two sands: Ottawa C-109 sand [17] and New Martin sand. The properties of these sands are given in Table 1, and their grain size distribution curves are presented in Figure 9.

**Table 1.** Properties of the soils used in the study.

|                                   | <b>C-109</b> | <b>New Martin</b> | <b>Sil-Co-Sil</b> |
|-----------------------------------|--------------|-------------------|-------------------|
| USCS Classification Symbol        | SP           | SP                | ML                |
| Median Grain Size, $D_{50}$ (mm)  | 0.33         | 0.70              | 0.023             |
| Coefficient of Uniformity, $C_u$  | 2.18         | 4.61              | 4.50              |
| Coefficient of Curvature, $C_c$   | 1.34         | 1.02              | 1.39              |
| Max. Index Void Ratio, $e_{\max}$ | 0.688        | 0.547             | 1.49              |
| Min. Index Void Ratio, $e_{\min}$ | 0.436        | 0.337             | 0.67              |
| Specific Gravity, $G_s$           | 2.65         | 2.63              | 2.61              |

The cyclic triaxial specimens were prepared by moist tamping to a relative density of either 40% or 65%. After saturation, they were isotropically consolidated to an effective stress of 100 kPa and loaded at an axial-strain level between 0.1% and 0.4% at a frequency of 0.1 Hz until initial liquefaction occurred.

The model was also evaluated using the results of 101 strain-controlled cyclic direct shear tests performed on nine mixtures of Ottawa C-109 sand and #106 Sil-Co-Sil silt [18]. The properties of these soils are given in Table 1, and their grain size distribution curves are shown in Figure 9.



**Figure 9.** Grain size distribution curves for the soils used in the study.

The mixtures of sand and silt had silt contents between 0% and 50%. These mixtures had a threshold fines content of 27.1%. The mixtures with silt contents below the threshold fines content were expected to show sand-like behavior (higher strength and lower compressibility when compared to the silts). Conversely, the mixtures with silt contents above the threshold fines content were expected to show silt-like behavior (lower strength and higher compressibility when compared to the sands).

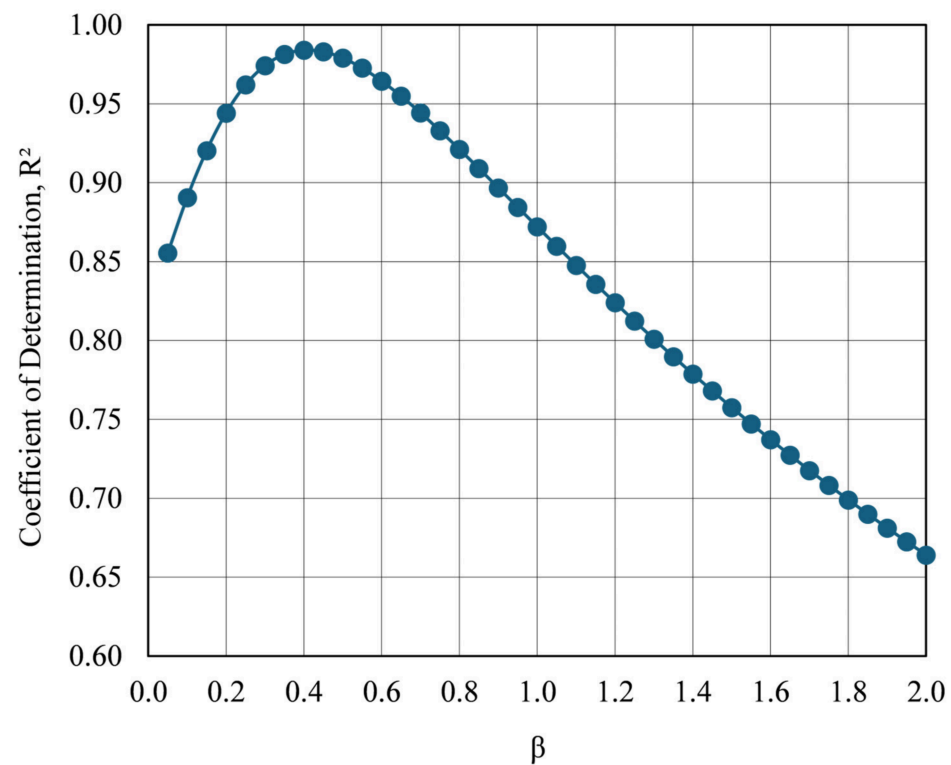
The cyclic direct simple shear specimens were prepared by moist tamping to a relative density of 40%. A normal stress of 105 kPa was applied to the specimens. They were then subjected to sinusoidal shear strain loading at a frequency of 0.1 Hz for five single-amplitude shear strain levels, continued until liquefaction was observed. Liquefaction was defined as the point at which the effective normal stress first reduced to 5 kPa.

Following the completion of testing, for each specimen, the residual pore pressure ratios and cycle ratios were calculated at each point where the applied strain equaled zero.

### 3. Evaluation of the Proposed Model

#### 3.1. Model Evaluation Methodology

Upon completion of the laboratory testing, the data set was processed through the new model in order to determine a best fit-value for the calibration parameter  $\beta$ . Values of  $\beta$  ranging from 0.1 to 2.0 in increments of 0.05 were used to predict the pore pressure response. These responses were then compared to the measured pore pressures and a coefficient of determination,  $R^2$ , was determined. For each test, the value of  $\beta$  that produced the largest  $R^2$  value was chosen as the optimum value. Figure 10 provides an example of how  $R^2$  varies with  $\beta$ . It is based on the soils with silt contents below the threshold fines content tested in cyclic triaxial tests.



**Figure 10.** Variation in  $R^2$  with  $\beta$  based on cyclic triaxial test for soils with silt contents below the threshold fines content.

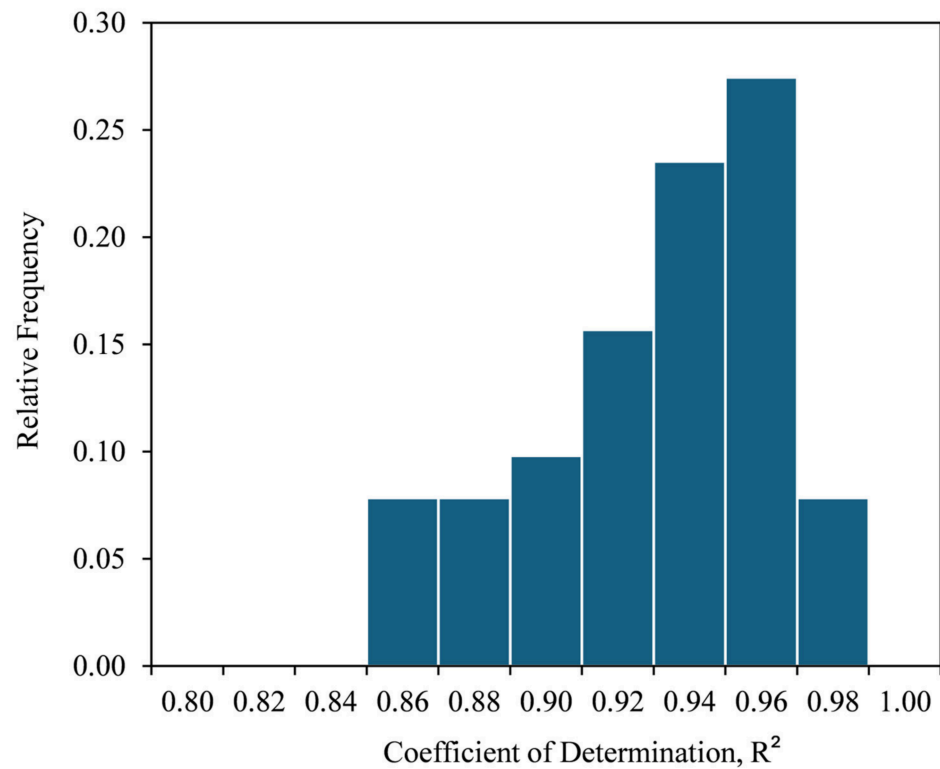
### 3.2. Results of the Model Evaluation on Cyclic Triaxial Tests

Fifty-one strain-controlled cyclic triaxial tests were performed on sand specimens. The pore pressure generation data from these tests were used to evaluate the effectiveness of the Modified Booker Model for predicting pore pressure generation under these test and soil conditions. For each test, the measured and predicted residual pore pressures were compared, and the goodness of fit of the model was evaluated using the coefficient of determination,  $R^2$ . A summary of the coefficients of determination for these tests is provided in Table 2, and a histogram of their distribution is provided in Figure 11.

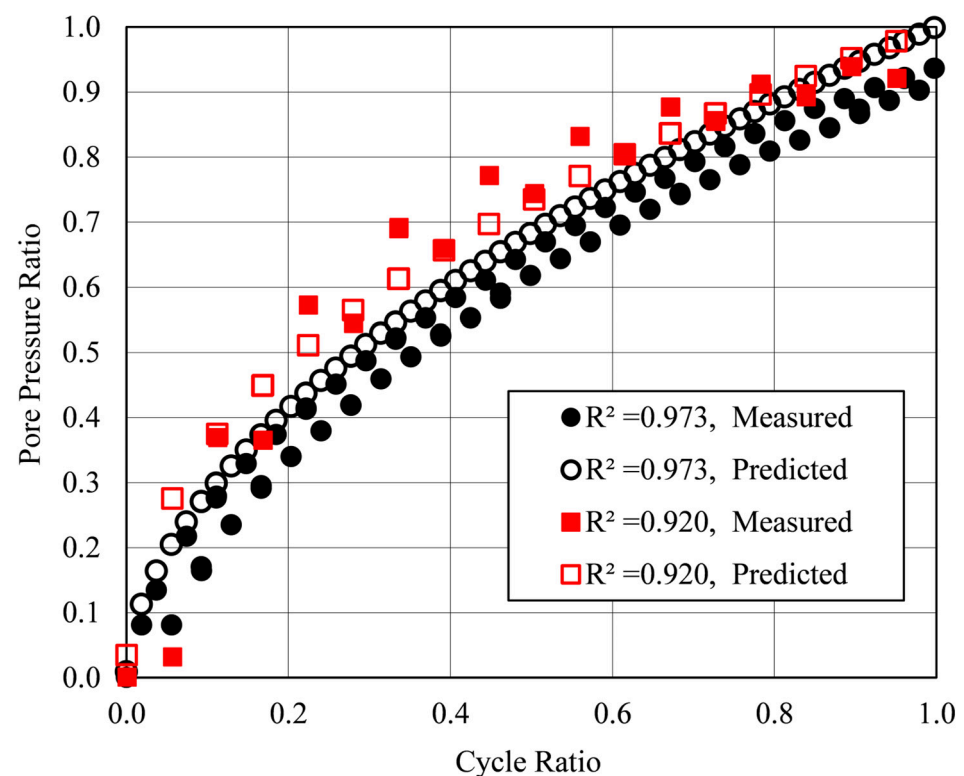
**Table 2.** Results of model application to cyclic triaxial tests.

| Test Type                   |       |
|-----------------------------|-------|
| Soil                        | Sand  |
| Number of Tests             | 51    |
| Average $R^2$               | 0.921 |
| Median $R^2$                | 0.930 |
| Maximum $R^2$               | 0.978 |
| Minimum $R^2$               | 0.846 |
| Standard Deviation of $R^2$ | 0.034 |

From the table and the figure, it is clear that the model proved excellent at predicting pore pressure generation in sand using cyclic triaxial data. The model results ranged from an  $R^2$  value of 0.846 to an  $R^2$  value of 0.979, with an average  $R^2$  value of 0.921. Additionally, 30 of the 51 tests (59%) yielded  $R^2$  values of 0.94 or higher. Figure 12 presents the measured and predicted pore pressure ratios for two typical cyclic triaxial tests.



**Figure 11.** Distribution of coefficients of determination for the 51 cyclic triaxial tests performed on sand specimens.



**Figure 12.** Typical test results for application of the Modified Booker Model to data from two strain-controlled cyclic triaxial tests.

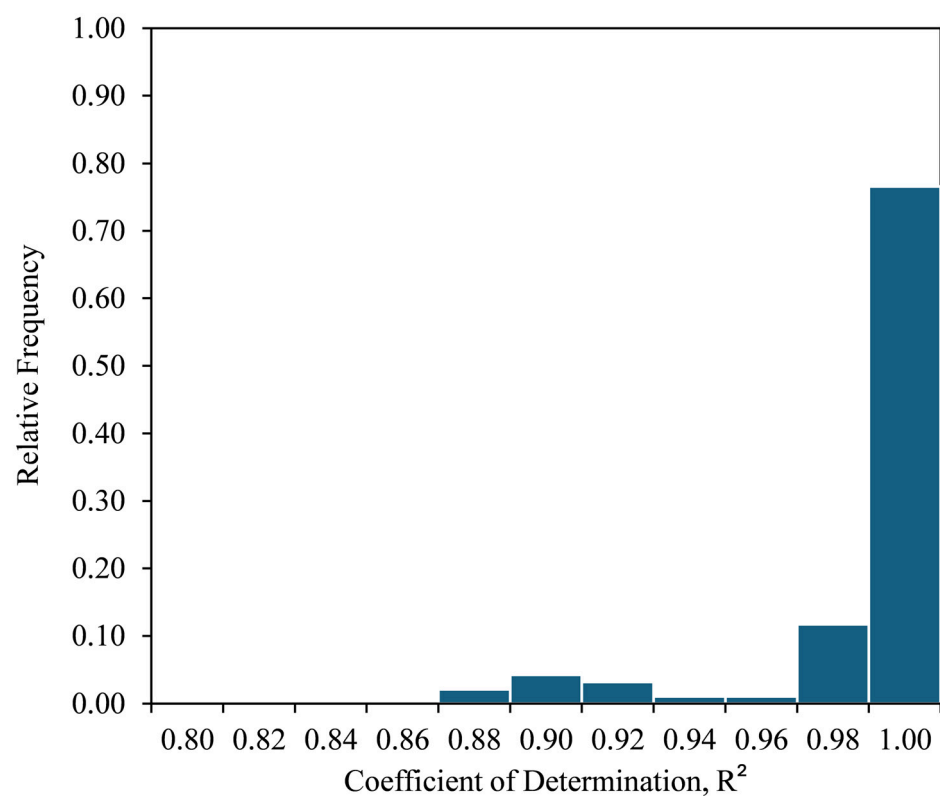
### 3.3. Results of the Model Evaluation on Cyclic Direct Simple Shear Tests

A total of 101 strain-controlled cyclic direct simple shear tests were conducted on sand specimens (38 tests) and sand–silt mixtures (63 tests). The pore pressure generation

data obtained from these tests were used to evaluate the effectiveness of the Modified Booker Model in predicting pore pressure generation under the investigated testing and soil conditions. For each test, the measured and predicted residual pore pressures were compared, and the goodness of fit of the model was assessed using the coefficient of determination  $R^2$ . A summary of the resulting  $R^2$  values is presented in Table 3, while the distribution of the coefficients is illustrated in Figure 13.

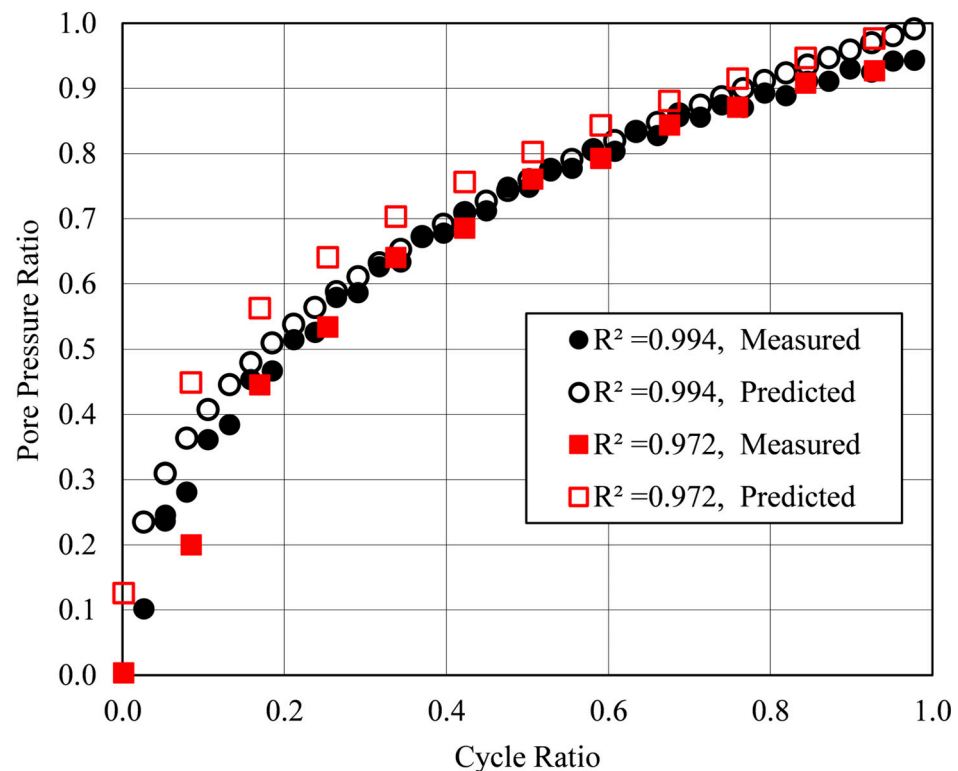
**Table 3.** Results of model application to cyclic direct simple shear tests.

| Test Type                   | Cyclic Direct Simple Shear Tests |                     |
|-----------------------------|----------------------------------|---------------------|
|                             | Soil                             |                     |
|                             | Sands                            | C-109 Sand and Silt |
| Number of Tests             | 38                               | 63                  |
| Average $R^2$               | 0.959                            | 0.989               |
| Median $R^2$                | 0.988                            | 0.991               |
| Maximum $R^2$               | 1.000                            | 0.998               |
| Minimum $R^2$               | 0.805                            | 0.963               |
| Standard Deviation of $R^2$ | 0.050                            | 0.006               |



**Figure 13.** Distribution of coefficients of determination for the 101 cyclic direct simple shear tests performed on sand specimens.

The results presented in Table 3 and Figure 13 indicate that the model provided highly accurate predictions of pore pressure generation in sand using cyclic direct simple shear test data. The coefficient of determination values ranged from 0.805 to 1.00, with an average value of 0.959 and a median value of 0.988. Figure 14 compares the measured and predicted pore pressure ratios for two representative cyclic direct simple shear tests.



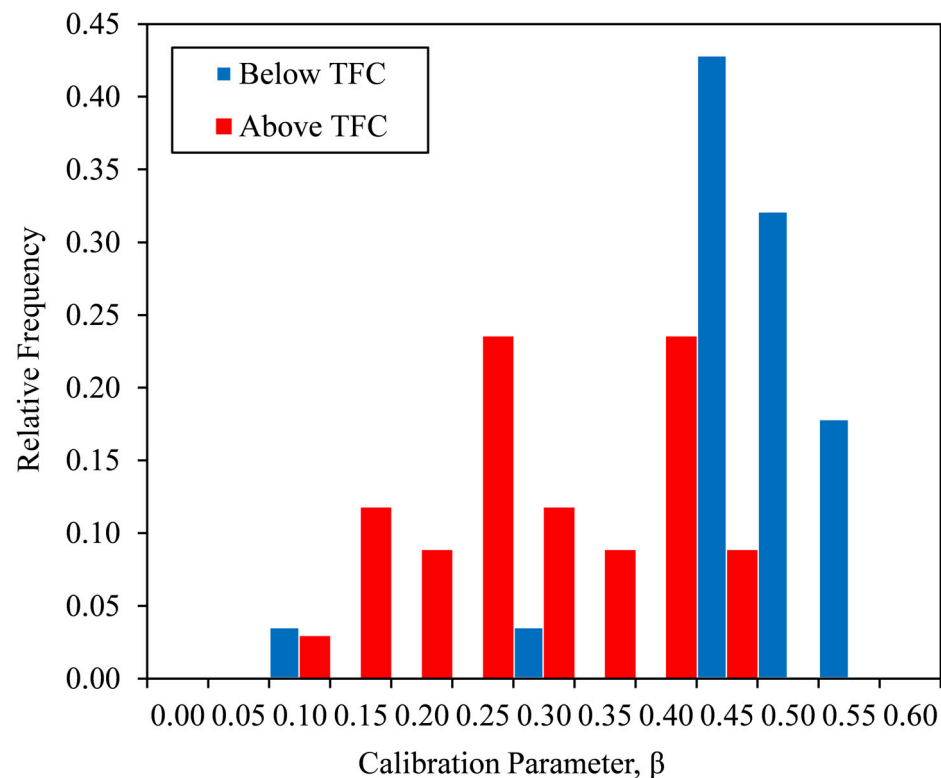
**Figure 14.** Typical test results for application of the Modified Booker Model to data from two strain-controlled cyclic direct simple shear tests.

The cyclic simple shear test data for the 63 sand–silt mixtures was next reexamined in order to evaluate the effects of the threshold fines content on model accuracy. The data was split into two groups: 28 tests on specimens with silt contents below the threshold fines content and 35 tests on specimens with silt contents above the threshold fines content. A summary of the resulting  $R^2$  values is presented in Table 4, while the distribution of the coefficients is illustrated in Figure 15.

**Table 4.** Results of model application to cyclic direct simple shear tests separated by fines content.

| Test Type                   | Cyclic Direct Simple Shear Tests |                               |
|-----------------------------|----------------------------------|-------------------------------|
|                             | C-109 Sand and Silt Below TFC    | C-109 Sand and Silt Above TFC |
| Number of Tests             | 28                               | 35                            |
| Average $R^2$               | 0.991                            | 0.987                         |
| Median $R^2$                | 0.992                            | 0.990                         |
| Maximum $R^2$               | 0.998                            | 0.996                         |
| Minimum $R^2$               | 0.981                            | 0.963                         |
| Standard Deviation of $R^2$ | 0.004                            | 0.007                         |

The results presented in Table 4 and Figure 15 indicate that the accuracy of the model is not significantly influenced by the transition between soils with silt contents below the threshold fines content, which generally exhibit sand-like behavior, and soils with silt contents above the threshold fines content, which typically exhibit silt-like behavior. The two groups exhibited nearly identical mean  $R^2$  values and comparable standard deviations, suggesting a consistent level of model performance across both soil classifications.



**Figure 15.** Distribution of coefficients of determination for the 63 cyclic direct simple shear tests performed on sand–silt combinations.

### 3.4. Model Validation

In addition to evaluating the new model using the original data set, the results of 12 strain-controlled cyclic triaxial tests published in the literature were used to validate the model using the methodology previously described. The tests were performed at George Washington University [19] on specimens of Ottawa F65 sand prepared to two relative densities.

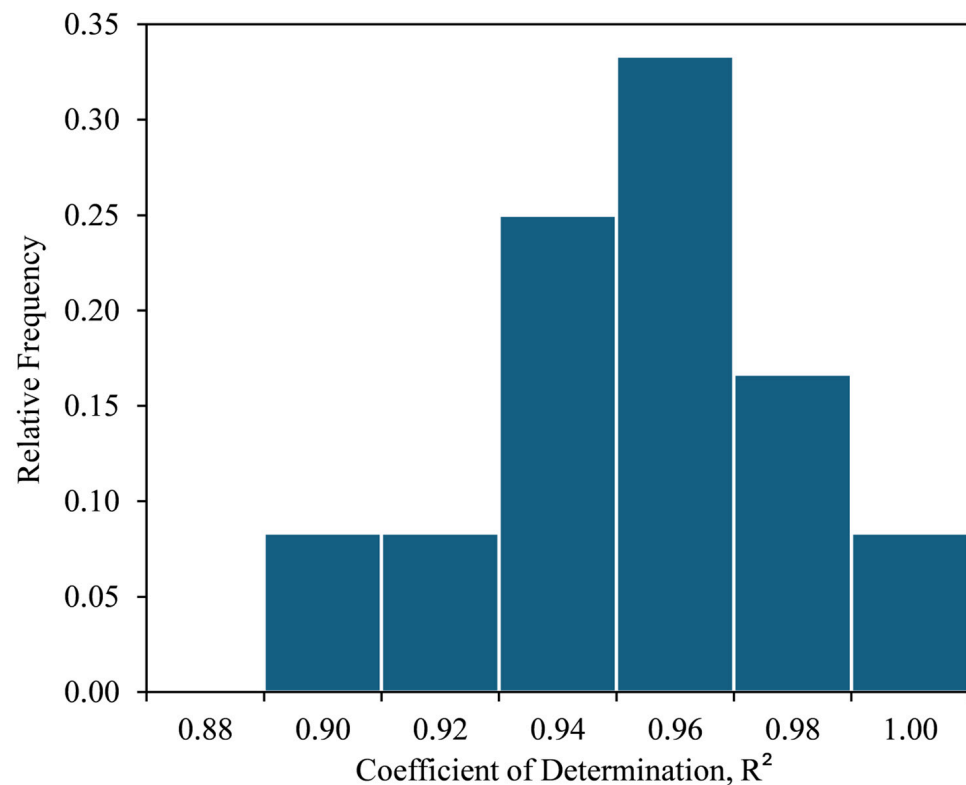
A summary of the resulting  $R^2$  values is presented in Table 5, while the distribution of the coefficients is illustrated in Figure 16. As shown in the table and the figure, the model proved excellent at predicting pore pressure generation in these tests, further validating the effectiveness of the model.

**Table 5.** Results of model application to cyclic triaxial tests from the literature.

| Test Type                   |       |
|-----------------------------|-------|
| Soil                        | Sand  |
| Number of Tests             | 12    |
| Average $R^2$               | 0.946 |
| Median $R^2$                | 0.948 |
| Maximum $R^2$               | 0.983 |
| Minimum $R^2$               | 0.898 |
| Standard Deviation of $R^2$ | 0.025 |

In addition to the coefficients of determination, the predictive performance of the model was evaluated for the 12 cyclic triaxial tests using the normalized mean absolute error (NMAE) [20]. For each test, the absolute error was calculated as the absolute difference

between the measured and predicted pore pressure ratios at each data point. The mean absolute error (MAE) was then obtained by averaging these absolute errors over all data pairs. The MAEs were then normalized by the range of the measured pore pressure ratios rather than by their mean because many of the measured values were close to zero. Normalization by the mean can produce unstable or misleading normalized errors when observations approach zero; therefore, normalization by the range provides a more robust basis for comparing model performance across tests [21].

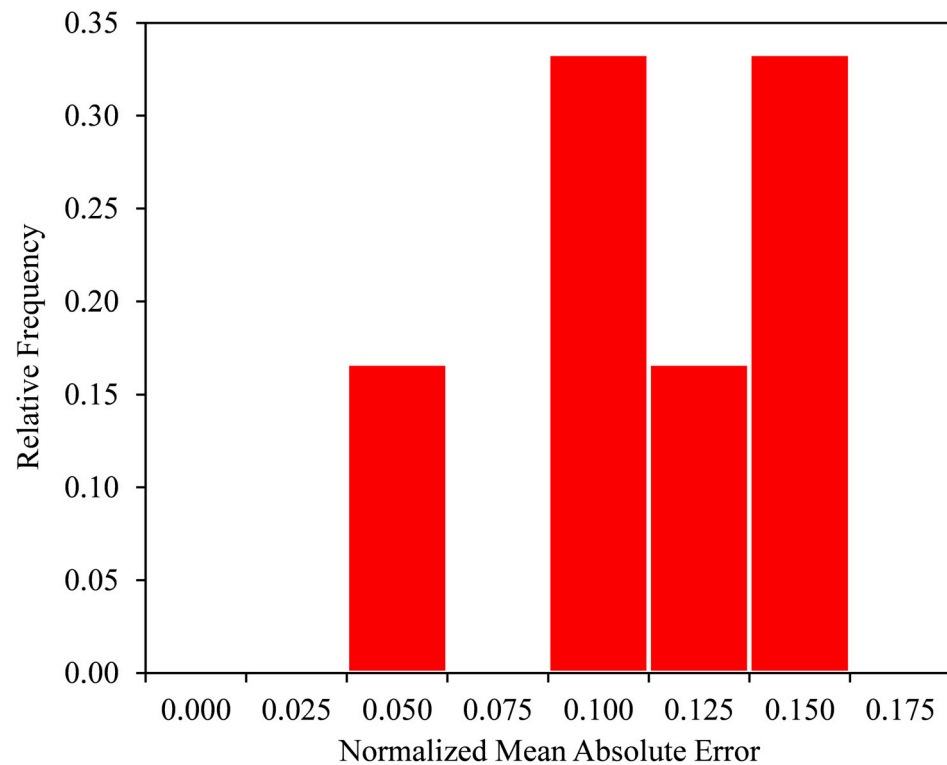


**Figure 16.** Distribution of coefficients of determination for the 12 cyclic triaxial tests reported in the literature.

The resulting NMAE values are presented in Table 6, and their distribution is shown in Figure 17. As indicated by both the table and the histogram, the model predicted excess pore pressure generation with high accuracy, with NMAE values ranging from 4% to 15%.

**Table 6.** Results of NMAE analysis for the cyclic triaxial tests from the literature.

| Test Type                  |       |
|----------------------------|-------|
| Soil                       | Sand  |
| Number of Tests            | 12    |
| Average NMAE               | 0.097 |
| Median NMAE                | 0.103 |
| Maximum NMAE               | 0.144 |
| Minimum NMAE               | 0.038 |
| Standard Deviation of NMAE | 0.033 |



**Figure 17.** Distribution of the normalized mean absolute errors for the 12 cyclic triaxial tests reported in the literature.

#### 4. Discussion

The results presented in this study demonstrate that the Modified Booker Model provides an excellent prediction of pore pressure generation during strain-controlled cyclic loading tests. The model was evaluated using 51 cyclic triaxial tests and 101 cyclic direct simple shear tests performed on sands and sand–silt mixtures. For the cyclic triaxial tests, the model achieved coefficients of determination ( $R^2$ ) ranging from 0.846 to 0.978, with an average value of 0.921, indicating a consistently strong agreement between measured and predicted pore pressure responses. Additionally, nearly 60% of the triaxial tests produced  $R^2$  values greater than 0.94, further confirming the reliability of the proposed model. The cyclic direct simple shear test results demonstrated even stronger predictive capability, with average  $R^2$  values of 0.959 for clean sands and 0.989 for sand–silt mixtures.

The superior performance of the Modified Booker Model for strain-controlled loading conditions is directly related to the fundamental differences between stress-controlled and strain-controlled cyclic soil behavior. In stress-controlled loading, the applied cyclic stress remains constant while shear strains progressively increase as excess pore pressures reduce the effective stress and stiffness of the soil. Consequently, pore pressure generation is initially slow and accelerates as liquefaction is approached. In contrast, strain-controlled loading imposes a constant deformation amplitude throughout the test. Under these conditions, relatively large resisting stresses are required during the early stages of loading when effective stresses are high, resulting in rapid early pore pressure generation. As excess pore pressures increase and soil stiffness decreases, progressively smaller stresses are needed to maintain the imposed strain amplitude, causing the rate of pore pressure generation to decrease as liquefaction is approached. The Modified Booker Model more accurately captures this reversed curvature in the pore pressure–cycle ratio relationship.

The findings also have important implications for earthquake engineering and liquefaction assessment. Many geotechnical problems involve loading conditions in which strains, rather than stresses, may govern localized soil response, particularly in areas adjacent to

foundations, retaining structures, buried pipelines, embankments, and other deforming systems. Under these conditions, traditional stress-based pore pressure generation models may not accurately represent the progression of excess pore pressure buildup. The Modified Booker Model provides engineers with a relatively simple and computationally efficient method for evaluating pore pressure development in these situations without requiring advanced constitutive modeling or extensive numerical analyses. Because the model performed well for soils both above and below the threshold fines content, it may also be applicable to a wide range of liquefiable natural soil deposits containing non-plastic silts.

## 5. Engineering Applications and Practical Implications

The findings of this study have important implications for geotechnical and earthquake engineering practice, particularly in the evaluation of liquefaction potential and cyclic soil behavior under strain-controlled loading conditions. The results demonstrate that the Modified Booker Model can accurately predict excess pore pressure generation in sands and sand–silt mixtures subjected to cyclic triaxial and cyclic direct simple shear loading. The consistently high coefficients of determination obtained across a wide range of soils and testing conditions indicate that the model provides a practical and reliable means of estimating pore pressure buildup without requiring advanced constitutive models or computationally intensive numerical analyses.

A significant advantage of the Modified Booker Model is its ability to accurately represent pore pressure generation in soils both above and below the threshold fines content. This broad applicability suggests that the model can be used for a wide range of natural soil deposits containing non-plastic fines, including many liquefiable sands and silty sands encountered in earthquake-prone regions. The simplicity of the model makes it particularly well suited for spreadsheet-based calculations, simplified numerical analyses, and preliminary design evaluations where computational efficiency and ease of implementation are important considerations.

The model also provides a useful framework for interpreting strain-controlled laboratory tests, including cyclic direct simple shear and cyclic triaxial tests commonly used to evaluate liquefaction resistance and cyclic degradation. By accurately representing the evolution of excess pore pressure throughout the loading process, the model enables engineers and researchers to better assess the progression toward liquefaction and the associated reduction in effective stress. In addition, the model may be incorporated into finite element and finite difference analyses that require a computationally efficient pore pressure generation relationship, making it valuable for parametric studies, sensitivity analyses, and large-scale simulations.

These capabilities have direct relevance to the evaluation and design of geotechnical systems subjected to cyclic deformations, including foundations, embankments, retaining structures, offshore facilities, and other earth structures exposed to earthquake loading. Overall, the Modified Booker Model offers a cost-effective and computationally efficient tool for improving liquefaction assessments and evaluating cyclic pore pressure generation in practical engineering applications.

## 6. Limitations

Although the Modified Booker Model demonstrated excellent predictive capability for strain-controlled cyclic loading conditions, several limitations should be recognized. First, the model is empirical in nature and was developed through curve fitting of laboratory test data. Consequently, its predictive accuracy is dependent on the range of soil types, stress conditions, and loading histories represented in the calibration database. While the model performed well for the sands and sand–silt mixtures evaluated in this study, its applicability

to soils containing significant plastic fines, cementation, aging effects, crushable particles, or pronounced fabric anisotropy remains uncertain.

A second limitation is that the model was validated using laboratory cyclic triaxial and cyclic direct simple shear tests conducted under controlled boundary conditions and relatively simple sinusoidal loading histories at low frequencies. Field loading conditions during earthquakes are substantially more complex and may involve irregular loading amplitudes, multidirectional stress paths, higher loading frequencies, varying drainage conditions, and spatially heterogeneous soil deposits. These factors may influence pore pressure generation behavior in ways not explicitly represented by the proposed model.

The study is also limited by the range of confining stresses, relative densities, and strain amplitudes investigated. Additional testing over a broader range of stress states and soil conditions would provide greater confidence regarding the robustness and general applicability of the model. Furthermore, model performance was evaluated primarily using the coefficient of determination ( $R^2$ ). Although the resulting values indicate strong agreement between measured and predicted responses, additional statistical measures could provide a more comprehensive assessment of prediction error.

Finally, the model focuses exclusively on the prediction of excess pore pressure generation and does not address related phenomena such as cyclic strain accumulation, post-liquefaction deformation, lateral spreading, or post-earthquake reconsolidation behavior. As a result, the model should be viewed as a simplified tool for pore pressure prediction rather than a comprehensive constitutive model of soil behavior under cyclic loading.

## 7. Conclusions

This study evaluated the effectiveness of a newly proposed Modified Booker Model for predicting pore pressure generation during strain-controlled cyclic loading of sands and sand–silt mixtures. The original Booker et al. model has long been recognized as a practical and computationally efficient tool for stress-controlled cyclic loading problems; however, it was shown to inadequately capture the shape and magnitude of pore pressure development observed in strain-controlled cyclic tests. To address this limitation, the Modified Booker Model was developed through curve fitting of strain-controlled cyclic triaxial and cyclic direct simple shear test data.

The model was evaluated using 51 cyclic triaxial tests and 101 cyclic direct simple shear tests performed on clean sands and sand–silt mixtures. The results demonstrated excellent agreement between measured and predicted pore pressure ratios, with average coefficients of determination ( $R^2$ ) of 0.921 for cyclic triaxial tests and up to 0.989 for cyclic direct simple shear tests. Statistical evaluation further showed that the model performed equally well for soils with fines contents both above and below the threshold fines content, indicating that the model is robust across soils exhibiting both sand-like and silt-like behavior.

Overall, the Modified Booker Model provides a practical, computationally efficient, and accurate method for estimating pore pressure generation under strain-controlled cyclic loading conditions. The model's simplicity and strong predictive capability make it well suited for routine engineering analyses, liquefaction evaluations, and preliminary geotechnical design applications where advanced constitutive modeling may not be practical.

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## Nomenclature

|          |                                                                                                                |
|----------|----------------------------------------------------------------------------------------------------------------|
| $r_u$    | Pore pressure ratio: The ratio of the excess pore pressure generated to the initial effective confining stress |
| $N$      | The number of the current cycle of loading                                                                     |
| $N_L$    | The number of cycles of loading required to trigger liquefaction                                               |
| $\Theta$ | The calibration coefficient for the original Booker et al. model                                               |
| $\beta$  | The calibration coefficient for the Modified Booker model                                                      |

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