Experimental and Numerical Study of Cyclic Stress–Strain Response and Fatigue Crack Initiation Life of Mid-Carbon Steel under Constant and Multi-Step Amplitude Loading

Kasumi Morita 1,2,*, Masashi Mouri 2, Riccardo Fincato 1 and Seiichiro Tsutsumi 1

Abstract: This paper investigates the fatigue cyclic deformation behavior of mid-carbon steel. Uniaxial tensile loading tests and fatigue tests under constant and multi-step amplitude loading steps are performed to characterize the influence of loading history. The material is shown to exhibit different uniaxial ratcheting behavior depending on loading history. A smooth and gradual increase in cyclic softening is observed under smaller stress/strain conditions. Based on experimental characterization, numerical investigations are carried out to reproduce the cyclic stress–strain behavior under different variable amplitude load ranges. The nonlinear material behavior is reproduced by means of an elastoplasticity model called the Fatigue SS Model (hereafter, FSS model). The main feature of the FSS model is the ability to describe the cyclic softening behavior within a macroscopically elastic stress state. The good agreement between experimental and numerical results proves the reliability of the model to catch a realistic material response in fatigue problems. Furthermore, the present study introduces a method for the prediction of fatigue crack initiation life under variable loading conditions based on cumulative plastic work.

Keywords: fatigue crack initiation life; cyclic plasticity; variable amplitude load

1. Introduction

Steel bridges are subjected to cyclic loading conditions during their service life and, to fulfill safety criteria, they are required to perform within a local stress or strain elastic regime, below the pertinent fatigue limit [1]. However, it is known that during fatigue tests, performed under a stress regime lower than the macroscopic yield stress, the initial elastic cyclic response smoothly transforms into an elastoplastic response with a significant increase in the strain amplitude due to the generation of irreversible deformations. The phenomenon is known as cyclic softening behavior and is typical not only of carbon steels, but has been experimentally observed in various structural materials over the past decades [2–8].

Conventional plasticity models [9] are unsuitable for the description of fatigue problems characterized by stress states lower than the macroscopic yield stress, as they assume the pure elastic response of materials. On the other hand, several unconventional plastic models have been proposed [10–14]; however, those models were originally designed to simulate inelastic deformation under low-cycle fatigue problems, where the applied cyclic stress is relatively large and exceeds the initial yield stress.

In recent years, an increasing number of models have been developed to describe the damage process during high-cycle fatigue loading. Based on a fatigue continuum damage model framework [15,16], Barbu et al. [17] proposed a stepwise load-advancing strategy for the prediction of high cycle fatigue performance. Furthermore, Van Do [18] developed continuum a damage mechanics model for multiaxial high cycle fatigue to describe the welding residual stresses of welded joints. In addition, Zhu et al. [19] proposed...
a damage accumulation model based on Miner’s rule to consider the coupled damage due to HCF–LCF interaction by introducing new load parameters. One common aspect of these models is that the damage becomes a function of the number of cycles, adapting a phenomenological description of the continuum damage variable to the high-cycle fatigue theory. However, the cyclic softening process, depending on the loading history, is generally neglected with these constitutive models.

Cumulative fatigue damage analysis plays a key role in life prediction of components and structures subjected to field load histories [20]. The increasing interest in the cumulative fatigue damage concept led to the formulation of life prediction models. In order to consider the accumulation of plastic deformation with cyclic loading conditions lower than the macroscopic yield stress, cyclic stress–strain curves (i.e., CSSCs) have been taken into fatigue design [21,22] and several methods have been proposed to describe CSSC in previous works [23–26].

In order to describe the softening response observable in steels under cyclic loading conditions lower than the macroscopic yield stress, the authors have proposed an elastoplastic model [27,28], the Fatigue SS model (hereafter, the FSS model). The FSS model is based on an unconventional plasticity theory [10], further enriched by including the elastic boundary concept [29] and cyclic damage concept [27], for the description of strain-softening behavior within a macroscopically elastic stress state [27,28]. However, previous study of the FSS model focused only on constant amplitude loading, whereas the applicability on more general loading conditions, such as multi-step loading, remains unclear.

In order to contribute to the reasonable and safe design of bridges, the first goal of this work is to prove the reliability of the FSS model to provide a realistic material description under variable fatigue loading conditions. Both experimental and numerical campaigns were carried out to clarify the evolution of cyclic stress–strain responses of mid-carbon steel under relatively low-stress regimes. Firstly, the static mechanical properties of the material were experimentally characterized by performing monotonic tensile tests at room conditions. Subsequently, fatigue experiments were conducted to investigate the uniaxial ratcheting behavior under constant loading, two-step amplitude loading, and repeated increasing/decreasing amplitude loading conditions. Based on the experimental campaign, the numerical analyses were carried out by means of the FSS model. The FSS model material constants were calibrated by minimizing the differences between the experimental and numerical cyclic stress–strain curves to give a realistic description of the ratcheting and softening behaviors. The second aim of this paper is to propose a fatigue crack initiation life prediction method that can consider the loading history based on the FSS model. In this study, two approaches, using the total strain range and the cumulative plastic strain damage, are compared and the criterion of cumulative plastic strain damage for fatigue crack initiation life prediction is proposed based on the fatigue tests result.

2. Experiment

2.1. Uniaxial Tensile Tests

A series of uniaxial tensile tests was performed to characterize the mechanical and stress–strain behavior of mid-carbon steel. The specimens were machined from a 16 mm thick mid-carbon steel plate named SM490A, as specified by the JIS G 3106 2008 [30]. The loading axis was assumed parallel to the rolling direction of the plate. Figure 1 displays the microstructure of a specimen.

As can be seen, the microstructure of the material clearly shows microstructural bands parallel to the rolling direction. The present work does not investigate material anisotropy; future works will consider this aspect by analyzing the different material response depending on the loading direction and by adopting an anisotropic yield criterion for the numerical analyses (e.g., Hill48 [31], Yld-2004-18p [32], CB2001 [33], among others). The details of the chemical composition and the geometric dimensions of the specimen are reported in Figure 2 and Table 1, respectively.
Following the standard method proposed by JIS 2241:2011 2009 [34], uniaxial tensile tests were carried out by a servo-hydraulic testing machine. All tensile tests were conducted at a displacement rate of 0.06 mm/min before specimen yield, and then at a crosshead rate of 0.9 mm/min after the yield behavior was recorded. Four specimens were tested at room temperature and the mean values of the static mechanical properties were computed. Figure 4 presents one of the uniaxial tensile stress–strain curves, whereas the averaged static mechanical constants are shown in Table 2.
Table 2. Mechanical properties of averaged material obtained by monotonic tensile tests.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Upper yield stress/MPa</td>
<td>402</td>
</tr>
<tr>
<td>Lower yield stress/MPa</td>
<td>336</td>
</tr>
<tr>
<td>Young’s modulus/GPa</td>
<td>208</td>
</tr>
</tbody>
</table>

2.2. Fatigue Tests

Uniaxial fatigue tests under various loading conditions were performed to characterize the fatigue behavior of the mid-carbon steel. The geometry and heat treatment conditions of the specimens tested in this section were the same as those used in uniaxial tensile tests, as shown in Figure 2. Before the fatigue test, the gauge section was polished along the loading axis with emery papers from a grit size of #220 to #1500. All the tests were conducted with fully reversed loading conditions (i.e., stress ratio \( R = -1 \)) at room temperature and the fatigue failure was defined when the displacement of the load cell exceeded 2 mm. Table 3 shows the test conditions applied in experiments using a servo-hydraulic testing machine.

Table 3. Fatigue test conditions.

<table>
<thead>
<tr>
<th>Name</th>
<th>Control</th>
<th>Stress Ratio ((R = \sigma_{\text{min}}/\sigma_{\text{max}}))</th>
<th>Cyclic Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant amplitude loading test</td>
<td>Strain</td>
<td>(-1)</td>
<td>(\varepsilon_a = 0.0014/0.005/0.008)</td>
</tr>
<tr>
<td></td>
<td>Load</td>
<td>(-1)</td>
<td>(\sigma_a = 210/230/250/270/290) MPa</td>
</tr>
<tr>
<td>Two-step amplitude loading test</td>
<td>Load</td>
<td>(-1)</td>
<td>(\sigma_{a1} = 240) (2000 cycles)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(\rightarrow \sigma_{a2} = 280) MPa (2000 cycles)</td>
</tr>
<tr>
<td>Repeated increasing and decreasing amplitude loading test</td>
<td>Load</td>
<td>(-1)</td>
<td>(\sigma_a = 160 \rightarrow 200 \rightarrow 240 \rightarrow 280 \rightarrow 240 \rightarrow 200 \rightarrow 160 \rightarrow 200) MPa () (Every 100 cycles, changing the amplitude)</td>
</tr>
</tbody>
</table>

To monitor the diameter of the gauge section and control the strain range during tests, a displacement meter and a clip gauge were attached to the specimen, as shown in Figure 3. All specimens were assumed as run-outs at 2,000,000 cycles.

Uniaxial cyclic loading tests were carried out with constant strain amplitude ranges from \(\varepsilon_a = 0.0014\) to \(0.008\) to investigate the strain range dependence. The stress–strain hysteresis loops for each amplitude are presented in Figure 5a. Comparing the stabilized loop sizes under different strain amplitudes, it is possible to notice that the stable loops do not overlap due to the increase in the elastic domain induced by material hardening. The evolution of the stress response versus the number of cycles is shown in Figure 5b. Cyclic
overlap due to the increase in the elastic domain induced by material hardening. The evolution of the stress response versus the number of cycles shows different tendencies under different strain range loadings. Under a low strain range regime, it shows a smooth and gradual increase with the number of cycles, whereas the increase is faster for larger strain range loadings.

Two-step amplitude loading tests were conducted by changing the stress amplitude, $\sigma_{\alpha 1} = 240 \text{ MPa}$, and $\sigma_{\alpha 2} = 280 \text{ MPa}$ for 2000 cycles, to investigate the material response under variable loading conditions. Figure 6a presents the stress–strain hysteresis loops, whereas the evolutions of the stress response versus the number of cycles are displayed in Figure 6b. As can be seen in the two graphs, the plastic deformation is almost negligible during the first step ($\sigma_{\alpha 1} = 240 \text{ MPa}$). However, as soon as the second step ($\sigma_{\alpha 2} = 280 \text{ MPa}$) starts, cyclic softening is detected with the subsequent saturation of the stress range.

A repeated increasing and decreasing amplitude loading test was also performed to study the influence of the loading history. In this test, the stress amplitude $\sigma_{\alpha}$ was increased...
by 40 MPa every 100 cycles, starting from a lower value of 160 MPa up to an upper limit of 280 MPa. After reaching the upper value of 280 MPa, the stress was decreased in the same manner. The aforementioned loading history was performed until fatigue failure was detected. Figure 7 shows the evolution of the strain response versus the number of cycles. Almost elastic behavior is detected within the first 3000 cycles, and subsequently, an increase in the strain ranges is recorded between 3000 and 6000 cycles. After 6000 cycles, the saturation of the strain ranges is observed.

Figure 7. Experimental relationships between $\Delta \varepsilon$ and the number of cycles under increasing and decreasing amplitude loading test.

3. Numerical Simulation of the Cyclic Stress–Strain Response

3.1. The FSS Model

The FSS model was used to investigate the inelastic response of the material under cyclic loading conditions. The FSS model [27,28] describes the generation of plastic strain within the yield surface, which can be obtained through a similarity transformation from the conventional yield surface [10,35,36]. Classical theories distinguish the elastic and plastic regions, allowing an irreversible stretch only in the plastic region whenever the stress increment satisfies the loading criterion. In contrast, the FSS model abolishes the separation into domains, stating that a plastic response can be realized for every change in the stress state that satisfies the loading criterion. Furthermore, the use of a mobile similarity center, which is a function of the plastic strain, makes this theory particularly suitable for studying cyclic mobility problems. A detailed explanation of the model is beyond the scope of this paper; the reader can refer to references [27–29,37–46] for a more complete discussion.

3.2. Calibration of the FSS Model

The material parameters for the FSS model were calibrated against the experimental cyclic stress–strain curves obtained in the fatigue tests. Numerical analyses were performed by implementing the constitutive equations of the FSS model via a single-quadrature point in in-house numerical code. The simulations were carried out under the same loading conditions in experiments (see in Table 3) and by adopting the elastoplastic parameters of the FSS model listed in Table 4.

Table 4. Set of material constants obtained from the calibration of the numerical results against the experimental data.

<table>
<thead>
<tr>
<th>Poisson’s Ratio</th>
<th>Isotropic Hardening Parameter</th>
<th>Kinematic Hardening Parameter</th>
<th>High-Cycle Fatigue Parameter for the Computation of Damage</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>$h_1$</td>
<td>$h_2$</td>
<td>$a_1$</td>
</tr>
<tr>
<td>0.3</td>
<td>0.1</td>
<td>4.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>
The description of material parameters can be found in references [28,29]. Figures 8a and 9a display the stress–strain hysteresis loops with constant strain amplitudes and two-step amplitude loading. As can be seen from Figures 8a and 9a, the size of the loops is realistically described by the model, even though the experimental hollow markers show a slight shift, approximately 7% difference, toward the compressive side in Figure 9a. The possible reason for this behavior is initial imperfections of the specimen and it was neglected in the calibration. The evolution of the stress response versus the number of cycles is shown in Figures 8b and 9b. Hollow markers report the experimental results discussed in Section 2.2. The solid continuous lines represent the numerical solutions. In Figures 8b and 9b, the variations in strain and stress against the number of cycles show good agreement with the experiments, including the plastic deformations.

Figure 8. (a) Numerical stress and strain curves under constant amplitude loading; (b) numerical relationships between $\sigma_{\text{max}} - \sigma_{\text{min}}$ and the number of cycles under constant amplitude loading.

Figure 9. (a) Numerical stress and strain curves under two-step amplitude loading; (b) numerical relationships between $\varepsilon_{\text{max}} - \varepsilon_{\text{min}}$ and the number of cycles under two-step amplitude loading.

Figure 10 compares the numerical and experimental results under repeated increasing and decreasing amplitude loading. The strain amplitude behavior and the evolution of plastic deformation are similar to the experimental behavior. Overall, the trends in the numerical simulations are in good agreement with the experimental data under several loading conditions.
when it comes to variable and multi-axial loading problems. well under constant amplitude loading conditions, as reported in previous works [49–51].

Numerical relationships between \( \Delta \varepsilon \) and the number of cycles under the repeated increasing and decreasing amplitude loading condition.

4. Prediction of Crack Initiation Life

The prediction of the fatigue crack initiation life of the material was carried out adopting the two methods discussed in this section, namely strain range-based and cumulative plastic damage-based criteria. The first approach was investigated utilizing Iida’s equation (see in Equation (1), [47]), which formulates a total strain range-based crack initiation life prediction. Figure 11 graphically reports the results of the fatigue tests, which are converted from fatigue failure lives into fatigue crack initiation lives [48], and the crack initiation life is predicted by Equation (1).

\[
\frac{\Delta \varepsilon_l}{2} = 0.415N_c^{-0.606} + 0.00412N_c^{-0.115}
\]  

(1)

Rhombooidal hollow markers indicate strain-controlled tests, whereas circular hollow markers indicate load-controlled tests. As can be seen from Figure 11, Iida’s prediction shows a good agreement with the experimental results obtained in this study. In general, it is well-known that strain range-based methods predict the fatigue (crack initiation) life well under constant amplitude loading conditions, as reported in previous works [49–51]. On the other hand, it is complicated to consider the dependency on the loading history when it comes to variable and multi-axial loading problems.

The second approach was investigated using the FSS model. The effect of the accumulation of plastic strain on damage evolution can be measured by the following functions.

\[
D(H_d) = (1 - d_2) \left[ 1 + \left( \frac{d_1}{H_d} \right)^{d_3} \right]^{-1} ; \quad H_d = \int \sqrt{\frac{7}{3}} |D^p|Ddt
\]  

(2)
\[
D(R) = (1 - k_2) \left[ 1 + \left( \frac{k_1}{R} \right)^{k_3} \right]^{-1}
\] 

(3)

The damage \( D \) in Equation (2) is representative of the damage accumulation in the material model [28,29] contributing to the opening of the hysteresis loops. However, it does not affect the elastic response of the material. In detail, \( d_1, d_2, \) and \( d_3 \) are material constants to be calibrated against experimental tests. The scalar plastic internal variable \( H_d \) represents the amount of accumulated damage in the form of accumulated plastic work, with \( |D'| \) being the norm of the plastic strain rate tensor. The variable \( D \) helps to regulate the amount of damage induced for stress states below the macroscopic yield stress. The variable \( R \) in Equation (3) is introduced as a measure of the magnitude of the stress state within the conventional elastic domain. It assumes a null value for a null stress state and it is at a maximum when the stress reaches the macroscopic yield stress. As can be seen from Equation (3), for stress states close to the plastic potential, \( D \) tends to its maximum value \( D = (1 - k_2) \), whereas \( D \) tends to be zero for lower stress regimes. An in-depth discussion of the role of the material constants \( d_1, d_2, \) and \( d_3 \) in Equation (2) and \( k_1, k_2, \) and \( k_3 \) in Equation (3) can be found in [29]. In this study, the cumulative plastic work, \( H_d \), is used to investigate the crack initiation life. The relation between cumulative plastic work (i.e., \( H_d \)) versus the number of cycles under constant amplitude loading is illustrated in Figure 12. \( H_d \) increases with the number of cycles, and larger loading conditions return higher \( H_d \) rates.

![Figure 12](image1.png)

Figure 12. Relationship between cumulative damage plastic strain, \( H_d \), and number of cycles under constant amplitude loading.

Figure 13 displays the relationships of strain amplitudes against the number of cycles, assuming the number of cycles when the variable \( H_d \) reaches a value of 2.0 as the criterion for the crack initiation life.

![Figure 13](image2.png)

Figure 13. Relationship between cumulative damage plastic strain, \( H_d \), and number of cycles under constant amplitude loading.
As indicated in the graph, the proposed crack initiation life criterion based on the cumulative plastic work agrees well with the experimental data. Finally, Figure 14 compares the experimental and numerical predicted number of cycles to failure under same loading conditions.

![Image](image_url)

**Figure 14.** The experimental versus the numerical predicted number of cycles to crack initiation under the same loading conditions.

The difference between the number of cycles to crack formation obtained in the experiments and the one predicted by the two aforementioned criteria is equivalent adopting the proposed criterion (i.e., based on the $H_d$ variable) compared against the strain range-based method. The applicability of the FSS model and cumulative plastic damage method to variable and multiaxial loading conditions will be investigated in future work.

5. Conclusions

The paper investigated the applicability of the FSS model to mid-carbon steel (SM490A) under variable uniaxial loading conditions and proposed a novel criterion for the evaluation of the fatigue crack life. The main aspects of the present work can be summarized as follows:

1. Uniaxial tensile tests were carried out to obtain the mechanical properties of SM490A.
2. Fatigue tests with various loading conditions were carried out to investigate the cyclic response of SM490A. Cyclic softening/hardening and cyclic shakedown were observed under different loading conditions.
3. The calibration of the material constants using the FSS model was performed under constant and two-step amplitude loading and repeated increasing and decreasing conditions. The numerical results in terms of stress and strain responses showed good agreement with the experimental results.
4. Fatigue crack initiation life prediction using Iida’s equation, based on the total strain range, agreed quite well with the experiment under constant amplitude loading. The criterion based on the cumulative plastic damage strain calculated by the FSS model predicted a realistic fatigue crack initiation life as well as Iida’s equation.

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References


