



Article

Research on the Formation Mechanism and Distribution Characteristics of Surface Roughness in End Milling

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Abstract

The cutting variables of end milling are positively correlated with the machined surface roughness, but their mechanism of action remains unclear. Since the cutting tool used in end milling is a three-dimensional solid, under the action of horizontal cutting force, there exist both axial tensile strain and axial compressive strain in the cutting tool at the same time, thus putting forward an assumption about the formation mechanism of machined surface roughness: cutting variables affect the axial tensile strain of the cutting edge through the horizontal cutting force, and cause the cutting-edge tip to vertically wedge into the machined surface, thereby influencing the unevenness of the processed surface. Based on the cutting-tool deflection model of a two-segment cantilever beam and the horizontal cutting force formula, the mathematical expression for the axial strain at the sharp tip of the cutting edge was derived. Groove milling and half-groove milling experiments were done, and the experimental results show that the roughness R_a value in the central area is significantly higher than that in the cut-in area, with its maximum average value being 1.32 times that of the cut-in area. The surface roughness rises following the rise in depth of machining and feed per cutting tooth, but this relationship is significant. The laboratory findings are consistent with the theoretical analysis results, indicating that the assumption about the formation mechanism of surface roughness should be reasonable, and the surface roughness in the central area is greater than that in the cutting tool cut-in area. The research results provide a kind of new insight into the formation mechanism of machined surface roughness, which can serve as a reference for relevant research and cutting practices.

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1. Introduction

In metal-cutting processing, the processed surface quality is a significant indicator of cutting standard. The machined surface roughness is a commonly used surface-quality parameter, and reducing the machined surface roughness is a crucial approach to improve the cutting quality.

Many research results indicate that the machined surface roughness is related to cutting variables. Chen et al. [1] utilized the orthogonal experiment method to analyze the

effect of cutting variables on machining force and machined surface roughness. It was found that the per-tooth feed rate and axial milling depth have a significant effect on the machining force and machined surface roughness, while the spindle speed has a relatively small effect. Özdemir et al. [2] utilized the orthogonal experiment method and found that the variables with the maximum effect on the surface roughness in turning are represented by the feed speed and tool-nose radius of the turning cutting tool. Zhen et al. [3] investigated the effect of milling variables on surface quality during high-speed milling of 16Co14Ni10Cr2Mo ultra-high-strength steel and found that cutting speed and per-tooth feed rate have a significant impact on machined roughness, while the effect of milling depth is minor. Gandjar et al. [4] researched the effect of milling variables on the surface roughness of titanium alloy Ti-6Al-4V in micro high-speed milling. The results show that the higher the feed rate and spindle speed, the better the machined surface roughness.

There is also a correlation between machining force and machined surface roughness. The studies of Shen et al. [5] showed that the thickness of undeformed cutting material during the cutting-tool entry and exit processes had an impact on both machining force and processed surface topography. Wang et al. [6] studied the surface roughness of TC4 titanium alloy in milling. The results showed that a larger helix angle of the milling cutting tool led to greater machining force and higher machined surface roughness. Fuqiang et al. [7] evaluated the resultant effects of the milling process variables on the machined surface roughness and machining force using an experimental procedure. The results show that the order of the degree of effect on the machined surface roughness and machining force is spindle speed, per-tooth feed rate, and depth of cut. Jinfu et al. [8] studied the machining force and machined surface roughness of GH4145 superalloy in milling. The results show that the depth of cut and per-tooth feed rate have a very significant effect on the machining force and machined surface roughness.

These research results confirm that there is a positive correlation between cutting variables, machining force, and machined surface roughness. However, most of these results are based on laboratory data and fail to explain the intrinsic connection between them from a mechanistic perspective; therefore, it is difficult to apply them to further solve the problem of improving surface quality.

The cutter geometry in cutting has an impact on the machined surface roughness. Studies by Wang et al. [9] show that runout of the cutting tool in milling has a significant impact on the machined surface roughness. Studies by Gao et al. [10] show that the vibration and wear of the cutting tool can lead to an increase in the waviness of the processed surface. Studies by Yuan et al. [11] show that the geometric shape, runout, and vibration induced by the cutting tool have an impact on the machined surface roughness. Studies by Sung et al. [12] show that the rake angle of the turning cutting tool has an impact on the machined surface roughness. Wenjun et al. [13] constructed a model of milling surface roughness based on the geometric variables of the cutting tool, which can predict the surface roughness with relatively high accuracy. Studies by Dang et al. [14] show that for milling with a highly flexible cutting tool, the prediction model based on deformation of the cutting tool can estimate the topography of the processed surface. Yin et al.'s [15] study demonstrates that the tool holder strain signal exhibits a strong correlation with the cutting force, and can therefore serve as a reliable indicator for optimizing machining parameters.

During the machining process, the cutting tool is subjected to dynamic strain as a result of the applied cutting process. Baci et al. [16] investigated cutting-edge deformation during the milling of complex surfaces and reported a positive correlation between this deformation and the magnitude of the cutting edge, which is positively correlated with the milling force. Smith et al. [17] examined the axial strain behavior of cutting tools through a combination of experimental testing and numerical simulation. Their findings

reveal that dynamic axial strain is predominantly governed by cutting force variations and mechanical vibrations during machining.

During the end milling process, the cutting tool experiences radial deflection as a consequence of the horizontal milling force. One end of the milling cutting tool is fixed, while the other end is free, which can be equivalent to a cantilever beam. Ni et al. [18] simplified the milling cutting tool into a two-segment cantilever beam model consisting of a shank part and an edge part, and a formula for calculating the radial deflection of the cutting tool was derived accordingly. Yuan et al. [19] divided a ball-nose milling cutting tool into a part of the shank, a part of the conical cutting edge, and a part of the ball head, and constructed a cutting tool model of a three-segment cantilever beam, and derived the calculating formula of radial deflection for the cutting tool. Chongyan et al.'s [20] previous work has revealed that the tool deflection angle plays a critical role in determining the Z-directional height of transition edges in front- and back-cutting, which in turn strongly affects the formation of right- and left-convex surface textures in end milling.

The cutting tool of end milling has a cylindrical three-dimensional structure: in radial deflection deformation, one side of the cutting tool undergoes tensile strain, while the other side undergoes compressive strain; the tip of the cutting edge would wedge into the processed surface, thereby affecting the uneven topography of the surface. According to existing research findings, cutting variables can affect the machining force. The level component of this machining force can cause radial deflection in the cutting tool. If it were possible to reveal the impact of this radial deflection on the surface roughness of the workpiece, it would be possible to theoretically explain the mathematical relationship between cutting variables and surface roughness.

Research Content: The present work focuses on the axial strain behavior of the cutting tool induced by horizontal milling forces, and formulates a quantitative relationship to describe the strain distribution along the cutting edge. The present study aims to elucidate the formation mechanism of surface roughness in vertical end milling processes.

The remainder of this paper is organized as follows:

- (1) **Introduction:** The existing literature concerning the influence of cutting parameters, cutting forces, and tool deformation on surface roughness is critically examined.
- (2) **Analysis:** This section presents a theoretical investigation of the axial strain at the cutting edge under horizontal cutting forces, leading to the formulation of a quantitative strain equation. Based on this model, the correlation between milling parameters and resulting surface roughness is systematically examined.
- (3) **Milling Experiments:** A series of milling tests, including groove milling and half-groove milling, were carried out. Surface roughness profiles were characterized by means of a white light interferometer, followed by a comprehensive analysis of the obtained data.
- (4) **Conclusions:** An analytical expression describing cutting-edge axial strain is established and applied to interpret variations in surface roughness. Under low cutting loads, higher feed rates result in larger horizontal forces, which in turn increase axial tensile strain, deepen the tool imprint on the machined surface, and consequently lead to an increase in roughness.

2. Theoretical Derivation

2.1. Components of Machining Force in End Milling

The machining force in end milling can be divided into three components, namely the axial, radial, and tangential components, among which the tangential component is the largest in magnitude. The axial component acts along the cutting tool's axis, while the radial and tangential components act on a plane normal to the rotational axis of the cutting tool; this plane can be referred to as the level plane. During end milling, the geometric positions of the cutting tool and workpiece are as shown in Figure 1, where the cross-

section of the cutting tool is circular, point O is the center of the circle, θ represents the rotation angle of the cutting tool, point E is the equivalent point of the cutting edge, and the line segment OE represents the radius of the cutting tool. $F_R(\theta)$ denotes the instantaneous radial component, which is directed along EO. $F_T(\theta)$ represents the instantaneous tangential component of force, which is tangential to the cutting tool. $F_C(\theta)$ is the instantaneous horizontal cutting force and represents the resultant of $F_R(\theta)$ and $F_T(\theta)$.

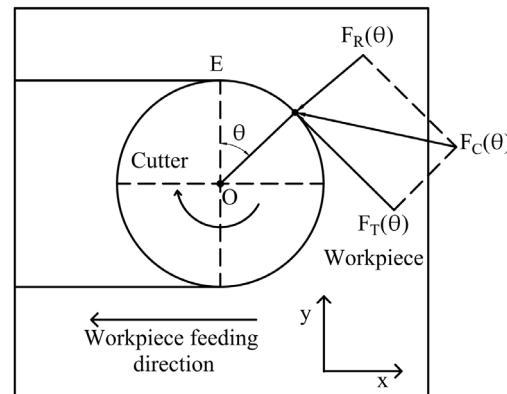


Figure 1. Cutting component force in the level plane.

The mathematical formula for the instantaneous horizontal cutting force is:

$$F_C(\theta) = K_C h(\theta) a_z \quad (1)$$

where $F_C(\theta)$ is the instantaneous horizontal cutting force, θ is the cutting tool rotation angle, K_C is the horizontal cutting force coefficient, $h(\theta)$ is the instantaneous cutting thickness, and a_z is the depth of cut.

The mathematical formula for instantaneous cutting thickness is:

$$h(\theta) = f_z \sin \theta \quad (2)$$

where f_z is the feed rate per tooth.

According to previous studies [12], within the commonly used helix angle range (30–45°), the influence of helix angle on the mean chip thickness is typically less than 5–8%, and this variation mainly affects high-frequency force fluctuations, with limited impact on the mean cutting force and axial strain. Therefore, neglecting the helix angle is acceptable for the purposes of this study.

Substituting Equation (2) into Equation (1):

$$F_C(\theta) = K_C f_z a_z \sin \theta \quad (3)$$

2.2. Deflection of Cutting Tool Under Horizontal Cutting Force

During the milling process, the cutting tool is equivalent to a cantilever-type beam. In this study, the cutting tool model of a two-segment cantilever beam was used, which divides the cutting tool into a portion near the shank and a portion near the end face; only the portion near the end face undergoes deflection deformation.

Figure 2 is an equivalent diagram of a two-segment cutting tool model, where O represents the constraint point of the cutting tool, B is the end face of the cutting tool, C is the starting point of the cutting tool's deflection deformation, the part of OC remains unchanged, and the part of CB is deformed. L is the length of the cutting tool extending from the handle, and the length of the part that undergoes deflection deformation is $6L/7$ [14]. F_C is the instantaneous horizontal cutting force, and x_i is the distance between the acting point of $F_C(\theta)$ and the constraint point. Point A is a point in the deflected part of the cutting

tool, and x is the distance from point A to point O. After the cutting tool undergoes deflection deformation, point A moves to point A_1 , and point B moves to point B_1 ; $\alpha(x)$ represents the bending deflection at point A.

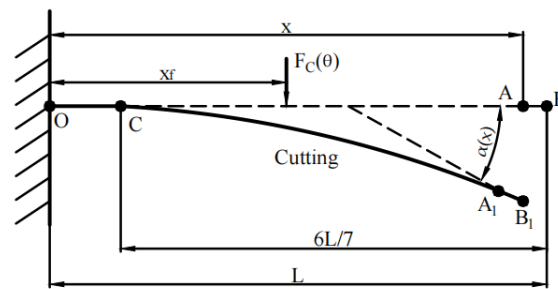


Figure 2. Cutting tool model of a two-segment cantilever beam.

The calculating formula for the bending deflection $\alpha(x)$ is as follows [18]:

$$\alpha(x) = -\frac{F_c x_f^2 (3x - x_f)}{6EI} \quad x_f \leq x \leq L \quad (4)$$

where $\alpha(x)$ is the bending deflection at a point of cutting tool; E is the elastic modulus of cutting tool (MPa); and I is the second moment of area (moment of inertia) of the cutting tool's cross-section.

When the depth of cut is small, the part of one cutting edge that simultaneously cuts into the workpiece can be simplified to a point on the end face of the cutting tool. It is point B in Figure 2. The instantaneous horizontal cutting force can then be simplified as a concentrated load acting at point B, as for point B, there is: $x = x_f = L$. Substituting this into Equation (4), the deflection at point B is:

$$\alpha(L) = \frac{F_c L^3}{3EI} \quad (5)$$

2.3. Axial Strain of the Cutting Edge Subjected to Action of Horizontal Cutting Force

The equivalent model illustrated in Figure 2 takes into account only the length of the cutting tool, while its width is neglected. In real practice, however, the cutting tool possesses a cylindrical geometry with a circular cross-section. According to the mechanics theory of force deformation in cantilever beams, when the tool is subjected to an instantaneous horizontal cutting force, the half of the cutting tool facing the force experiences tensile stress and strain, whereas the opposite half is subjected to compressive stress and strain.

To simplify the analysis, it is assumed that only a single cutting edge engages the specimen at any instant, as illustrated in Figure 3. Before the cutting tool is deformed under force, the cutting edge's tip is point A_1 , which also serves as the equivalent point for the simultaneously cutting part of the edge. Point O_1 represents the center of the cutting tool's end face, L denotes the length of the cutting tool protruding from the handle, and a_z signifies the depth of cut. $F_c(\theta)$ represents the instantaneous horizontal cutting force, acting at the tip of the cutting edge. Under the effect of $F_c(\theta)$, the portion of the cutting tool below point B undergoes deflection, with point O_1 shifting to point O_2 and point A_1 moving to A_2 . Consequently, the cutting tool's centerline transforms from a straight line BO_1 to a curved line BO_2 . Taking curve BO_2 as the boundary, axial compressive strain occurs on the left-hand side of the cutting tool, while axial tensile strain occurs on the right side.

As depicted in Figure 3, under the action of $F_c(\theta)$, the axial tensile strain of the cutting edge is denoted as Δa_z . During this process, the tip of the edge moves from point A_1 to point A_2 and wedges beneath the machined surface of the workpiece. This action induces microscopic unevenness on the processed surface, commonly referred to as surface roughness. The deeper the point A_2 wedges into the surface, the greater the resulting surface

unevenness, and consequently the higher the roughness value. Therefore, the axial tensile strain Δa_z of the cutting edge can be employed to characterize the roughness of the machined surface.

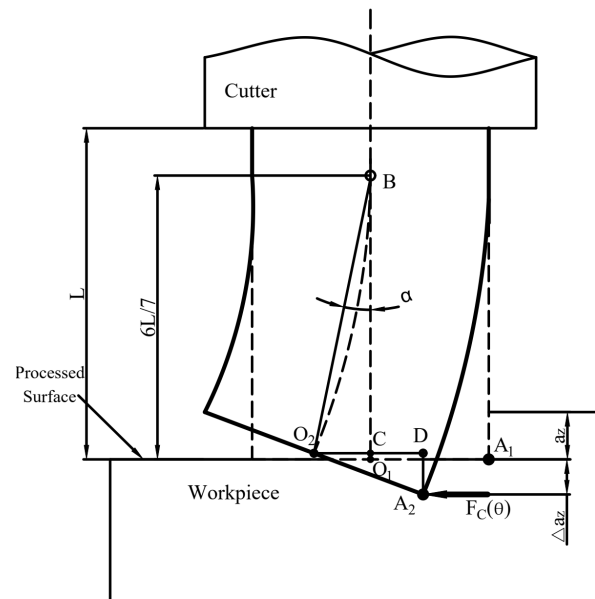


Figure 3. Axial strain of the cutting tool under the action of instantaneous horizontal cutting force.

In order to derive the mathematical expression of Δa_z , in Figure 2, there is the following:

$$\Delta a_z = \overline{DA_2} - \overline{CO_1} \quad (6)$$

$$\overline{CO_1} = \overline{BO_1} - \overline{BC} \quad (7)$$

In $\triangle O_2A_2D$, there is the following:

$$\overline{DA_2} = \overline{O_2A_2} \times \sin \angle A_2O_2D \quad (8)$$

where $\overline{O_2A_2}$ is the radius of the cutting tool, there is the following:

$$\overline{O_2A_2} = R \quad (9)$$

where R is the radius of the cutting tool.

In $\triangle BO_2C$, there is the following:

$$\overline{BC} = \overline{BO_2} \times \cos \angle O_2BC \quad (10)$$

The straight-line segment BO_1 and the curved segment BO_2 represent the axes of the cutting tool before and after deflection deformation, respectively. As the length of the axis remains constant after the deflection deformation, the length of the curve segment BO_2 is equal to that of the straight-line segment BO_1 . Because the degree of deflection is very small, the curved segment BO_2 can be approximately replaced by the straight-line segment, leading to the equation as follows:

$$\overline{BO_2} \approx \overline{BO_1} = \frac{6}{7}L \quad (11)$$

Because: $BO_2 \perp O_2A_2$, $O_2C \perp BC$

Therefore:

$$\angle A_2O_2D = \angle O_2BC = \alpha \quad (12)$$

By substituting Equations (7)–(12) into Equation (6), it is obtained that:

$$\Delta a_z = R \sin \alpha - \frac{6}{7} L (1 - \cos \alpha) \quad (13)$$

where R denotes the radius of the cutting tool, and L represents the length of the cutting tool extending out of the handle.

Since the deflection of the cutting tool is very small, it can be approximated as follows:

$$\alpha \approx \alpha(L) \quad (14)$$

Substituting Equations (5) and (14) into Equation (13), there is the following:

$$\Delta a_z = R \sin \left(\frac{F_c(\theta)L^3}{3EI} \right) - \frac{6}{7} L \left[1 - \cos \left(\frac{F_c(\theta)L^3}{3EI} \right) \right] \quad (15)$$

Both Equations (3) and (15) establish the relationship between the machined surface roughness and cutting variables such as the per-tooth feed rate f_z and the depth of cut a_z .

2.4. The Distribution Law of Machined Surface Roughness

2.4.1. The Relationship Between Machined Surface Roughness and Cutting Variables.

From Equation (15), it is evident that Δa_z does not exhibit a linear relationship with $F_c(\theta)$; that is, a rise in $F_c(\theta)$ may result in either a rise or a decrease in Δa_z . To elucidate the relationship between Δa_z and $F_c(\theta)$, a special case was considered to plot a relational curve between Δa_z and $F_c(\theta)$.

Taking, for instance, that the end mills are integral cemented carbide with overhang length L equal to 60 mm and diameter equal to 8 mm, and that the elastic modulus E of the cutting tool is 2.1×10^5 MPa, then the section moment of inertia for the cutting tool is:

$$I = \frac{\pi d^4}{64} = 64\pi \quad (16)$$

By substituting the values of R , E , I , and L into Equation (15), a mathematical relationship between Δa_z and the instantaneous horizontal cutting force $F_c(\theta)$ is obtained. This relational curve is plotted as Figure 4.

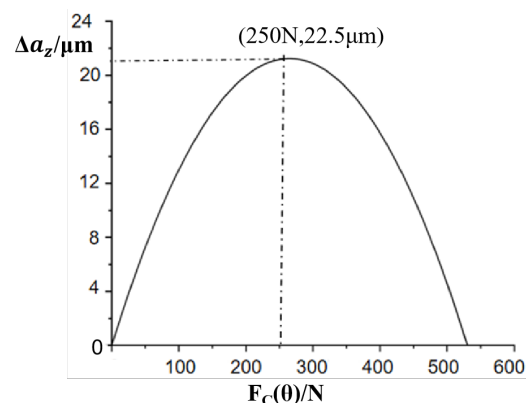


Figure 4. The relational curve between Δa_z and $F_c(\theta)$.

As depicted in Figure 4, when $F_c(\theta) \leq 250$ N, Δa_z rises as $F_c(\theta)$ rises; however, when $F_c(\theta) > 250$ N, Δa_z decreases as $F_c(\theta)$ rises.

During finish machining, a depth of cut of $a_z \leq 0.5$ mm and a feed rate of $f_z \leq 0.2$ mm/z are generally applied. Within this cutting parameter range, research results [14] indicate that $F_c(\theta)$ remains below 250 N. Consequently, during finish machining, a rise in the instantaneous machining force results in a corresponding rise in the axial strain Δa_z of the cutting edge, which in turn leads to an elevation in the machined surface roughness.

From Equation (3), it is evident that a rise in either the per-tooth feed rate or the depth of cut would lead to a rise in the instantaneous horizontal cutting force $F_c(\theta)$.

Based on these analytical results, the following conclusion can be drawn: when the values of cutting variables are small, with the feed rate per tooth or the depth of cut rising, the instantaneous horizontal cutting force $F_c(\theta)$ would also increase; this in turn would cause an increase in the axial strain Δa_z of the cutting edge, thereby raising the value of machined surface roughness. This conclusion offers a mechanistic explanation for the observed results, which show that machined surface roughness rises following increases in feed rates per tooth or depth of cuts.

2.4.2. Relationship Between Machined Surface Roughness and Rotational Angle of Cutting Tool

Equation (3) indicates that the instantaneous horizontal cutting force $F_c(\theta)$ is directly proportional to $\sin(\theta)$. During a single cutting stroke, while all other variables remain constant, the cutting tool rotational angle θ varies. For a single cutting edge in groove cutting, from the cutting tool's entry into to its exit from the workpiece, the value of its rotational angle θ ranges from 0 to π .

Within the range of $\theta \in \left[0, \frac{\pi}{2}\right]$, with the value of θ increasing, the value of $\sin(\theta)$ rises, so the instantaneous horizontal cutting force $F_c(\theta)$ will also rise accordingly. From Equation (15), it can be seen that the axial strain Δa_z of the cutting edge will rise as well, leading to an elevation in the machined surface roughness.

Taking groove milling into account, it is better to segment the processed surface into three distinct regions along the cutting width direction: the region machined in the time interval after the cutting teeth initially entered into the workpiece, which is termed as the "cut-in area"; the region machined when the rotation angles of the cutting teeth are near $\pi/2$, which is termed as the "central area"; and the region machined in the time interval before the cutting teeth finally exit from the workpiece, which is termed as the "cut-out area".

As depicted in Figure 5, when the rotation angle θ is 0, the cutting tooth starts to cut into the workpiece at point A; as θ is equal to θ_1 , the cutting tooth moves to point A_1 , and the surface region of the workpiece cut by all teeth within this range is the cut-in area. When θ reaches θ_2 , the cutting-tool tooth move to point A_2 , and the surface region of the workpiece cut by all teeth within this range is the central area. As θ varies from θ_2 to π , the surface region cut by all teeth within this range is the cut-out area.

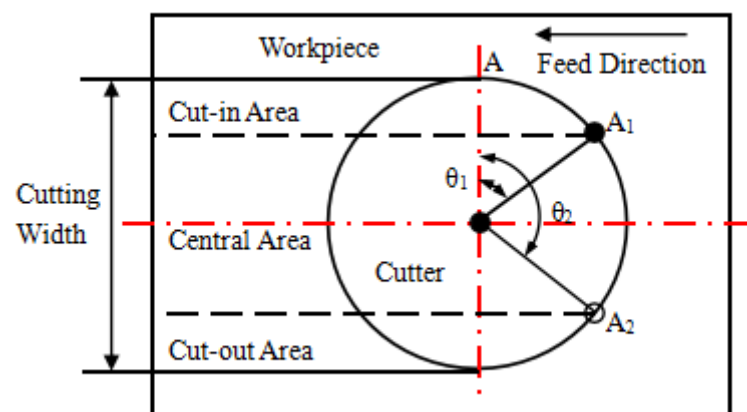


Figure 5. Division of three regions on the processed surface.

Comparing the roughness of the cut-in area with that of the central area, from the conclusion of the above analysis, it can be known that the machined surface roughness will be greatest when the rotation angle θ is $\pi/2$; therefore, the surface roughness of the central area is greater than that of the cut-in area.

3. Milling Experiments

Machine: The five-axis milling center of Jieyongda V545-5AXIII (Anhui Joint CNC Machine Corporation Limited, Hefei, China) with a maximum rotational speed of 12,000 rpm. Milling operations were carried out with the application of the coolant HOFFMAN 8004-406 (Jiangxi Hoffman New Energy Co., Ltd. Ganzhou, China)

Measuring equipment for surface roughness: All surface topography measurements were carried out using a Contour X-200 white light interferometer (Bruker Corporation, USA), which was integrated with a model 150–1280 vibration isolation/control console supplied by the same manufacturer. 50× lens, measuring distance of 2.4 mm, scanning range of $165\ \mu\text{m} \times 138\ \mu\text{m}$, as shown in Figure 6.

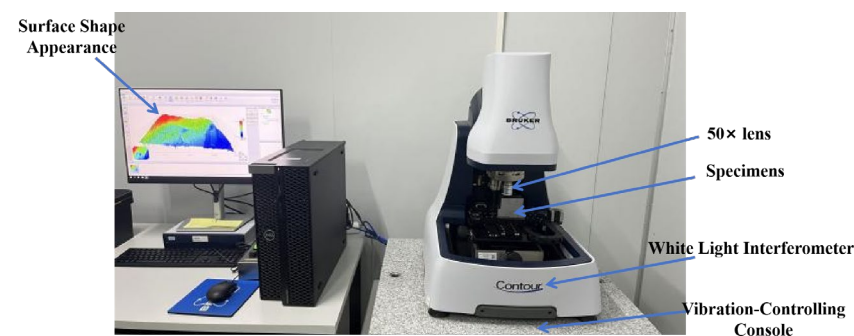


Figure 6. On-site measurement of surface roughness.

3.1. Experiments of Groove Milling

- (1) Workpiece: #45 steel, with dimensions of 80 mm × 70 mm × 40 mm.
- (2) Cutting tool, material: Tungsten carbide cutter, model: PM-2E-D8.0; manufacturer: ZCC.CT, made in China.
- (3) Cutting variables: Spindle speed of 900 rpm, cutting width of 8 mm, feed rate of 0.12 mm/z, depths of cut of 0.3 mm and 0.5 mm, as detailed in Table 1.

Table 1. Cutting variables for groove milling.

Experiment No.	Spindle Speed $n(\text{rpm})$	Per-Tooth Feed Rate f_z (mm/z)	Cutting Width a_e (mm)	Depth of Cut a_z (mm)
#1	900	0.12	8	0.3
#2	900	0.12	8	0.5

(4) Experimental Procedure

The workpiece was clamped onto a jig, and the jig was attached to the worktable of the machine. A CAM cutting program was meticulously crafted, and after the cutting tool was precisely positioned, the cutting process was commenced. The cutting operation is depicted in Figure 7.

After finishing cutting, the workpiece was detached, and the white light interferometer was employed to measure the surface roughness of both the cut-in area and cut-out area of the machined groove; simultaneously, microscopic images of the surface morphology were captured. The locations of the measuring points are indicated in Figure 8. There were 25 measuring points each in both the cut-in area and the central area. The measuring points in the central area were positioned along the groove's centerline, whereas those in the cut-in area were aligned parallel to this centerline. Moreover, the measuring points in both the cut-in area and the central area shared identical coordinate positions along the cutting-feed direction. The total length of the workpiece was 40 mm; for the 25 measuring

points, points #1 and #25 were located at the left and right edges of the workpiece, respectively, From measuring point #1 and point #5, as well as from point #21 to point #25, the interval between successive measurement locations was 1 mm, while from point #5 to point #21, the separation between neighboring measurement locations was 2 mm.

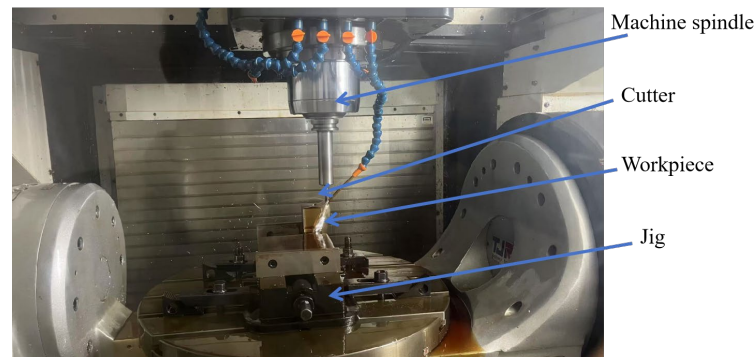


Figure 7. Milling site.

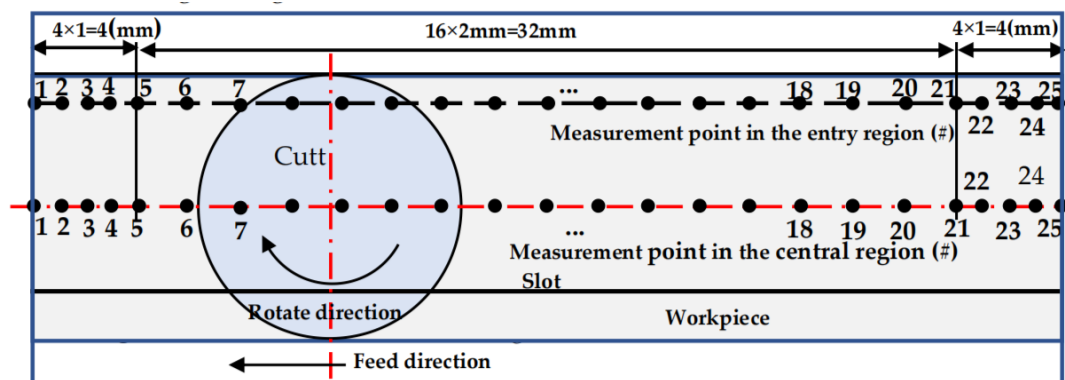


Figure 8. The positions of measuring points in the feed direction. Black dashed line: entry region; red dashed line: central region.

(5) Experimental results and analysis

The processed surface micro-morphology at the measured points is displayed in Figure 9. The corresponding roughness R_a values at each measuring point were measured, taking the distance of each measuring point from the left boundary of the workpiece in Figure 8 as the level axis, and the surface roughness R_a value at that point as the vertical axis. The distribution curves of machined surface roughness together with error values for the cut-in area and the central area are plotted in Figure 10. Surface roughness of the measurement points was measured three times; the average R_a value and the error were on the vertical axis.

The average values of surface roughness at the measuring points within both the cut-in area and the central area have been calculated and are presented in Table 2.

Table 2. Average values of surface roughness in the cut-in Area and central area.

Laboratory Number	Depth of Cut (mm)	Average Values of Surface Roughness (R_a) (μm)	
		Cut-in Area	Central Area
#1	0.3	0.787	1.039
#2	0.5	0.872	1.004

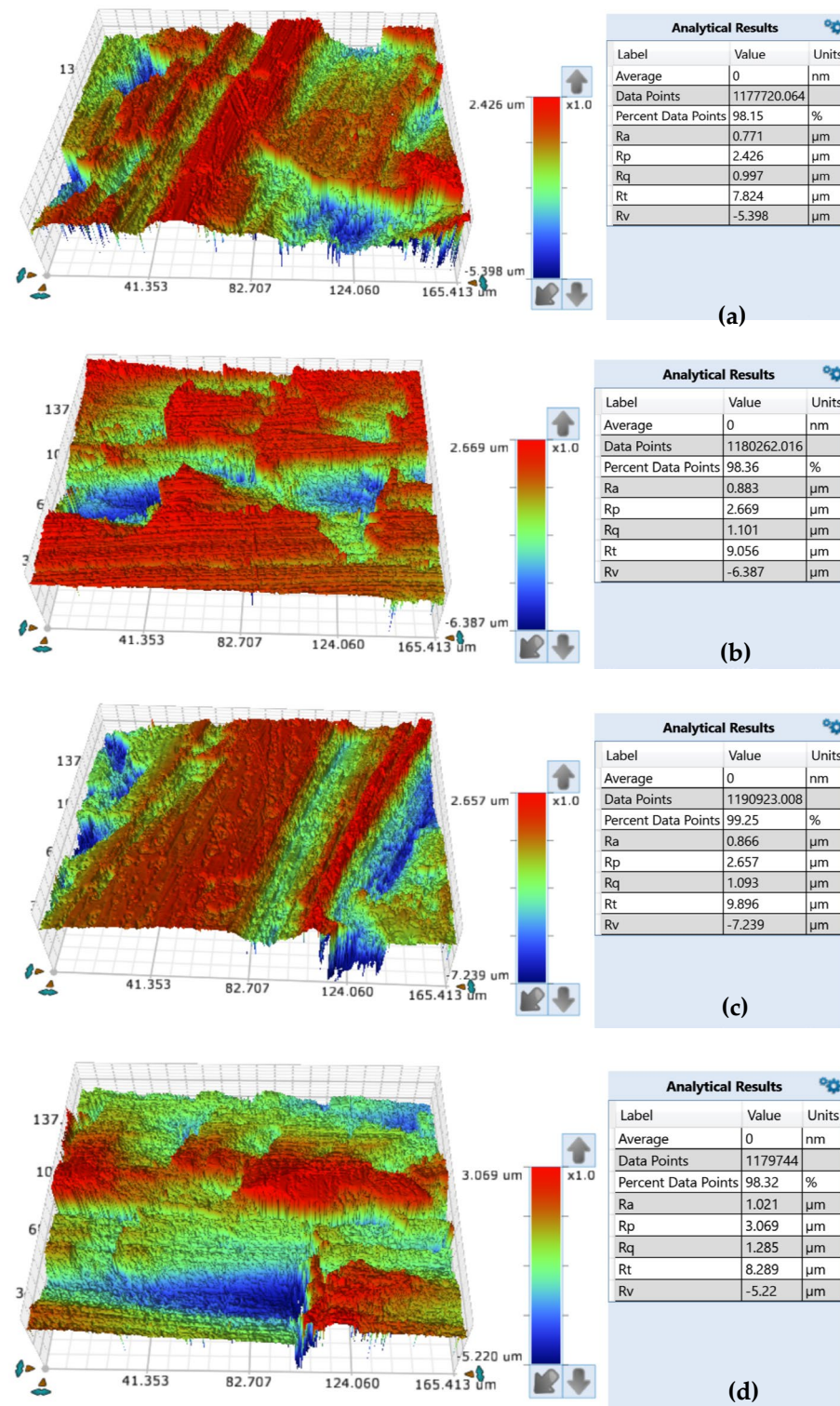


Figure 9. Surface morphology at the measuring point #17. (a) Cut-in area ($a_z = 0.3 \text{ mm}$); (b) central area ($a_z = 0.3 \text{ mm}$); (c) cut-in area ($a_z = 0.5 \text{ mm}$) and (d) central area ($a_z = 0.5 \text{ mm}$).

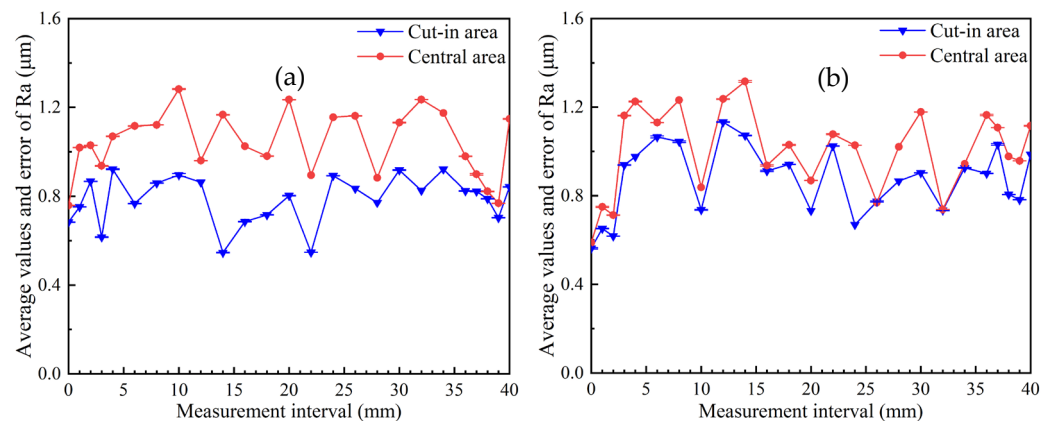


Figure 10. Distribution curves of surface roughness and effort values in the cut-in area and the central area. (a) $a_z = 0.3$ mm; (b) $a_z = 0.5$ mm.

The experimental results of groove milling reveal that:

(1) As indicated by the data on average roughness listed in Table 2, in the cut-in area, the average Ra values of surface roughness at a depth of cut of 0.5 mm are roughly 11% higher than that at a depth of cut of 0.3 mm, which is consistent with the conclusions of the theoretical analysis presented in Section 2.4.1.

(2) At depths of cuts of 0.3 mm and 0.5 mm, the Ra values of surface roughness in the central area are notably higher than those in the cut-in area; specifically, at the depth of cut of 0.3 mm, the surface roughness in the central area is about 32% greater than that in the cut-in area. The rise is significant. This corroborates the theoretical analysis in Section 2.4.2.

(3) At a depth of cut of 0.5 mm, the surface roughness in the central area is approximately 15% higher than in the cut-in area; this increase is obviously lower than that observed at a depth of cut of 0.3 mm. In addition, the surface roughness in the central area at the depth of cut of 0.5 mm is about equal to that at the depth of cut of 0.3 mm; these observations are consistent with the relationship between the axial strain of the cutting-edge tip and the level of machining force (horizontal cutting force) depicted in Figure 4. A reasonable explanation for this might be that: 0.5 mm is relatively great for the depth of cut. When the cutting-tool tooth cuts into the central area, the instantaneous horizontal cutting force $F_c(\theta)$ would be relatively great. Therefore, although the $F_c(\theta)$ rises a lot, the axial strain Δa_z of the tip rises a little, or even decreases slightly.

3.2. Experiments of Half-Groove Milling

(1) Workpiece: #45 steel, with dimensions of 80 mm × 70 mm × 40 mm.

(2) Cutting tool, material: Tungsten carbide cutter, model: PM-2E-D8.0, manufacturer: ZCC.CT, made in China.

The layout of the measuring points is depicted in Figure 11, where the line of measuring points intersected the upper boundary of the semi-groove at point O. A total of 20 measuring points were set up, with point #1 located at 0.2 mm from point O, and point #20 at the lower boundary of the semi-groove. The distance between two adjacent measuring points was 0.2 mm.

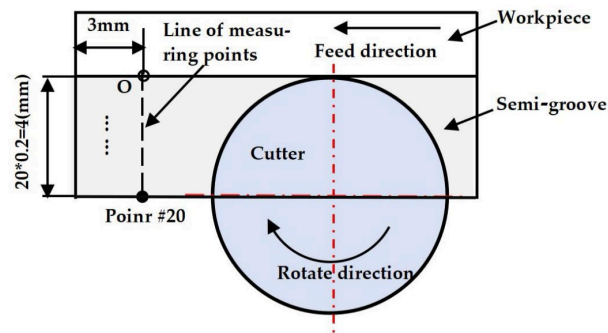


Figure 11. Positions of measuring points in the direction of cutting width.

3.2.1. Experiments of Variable Depth of Cut

(1) Cutting variables

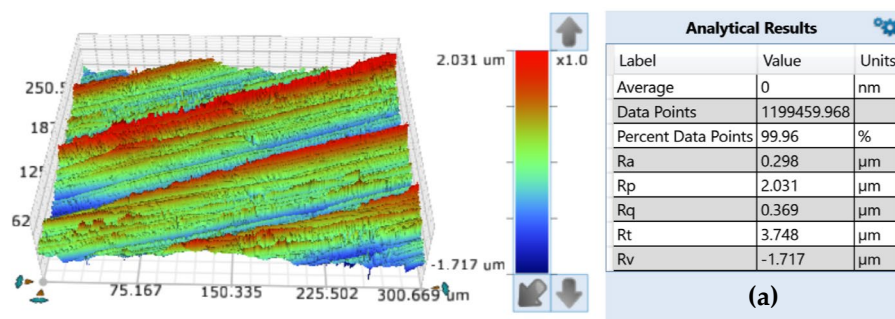
The spindle speed was set to 900 rpm, the per-tooth feed rate was 0.2 mm/z and the cutting width was 4 mm, while the depth of cut ranged from 0.1 to 0.3 mm. These cutting variables are detailed in Table 3.

Table 3. Cutting variables for half-groove milling with a variable depth of cut.

Laboratory Number	Spindle Speed n (rpm)	Per-Tooth Feed Rate f_z (mm/z)	Cutting Width (mm)	Depth of Cut a_z (mm)
1#	900	0.2	4	0.1
2#	900	0.2	4	0.2
3#	900	0.2	4	0.3

(2) Experimental Results and Analysis

The surface micro-morphology at the measuring points is depicted in Figure 12, and the R_a values of surface roughness at each point were measured. Taking the distance of the measuring point from point O in Figure 11 as the level axis, the surface roughness of the measurement points was measured three times; the average R_a value and the error were on the vertical axis, and the variation curve of surface roughness along the direction of cutting width is plotted, as depicted in Figure 13. Surface roughness of the measurement points was measured three times; the average R_a value and the error were on the vertical axis.



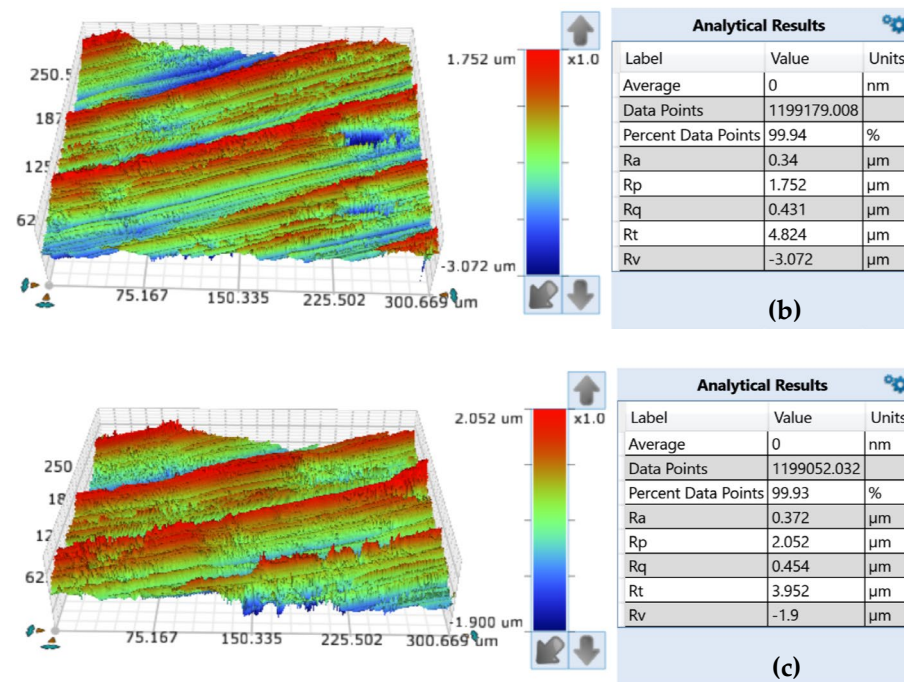


Figure 12. Surface morphology at the measuring point #2. (a) $a_z = 0.1$ mm; (b) $a_z = 0.2$ mm and (c) $a_z = 0.3$ mm.

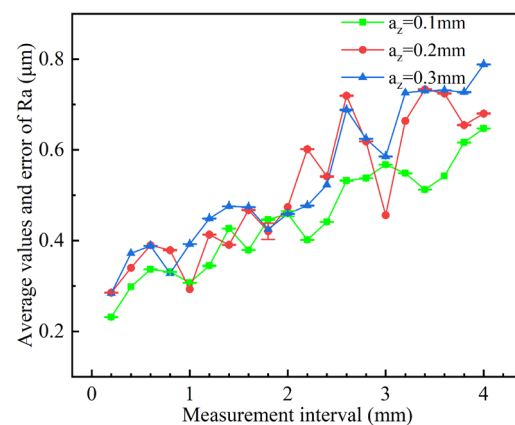


Figure 13. Curves of the mean value and error of surface roughness varying along the cutting width direction (for different depths of cuts).

The average values of surface roughness for the measuring points were calculated and are listed in Table 4.

Table 4. Average values of surface roughness at different depths of cuts.

Per-tooth feed rate f_z (mm/z)	0.1	0.2	0.3
Average Values of R_a (μm)	0.445	0.513	0.532

The experimental results of half-groove milling with variable depth of cut reveal that:

- (1) As depicted in Figure 13, from the cut-in area to the central area, the increasing trend of the machined surface roughness is significant, which is consistent with the conclusions of the theoretical analysis presented in Section 2.4.2.

- (2) According to Table 4, the average values of surface roughness rise following the rise in depth of cut; particularly, the average value of surface roughness at the depth of cut 0.2 mm is approximately 15.6% higher than that at the depth of cut 0.1 mm, which is

consistent with the conclusions of the theoretical analysis presented in Section 2.4.2. However, the average value of surface roughness for the depth of cut 0.3 mm is close to that for the depth of cut 0.2 mm.

3.2.2. Experiments with Variable Feed

(1) Cutting variables

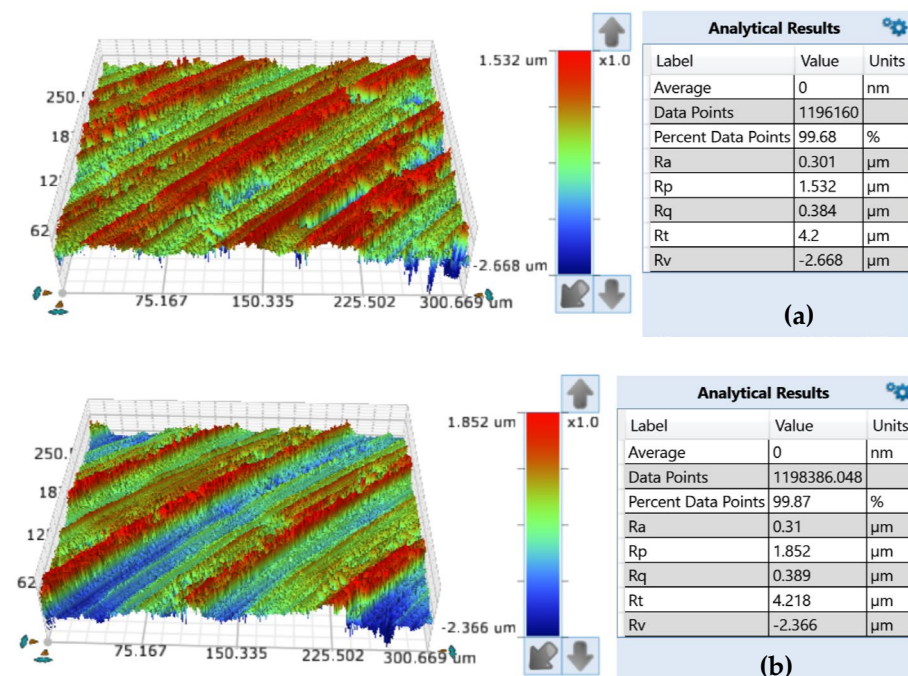
The spindle speed was set to 8000 rpm, with a depth of cut of 0.3 mm and a cutting width of 4 mm, and the per-tooth feed rate was varied at 0.08 mm/z, 0.12 mm/z, and 0.16 mm/z. The cutting variables are summarized in Table 5.

Table 5. Cutting variables for half-groove milling with variable feed.

Laboratory Number	Spindle Speed (n, rpm)	Per-Tooth Feed Rate f_z (mm/z)	Cutting Width a_c (mm)	Depth of Cut a_z (mm)
1#	8000	0.08	4	0.3
2#	8000	0.12	4	0.3
3#	8000	0.16	4	0.3

(2) Experimental Results and Analysis

The surface micro-morphology at the measuring points is presented in Figure 14, and the Ra values of surface roughness at each point were measured. Taking the distance of the measuring point from point O in Figure 11 as the level axis, surface roughness of the measurement points was measured three times; the average Ra value and the error were on the vertical axis, and the variation curve of surface roughness along the direction of cutting width is plotted, as depicted in Figure 15. Surface roughness of the measurement points was measured three times; the average Ra value and the error were on the vertical axis.



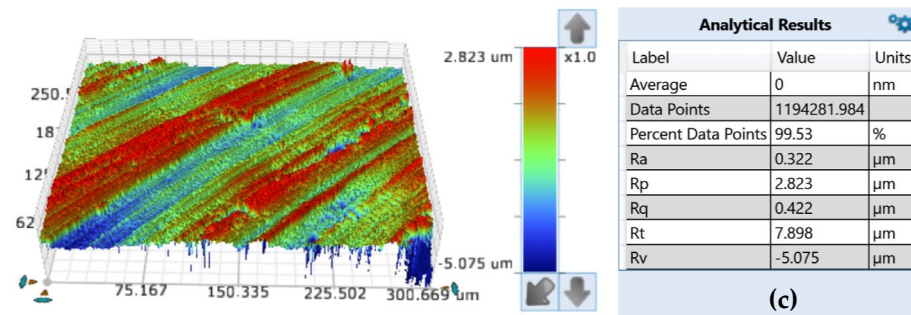


Figure 14. Surface morphology at the measuring point #6. (a) $f_z = 0.08$ mm; (b) $f_z = 0.12$ mm and (c) $f_z = 0.16$ mm.

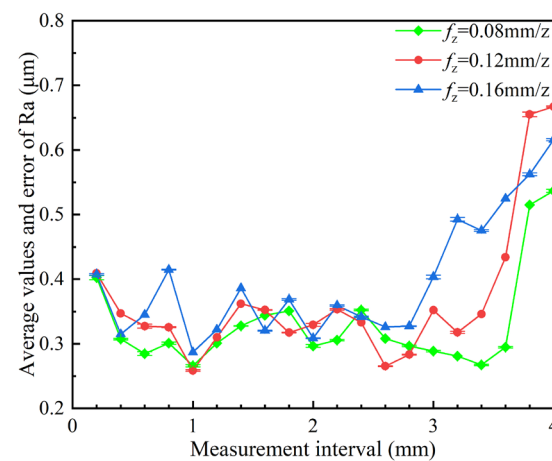


Figure 15. Curves of the mean value and error of surface roughness varying along the cutting width direction (for different feeds).

The average values of surface roughness for the measuring points were calculated and are listed in Table 6.

Table 6. Average values of surface roughness at different feeds.

Depth of cut a_z (mm)	0.08	0.12	0.16
Mean surface roughness R_a (μm)	0.331	0.367	0.395

The experimental results of half-groove milling with variable feeds reveal that:

(1) As depicted in Figure 15, the surface roughness in the central area is greater than that in the cut-in area, which is consistent with the conclusions of the theoretical analysis presented in Section 2.4.2.

(2) According to Table 6, the average values of surface roughness rise following the rise in per-tooth feed rate; the average value of surface roughness at the feed of 0.12 mm/z is approximately 10.8% higher than at the feed of 0.08 mm/z, though this increasing trend is not significant.

3.3. Chapter Summary

As demonstrated in Tables 5 and 6, the positive correlations between machined surface roughness and both depth of cut and per-tooth feed rate are not notably strong, which may stem from the machinability characteristics of #45 steel. In addition, the variation curves of surface roughness in Figures 10, 13, and 15 exhibit significant fluctuations, indicating that there might be substantial randomness in the roughness values, so the selected measuring points might also be a contributing factor to these weak correlations.

4. Conclusions

This study develops a cantilever-beam-based analytical model to evaluate the axial strain at the cutting tooth, yielding an explicit mathematical expression for the strain. Using this model, the dependencies of surface roughness on milling parameters and tool rotation angle are systematically examined. Moreover, the axial strain is adopted as a quantitative metric to interpret the effects of the aforementioned parameters on surface quality. Slot and semi-slot milling trials were carried out to obtain machined surfaces for roughness evaluation; based on the above results of theoretical analysis and laboratory research, the following conclusions may be drawn:

(1) When the cutting variables are small, the machined surface roughness would rise following the depth of cut or per-tooth feed rate increasing, though not proportionally; this rise trend is not significant, and this is valuable for further exploration.

(2) Under the effect of instantaneous horizontal cutting forces, the axial strains in different parts of the cutting tool differ; the tip of the cutting edge wedges into the processed surface of the workpiece, which is a key cause of machined surface roughness. While the cutting variables are small, with the per-tooth feed rate increasing, the instantaneous horizontal cutting force rises, leading to greater axial tensile strain on the tip of the cutting edge. This, in turn, causes the tip of the cutting edge to wedge deeper into the processed surface, resulting in an increase in machined surface roughness. This explains the reasons for machined surface roughness increasing following an increase in cutting variables.

(3) In this study, a mathematical expression was established to determine axial strain on the tip of the cutting edge under horizontal cutting force, and this axial strain was used to represent the machined surface roughness, which is demonstrated to be rational and feasible.

(4) In end milling, the roughness of the same machined surface is not uniform within a single tool path, and along the cutting width, the machined surface roughness rises with the rotating angle of the cutting tool increasing, and the machined surface roughness of the central area is greater than that of the cut-in area where the teeth of the cutting tool begin to cut into the workpiece.

Study limitations:

- (1) The present work does not incorporate finite element simulation to verify whether the observed R_a trends in the tool entry and central regions are consistent with the experimental findings.
- (2) Owing to the absence of real-time cutting force measurement, the correlation between Δa_z and $F_c(\theta)$ was established via numerical evaluation derived from the theoretical axial strain model. Further investigations will be carried out to resolve these issues.

This study establishes an analytical model to quantify the axial strain at the cutting-edge tip induced by horizontal cutting forces. Using this strain as a representative indicator, the influence on machined surface roughness is systematically investigated. The principal contributions of this model are reflected in the following two aspects, as detailed below:

- (1) From a theoretical perspective, this work elucidates the underlying mechanism through which cutting-force-induced axial micro-deformation of the tool is transferred to the resulting workpiece surface topography. A quantitative correlation between mechanical loading and surface quality is thereby established, which advances the theoretical foundation for roughness prediction in milling operations.
- (2) From a practical standpoint, cutting-edge strain enables the rapid prediction of surface roughness, thereby facilitating reverse optimization of cutting parameters and tool geometry. This approach enhances machining efficiency while ensuring surface accuracy, reducing redundant trial cuts, and lowering production costs.

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