



Article

From Rotary Burnishing Parameters to Joint Stiffness: A Two-Level Model for Surfaces with Regular Dimples

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Abstract

Normal contact stiffness is a key property of mechanical surface joints because it governs load transfer, local approach, vibration response, and positioning stability. Rotary burnishing can generate regular dimple patterns without superimposed tool vibration; however, existing process models mainly describe texture geometry and surface-layer modification rather than the mapping from burnishing parameters to normal joint stiffness. This paper develops a two-level semi-analytical model that couples deterministic dimple geometry and load redistribution with a Greenwood–Williamson-type response of the load-bearing land. The main geometrical and contact contributions are expressed explicitly, whereas the spectral summation, micro-roughness quadrature, and nonlinear closure are evaluated numerically. The process parameters define the nominal unit-cell geometry, whereas the residual dimple dimensions and measured post-burnishing descriptors are post-burnishing inputs. Published data for related regular microreliefs are used to benchmark the predicted trends and order of magnitude. For the reference regime, the explicit spectral displacement remains below 1% of the micro-roughness contribution. Periodic BEM predicts an additional approach 14–15% lower than the analytical term, while pressure heterogeneity changes the averaged micro-roughness approach by less than 1.7%. The model is also used for sensitivity analysis and inverse design of rotary-burnishing parameters to generate design maps for preliminary process selection. Within the stated validity domain, the model serves as a fast screening tool for the stabilized repeated-loading stiffness of dimpled joints.

Keywords: rotary burnishing; joint stiffness; normal contact stiffness; regular dimples; surface texture; relative bearing area; micro-roughness; semi-analytical model



Academic Editors: Feijun Qu and Zhengyi Jiang

Received: 8 July 2026

Revised: 27 July 2026

Accepted: 30 July 2026

Published: 31 July 2026

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1. Introduction

Normal contact stiffness and the associated contact compliance are key characteristics of mating surfaces in machine assemblies. They influence load sharing, local displacements, vibration response, positioning stability, and the service life of assembled joints. In slideways, bolted joints, supports, hydraulic-cylinder surfaces, and other tribological and mechanical interfaces, the finished surface is not merely a geometrical boundary; it is a functional layer that controls the real contact area, lubricant-retention paths, and initial normal contact stiffness [1,2]. A machined surface typically contains two distinguishable levels: deterministic mesotexture induced by the tool path and stochastic micro-roughness associated with local asperities. Estimating normal contact stiffness must therefore account not only for roughness-height parameters but also for the geometry of the regular surface texture [3].

Surface texture can be controlled by plastic deformation processes, including ball, roller, ultrasonic, and vibratory burnishing. In these processes, a deforming element smooths surface asperities, modifies the surface layer, and forms prescribed surface-relief features without material removal. Reviews and application studies indicate that burnishing can improve surface integrity and mechanical performance [4,5]. Burnishing can form deterministic surface patterns consisting of grooves, dimples, or their combinations on an initially rough surface. Experimental studies of ball vibratory burnishing relate process parameters to feature width and depth, roughness, microhardness, and the work-hardened surface layer [6,7]. Considerable knowledge has been accumulated on surface formation and surface-layer modification, whereas the relationship between burnishing parameters and normal contact stiffness remains insufficiently characterized.

This study examines dimpled surfaces formed by rotary burnishing. Unlike vibratory burnishing, which typically creates continuous or intersecting grooves, rotary burnishing does not require superimposed tool vibration and can form isolated elliptical dimples. This simplifies tool design, improves process stability, and produces a surface relief that can be described more compactly than the relief generated by vibratory burnishing. The surface is described by two levels: a regular dimple array, which specifies the deterministic mesotexture within a repeating unit cell, and a load-bearing land between dimples, where contact is carried by stochastic asperities. In this context, the dimple area fraction denotes the fraction of the nominal surface occupied by dimples, whereas the relative bearing area denotes the complementary load-bearing fraction between them. Because dimples exclude part of the nominal surface from direct contact over the loading range considered, they must be treated as distinct geometrical objects rather than represented only through an averaged roughness parameter. Prior studies on rotary burnishing have examined tool trajectories, dimple area fraction, and imprint area produced by combined rotational and translational motions [8–10]. These relations describe surface formation but do not derive normal contact stiffness from dimple shape, pitch, overlap, and relative bearing area.

Classical studies of regular microrelief show that the performance of such surfaces cannot be described by a single roughness-height parameter. For working surfaces of hydraulic cylinders, slideways, and sliding interfaces, important factors include the area fraction of relief elements, their arrangement and pitch, the shape of the load-bearing area, and lubricant-retention capacity [8]. Shneyder's studies established an experimental basis for evaluating the contact stiffness of surfaces with regular microrelief after different finishing methods [11,12]. Subsequent work on the optimization and oil-retention behavior of partially regular microrelief confirms the technological importance of relative area and relief geometry [13,14]. However, these results are mainly limited to experimental data on surface formation and geometrical area relations. They do not provide an analytical transition suitable for predicting normal contact stiffness from prescribed rotary-burnishing parameters.

The shape of regular texture elements is also important. Studies of microindented 100Cr6 steel surfaces show that lubrication behavior depends on dimple size and relative bearing area [15], while micro-grooved textures in high-speed train brake systems are used to modify contact behavior and suppress stick-slip vibrations [16,17]. Elliptical dimples formed by rotary burnishing are especially relevant because comparative studies have reported favorable hydrodynamic behavior for elliptical dimples under selected operating conditions [18–20]. In addition, asperities with different initial shapes tend to evolve toward an elliptical shape during wear [11,21]. The surface relief formed by rotary burnishing is close to an equilibrium geometry and may help reduce friction and wear. Experimental studies of textured interfaces also show that the effect of texture on contact stiffness depends on load, initial roughness, feature shape, and real contact area [22,23].

Existing contact-stiffness models do not close this process-to-property gap. Empirical and finite-element models reproduce pressure–approach relations for measured or reconstructed random surfaces, but they usually require measured topography, case-specific numerical contact analysis, or fitted parameters [1,2]. Asperity-based, fractal, and scale-coupled models describe rough contact and reduce scale dependence, yet their texture descriptors are not obtained from rotary-burnishing kinematics [24–27]. Recent homogenized models distinguish macrogeometry, mesotexture, and micro-roughness and allow complex interfaces to be analyzed at lower computational cost [3,27], but they do not provide a direct parametric mapping from dimple shape, pitch, overlap, and relative bearing area to normal contact stiffness.

The most directly relevant theoretical framework is periodic elastic contact theory. Goryacheva and Tsukanov developed formulations for regular microgeometry and the localization principle, which accounts for the interaction of neighboring contact spots [28]. Extended asymptotic analysis of three-dimensional wavy surfaces gives relations for mean contact characteristics in limiting loading regimes [29]. These results provide a foundation for analyzing regular asperities, depressions, and waviness. Rotary-burnished surfaces differ from idealized regular-asperity systems because their texture consists of concave dimples and the load is carried by the load-bearing land between them. Direct application of a full periodic contact formulation would require determination of the unknown contact spots, including possible contact inside each dimple as the approach increases. Deriving an engineering relationship between process parameters and normal contact stiffness from the full unilateral formulation would be computationally expensive for parametric design. In the present formulation, the dimple region is treated as excluded from direct contact at the loading levels considered here, and the contribution of the regular pattern to normal approach is represented as an additional normal displacement associated with dimple geometry.

A computationally efficient process-to-property model is still lacking for cylindrical surfaces with isolated elliptical dimples. Geometrical models describe dimple shape and area but not the normal-load response, whereas rough-contact and periodic-contact models do not derive the texture descriptors directly from rotary-burnishing kinematics.

Accordingly, this paper develops a two-level semi-analytical model for the normal contact stiffness of a cylindrical surface with a regular array of elliptical dimples formed by rotary burnishing. The model links rotary-burnishing kinematics to overlapping-dimple geometry, separates deterministic mesotexture from the load-bearing land, and couples area-averaged pressure redistribution with the micro-roughness response and the additional elastic displacement of the dimple array. The process kinematics define the nominal unit-cell geometry, whereas measured residual dimple dimensions and roughness descriptors characterize the post-burnishing surface. The result is a computationally efficient screening model that maps nominal process geometry and measured post-burnishing surface descriptors to stabilized normal joint stiffness. It also supports an inverse design problem in which burnishing parameters are selected for a required stiffness.

2. Materials and Methods

2.1. Problem Statement and Scope

This study examines a mechanical joint in which one mating surface contains a regular dimple pattern produced by rotary burnishing. The model predicts the normal contact response and specific normal contact stiffness associated with the textured interface. Quantities reported in MPa/ μm are termed *specific normal contact stiffness*; multiplying them by the nominal contact area gives the area-integrated normal joint stiffness in N/ μm .

The model describes the stabilized branch of repeated normal loading after dimple formation and initial asperity accommodation. The first loading cycle, transient shakedown, and surface evolution during service are not simulated; residual dimple geometry and descriptors of the load-bearing land are post-burnishing inputs. The calculated response is the normal mechanical stiffness of this stabilized post-burnishing interface. In the loading range considered here, dimple bottoms remain outside contact and the load is transmitted through the land between dimples.

2.2. Rotary Burnishing and Formation of Regular Dimples

Figure 1 introduces the mechanical setting used throughout the paper.

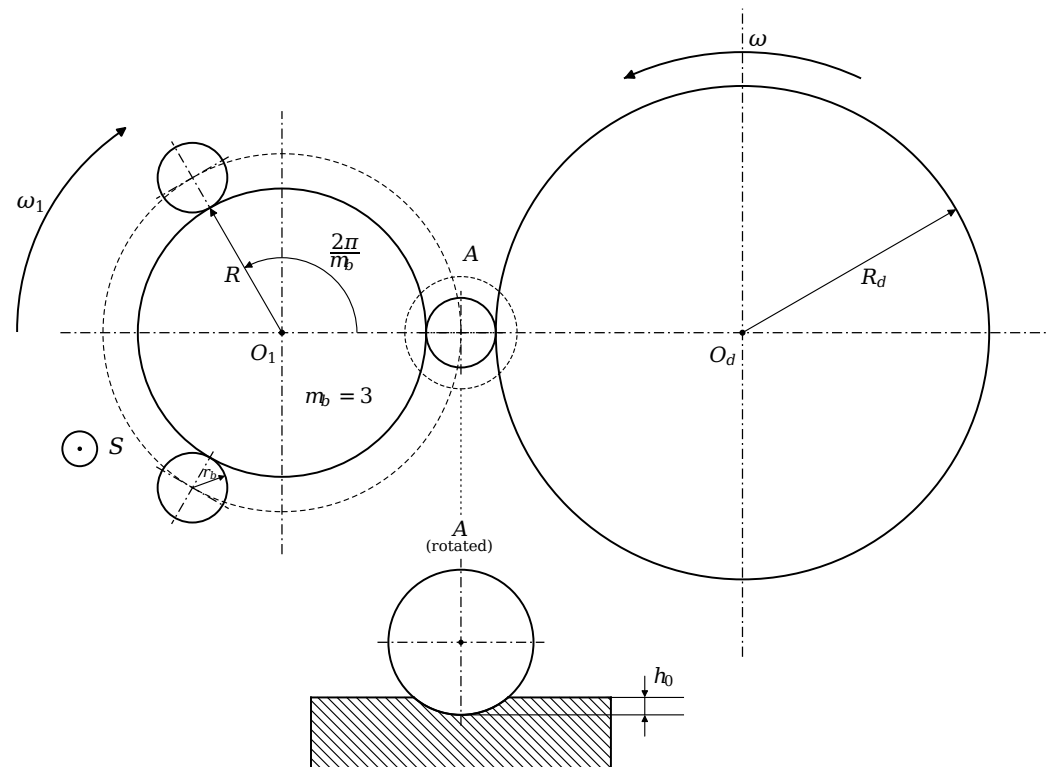


Figure 1. Problem setting for rotary burnishing and a dimpled surface joint. Here, ω_1 is the angular velocity of the burnishing head, ω is the angular velocity of the workpiece, R is the head radius, R_d is the workpiece radius, m_b is the number of active angular positions in the burnishing head, r_b is the ball radius, h_0 is the penetration depth, and S is the axial feed per workpiece revolution. O_1 and O_d are the centers of the burnishing head and workpiece, respectively. Point A marks the active ball–workpiece contact; the lower sketch shows the local normal section through A after rotation into the drawing plane. Solid outlines denote the bodies, the large dashed circle is the trajectory of the ball centers, and dash-dotted lines are axes and centerlines.

In rotary burnishing, a deforming element follows a combined rotational and feed motion and produces surface relief by local plastic deformation rather than by material removal. In the configuration considered here, the process forms repeated elliptical imprints (dimples) on the nominal surface.

The process parameters determine dimple size, pitch, and overlap. The main geometrical inputs are the ball radius r_b , penetration depth h_0 , axial feed S , workpiece radius R_d , number of active elements or angular positions m_b , and phase coefficient k_ϕ . Compared with vibratory burnishing, the rotary-burnishing configuration does not require superimposed tool vibration. This simplifies tool design and yields a deterministic dimple pattern that can be represented by a repeating unit cell.

The transverse semi-axis of an individual imprint follows from the chord of a ball penetrating the surface to a depth h_0 ,

$$b = \sqrt{h_0(2r_b - h_0)}. \quad (1)$$

The major semi-axis a is an effective along-track half-length of the imprint,

$$a = \frac{\ell_s}{2} = \kappa_a b, \quad (2)$$

where ℓ_s is the along-track imprint length and κ_a is the footprint aspect factor determined by the local kinematics of the deforming element or measured from the formed imprint. Unlike b , the semi-axis a is not obtained from the transverse ball chord; the elliptical shape reflects anisotropy between the along-track and transverse directions. In the reference calculation, $a = 0.600$ mm and $b = 0.631$ mm; the sensitivity analysis varies a to assess how the assumed along-track imprint length affects benchmark-matched regime B . The following subsection derives the unit-cell dimensions, dimple overlap area, open-area fraction, and relative bearing area from these parameters, which are then used in the contact-stiffness model.

Equations (1) and (2) provide a first-order nominal imprint estimate; the residual semi-axes and depth may differ because of plastic flow, pile-up or sink-in, and elastic recovery, and measured post-burnishing dimensions should replace the nominal estimates when available.

2.3. Geometry of the Dimple Pattern and Unit Cell

Figure 2 shows the regular dimple pattern on the unwrapped cylindrical surface.

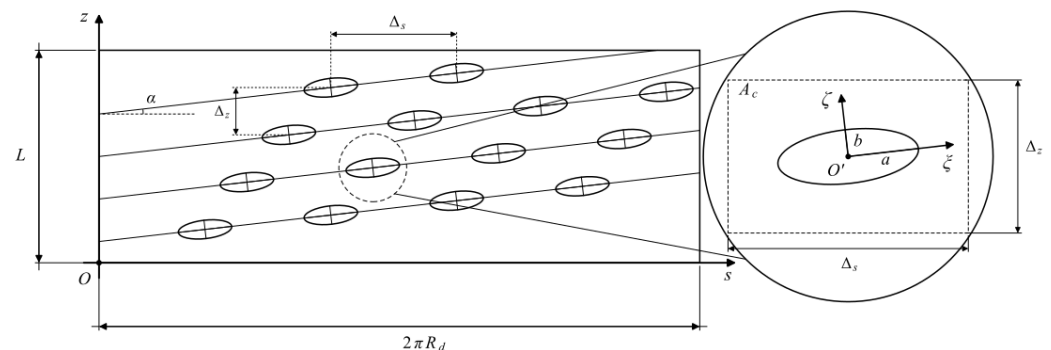


Figure 2. Unwrapped layout of the regular dimple pattern formed by rotary burnishing. The local cylindrical surface is unwrapped onto the (s, z) plane; s is the circumferential coordinate, z is the axial coordinate, L is the axial length, $2\pi R_d$ is the circumferential length of the unwrapped surface, α is the inclination of the dimple rows, Δ_s and Δ_z are the unit-cell dimensions, A_c is the unit-cell area, and a and b are the semi-axes of an elliptical dimple footprint. O is the origin of the unwrapped coordinates, O' is the selected dimple center, and (ξ, ζ) are the local dimple axes. Solid oblique lines connect centers in the same dimple row; the dashed rectangle denotes the unit cell A_c ; the dashed circle and connector lines identify the cell magnified in the inset.

The local cylindrical surface is unwrapped onto a plane with circumferential coordinate s and axial coordinate z . A repeating unit cell has dimensions Δ_s and Δ_z and the area

$$A_c = \Delta_s \Delta_z. \quad (3)$$

For the rotary layout used in the calculations, the unit-cell area can also be written as

$$A_c = \frac{S 2\pi R_d}{m_b k_\phi}, \quad (4)$$

where S is the axial feed per workpiece revolution, R_d is the workpiece radius, m_b is the number of angular positions in the head, and k_ϕ is the phase coefficient of the pattern. The cell dimensions are then chosen to preserve the aspect ratio of the elliptical footprint:

$$\Delta_s = \sqrt{A_c \frac{a}{b}}, \quad \Delta_z = \frac{A_c}{\Delta_s}. \quad (5)$$

A single dimple is approximated by an elliptical footprint with semi-axes a and b . For an isolated dimple, the footprint area is

$$A_d = \pi ab. \quad (6)$$

The isolated-area estimate A_d/A_c is valid only when footprints do not overlap. In dense patterns, it may overestimate the excluded area and can even yield a nonphysical fraction greater than one. Therefore, the dimple open-area fraction is calculated as the area of the union of periodic elliptical footprints in the unit cell,

$$F_d = \frac{1}{A_c} \iint_{\Omega_c} \mathbf{1} \left[\bigcup_{i,j \in \mathbb{Z}} \left(\frac{s - i\Delta_s}{a} \right)^2 + \left(\frac{z - j\Delta_z}{b} \right)^2 \leq 1 \right] ds dz, \quad (7)$$

where Ω_c is the rectangular unit cell and $\mathbf{1}[\cdot]$ is the indicator function. Equation (7) is evaluated by direct area integration over the unit cell with the periodic copies $i, j = -1, 0, 1$; for the footprint sizes considered, no more distant copy intersects the unit cell. The value at zero approach is denoted F_{d0} ; otherwise, F_d without an argument refers to this zero-approach value.

The relative bearing area is the complementary load-bearing fraction,

$$\varphi_0 = 1 - F_{d0}. \quad (8)$$

Thus, *open-area fraction* F_{d0} denotes the part of the nominal surface occupied by dimples, whereas *relative bearing area* denotes the part of the nominal surface through which normal load can be transmitted at the loading levels considered here. The unit-cell definitions and the overlap correction are shown schematically in Figure 3.

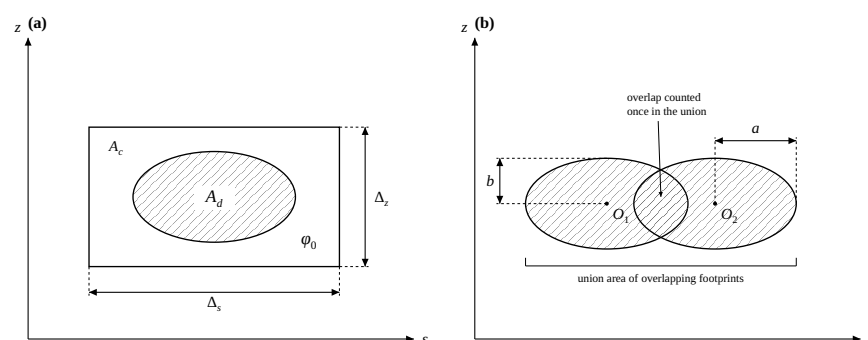


Figure 3. Unit cell of the regular dimple pattern and union-area evaluation for overlapping elliptical footprints: (a) unit cell with dimensions Δ_s and Δ_z , unit-cell area A_c , zero-approach open-area fraction F_{d0} , and relative bearing area $\varphi_0 = 1 - F_{d0}$; (b) overlap correction, where intersecting elliptical footprints are counted once in the union area used to calculate F_{d0} .

2.4. Two-Level Surface Representation

The model separates the deterministic dimple pattern from the stochastic micro-roughness of the load-bearing land. Burnishing kinematics defines the first level, whereas an asperity contact law describes the second.

Figure 4 illustrates the separation.

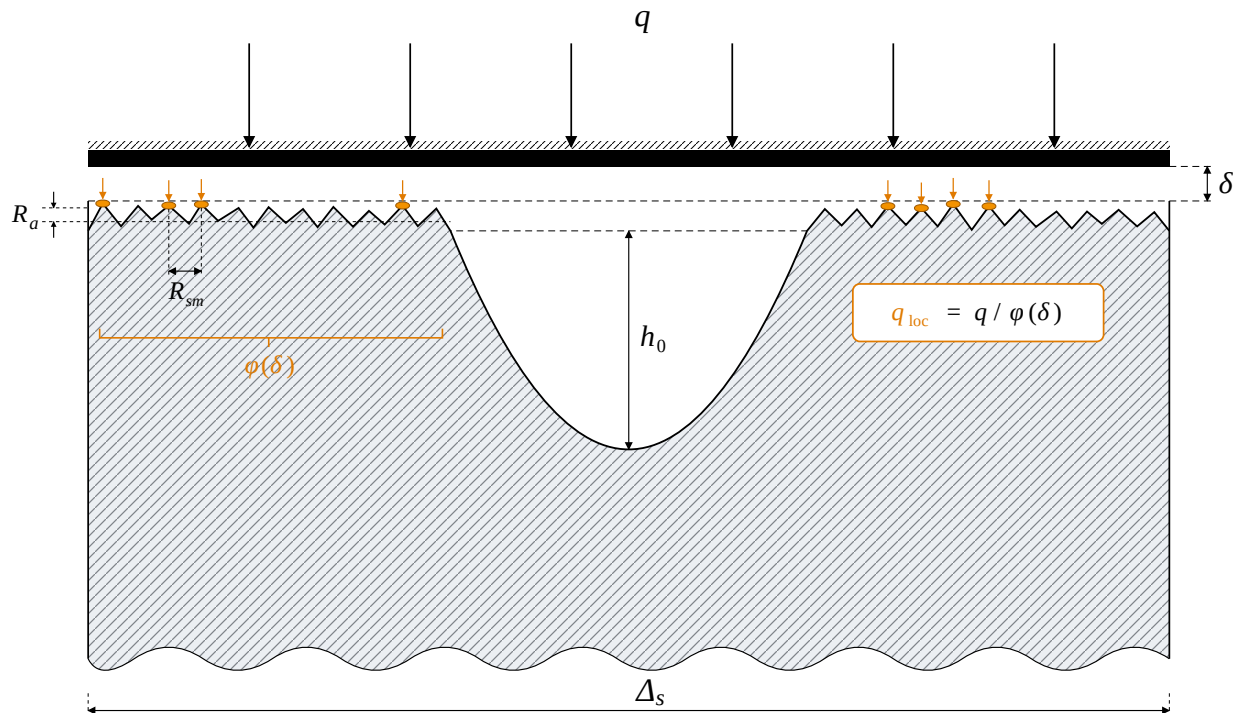


Figure 4. Two-level representation of the textured surface: regular dimple pattern and load-bearing land between dimples. The black horizontal bar represents the nominally flat counterface; black arrows denote the applied nominal pressure q , and orange arrows and contact spots denote load transmission through asperities on the land. Dashed horizontal lines are the reference levels used to define the normal approach δ and dimple depth h_0 ; R_a and R_{sm} characterize the micro-rough land, $\varphi(\delta)$ is the current load-bearing fraction, and $q_{loc} = q / \varphi(\delta)$ is the corresponding area-averaged land pressure.

2.5. Load Redistribution over the Relative Bearing Area

As the normal approach increases but remains below the dimple depth h_0 , the part of the dimple region excluded from direct contact becomes smaller. In an elliptical-paraboloid approximation, the semi-axes of the still-open part of a dimple are

$$a_\delta = a \sqrt{1 - \frac{\delta}{h_0}}, \quad b_\delta = b \sqrt{1 - \frac{\delta}{h_0}}, \quad 0 \leq \delta < h_0. \quad (9)$$

The relative bearing area is therefore written as

$$\varphi(\delta) = 1 - F_{d0} \left(1 - \frac{\delta}{h_0} \right), \quad 0 \leq \delta < h_0, \quad (10)$$

and $\varphi(\delta) = 1$ when the dimple depth is fully closed in the simplified geometrical sense. The applied nominal pressure q is transmitted through the load-bearing land; therefore, the average pressure over that area is computed as

$$q_{loc}(\delta) = \frac{q}{\varphi(\delta)}. \quad (11)$$

The quantity q/φ is the area-averaged pressure required by force equilibrium over the load-bearing land; it does not imply that the local field $p(x, y)$ is uniform. This averaged redistribution couples the regular dimple pattern to the micro-roughness response without resolving the local contact problem inside each dimple. Stress concentrations can occur near dimple edges, but, in the sense of Saint-Venant's principle, their disturbance is expected to remain localized relative to the bridge width. Quantitative confirmation is given in Section 3.6.

The linear approximation in Equation (10) is adopted for computational efficiency. A direct union-area calculation in Section 3.1 shows that the approximation error remains below 0.011% for regime B.

2.6. Fourier Coefficient of the Elliptical Dimple Region

The coefficient derived in this subsection is the dimensionless Fourier component of the open-dimple indicator. It is used in Section 2.7 to evaluate the spectral length scale associated with the regular dimple pattern.

Let $\chi_d(x, y; \delta)$ be the indicator of the open elliptical region with semi-axes a_δ and b_δ . For a reciprocal-lattice vector $\mathbf{k} = (k_s, k_z)$, the unnormalized transform of this indicator is

$$\hat{\chi}_d(\mathbf{k}; \delta) = \iint_{x^2/a_\delta^2 + y^2/b_\delta^2 \leq 1} \exp[-i(k_s x + k_z y)] dx dy. \quad (12)$$

The coordinate transformation and Bessel-function integration are given in Appendix A. After normalization by the unit-cell area, the required Fourier coefficient is

$$H_{\mathbf{k}}(\delta) = F_{d0} \left(1 - \frac{\delta}{h_0}\right) \frac{2J_1(Q_{\mathbf{k}})}{Q_{\mathbf{k}}}, \quad Q_{\mathbf{k}} = \sqrt{(k_s a_\delta)^2 + (k_z b_\delta)^2}. \quad (13)$$

Here, J_1 is the Bessel function of the first kind, and the limiting value $2J_1(Q)/Q \rightarrow 1$ is used when $Q \rightarrow 0$.

2.7. Additional Normal Displacement from the Regular Dimple Pattern

The pressure deficit over each dimple is represented as a periodic traction field. The elastic energy of its nonzero Fourier modes defines the spectral length scale $\Lambda(\delta)$, from which the additional approach is obtained.

For a linear elastic half-space, a nonzero pressure harmonic with wave number $|\mathbf{k}|$ produces a normal-displacement amplitude proportional to $2/(E^*|\mathbf{k}|)$. The modulation associated with the dimple indicator has amplitude proportional to $qH_{\mathbf{k}}/\varphi(\delta)$: $H_{\mathbf{k}}$ describes the excluded region, whereas $1/\varphi(\delta)$ accounts for the increased mean pressure on the bearing area. The resulting cell-averaged compliance contribution is proportional to $|H_{\mathbf{k}}|^2/[\varphi^2(\delta)|\mathbf{k}|]$. After the mean component is removed, the nonzero reciprocal-lattice vectors $\mathbf{k}_{mn} = (2\pi m/\Delta_s, 2\pi n/\Delta_z)$ define

$$\Lambda(\delta) = \frac{2}{\varphi^2(\delta)} \sum_{\mathbf{k} \neq 0} \frac{|H_{\mathbf{k}}(\delta)|^2}{|\mathbf{k}|}, \quad (14)$$

and the displacement contribution is

$$\delta_{\text{reg}}(q, \delta) = 1000 \frac{q}{E^*} \Lambda(\delta), \quad (15)$$

where E^* is the reduced elastic modulus, q is expressed in MPa, Λ is in mm, and the factor 1000 converts millimeters to micrometers. The coefficient $H_{\mathbf{k}}$ is dimensionless and $|\mathbf{k}|$ is in mm^{-1} ; hence, the summation in Equation (14) has the dimension of length. The ratio q/E^* is dimensionless when both quantities are expressed in MPa.

Physically, the factor φ^{-2} reflects load balance under a reduced bearing fraction: the mean pressure on the bearing area grows as $1/\varphi$, and the elastic energy scales with the pressure squared. Equation (15) therefore represents the elastic approach induced by the periodic pressure modulation at a prescribed nominal pressure, coupled to the micro-roughness response through the same local pressure q_{loc} .

2.8. Micro-Roughness Response of the Load-Bearing Area

The load-bearing land is described by a Greenwood–Williamson-type elastic asperity response [24,30]. Let σ denote the standard deviation of summit heights, R_s the effective summit radius, and η the summit density. For two identical contacting bodies, the reduced modulus is $E^* = E/[2(1 - \nu^2)] = 59.0$ GPa. The pressure carried by the load-bearing land at micro-roughness approach δ_μ is

$$p_c(\delta_\mu) = \eta \frac{4}{3} E^* \sqrt{R_s} \int_{h_{\max} - \delta_\mu}^{h_{\max}} [h - (h_{\max} - \delta_\mu)]^{3/2} \frac{\exp[-h^2/(2\sigma^2)]}{\sigma \sqrt{2\pi}} dh. \quad (16)$$

The model conventions are

$$\sigma = R_q = R_a \sqrt{\frac{\pi}{2}}, \quad h_{\max} = n_\sigma \sigma, \quad \eta = \frac{c_\eta}{R_{sm}^2}, \quad (17)$$

with

$$R_s = \frac{R_{sm}^2}{4\pi^2 \sqrt{2} R_q}. \quad (18)$$

The reference values of R_a and R_{sm} are taken from the literature ranges representative of the rough load-bearing land, whereas n_σ and c_η are prescribed modeling parameters [8,31]. The value R_q is estimated from R_a under the Gaussian summit-height assumption, and $\sigma = R_q$ is used in the Greenwood–Williamson-type integral [24,30]. The summit radius R_s is estimated by Equation (18), which approximates a representative summit as a sinusoidal cap with wavelength R_{sm} and amplitude $\sqrt{2}R_q$. Here, $n_\sigma = 3.0$ sets the upper tail $h_{\max} = n_\sigma \sigma$, and $c_\eta = 1.0$ gives one effective summit per R_{sm}^2 cell. Both parameters are fixed before comparison with external data and varied in the sensitivity analysis.

The estimated plasticity index supports the use of an elastic Greenwood–Williamson closure: with $E^* = 59.0$ GPa, $\sigma = 0.351$ μm , and $R_s = 12.76$ mm, $\psi = (E^*/H_{\text{land}})\sqrt{\sigma/R_s} = 0.21\text{--}0.12$ for $H_{\text{land}} = 1.5\text{--}2.6$ GPa, below the commonly used threshold $\psi = 0.6$. Burnishing-induced work hardening increases H_{land} and thus decreases ψ . Within the present model, burnishing-related changes are represented through the measured post-burnishing descriptors R_a and R_{sm} ; hardness is used only to assess the elastic-asperity assumption. The scale ratios are $\sigma/h_0 = 8.8 \cdot 10^{-3}$ and $R_{sm}/\min(\Delta_s, \Delta_z) \approx 0.29$, supporting the separation between the regular-texture scale and the load-bearing-land scale for regime B.

2.9. Nonlinear Coupling and Specific Contact Stiffness Evaluation

The two levels are coupled by requiring the total normal approach δ to be the sum of the micro-roughness approach and the additional displacement associated with the regular dimple pattern:

$$F(\delta; q) = \delta - \delta_{\text{reg}}(q, \delta) - \delta_\mu \left(\frac{q}{\varphi(\delta)} \right) = 0. \quad (19)$$

Here, $\delta_\mu(q_{\text{loc}})$ is obtained by inverting Equation (16). Both δ_{reg} and δ_μ depend on the same area-averaged land pressure. The differential specific normal contact stiffness is then evaluated as

$$K_n(q) = \left(\frac{d\delta(q)}{dq} \right)^{-1}. \quad (20)$$

For comparison with empirical pressure–approach relations, the secant specific normal contact stiffness $j_s(q) = q/\delta(q)$ is also reported. The spectral summation is truncated, the micro-roughness integral is evaluated by Gauss–Legendre quadrature, and the nonlinear Equation (19) is solved numerically using Brent’s method.

2.10. Periodic BEM Procedure

An independent periodic BEM calculation was performed for regime *B* using the elastic half-space kernel and a conjugate-gradient contact solver [32]. The periodic cell was $\Delta_s \times \Delta_z = 1.725 \times 1.814$ mm, with a 0.600×0.631 mm dimple and $h_0 = 40$ μm . Unilateral frictionless normal contact with a flat counterface was imposed on 256^2 and 512^2 grids at $q = 0.5, 1.0$, and 2.5 MPa; the same reduced modulus E^* as in Sections 2.7 and 2.8 was used in the half-space kernel, so that $\Delta\delta_{\text{BEM}}$ is directly comparable with δ_{reg} . The calculation yielded the pressure field over the load-bearing land and the cell-averaged approach; $\Delta\delta_{\text{BEM}}$ denotes the excess approach of the dimpled cell relative to a smooth cell at the same nominal pressure. A pixelwise inversion of the Greenwood–Williamson relation was then applied to $p(x, y)$ to assess the influence of pressure heterogeneity on the micro-roughness closure on the load-bearing land. This comparison assesses the area-averaged load redistribution and the additional compliance assigned to the regular dimple pattern for regime *B*.

2.11. Numerical Implementation

The numerical implementation follows the calculation sequence outlined above:

- The rotary-burnishing parameters define the dimple geometry;
- Dimple overlap gives the open-area fraction and relative bearing area;
- The local pressure on the load-bearing land is obtained by averaged load redistribution;
- The regular dimple pattern adds a finite spectral contribution to normal approach;
- The response of the load-bearing land follows from the micro-roughness model;
- The nonlinear closure is solved and differentiated numerically to obtain the specific normal contact stiffness.

All analytical calculations, root-finding operations, data processing, and plotting were performed in Python 3.12.2 using NumPy 2.4.6, SciPy 1.17.1 and Matplotlib 3.10.8. The direct periodic boundary-element calculations used a custom Python driver with Tamaas 2.9.0 and the Polonsky–Keer–Rey contact algorithm.

The spectral summation is truncated at $N_{\text{max}} = 6$, giving 168 nonzero reciprocal-lattice vectors. Increasing the truncation from $N_{\text{max}} = 6$ to $N_{\text{max}} = 12$ changes δ_{reg} from 0.010865 to 0.010914 μm at $q = 2.5$ MPa, i.e., by less than 0.5% of its value (Figure 5). The micro-roughness integral is evaluated with 48-point Gauss–Legendre quadrature, and doubling the order changes the computed pressure by less than 0.001%. Brent’s method solves Equation (19) with a residual below 10^{-6} μm .

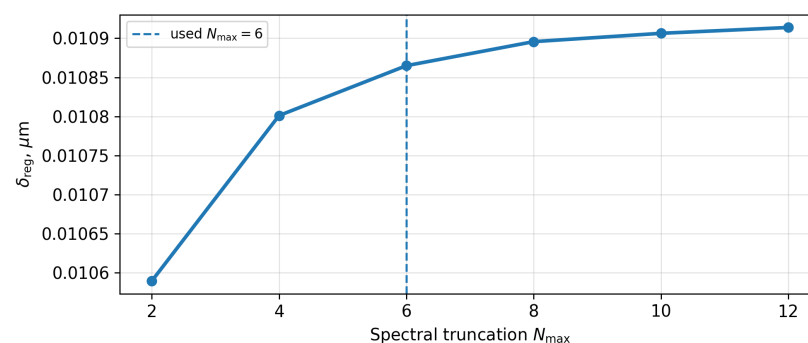


Figure 5. Convergence of the additional normal displacement δ_{reg} with respect to the spectral truncation N_{max} for benchmark-matched regime *B* at $q = 2.5$ MPa.

The reference calculations use the parameters summarized in Table 1.

Table 1. Parameters of the reference calculation.

Parameter	Value	Status and Source
R_a	0.28 μm	Central value in the 0.24–0.32 μm range reported for ball-burnished regular-relief surfaces; used as a prescribed reference value for the load-bearing land [31].
R_{sm}	0.50 mm	Central value in the 0.40–0.60 mm pitch-parameter range; identified with R_{sm} as an engineering approximation [8].
F_d	38%	Center of the 36–40% area range reported by Shneyder–Ambaryan for partially regular microrelief [12].
R_q/R_a	$\sqrt{\pi/2}$	Consequence of a Gaussian summit-height distribution.
n_σ	3.0	Dimensionless multiplier for the upper tail of the summit-height distribution, $h_{\max} = n_\sigma\sigma$; fixed in the reference calculation and varied in the sensitivity analysis.
c_η	1.0	Dimensionless summit-density factor in $\eta = c_\eta/R_{sm}^2$; fixed in the reference calculation and varied in the sensitivity analysis.
R_s	12.76 mm	Derived from R_a and R_{sm} using Equation (18); not an independent constant.
η	4.0 mm^{-2}	Computed as c_η/R_{sm}^2 for $c_\eta = 1.0$.
E, ν	110 GPa; 0.26	Fixed properties of gray cast iron in the engineering range.
h_0	40 μm	Process dimple depth; sensitivity analysis shows a weak influence on the Shneyder–Ambaryan comparison when F_d is fixed.
r_b	5 mm	Radius of the deforming ball.
a, b	0.600; 0.631 mm	Semi-axes of the elliptical footprint; b follows from Equation (1), while a follows from the effective along-track length in Equation (2).
$N_{\max}$	6	Spectral truncation level.

Table 2 summarizes the validity domain adopted for the comparisons and design calculations.

Table 2. Validity domain used for the present comparisons and design calculations.

Quantity or Assumption	Range or Condition	Comment
Nominal pressure q	0.1–2.5 MPa for the Shneyder–Ambaryan comparison; $q_0 = 0.5$ MPa for the inverse-design example	The external benchmark covers the stated pressure interval.
Approach ratio δ/h_0	$\delta/h_0 < 0.03$ for the regime B over 0.1–2.5 MPa	The dimple bottoms remain outside direct contact in the reported comparisons.
Open-area fraction F_{d0}	36–40% for the main comparison; 36.16% for the inverse-design illustration	The dense regime A with $F_{d0} = 93.35\%$ is used as a geometric overlap illustration, not as the quantitative benchmark.
Relative bearing area φ_0	60–64% for the main comparison; 63.84% in the inverse-design illustration	The local pressure is evaluated as $q_{\text{loc}} = q/\varphi(\delta)$; cases with very small bearing fractions require a separate check.
Dimple depth h_0	Central value 40 μm ; inverse-design example 35 μm	The mechanical model assumes that closure remains far below the dimple depth.
Load-bearing land	$R_a = 0.24\text{--}0.32$ μm , $R_{sm} = 0.40\text{--}0.60$ mm in the range of literature parameters	Other surfaces require re-identification of R_a , R_{sm} , and the derived micro-roughness parameters.
Material parameters	Gray-cast-iron properties represented by $E = 110$ GPa and $\nu = 0.26$	Different materials enter through the reduced modulus E^* .

2.12. Inverse Design Formulation

For process design, the forward relation can be inverted. Given a target secant specific normal contact stiffness $j_{s,\text{target}}$ at nominal pressure q_0 , the design variables are selected from the admissible process range so that

$$\min_{\mathbf{u}} |j_s(q_0; \mathbf{u}) - j_{s,\text{target}}|, \quad (21)$$

where $\mathbf{u}$ may include feed, phase coefficient k_ϕ , ball radius, penetration depth, and the corresponding dimple open-area fraction. The inverse calculation is applied to the established forward model to identify feasible process-parameter combinations.

3. Results

3.1. Geometry Generated by Rotary Burnishing Parameters

The geometry module maps the rotary-burnishing parameters to the dimple footprint, overlap state, open-area fraction, and bearing-area fraction. These quantities control the area-averaged pressure acting on the load-bearing land through Equation (11).

For dense regime *A*, with $S = 0.085$ mm/rev, Equation (7) gives $F_d = 93.35\%$. For benchmark-matched regime *B*, with $S = 0.239$ mm/rev, it gives $F_d = 38.01\%$. Regime *B* is used for comparison with the Shneyder–Ambaryan data because the reported partially regular microrelief has a groove-area range of approximately 36–40% [12].

The linear approximation in Equation (10) was verified by recalculating the union area after the semi-axes were scaled according to Equation (9). Table 3 compares the direct numerical integration of the union area with the linear approximation. For the regime *B*, used in the Shneyder–Ambaryan comparison and in the reference calculation, the relative error remains below 0.011% over the tested range. Dense regime *A* illustrates a strongly overlapping pattern, for which the linear approximation is less accurate; therefore, quantitative stiffness comparisons are not performed for this regime.

Table 3. Verification of the linear approximation in Equation (10) for $F_d(\delta)$ against direct numerical integration of the union area. Values are given in percent.

δ/h_0	Regime A, Dense Pattern			Regime B, Benchmark-Matched Pattern		
	Direct Union	Linear Equation (10)	Rel. Err.	Direct Union	Linear Equation (10)	Rel. Err.
0.05	91.545	88.685	3.12	36.113	36.115	0.007
0.10	89.400	84.017	6.02	34.216	34.214	0.004
0.20	83.796	74.682	10.88	30.410	30.413	0.011
0.50	53.462	46.676	12.69	19.009	19.008	0.004
0.80	21.386	18.670	12.70	7.602	7.603	0.011

For a dimple pattern with low overlap, the open-area fraction is close to the isolated value A_d/A_c . For dense patterns, the difference between the isolated-area sum and the union area becomes important. Treating the dimples as non-overlapping in such cases overestimates the excluded area and underestimates the relative bearing area. The corresponding feed trend for the reference process study is summarized in Figure 6.

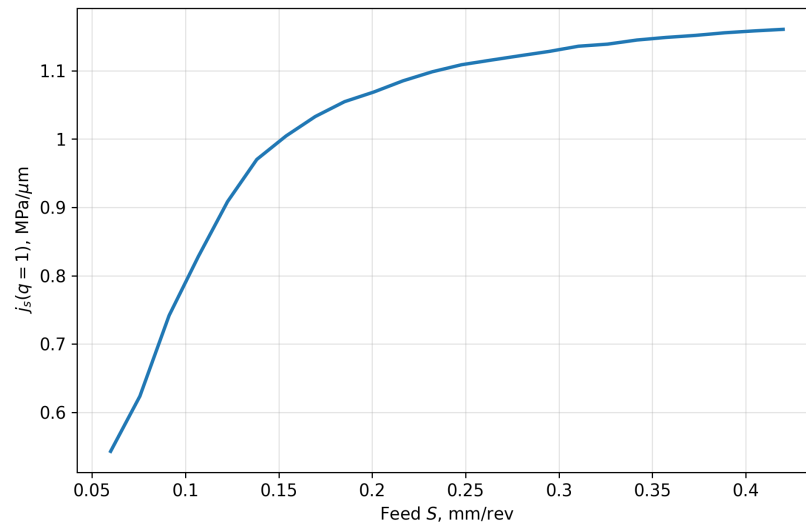


Figure 6. Effect of feed S on the secant specific normal contact stiffness j_s at $q = 1$ MPa in the reference process-parameter study.

3.2. Normal Approach and Specific Normal Contact Stiffness Predicted by the Model

The model predicts the normal approach as a nonlinear function of nominal pressure. The nonlinearity is governed mainly by the micro-roughness response of the load-bearing area, while the regular dimple pattern modifies this response through the relative bearing area and the additional normal displacement.

The corresponding secant specific normal contact stiffness $j_s = q/\delta$ increases with nominal pressure as more asperities carry load and the local microcontacts stiffen. The pressure–approach and pressure–stiffness curves are summarized in Figure 7. The differential specific normal contact stiffness $K_n = (d\delta/dq)^{-1}$ is reported separately in Table 4.

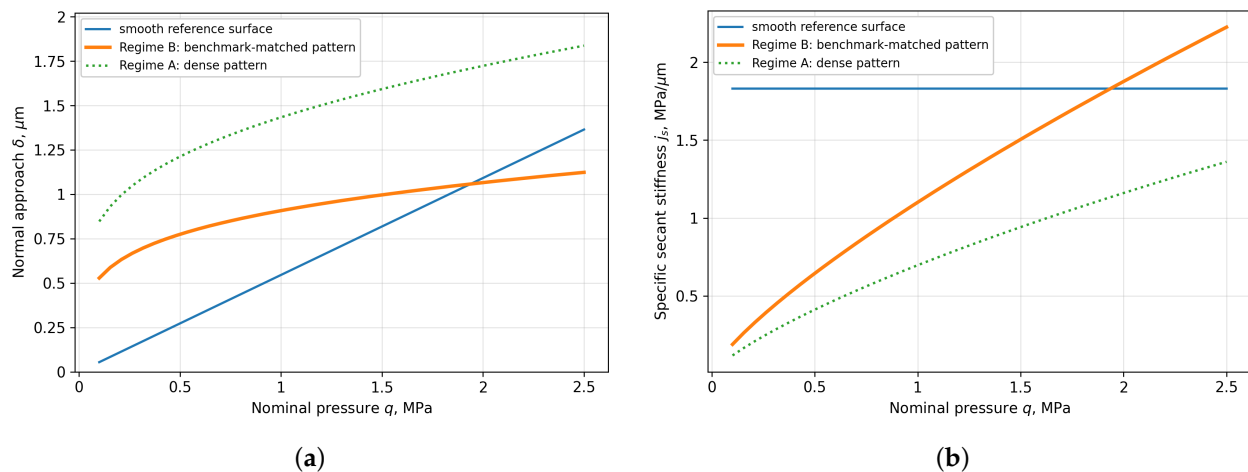


Figure 7. Predicted normal approach (a) and secant specific normal contact stiffness (b) as functions of nominal pressure.

Table 4. Selected numerical values for regime B. Both secant and differential specific normal contact stiffnesses are reported in MPa/μm.

q , MPa	δ , μm	δ_μ , μm	δ_{reg} , μm	$\varphi(\delta)$	q_{loc} , MPa	$j_s = q/\delta$	K_n
0.5	0.775	0.773	0.0022	0.627	0.797	0.645	2.748
1.0	0.908	0.903	0.0044	0.629	1.591	1.102	4.720
2.0	1.066	1.057	0.0087	0.630	3.174	1.877	7.848
2.5	1.123	1.113	0.0109	0.631	3.964	2.225	9.185

3.3. Contributions of the Dimple Pattern and Micro-Roughness

The two-level representation allows the compliance to be decomposed into a micro-roughness contribution and an additional contribution associated with the regular dimple pattern. The contributions are not independent linear springs: the dimple pattern changes the local pressure acting on the load-bearing land, and the micro-roughness response is nonlinear with respect to that local pressure.

For the regime *B* at $q = 2.5$ MPa, the micro-roughness contribution of the load-bearing land is $1.113 \mu\text{m}$, whereas the regular spectral contribution is $0.0109 \mu\text{m}$, i.e., approximately 0.98% of the micro-roughness contribution. Although δ_{reg} is numerically small, the regular pattern remains essential because it sets the relative bearing area and thus controls the local pressure driving the micro-roughness response. The physical origin of this scale separation is discussed in Section 4.1. Figure 8 shows this decomposition. The micro-roughness response and the comparison with a smooth reference surface are shown in Figure 9 and in Figure 7b, respectively.

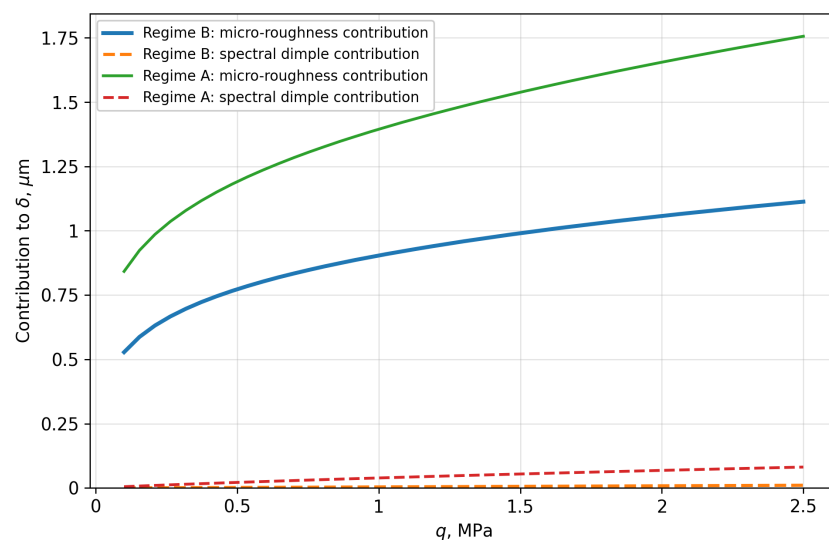


Figure 8. Normal-approach contributions of the regular dimple pattern and the micro-roughness response. The two curves are coupled through the local pressure and are not independent linear springs.

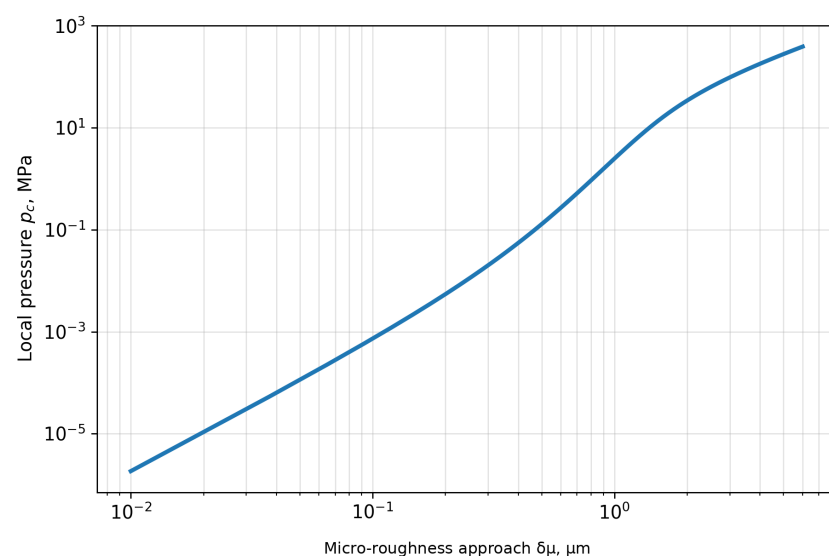


Figure 9. Greenwood–Williamson-type response of the load-bearing land; the nonlinear initial contact dominates the shape of the pressure–approach curve.

3.4. External Benchmark Against Shneyder–Ambaryan Contact-Stiffness Data

The literature comparisons in Sections 3.4 and 3.5 are external benchmarks for predicted trends and order of magnitude. Because the manufacturing routes and surface-layer states differ from the present rotary-burnishing configuration, these datasets do not constitute process-specific experimental validation.

The Shneyder–Ambaryan data combine the effects of regular microgeometry, finishing history, and surface-layer strengthening and therefore provide an external benchmark for the predicted range and trend [12]. Because the reference surfaces contain continuous grooves rather than isolated dimples, the comparison addresses trend and order of magnitude at a comparable open-area fraction; the effect of texture shape is a separate question.

The four empirical relations used for the error calculation are listed in Table 5.

Table 5. Empirical Shneyder–Ambaryan relations used for the error calculation.

Finishing Condition	Formula for Normal Approach, μm
Grinding	$\delta = 1.07 q^{0.44}$
Vibratory burnishing after grinding	$\delta = 0.64 q^{0.56}$
Smooth burnishing	$\delta = 0.58 q^{0.50}$
Vibratory burnishing after smooth burnishing	$\delta = 0.72 q^{0.55}$

The model is compared with the curve for regular microrelief produced by vibratory burnishing after smooth burnishing. The dimple open-area fraction is selected to match the reported 36–40% range for partially regular microrelief [12]. With the reference parameter set, the predicted normal approach remains within the same order of magnitude as the external reference (Figure 10).

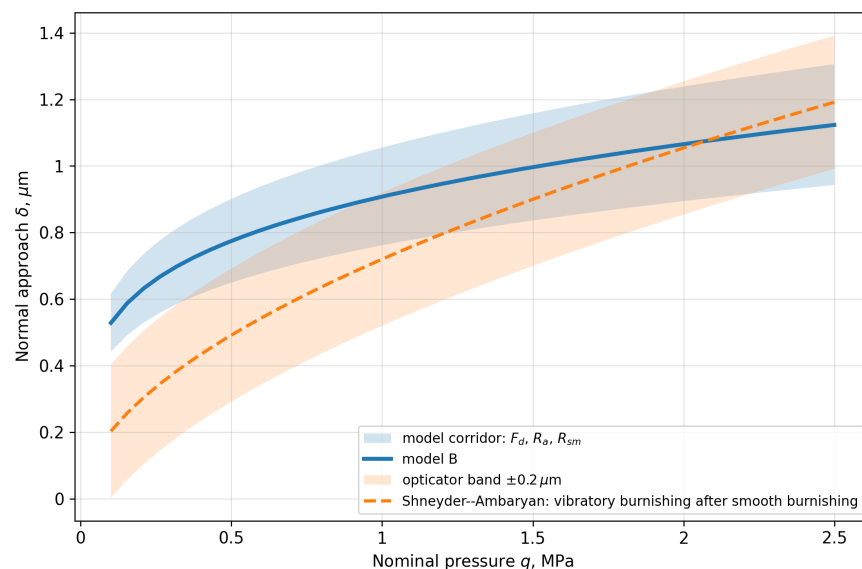


Figure 10. External benchmark of the predicted normal-approach trend against Shneyder–Ambaryan data for surfaces with regular microrelief [12].

The corresponding error metrics for the four Shneyder–Ambaryan relations are summarized in Table 6.

Table 6. Descriptive RMSE and MAE of regime *B* against four external Shneyder–Ambaryan reference curves ($n = 7$ points, $q = 0.1\text{--}2.5$ MPa). Bold indicates the lowest RMSE and MAE.

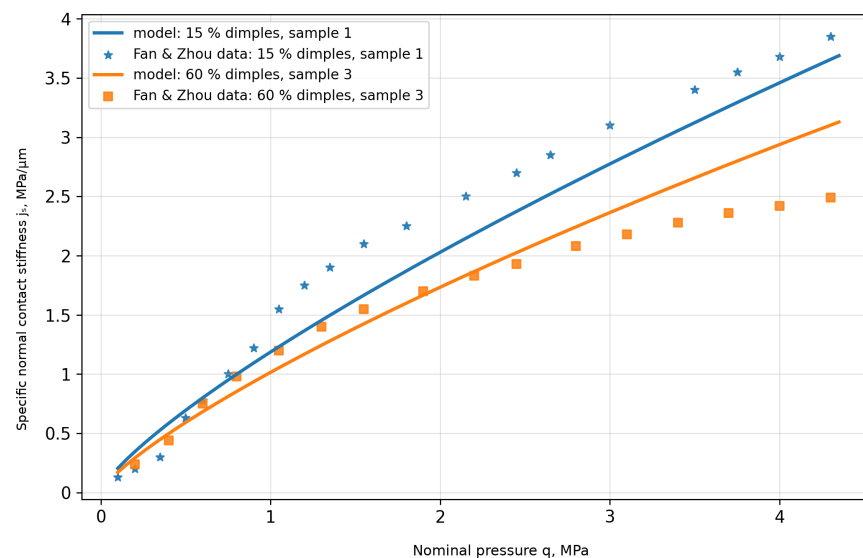
Curve	RMSE, μm	MAE, μm
Grinding	0.271	0.223
Vibratory burnishing after grinding	0.267	0.242
Smooth burnishing	0.311	0.306
Vibratory burnishing after smooth burnishing	0.221	0.186

When the empirical coefficient 0.64 in Table 5 is used, the vibratory-burnishing-after-grinding curve approaches the model predictions. For the reference parameter set, however, the closest external reference remains the vibratory-burnishing-after-smooth-burnishing curve.

A robustness sweep over 27 corner combinations, $R_a \in \{0.24, 0.28, 0.32\} \mu\text{m}$, $R_{sm} \in \{0.40, 0.50, 0.60\} \text{ mm}$, and $F_d \in \{36, 38, 40\} \%$, identifies that curve as the closest in 15 cases, grinding in 11 cases, and vibratory burnishing after grinding in one case. The ranking is descriptive and is not used as process-specific validation.

3.5. Independent Comparison with Fan & Zhou Dimple-Density Data

The Fan and Zhou data provide a qualitative check of texture-area effects for a different manufacturing route [23]. Their polished cast-iron specimens had circular pits of $d = 100 \mu\text{m}$ and $h = 20 \mu\text{m}$ at pit densities of 15%, 30%, and 60%, together with a smooth reference. The laser-textured specimens also contained hardened remelted rims, a technology-specific mechanism not represented by the present dimple-geometry model. Samples 1 (15%) and 3 (60%) are therefore retained as an external order-of-magnitude check of the stiffness change associated with texture area, while the 30% case and the smooth sample are not used for direct model comparison. The selected benchmark is shown in Figure 11.

**Figure 11.** External benchmark against the Fan and Zhou dimple-density data [23]. Samples 1 (15%) and 3 (60%) provide a qualitative check of texture-area effects for a different manufacturing route.

For samples 1 and 3, the root-mean-square error between the model predictions and the digitized experimental data is approximately $0.270 \text{ MPa}/\mu\text{m}$, and the mean absolute error is approximately $0.224 \text{ MPa}/\mu\text{m}$ over 36 digitized pressure points. These metrics summarize the external order-of-magnitude benchmark.

3.6. Periodic BEM Assessment

The contact fraction remained $\varphi_0 = 0.620$ at all investigated nominal pressures. The edge concentrations decayed to 1.1 times the mean land pressure within approximately $77\text{ }\mu\text{m}$ from the dimple boundary, compared with bridge widths of approximately $0.5\text{--}0.6\text{ mm}$. A band of width $2h_0$ adjacent to the edges carried 31% of the load, and the 95th-percentile pressure was $p_{95} = 1.67 (q/\varphi_0)$. The maximum and 99th-percentile pressures were mesh-sensitive because of the sharp idealized edge, so percentile measures are reported instead.

The normalized pressure field, bridge profiles, and additional compliance are summarized in Figure 12, while the corresponding numerical values are listed in Table 7.

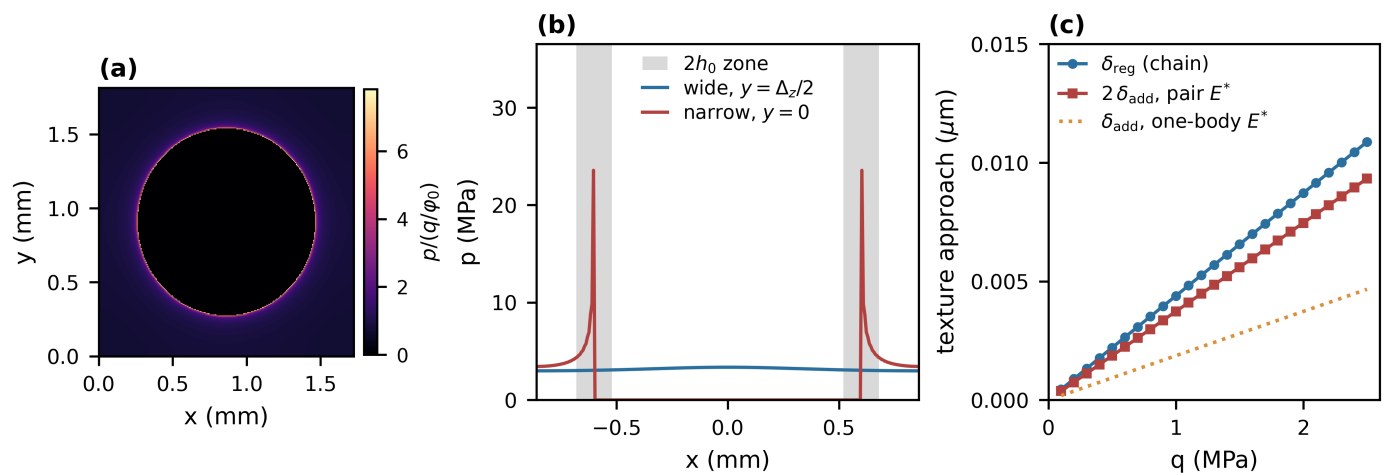


Figure 12. Periodic BEM assessment for regime B: (a) normalized pressure map; (b) pressure profiles across the load-bearing bridges; and (c) additional approach predicted by the analytical spectral model and periodic BEM.

Table 7. Periodic BEM assessment of the additional compliance and pressure redistribution for regime B.

q , MPa	$\Delta\delta_{\text{BEM}}$, μm	δ_{reg} , μm	Difference	$p_{95}/\bar{p}$	Edge-Band Load
0.5	0.00187	0.00221	−15%	1.68	0.31
1.0	0.00373	0.00439	−15%	1.67	0.31
2.5	0.00933	0.01088	−14%	1.67	0.31

Increasing the grid from 256^2 to 512^2 decreased $\Delta\delta_{\text{BEM}}$ by 0.26%. The numerical excess approach was 14–15% below δ_{reg} , which is consistent with the uniform-pressure idealization of the analytical spectral model relative to the unilateral contact solution. Pixelwise Greenwood–Williamson inversion on the calculated $p(x, y)$ field changed the area-averaged micro-roughness approach of the load-bearing land by less than 1.7% relative to closure at the mean land pressure, with the difference decreasing monotonically with q . This result supports the use of the area-averaged land pressure for regime B over the investigated pressure range.

In the rough-contact reference frame, geometrical closure is driven by the penetration below the mean land plane, $u_d = \max(0, \delta_\mu - h_{\text{max}})$. At $q = 2.5\text{ MPa}$, $u_d \approx 0.060\text{ }\mu\text{m}$ and $u_d/h_0 \approx 1.5 \cdot 10^{-3}$. Using u_d instead of δ in Equation (10) gives $|\Delta\varphi| < 0.012$ ($0.631 \rightarrow 0.620$) and changes q/φ by less than 2%. The remaining dimple-bottom gap is $g_{\text{bottom}} \approx 39.9\text{ }\mu\text{m}$, so the bottom remains outside contact throughout the stated loading range.

3.7. Sensitivity to Dimple Geometry and Micro-Roughness Parameters

The sensitivity analysis separates parameters that primarily affect the deterministic dimple pattern from parameters that primarily affect the load-bearing land. The dimple size, pitch, and overlap control the open-area fraction and relative bearing area. The roughness parameters R_a , R_{sm} , c_η , and n_σ control the nonlinear micro-roughness response.

The analysis shows that the relative bearing area and micro-roughness parameters are the quantities most relevant to design. Dimple depth influences closure of the non-contact region, whereas its effect on the comparison at low normal approach is weaker when the open-area fraction is fixed. The sensitivity trends are summarized in Figure 13.

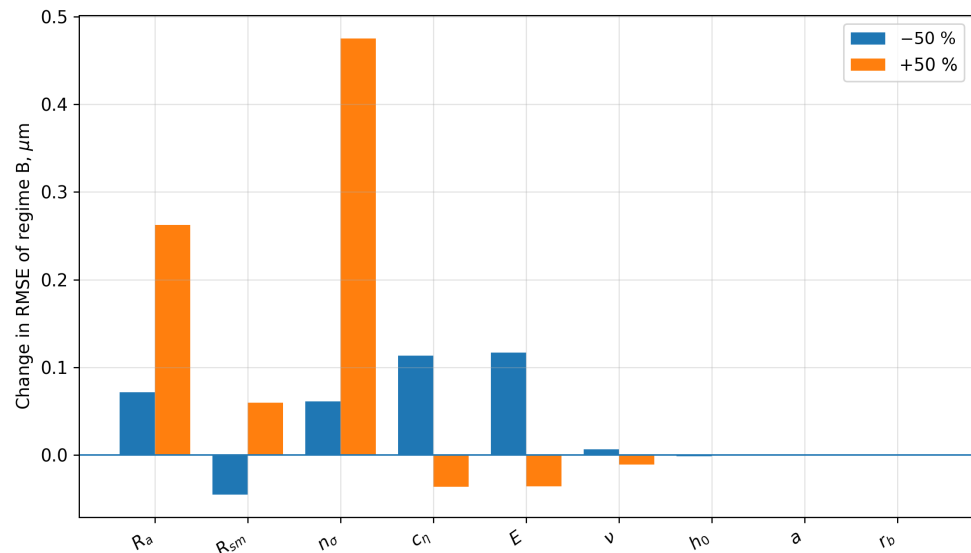


Figure 13. Sensitivity of specific normal contact stiffness to dimple geometry, relative bearing area, and micro-roughness parameters. The bars correspond to changing one parameter at a time to $0.5\times$ and $1.5\times$ of its reference value while keeping the remaining parameters fixed.

The extended sensitivity results are summarized in Table 8. They show that the response to h_0 , a , and r_b is weak for regime B when F_d is fixed, whereas the micro-roughness conventions have a stronger effect on the Shneyder–Ambaryan comparison.

The variation of E in the sensitivity analysis represents uncertainty in elastic properties and material selection; it is not a model of burnishing-induced work hardening. In the present model, burnishing-induced changes are represented through the measured post-burnishing microgeometry R_a and R_{sm} , whereas H_{land} enters only the plasticity-index check. An increase in H_{land} decreases ψ and supports the elastic-asperity assumption.

Table 8. Extended sensitivity summary for regime B against the target Shneyder–Ambaryan curve.

Parameter Varied by $\pm 50\%$	RMSE at $0.5\times$, μm	RMSE at $1.5\times$, μm	Main Interpretation
R_a	0.293	0.484	Roughness amplitude controls micro-roughness approach.
R_{sm}	0.177	0.281	Summit spacing affects density and radius.
n_σ	0.283	0.697	Upper-tail multiplier controls the initial branch.
c_η	0.335	0.185	Density factor controls stiffness.
E	0.338	0.186	Elastic modulus scales compliance.
ν	0.228	0.211	Poisson's ratio has a secondary effect.
h_0	0.220	0.222	Dimple depth is weak for $\delta \ll h_0$.
a	0.221	0.222	Footprint size is weak at fixed F_d .
r_b	0.221	0.222	Ball radius has a weak effect here.

3.8. Inverse Design Maps for Target Specific Contact Stiffness

The inverse-design calculation applies the forward model to identify process parameters that achieve a prescribed secant specific normal contact stiffness at a given nominal pressure. The maps identify feed–depth combinations that satisfy the target stiffness and can therefore be prioritized for experimental testing.

The design map in Figure 14 shows the secant specific normal contact stiffness $j_s(q_0) = q_0/\delta(q_0)$ at $q_0 = 0.5$ MPa. For illustration, a target value $j_{s,\text{target}} = 0.65$ MPa/ μm gives a feasible design with $h_0 = 35$ μm and $S = 0.235$ mm/rev at $F_d = 36.16$ %.

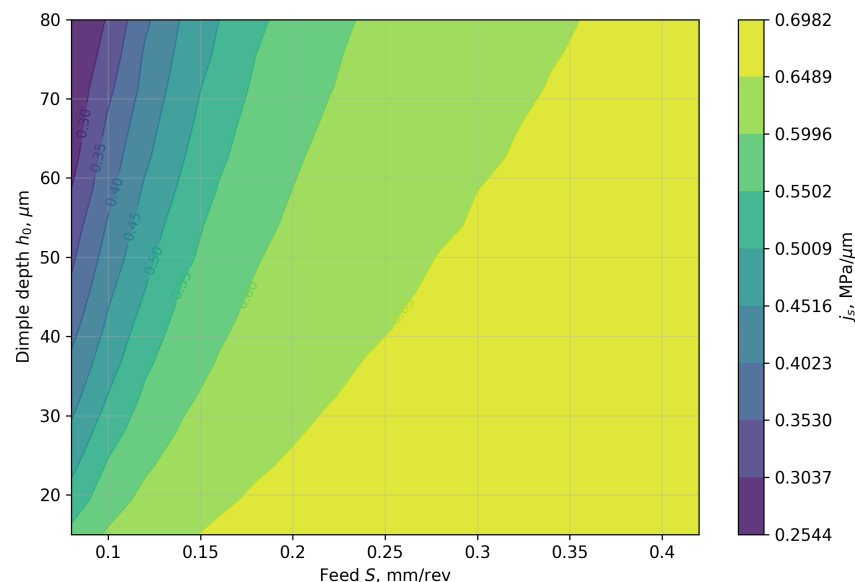


Figure 14. Inverse design map for selecting rotary-burnishing parameters to achieve a target secant specific normal contact stiffness $j_s(q_0)$ at $q_0 = 0.5$ MPa. The color scale shows $j_s = q_0/\delta(q_0)$ in MPa/ μm .

4. Discussion

4.1. Physical Interpretation of the Two-Level Mechanism

Regular dimples influence the stabilized normal stiffness through two coupled mechanisms. Their direct contribution is the elastic displacement generated by the periodic pressure modulation, while their dominant indirect contribution is the reduction of the load-bearing fraction and the resulting increase in the mean pressure acting on the load-bearing land. The periodic contact calculation confirms that the local pressure is nonuniform but that its heterogeneity changes the averaged Greenwood–Williamson closure by less than 1.7% for regime B; the averaged pressure therefore captures the cell-scale compliance even though it does not reproduce the edge field.

The small direct contribution follows from the elastic scale $\delta_{\text{reg}} \sim (q/E^*)\Lambda$. At $q = 2.5$ MPa, $E^* = 59.0$ GPa, and $\Lambda \approx 0.257$ mm, this estimate gives approximately 0.011 μm . The micro-roughness response of the load-bearing land is much more compliant because asperity summits enter contact sequentially. Consequently, the principal dimple effect is indirect: $\varphi_0 \approx 0.62$ raises the mean land pressure by $1/\varphi_0 \approx 1.61$ times and drives the nonlinear micro-roughness response.

4.2. Relation to Periodic Contact Theory

The Goryacheva–Tsukanov framework provides the most directly relevant analytical framework because it treats periodic microgeometry and interaction between neighboring contact regions [28,29,33]. The present formulation retains a spectral description of the

regular geometry but replaces the full unilateral problem by an averaged land pressure. The independent periodic boundary-element solution quantifies the consequence of that replacement: $\Delta\delta_{\text{BEM}}$ is 14–15% below δ_{reg} , while the edge disturbance decays over approximately 77 μm compared with bridge widths of approximately 0.5–0.6 mm. The difference is consistent with the uniform-pressure idealization in the analytical spectral term, whereas the localization supports the use of the mean land pressure for the integral response.

4.3. Comparison with Numerical and Homogenized Contact Models

Full finite-element, boundary-element, and homogenized multiscale models can resolve detailed pressure and material fields, but each texture layout requires a corresponding discretization or topographic representation. The present two-level model occupies an intermediate role: the periodic boundary-element calculation assesses the central homogenization and compliance assumptions for the primary cell, while the semi-analytical formulation remains inexpensive enough for parameter sweeps and inverse screening. This division preserves rapid parameter exploration while retaining a targeted full-field check of the governing contact assumptions.

4.4. Engineering Implications for Rotary Burnishing Design

The engineering pathway is therefore nominal process geometry $\rightarrow$ residual dimple geometry and state of the load-bearing land $\rightarrow$ relative bearing area and local pressure $\rightarrow$ stabilized normal stiffness. Rotary-burnishing kinematics supplies the nominal unit cell, whereas measured post-burnishing dimple dimensions, R_a , and R_{sm} should be used when available. This separation preserves a transparent process link without treating the forming operation itself as part of the service-contact calculation. The inverse maps can then narrow the process window before process-specific testing.

4.5. Limitations

The model is limited to stabilized normal mechanical loading of a post-burnishing dimpled surface and does not resolve first-cycle plastic forming or surface evolution during service. Depth-dependent hardening, residual-stress gradients, friction, lubrication, wear, and local edge stress maxima are not modeled explicitly. Very dense patterns with edge-dominated load transfer fall outside the range in which the area-averaging assumption is valid; in regime A, the edge band carries 84% of the load. Contact with dimple bottoms also lies outside the stated loading range. Process-specific experiments on rotary-burnished dimples, with measured texture geometry and joint stiffness, are planned as the next stage.

5. Conclusions

A two-level model was developed for the stabilized normal joint stiffness of surfaces with regular dimples. The model provides an analytical link from nominal process geometry and post-burnishing surface state to stabilized normal joint stiffness, numerically assessed and externally benchmarked against data for related surface types. The main conclusions are as follows:

1. The geometry module links rotary-burnishing parameters to the unit-cell geometry, elliptical dimple footprint, dimple overlap, open-area fraction, and relative bearing area. The union-area formulation prevents nonphysical open-area estimates for dense overlapping patterns.
2. The contact module separates the deterministic regular dimple pattern from the load-bearing land and couples them through averaged load redistribution.

3. The regular dimple pattern influences normal stiffness primarily by reducing the relative bearing area and thereby increases the local pressure on the rough contact spots. In the investigated range, the explicit spectral contribution δ_{reg} remains below 1% of the micro-roughness contribution; therefore, the dominant physical mechanism is load redistribution over the bearing area.
4. Sensitivity analysis shows that the stiffness prediction is governed mainly by the micro-roughness parameters and modeling assumptions, particularly R_a , R_{sm} , and n_σ . The dimple depth and footprint geometry have a weaker influence when the open-area fraction is fixed.
5. The periodic BEM assessment supports the area-averaged closure for regime B: pressure heterogeneity changes the area-averaged micro-roughness approach of the load-bearing land by less than 1.7%, while the analytical spectral term differs from the BEM value by 14–15%. The closest Shneyder–Ambaryan reference curve gives RMSE = 0.221 μm and MAE = 0.186 μm .
6. Within the stated validity domain, the formulation can support preliminary screening and inverse parameter selection for a prescribed stabilized normal joint stiffness.

Author Contributions: Conceptualization, K.A.B. and A.V.Z.; methodology, K.A.B. and A.V.Z.; software, K.A.B. and A.V.Z.; validation, K.A.B. and A.V.Z.; formal analysis, K.A.B. and A.V.Z.; investigation, K.A.B. and A.V.Z.; writing—original draft preparation, K.A.B. and A.V.Z.; writing—review and editing, K.A.B. and A.V.Z.; visualization, K.A.B. and A.V.Z.; supervision, K.A.B. and A.V.Z. All contributions were shared equally between the authors. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: The datasets and source code generated during the current study are available from the corresponding author upon reasonable request.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A. Fourier Coefficient Derivation

With the substitution $x = a_\delta r \cos \theta$, $y = b_\delta r \sin \theta$, $0 \leq r \leq 1$, the Jacobian is $a_\delta b_\delta r$. Defining

$$Q_{\mathbf{k}} = \sqrt{(k_s a_\delta)^2 + (k_z b_\delta)^2}, \quad (\text{A1})$$

the angular integration of Equation (12) gives $2\pi J_0(Q_{\mathbf{k}} r)$, and the radial integration gives

$$\hat{\chi}_d(\mathbf{k}; \delta) = 2\pi a_\delta b_\delta \int_0^1 J_0(Q_{\mathbf{k}} r) r \, dr = \pi a_\delta b_\delta \frac{2J_1(Q_{\mathbf{k}})}{Q_{\mathbf{k}}}. \quad (\text{A2})$$

Normalizing by the unit-cell area and using $\pi a_\delta b_\delta / A_c = F_{d0}(1 - \delta/h_0)$ yields Equation (13).

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