Prefiltered Striation-Based Beamforming for Range Estimation of Multiple Sources

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Abstract: The element–frequency acoustic intensity of a horizontal line array with a sufficient aperture exhibits interference striation patterns, which can be used for source range estimation without prior environmental information. Under multisource scenarios, the interference striations of the sources overlap with each other, leading to great difficulty in utilizing the information of striations. In this paper, the wavenumber filtering method is applied to each sensor of the horizontal line array to extract the surface-reflected–bottom-reflected modes and reconstruct the recognizable interference spectrogram for each source. Then, via beamforming along the striations, the source ranges can be estimated individually with little prior environmental information and without the long-time observation of moving sources. The required sensor spacing is analyzed, and the spatial filtering capabilities for a single source from different bearings and two sources for which azimuth angles are close to each other have also been investigated. The simulation results indicate that the proposed algorithm can estimate ranges of multiple sources within 25 km, with relative errors of less than 4%.

Keywords: waveguide invariant; passive ranging; wavenumber filtering; horizontal line array; multiple sources

1. Introduction

In shallow water, the spectrograms of acoustic intensity often exhibit interference striation patterns in the time-frequency or range-frequency plot [1–4]. Chuprov introduced the concept of the waveguide invariant $\beta$ to describe this interference phenomenon [5,6], relating the striation slopes to range and frequency. The waveguide invariant has been widely used in underwater acoustics, such as with a time reversal mirror [7], array signal processing [8], geoacoustic inversion [9], and active sonar detection [10,11].

Passive source ranging is an important application for the waveguide invariant. By analyzing the interference striations in the LOFARgram, the source range can be estimated if the waveguide invariant is provided and vice versa. Although the value of $\beta$ equals 1 for an ideal waveguide with perfect reflection boundaries, obtaining an accurate estimation of $\beta$ can effectively improve the ranging accuracy in an arbitrary waveguide in practice. Common methods for calculating the waveguide invariant require very accurate knowledge of the environment [12] or a guide source at a known range [13]. Moreover, the traditional ranging methods based on the waveguide invariant require the long-time continuous observation of the target and assume that the target moves uniformly in a line. All these requirements limit the application of the waveguide-invariant-based ranging methods.

Using a horizontal line array (HLA) with a sufficient aperture, the acoustic intensity as a function of the element and frequency also displays the interference pattern, which is referred to as the element–frequency spectrogram. These spectrograms can be obtained using a snapshot of the HLA data, without requirements of long-time observation or target motion [14,15]. Zurk et al. [16] proposed the striation-based beamforming (SBF) method, which conducts beamforming along the interference striations to determine $\beta$. 


without any prior environmental information. But it is only applicable to targets within 30° of the array broadside direction. To eliminate the bearing limitation, Liu et al. [17] developed two modified striation-based beamformers: waveguide-invariant dimension striation-based beamforming (WISBF) and range-dimension striation-based beamforming (RSBF). The WISBF method can estimate \( \beta \), while the RSBF method can estimate the source range directly.

However, when multiple sources at different bearings exist in the waveguide, the interference patterns of each source will overlap. Then, identifying and beamforming along the interference striations of one source is rather difficult. In this case, the RSBF method could not be applied to estimate the source ranges. To overcome the problem of identifying the interference striations, one can beamform the HLA data using the bearing of a target of interest (TOI) [18–20]. Yang [18] showed that the output of a beamformer has the same striation patterns as that presented by a single hydrophone while rejecting noise from directions other than that of the TOI. Turgut et al. [19] localized an acoustic source with experimental data successfully using the methods proposed by Yang. But the HLA dataset are synthesized into one data after beamforming, which means that sufficient long-time observation is required to obtain the integral spectrogram. Moreover, the RSBF method cannot be used anymore because a spectrogram in the element–frequency domain is not available.

In shallow water with a thermocline, the waveguide invariant \( \beta \) becomes diffuse when the sound field for the source and the receiver below the thermocline is dominated by the non-surface-reflected–bottom-reflected (non-SRBR) modes, resulting in some difficulties in describing the striation slopes [21,22]. To solve this problem, many methods have been proposed to extract the surface-reflected–bottom-reflected (SRBR) modes, in which \( \beta \approx 1 \) [20,23,24]. Song et al. [25] proposed the wavenumber–frequency domain filtering technique to separate the SRBR and non-SRBR modes of a source at the end fire of the HLA. In the present work, we extend the filtering technique to separate the SRBR modes of different sources under the scenarios of multiple sources at different bearings. The source bearings are obtained by conventional beamforming (CBF) first. The wavenumber filter is applied at each sensor of the HLA to reconstruct clear interference patterns for individual sources, and then the RSBF method conducts beamforming along the interference striations for source ranging. The proposed method could achieve passive source ranging with little environmental information. In addition, the long-time observation of the target is unnecessary since the spatial sampling of the acoustic field using a long HLA provides sufficient information for source ranging.

The rest of the paper is organized as follows: In Section 2, the basic theory of the prefILTERed RSBF method is developed. Section 3 presents the simulation results to verify the proposed method. The required sensor spacing and the bearing relationship for the source reference to the array and between the multiple sources are analyzed in Section 4. Section 5 concludes the study.

2. The Prefiltered Striation-Based Beamforming

2.1. Range Estimation Using Striation-Based Beamforming Method

In shallow water waveguides, the acoustic field can be expressed as the sum of a finite number of modes. The acoustic pressure received by the hydrophone at the range \( r \) emitting from the source of frequency \( \omega \) can be expressed as

\[
p(r, \omega) = \sum_{m} (k_{rm}r)^{-1/2} \Psi_{m}(z_0) \Psi_{m}(z_a) \exp(jk_{rm}r) = \sum_{m} A_m \exp(jk_{rm}r) \tag{1}
\]

where \( \Psi_{m} \) and \( k_{rm} \) denote the eigenfunction and the horizontal wavenumber of the \( m \)th mode; \( z_0 \) and \( z_a \) are the depths of the source and the receiver, respectively; \( A_m \) is the amplitude of the \( m \)th mode, \( A_m = (k_{rm}r)^{-1/2} \Psi_{m}(z_0) \Psi_{m}(z_a) \).
The acoustic intensity is given by

\[ I(r, \omega) = E[pp^*] = \sum_m A_m^2 + 2 \sum_{m,n} A_m A_n \cos(\Delta k_{mn}(\omega)r) \]  

(2)

where \( \Delta k_{mn} = k_{rm} - k_{rn} \) is the difference between the horizontal wavenumbers of the \( m \)th and \( n \)th mode. In Equation (2), the first item changes slowly with range and frequency, while the second item generates the constructive and destructive interference exhibited in the range-frequency plot of acoustic intensity.

Chuprov [5,6] defined the waveguide invariant \( \beta \) as

\[ \beta = \frac{d\omega}{dr} \]  

(3)

where \( \frac{d\omega}{dr} \) is the slope of the interference striation.

As shown in Figure 1, assuming that an HLA with a length of \( L \) is placed along the \( y \)-axis, \(-L/2 < y < L/2\), the location of the source can be expressed as \( r_0 = [r_0 \cos \phi_0, r_0 \sin \phi_0]^T \), where \( r_0 \) and \( \phi_0 \) represent the range and the bearing from the source to the center of the array, respectively. For horizontal arrays located in shallow water, the elevation angles (or grazing angles) of the arrivals are very small, and the source bearings correspond approximately to their azimuth angles in the horizontal direction [26,27]. Using the far field approximation, the horizontal range from any one element to the source is given by [18]

\[ r = [(r_0 \cos \phi_0)^2 + (y - r_0 \sin \phi_0)^2]^{1/2} \approx r_0 - y \sin \phi_0 + y^2 / (2r_0) \]  

(4)

![Figure 1. The schematic diagram for an HLA. The reference element is at \( y = 0 \).](https://example.com/figure1.png)

When the array aperture is large enough, a certain interference structure can be observed along the direction of the array, which is referred to as the interference structure of the acoustic intensity in the array element–frequency domain [17]. Since \( r \) depends on the array element position \( y \) and the source bearing \( \phi_0 \), the acoustic pressure can be expressed as \( p(r, \omega) = p(y, \omega; \phi_0) \). In the element–frequency interference spectrogram, the relationship between \( \omega \) and \( y \) along the interference striation can be expressed as [17]

\[ \omega = \omega_0 - Sy, \text{ where } S = \frac{d\omega}{dy} = -\frac{d\omega}{dr} \frac{dr}{dy} = \omega_0 (\beta / r_0) \sin \phi_0 \]  

(5)

where \( \omega_0 \) is the center frequency corresponding to the striation, and \( S \) is the striation slope in the element–frequency domain.
Rewrite Equation (5) as

\[ r_0 = \frac{\omega y \beta \sin \phi_0}{S} \]  

(6)

Equation (6) provides the estimation of the source range once the three parameters \( S, \beta \) and \( \phi_0 \) are achieved. The effect of assuming an incorrect value of \( \beta \) can be seen in Equation (6). If the true value of \( \beta \) is \( \beta_{\text{true}} \), and the assumed value is \( \beta_{\text{assumed}} \), the range estimates will be incorrect by a factor of \( \beta_{\text{assumed}} / \beta_{\text{true}} \). The SBF method proposed by Zurk et al. [16] can estimate \( \beta \) without prior environmental parameters or a guide source. The beamforming is conducted along different frequencies of the striations. And the peak for this beamformer will shift away from the true bearing of the source. Combined with the peak of CBF, \( \beta \) can be measured independently of the source range. To eliminate the bearing limitation of the SBF method, Liu et al. [17] established the phase and amplitude relationships among elements and proposed the WISBF and RSBF methods. The basic theory of the RSBF method is reviewed here.

Equation (5) shows the constructive and destructive striations in the element–frequency domain. Along the striations, the phase of the \( c_0 \) is used for calculating \( S \). The source bearing is usually determined by CBF. The commonly used methods for estimating \( S \) include Radon transform [13,28,29] and two-dimensional discrete Fourier transform (2D-DFT) [22,30]. Among them, the 2D-DFT method has better noise resistance and can extract the slope more accurately. Therefore, the 2D-DFT method is used for calculating \( S \) in this paper.

When multiple sources at different bearings exist in the waveguide, the bearing of each source can be determined separately using CBF. However, the interference spectrograms will become blurry because all the sources contribute to the acoustic field in a non-linear manner. It is hard to calculate \( S \) directly by Radon transform or 2D-DFT. Therefore, in the next section, wavenumber-filtered range-dimension striation-based beamforming (WF-
RSBF) is proposed. Under the multisource scenarios, this method can calculate $S$ and then estimate the range for each source.

2.2. Wavenumber-Filtered Range-Dimension Striation-Based Beamforming

In shallow water waveguides, all the normal modes can be divided into SRBR modes (or reflective modes) and non-SRBR modes (mainly refractive modes). SRBR modes usually occupy significant components of the acoustic field and only come second to non-SRBR modes when there is a thermocline in the waveguide. The value of $\beta$ for SRBR modes is always around 1 regardless of whether a thermocline exists. Thus, it is theoretically effective to extract SRBR modes and then estimate the source range.

When the HLA used has a sufficient aperture, FFT can be performed on finite-length data to construct an interference spectrogram in the element–frequency domain, without requiring the source to move uniformly for a long time. Performing Fourier transform on the acoustic pressure $p(y, \omega; \phi_0)$ along the array yields

$$P(k_r, \omega) = \int p(y, \omega; \phi_0) e^{jk_r y} dy$$

Substituting Equations (1) and (7) into Equation (10) yields

$$P(k_r, \omega) = \int_{\frac{L}{2}}^{\frac{L}{2}} p(y, \omega; \phi_0) e^{jk_r y} dy$$
$$= \int_{\frac{L}{2}}^{\frac{L}{2}} \sum m A_m e^{jk_m c_\text{bottom} L (y)} e^{i(\omega - k_m c_\text{bottom}) y} dy$$
$$= \sum m A_m e^{jk_m c_\text{bottom} L} [(e^{\omega y} - e^{i(\omega - k_m c_\text{bottom}) y}) e^{i(\omega - k_m c_\text{bottom}) L / 2}]$$

In SRBR modes, $\omega / c_\text{bottom} < k_m < \omega / c_\text{max}$, where $c_\text{max}$ is the maximum sound speed in the water (not including the seafloor), and $c_\text{bottom}$ is the sound speed in the bottom halfspace. Correspondingly, for a single source at bearing $\phi_0$, $k_r$ needs to satisfy $|\sin \phi_0| \frac{\omega}{c_\text{bottom}} \leq |k_r| \leq |\sin \phi_0| \frac{\omega}{c_\text{max}}$. The effective modes for sources at different bearings can be separated by setting different windows in the wavenumber domain. The rectangular window filter is given by

$$W(k_r, \phi_0) = \begin{cases} 1, & |\sin \phi_0| \frac{\omega}{c_\text{bottom}} \leq |k_r| \leq |\sin \phi_0| \frac{\omega}{c_\text{max}} \\ 0, & \text{otherwise} \end{cases}$$

Then, the filtered horizontal wavenumber spectral function $P_{\text{win}}(k_r, \omega)$ is expressed as

$$P_{\text{win}}(k_r, \omega) = P(k_r, \omega) \cdot W(k_r, \phi_0) = \sum_{m_1}^{m_2} \sum_{m_1}^{m_2} A_m e^{jk_m c_\text{bottom} L} \sin c[(k_r - k_m c_\text{bottom} \sin \phi_0) L / 2]$$

where $m_1$ and $m_2$ are the mode index for the corresponding modes with wavenumbers $k_m = \frac{\omega}{c_\text{bottom}}$ and $k_m = \frac{\omega}{c_\text{max}}$, respectively. In this way, the SRBR modes of the source at the bearing $\phi_0$ are obtained. Applying the inverse Fourier transform on $P_{\text{win}}(k_r, \omega)$, the filtered acoustic signals can be expressed as

$$p_{\text{filter}}(y, \omega; \phi_0) = \frac{1}{2\pi} \int P_{\text{win}}(k_r, \omega) e^{-jk_r y} dk_r$$
$$= \frac{1}{2\pi} \int \sum_{m_1}^{m_2} A_m e^{jk_m c_\text{bottom} L} \sin c[(k_r - k_m c_\text{bottom} \sin \phi_0) L / 2] e^{-jk_r y} dk_r$$
$$= \sum_{m_1}^{m_2} A_m \exp \{jk_m (\omega) r(y) \}$$

Compared to the original acoustic pressure, the filtered acoustic field only contains the SRBR modes from the source at a specific bearing. Equation (14) shows that the filtered acoustic pressure is still a complex signal, and the phase term for each mode remains...
unchanged. We call this filtering process wavenumber filtering. Replacing \( p(y, \omega; \phi_0) \) in Equation (9) with \( p_{\text{filter}}(y, \omega; \phi_0) \) yields the beamforming output as

\[
B_{r-\text{filter}}(\omega_0, \hat{r}) \approx \left| \sum_{m_1} A_m \exp \left( \frac{i \omega_0 r_0}{c_0(\omega)} \right) \sin c \left[ \frac{SL(\hat{r} - r_0)}{2c_0} \right] \right|^2
\]

(15)

\( B_{r-\text{filter}} \) reaches the maximum when \( \hat{r} = r_0 \), and then the range of the TOI can be estimated. The merits of the WF-RSBF method are manifested in two aspects. On the one hand, the final approval for range estimation is RSBF, thus there is no need for many prior environmental parameters or a guide source. On the other hand, the methods proposed by Yang et al. [18] conduct beamforming towards the TOI to obtain a beamformed acoustic signal, which requires long-term observation to obtain the integral spectrogram, whereas the WF-RSBF method can obtain the spectrogram for each source using the limited duration signal.

Figure 2 shows the algorithm flowchart of the WF-RSBF method. The design of the filter requires the estimation of the source bearing through CBF first. After that, the pressure data are Fourier-transformed along the HLA into the wavenumber domain. Then, a rectangular window is constructed to extract the SRBR modes of the TOI. The filtered HLA data are obtained via inverse Fourier transforming with the SRBR modes of the TOI, and the striation slope is calculated using the 2D-DFT based on the constructed interference pattern. Lastly, the source range can be estimated using the RSBF method based on the filtered acoustic field.

![Algorithm flowchart for the WF-RSBF method.](image)

**Figure 2.** Algorithm flowchart for the WF-RSBF method.

### 3. Simulation Results

Simulations are conducted to verify the WF-RSBF method. The sound speed in the bottom is 1800 m/s, the bottom density is 1.8 g/cm³, and the absorption is 0.5 dB/λ (λ is the wavelength). The water depth is 70 m. There is a 20 m thermocline in the waveguide, from 10 m to 30 m, with the sound speed varying from 1510 m/s to 1490 m/s. The 128-element HLA with an interval of 1 m is placed at a depth of 60 m. Two sources coexist in the waveguide, the source bearings measured from the broadside of the HLA are \( \phi_1 = -45^\circ \) and \( \phi_2 = 60^\circ \). And the two sources are at the range of 10 km and 5 km, respectively, from the mid-point of the HLA, with a depth of 60 m, as shown by points “A” and “B” in Figure 3a. KrakenC code is used for simulation [31]. The processing frequency is 525–575 Hz, and the signal-to-noise ratio (SNR) is 0 dB. The SNR is defined as

\[
SNR = \frac{S_{\text{average}}}{N_{\text{average}}}
\]

(16)

where \( S_{\text{average}} \) is the average signal power of all elements, and \( N_{\text{average}} \) is the average noise power of all elements. As shown in Figure 4a, because the two sources contribute to the striation pattern in a non-linear manner, the interference spectrogram becomes so fuzzy that it is hard to distinguish the striations in the plot.
The wavenumber distributions in the spectrum, which means that it is possible to separate the interference features of the two sources so that the ranging results of the two sources are accurate.

In this simulation condition, the bearing of source 1 is a negative value, so the window function of source 1 satisfies \( \sin \hat{\phi}_1 - \frac{\omega}{c_{\text{max}}} \leq k_{r1} \leq \sin \hat{\phi}_1 - \frac{\omega}{c_{\text{bottom}}} \), and that of source 2 satisfies \( \sin \hat{\phi}_2 - \frac{\omega}{c_{\text{bottom}}} \leq k_{r2} \leq \sin \hat{\phi}_2 - \frac{\omega}{c_{\text{max}}} \). The azimuths of each of the two sources can also be both negative or both positive. In fact, the azimuth angles of two sources could be at any region but cannot be close to each other. The relationship that the azimuths should satisfy will be discussed as the anti-aliasing criteria in the next section.

![Figure 3](image1.png)

**Figure 3.** (a) The position and (b) range of the two sources. A and B denote the start position of source 1 and source 2, respectively.

![Figure 4](image2.png)

**Figure 4.** (a) The interference spectrogram of multiple sources. (b) The corresponding wavenumber spectrum at the frequency of 575 Hz, where the two black rectangles denote the window functions for the two sources.

Firstly, using CBF, the estimated bearings of the two sources are \( \hat{\phi}_1 = -45.1^\circ \) and \( \hat{\phi}_2 = 59.5^\circ \). Figure 4b shows the wavenumber spectrum of the acoustic signal at the frequency of 575 Hz. It can be found that sources with different bearings have diverse wavenumber distributions in the spectrum, which means that it is possible to separate SRBR modes of multiple sources using different window filters. The black lines in Figure 4b delimit the window function in the wavenumber domain. It is worth noting that the noise is uniformly distributed throughout the whole frequency band, and the noise in the stop band of the wavenumber axis could be filtered out in the filtering process, which is beneficial for improving the ranging accuracy.
Figure 5 shows the filtered interference spectrograms of the two sources. Compared to Figure 4a, the interference features of the two sources are well separated so that the striation slopes for the two sources are 0.0297 and −0.0769 Hz/m, calculated by conducting the 2D-DFT method. It is important to note that the phase relationship among elements remains unchanged after filtering. Then, the ranging results of the two sources are estimated as 9.93 km and 5 km.

![Figure 5](image)

**Figure 5.** The interference spectrograms after filtering and the extracted striation slopes shown in dashed lines for each source: (a) source 1 with the striation slope of 0.0629 Hz/m; (b) source 2 with the striation slope of −0.0857 Hz/m.

Furthermore, assuming the ranges for the two sources are 10–20 km and 5–25 km, the sources remain at the bearing of −45° and 60°, as shown in Figure 3. The results of CBF are shown in Figure 6; the azimuth angles of the two sources are estimated accurately. Figures 7 and 8 show the ranging results and the relative ranging errors of the two sources. The relative error (RE) is defined as

\[
RE = \frac{|\hat{r}_0 - r_0|}{r_0} \times 100\%
\]

where \(\hat{r}_0\) is the estimation of the source range. It can be seen that the WF-RSBF method estimates the ranges of two sources with the RE lower than 4%.

![Figure 6](image)

**Figure 6.** (a) The beamforming output and (b) estimated azimuths.
The simulation results in this subsection show that WF-RSBF is an effective method to achieve source ranging with multiple sources in real-time situations. The wavenumber filtering process along the HLA can extract the SRBR modes of each source and obtain recognizable interference striations. Then, the source ranges can be estimated using the RSBF method with reasonable errors.

4. Performance Analyses

In this section, the performance of the WF-RSBF method is discussed. Firstly, the sensor spacing required for the Fourier transform is analyzed. Next, the modal overlap due to the two sources being closely spaced in the bearing is discussed. Lastly, for a single source from different bearings, the spatial filtering capability between the RSBF and WF-RSBF methods is compared.

4.1. The Required Sensor Spacing

According to the Nyquist sampling theorem in the time and frequency domains, the maximum frequency should be less than half the frequency sampling rate. Correspondingly,
since the Fourier transform is performed along the HLA during the wavenumber filtering, the sensor spacing (i.e., the spatial sampling) of the HLA should be as follows:

\[
\frac{1}{2 \cdot \Delta r} \geq \frac{1}{2 \pi} \cdot \max(k_{rm} \cdot \sin \phi_i) = \frac{1}{2 \pi} \cdot \max(k_{rm})
\]  

(18)

where \( \Delta r \) is the spacing between array elements. Thus, the spacing between adjacent elements needs to satisfy

\[
\Delta r \leq \frac{c_{\min}}{2 \cdot f_{\max}}
\]  

(19)

Equation (19) gives the same criteria for beamforming and filtering with respect to sensor spacing. In the simulation cases of Section 3, the maximum spacing between adjacent elements should be 1.28 m at 575 Hz to satisfy the requirements of the sampling theorem. Figure 9 shows the wavenumber spectrum when \( \Delta r \) is taken as 0.5 m and 2 m, and Figure 10 shows the ranging errors in these conditions. The other simulation parameters are the same as those in Section 3.

![Figure 9](image_url)  
Figure 9. The wavenumber spectrum when (a) \( \Delta r = 0.5 \) m and (b) \( \Delta r = 2 \) m. The windows are shown in black rectangles.

![Figure 10](image_url)  
Figure 10. The ranging errors when (a) \( \Delta r = 0.5 \) m and (b) \( \Delta r = 2 \) m. The other simulation parameters are the same as those in Section 3.

When \( \Delta r \) is taken as 0.5 m while the number of elements is unchanged, the effective length of the array is actually reduced. Thus, the RE has increased compared to that...
when $\Delta r = 1$ m, as shown in Figure 10a. However, when the sensor spacing increases to 2 m, the excessive sensor spacing results in undersampling in the spatial domain. After Fourier transforming along the array, the wavenumber of the acoustic field cannot be fully presented in the wavenumber spectrum. In other words, the wavenumber part that is greater than $\frac{1}{2 \cdot \Delta r}$ will be aliased into the part that is less than $\frac{1}{2 \cdot \Delta r}$ in the wavenumber spectrum. As shown in Figure 9b, the SRBR modes of source 2 cannot be extracted through the window filter, which indirectly proves the existence of aliasing. There is no doubt that the WF-RSBF method cannot estimate the ranges of source 2, as shown in Figure 10b. Thus, the sensor spacing should satisfy Equation (19) to ensure the effectiveness of the WF-RSBF method.

4.2. Anti-Aliasing Criteria Due to Modal Overlap between Two Sources

As mentioned in Section 3, the azimuth angles of two sources could be at any region only if their angles satisfy a certain relationship. For example, as shown in Figure 11a, when $\phi_1 = 45^\circ$ and $\phi_2 = 85^\circ$, the window functions for the two sources can be separated from each other, and the two sources can be correctly estimated. However, when the azimuth angles of the two sources are close to each other, as shown in Figure 11b, the window functions overlap with each other. This may result in the failure of the ranging method. Thus, it is important to investigate the conditions to avoid the modal overlap of filters.

![Figure 11](image-url): The wavenumber spectrum for (a) $\phi_1 = 45^\circ$ and $\phi_2 = 85^\circ$; (b) $\phi_1 = 75^\circ$ and $\phi_2 = 85^\circ$.

As shown in Figure 4b, when one azimuth angle is negative and one is positive, the wavenumber band of two filters will never overlap with each other. In other words, the modal overlap will occur only when the azimuth angles are both positive or both negative. Assuming that $\phi_2 < \phi_1 < 0$ or $\phi_2 > \phi_1 > 0$, to avoid the overlap of the SRBR modes of the two sources in the wavenumber spectra, their bearings should satisfy the anti-aliasing criteria as follows:

$$|\sin \phi_1| \frac{\omega}{c_{\text{max}}} < |\sin \phi_2| \frac{\omega}{c_{\text{bottom}}}, \text{ when } \phi_2 < \phi_1 < 0 \text{ or } \phi_2 > \phi_1 > 0$$

(20)

Theoretically, as long as the bearings of the two sources satisfy the anti-aliasing criteria, the SRBR modes from the sources with different bearings can be separated after filtering. Then, it follows that

$$|\sin \phi_1| < |\sin \phi_2| \frac{c_{\text{max}}}{c_{\text{bottom}}} \leq \frac{c_{\text{max}}}{c_{\text{bottom}}}, \text{ when } \phi_2 < \phi_1 < 0 \text{ or } \phi_2 > \phi_1 > 0$$

Taking source 2 at the bearing of $90^\circ$ and range of 5 km, consider the situation that source 1 remains at the range of 15 km and that its bearing changes from $-90^\circ$ to $85^\circ$. The
environmental parameters are the same as those in Section 3. To reduce the impact of the noise on the subsequent analyses, no noise was added in this or the next subsection.

Figure 12 shows the positioning and ranging results. Considering the waveguide with $c_{\text{max}} = 1510 \text{ m/s}$ and $c_{\text{bottom}} = 1800 \text{ m/s}$, $\phi_1$ should satisfy $\phi_1 \leq 57^\circ$ theoretically. However, it is shown in Figure 12b that the range of source 1 can be estimated successfully using WF-RSBF when the bearing satisfies $\phi_1 < 70^\circ$. The results validate that the effective component of the acoustic field from the TOI can be extracted even though a slight modal overlap exists. Thus, Equation (20) provides a very strict constraint for the azimuth angles of multiple sources. We cannot determine the level of aliasing at which the WF-RSBF method will be effective, but this method should be effective as long as the source bearings satisfy the anti-aliasing criteria.

The results also proved that, as described in the previous section, the azimuth angles of the two sources could be at any region only if their angles satisfy the anti-aliasing criteria. However, from Figure 12, we can find that the WF-RSBF method fails when the azimuth angle is near the broadside of the array, so we analyze this phenomenon in the next subsection.

4.3. Spatial Filtering Capability for a Single Source from Different Bearings

In this subsection, the proposed WF-RSBF method is tested for a single source in different bearings. In this case, the RSBF method proposed by Liu et al. [17] is also workable for a single source. It is used for comparison with our method. As discussed in Section 3, the RSBF method will fail when multiple sources exist in the waveguide. Thus, to compare the WF-RSBF method presented in this paper with the RSBF method, we only simulate one source with a fixed source range of 15 km and different bearings, as shown by the red crosses in Figure 13a. The parameters of the environment and array are the same as those in Section 3.

Figure 13 shows the ranging results for the source at different bearings. It is seen that the RSBF method is effective for almost all the bearings, except for the case where the source is at the broadside of the array. However, the WF-RSBF method may also fail to estimate the source range when the source is near the broadside of the array. From Equation (12), it can be found that the bandwidth of the filter narrows as the source bearing decreases, which means that fewer modes will be extracted to reconstruct the acoustic field. Thus,
the wavenumber filtering process requires that the source bearing is not too close to the broadside of the array.

![Figure 13. (a) Positioning and (b) ranging results for a single source from different bearings.](image)

**5. Summary**

This paper proposed the WF-RSBF method to estimate the ranges of multiple sources. Combined with the source bearing estimated using CBF, the SRBR modes of multiple sources are separated in the wavenumber domain. The filter is then applied to bandpass the SRBR modes for the individual source, and the noise in the stop band of the wavenumber axis could be filtered out, which is beneficial for improving the ranging accuracy. Then, a recognizable intensity spectrogram contributed by one source is obtained to apply the RSBF method for source range estimation. The method can estimate the multisource ranges with few prior environmental parameters and without requiring the source motion. For multiple sources within 25 km, the proposed algorithm is verified via simulation research with errors of lower than 4%. And the performance of this method is discussed in the penultimate section. The analyses show that the sensor spacing of the HLA used needs to be less than half the wavelength corresponding to the detection frequency. The WF-RSBF method is applicable for situations where the bearings of the sources satisfy the anti-aliasing criteria, and the source bearing should not be too close to the broadside direction of the array.

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