Dynamic Modeling and Robust Trajectory Tracking Control of a Hybrid Propulsion-Based Small Underwater Robot

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Abstract: This paper proposes a hybrid propulsion-based small underwater robot for robust trajectory tracking control in a harsh and complex underwater environment. The robot is equipped with a Coanda-effect jet thruster and a pair of propeller-based reconfigurable magnetic-coupling thrusters, allowing it to traverse safely in confined or cluttered spaces as well as cruise efficiently in the open water. To investigate the robot dynamic modeling, we first formulated its simplified mathematical model and estimated the hydrodynamic coefficients by performing the planar motion mechanism using CFD (computational fluid dynamics) simulation. Then, a double-loop trajectory tracking control architecture was designed considering the model uncertainties and environmental disturbances. Based on Lyapunov theory, the outer-loop kinematic control produces the virtual velocity command, while the inner-loop dynamic control adopts the full-state feedback L1-adaptive control to match the command. The asymptotic convergence of the tracking errors and the stability of the whole closed-loop system are guaranteed. Finally, comparative simulations in the presence of unknown disturbances and the variation of model parameters were carried out to verify the robustness of our proposed trajectory tracking control, which is also suitable for the separated son robots.

Keywords: robust trajectory tracking; adaptive control; hydrodynamic coefficients; underwater robots; hybrid propulsion

1. Introduction

With rapid advancements in unmanned underwater vehicles (UUVs), a variety of propulsion systems are emerging for specific applications, including marine resource exploration, underwater structure inspection, environmental monitoring, etc. [1,2]. To conduct surveillance or marine biologic studies, biomimetic propulsion techniques are preferred, as they are relatively quiet, energy efficient and flexible in operation [3]. However, the classic rear propeller and control surface (rudder) architectures still dominate the propulsion systems of autonomous underwater vehicles (AUVs) for achieving high speeds at long-range cruising. In particular, the commercial AUVs of torpedo shapes, such as the RThys COMET 300 [4], WHOI REMUS-600 [5] and KONGSBERG HUGIN [6], play an important role in the fields of seabed mapping and search and rescue. This type of propulsion may result in low maneuverability because the control surfaces or rudders are incapable of generating sufficient torques when AUVs are operating at low speeds. Therefore, some researchers have developed over-actuated AUVs with redundant fixed propellers to realize full holonomic propulsion [7,8], while others focus on designing vectored thrusters [9] that can reorient the thrust vectors in more than one degree of freedom (DOF), making the underwater vehicles highly maneuverable with fewer thrusters.

Considering the pros and cons of the propulsion systems introduced above, hybrid propulsion-based underwater vehicles have been proposed, which combine different propulsion systems to improve the vehicle’s performance in terms of motion pattern, speed
or whatever is required. For example, a thermal-electric hybrid propulsion underwater glider was proposed in [10] due to the abundant storage of thermal energy in the ocean. The hybrid propulsion integrates both thermal propulsion and electric propulsion to drive the buoyancy systems for long endurance. To improve the performance of underwater tasks, a bio-inspired hybrid propulsion underwater robotic vehicle (HPURV) was conceptually designed as far as the minimum energy consumption in ocean observations is concerned [11]. The HPURV consists of a caudal fin, a pair of pectoral fins, a pitch-roll moving mass mechanism, and a dual buoyancy controlling module, allowing flexible operation strategies by combining any two propulsion modes. Also, through bio-inspired propulsion, Guo et al. presented a micro underwater robot with hybrid propulsion of a 3-DOF fishtail and a pair of 1-DOF propellers [12]. This hybrid propulsion has a lot of advantages in the cruise and tracking motion modes, respectively. In addition, underwater robots sometimes need to move fast but sometimes need to maintain a low speed and low noise in a complex environment. These task requirements motivated Gu et al. to develop a hybrid propulsion system with an integrated waterjet and propeller for the spherical underwater robot (SUR IV) [13]. Compared to a single waterjet thruster, the propeller generates powerful thrust but high noise, while the hybrid propulsion system provides the alternative of switching between the propeller, waterjet and hybrid modes for different motion control. Based on the above multiple propulsion modes, Li and Guo further introduced an adaptive multi-mode switching strategy for the SUR IV to achieve a smooth and stable mode of transition in the marine environment during operation [14].

Although a number of propulsion techniques along with propulsion modes have been extensively discussed, motion stability and accuracy, in terms of the robot trajectory tracking issue, remain challenging due to the model uncertainties and the unknown disturbances in the underwater environment. The recent control architectures are classified and summarized in [15,16]. To simplify the complexity of the controller, the work in [17,18] decoupled the underwater robot dynamic model and utilized the sliding mode control (SMC) to realize trajectory tracking but only in the horizontal plane, although the tracking tasks typically require AUVs to follow the time-parameterized path in 3D space. Based on the reduced 4-DOF kinematic and dynamic models, Karkoub et al. proposed a hierarchical nonlinear controller that employed the backstepping method and SMC to achieve asymptotic tracking performance [19]. To make tracking controllers less dependent on the accurate AUV model, a 6-DOF adaptive controller was presented for an underactuated AUV in the presence of parameter uncertainties [20]. Using Lyapunov’s direct method and backstepping method, the simulations prove that the adaptive nonlinear controller provides an asymptotic convergence of position and orientation tracking errors. For the fast convergence of adaption in working environmental disturbances, Wadi et al. devised a conditional adaptation law to tune the gains of kinematic and dynamic controllers in two ways: adaptive proportional control and universal adaptive stabilization-based control [21]. Moreover, intelligent methods, including fuzzy control [22], model predictive control [23], and reinforcement learning-based control [24], are combined with other control methods to improve the performance of trajectory tracking, taking into account complicated models, environments and system constraints.

Despite the above advance of trajectory control and some other high-level controls related to station-keeping [25] or position-attitude [26], harsh and hostile environments require underwater robots to be equipped with a flexible propulsion system to adapt to complicated marine situations or missions. Specifically, propeller-driven underwater robots are superior in speed and maneuverability when traveling in open water. However, they are not suitable for conducting underwater inspections in cluttered or confined environments, as spinning propellers carry the risk of becoming tangled, stuck or damaging wildlife when close interactions are necessary. Meanwhile, low-speed maneuvers are also essential for robots operating in narrow spaces. In this case, if the propeller-driven underwater robot is also available to employ a waterjet-based thruster, the hybrid propulsion mode of the propeller and waterjet provides the underwater robot with a promising way
to meet high demands in terms of adaptivity to the underwater environment, maneuvering capability as well as thruster failure.

The above rationale inspires the hybrid propulsion-based underwater robot in Figure 1. The main contributions are: (1) An underwater robot is conceptually designed with a multibody structure, equipped with a Coanda-effect jet thruster and a pair of propeller-based reconfigurable magnetic-coupling thrusters, which can work together or actuate each son robot independently after separation. (2) To track any smooth 3D trajectories, a planar motion mechanism (PMM)-based CFD simulation is performed to estimate the hydrodynamic coefficients, while a double-loop trajectory tracking control architecture is established, which consists of a Lyapunov-based outer-loop kinematic control and an inner-loop dynamic control using the full-state feedback L1-adaptive control to ensure fast adaption with guaranteed robustness. (3) The simulation results validate the good tracking performance despite the time-varying parameters in the model and the external unknown disturbance, even for separated son robots. This paper is organized as follows: Section 2 not only presents the hybrid propulsion-actuated underwater robot but also describes its simplified model and a CFD simulation technique to estimate the hydrodynamic coefficients. Section 3 establishes a double-loop control architecture of trajectory tracking. Finally, we illustrate the simulations of the underwater robot trajectory tracking under three different scenarios in Section 4, and we summarize the paper in Section 5.

Figure 1. The conceptual design of a hybrid propulsion-actuated underwater robot.

2. Dynamic Modeling

2.1. The Hybrid Propulsion-Actuated Small Underwater Robot

A hybrid propulsion-actuated small underwater robot is designed to cope with unpredictable and complex environments for safe underwater movement. As exhibited in Figure 2, the skeleton of the robot is a multibody design that consists of three streamlined hulls with separate thrusters mounted on the stern. For efficient open-water cruising, a pair of propeller-based reconfigurable magnetic-coupling thrusters on either side of the robot’s body is capable of generating vector thrusts of 2-DOF, giving the actuated robot high maneuverability to track trajectories. When attempting to traverse in a confined and cluttered environment, a Coanda-effect jet thruster, located at the rear of the robot’s main body, can change the jet direction to flexibly maneuver the robot at low speeds. Instead of using the propeller-based thruster, the waterjet thruster prevents the rotating propellers from becoming tangled in aquatic plants or even damaged for safety reasons.
Regarding the magnetic-coupling thruster, a propeller is connected with a spherical rotor, which sits in a cylindrical rotor. Using custom-designed permanent magnets mounted symmetrically in two rotors, the non-contact magnetic transmission between the rotors will spin the propeller when a brushless motor directly drives the cylindrical rotor. To reorient the propeller thrust, two arc-shaped rails are intersected orthogonally through a sliding block, which allows the propeller shaft to pass. In this way, the propeller can generate vectored thrust in 2-DOF when two waterproof servomotors drive the sliding rails independently.

The working principle of the above waterjet thruster refers to our previous work [27]. This multi-axis jet thruster is designed based on the physical phenomenon of the Coanda-effect, which can redirect a pump output of high-speed jets through a finely designed Coanda-effect valve. A servomotor is used to switch the control valve so that the vectored jets can be ejected from the desired outlet. We have already developed a robot prototype driven by the Coanda-effect jet thruster and verified its thrust performance, which can maneuver the robot moving in multi-DOFs [28].

To reduce the drag force, the robot’s multibody is of a simple, symmetrical and compact hull, and a revolution body of von Karman curve is chosen to form the front compartments of the multibody. Through the separation units beside the main body, two propeller-driven son robots are launched, and consequently, the hybrid propulsion-actuated underwater robot is divided into three son robots with independent thrusters. The investigation of multiple robots will be discussed in future work.

![Figure 2. The design of a hybrid propulsion-actuated underwater robot.](image)

### 2.2. Assumptions

To facilitate building the robot mathematical model and the following trajectory tracking control architecture, some assumptions are made:

- The origin of the robot’s body-fixed frame is located at the center of mass, which is slightly below the geometric center of the robot so that roll motion can be neglected due to self-stabilization.
- The underwater robot is of neutral buoyancy. Moreover, it is designed to be of small size and is lightweight. In this case, the robot has slow dynamics and moves at a comparatively low speed.
- The translational and angular velocities of the underwater robot are available for feedback. Such an assumption is reasonable since the robot is equipped with a gyro or IMU in practice.
All dynamic parameters of the underwater robot are bounded.

2.3. Underwater Robot Mathematical Model

According to the marine engineering conventions, Figure 3 shows the robot’s body-fixed frame \( \{B\} = \{x_b, y_b, z_b\} \) with respect to the earth-fixed inertial frame \( \{E\} = \{X, Y, Z\} \). Based on the above assumptions, the simplified 5-DOF kinematic model can be written in terms of a transformation matrix \( J(\eta) \).

\[
\eta = J(\eta) \nu
\]

(1)

\[
J(\eta) = \begin{bmatrix}
\cos \psi \cos \theta & -\sin \psi & \cos \psi \sin \theta & 0 & 0 \\
\sin \psi \cos \theta & \cos \psi & \sin \psi \sin \theta & 0 & 0 \\
-\sin \theta & 0 & \cos \theta & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 / \cos \theta
\end{bmatrix}
\]

(2)

where \( \eta = [x, y, z, \theta, \psi]^T \) and \( \nu = [u, v, w, q, r]^T \) present the attitude and velocity of the robot in vector forms. When the control input \( \tau \) is applied to the underwater robot, the dynamic model under the time-varying disturbance \( \tau_d \) is governed by

\[
M\ddot{\nu} + C(\nu)\nu + D(\nu)\nu + G = \tau + \tau_d
\]

(3)

where the inertial matrix \( M \) is the addition of the rigid-body mass matrix \( M_{RB} \) and the added mass matrix \( M_a \).

\[
M = M_{RB} + M_a = \begin{bmatrix}
m - X_u & 0 & 0 & 0 & 0 \\
0 & m - Y_v & 0 & 0 & 0 \\
0 & 0 & m - Z_w & 0 & 0 \\
0 & 0 & 0 & I_{yy} - M_q & 0 \\
0 & 0 & 0 & 0 & I_{zz} - N_s
\end{bmatrix}
\]

(4)

The Coriolis matrix is
The linear and quadratic drag matrix is

\[
C(v) = \begin{bmatrix}
  0 & 0 & m_w & -m_v \\
  0 & 0 & 0 & m_u \\
  0 & 0 & 0 & -m_u \\
  -m_w & 0 & m_u & 0 \\
  m_w & -m_u & 0 & 0 \\
\end{bmatrix}
\]  \tag{5}

The linear and quadratic drag matrix is

\[
D(v) = \begin{bmatrix}
  X_x + X_{ik} [u] & 0 & 0 & 0 \\
  0 & Y_y + Y_{ik} [v] & 0 & 0 \\
  0 & 0 & Z_z + Z_{ik} [w] & 0 \\
  0 & 0 & 0 & M_q + M_{ik} [q] \\
  0 & 0 & 0 & 0 & N_r + N_{i[i]} \\
\end{bmatrix}
\]  \tag{6}

Here, the restoring force \( \mathbf{G} \) is ignored, as the underwater robot can be designed with neutral buoyancy, and the notations for the mathematical model are explained in Table 1.

Table 1. Notations for items in the mathematical model.

<table>
<thead>
<tr>
<th>Motion</th>
<th>Control Input</th>
<th>Added Mass Coefficients</th>
<th>Linear Drag Coefficients</th>
<th>Quadratic Drag Coefficients</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surge</td>
<td>( X )</td>
<td>( \dot{X}_u )</td>
<td>( X_u )</td>
<td>( \dot{X}_{u[i]} )</td>
</tr>
<tr>
<td>Sway</td>
<td>( Y )</td>
<td>( Y_v )</td>
<td>( Y_v )</td>
<td>( \dot{Y}_{v[i]} )</td>
</tr>
<tr>
<td>Heave</td>
<td>( Z )</td>
<td>( Z_w )</td>
<td>( Z_w )</td>
<td>( \dot{Z}_{w[i]} )</td>
</tr>
<tr>
<td>Pitch</td>
<td>( M )</td>
<td>( M_q )</td>
<td>( M_q )</td>
<td>( \dot{M}_{q[i]} )</td>
</tr>
<tr>
<td>Yaw</td>
<td>( N )</td>
<td>( N_r )</td>
<td>( N_r )</td>
<td>( \dot{N}_{r[i]} )</td>
</tr>
</tbody>
</table>

2.4. Estimation of Hydrodynamic Coefficients Using CFD

The dynamic features of the mathematical model of an underwater robot are related to the hydrodynamic coefficients. Inaccurate coefficients have a great influence on the performance of the trajectory control. To predict these coefficients using CFD simulation, dynamic mesh technology is used to simulate the robot performing the planar motion mechanism (PMM) experiment.

Before the CFD analysis in the Fluent module of ANASYS, we first used the static mesh model to verify the grid’s independence in terms of the result of the variation of the refined grid resolution of the underwater robot model. As shown in Figure 4, the underwater robot is enclosed by a cuboid fluid domain of size 830 mm × 653 mm × 370 mm, with an inflow velocity of 0.2 m/s and a pressure-outlet of 0, while the rest are walls. The mesh grid was refined from 11 mm to 3 mm, and the k-\( \omega \) SST turbulence model was adopted to calculate the longitudinal drag. Table 2 summarizes the total number of nodes, mesh, average grid quality and drag force from the coarse grid to the fine grid. Overall, the grid qualities are all over 0.83, indicating that a good meshing performance is achieved from 11 mm to 3 mm. However, the drag force results vary slightly from 8 mm to 3 mm. Therefore, we chose 7 mm for the dynamic meshing to simulate the PMM of the underwater robot, considering the limited computational resources.
Table 2. Mesh sensitivity.

<table>
<thead>
<tr>
<th>Mesh Size</th>
<th>Node Number</th>
<th>Grid Number</th>
<th>Average Grid Quality</th>
<th>Drag Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>11 mm</td>
<td>187,906</td>
<td>1,017,927</td>
<td>0.83815</td>
<td>0.482653</td>
</tr>
<tr>
<td>10 mm</td>
<td>269,628</td>
<td>1,448,755</td>
<td>0.83799</td>
<td>0.509271</td>
</tr>
<tr>
<td>9 mm</td>
<td>397,663</td>
<td>1,598,702</td>
<td>0.83781</td>
<td>0.552812</td>
</tr>
<tr>
<td>8 mm</td>
<td>537,084</td>
<td>1,810,195</td>
<td>0.83873</td>
<td>0.560561</td>
</tr>
<tr>
<td>7 mm</td>
<td>800,690</td>
<td>4,312,718</td>
<td>0.83898</td>
<td>0.559806</td>
</tr>
<tr>
<td>6 mm</td>
<td>876,301</td>
<td>4,709,071</td>
<td>0.83836</td>
<td>0.561093</td>
</tr>
<tr>
<td>5 mm</td>
<td>1,017,830</td>
<td>5,466,527</td>
<td>0.83813</td>
<td>0.560211</td>
</tr>
<tr>
<td>4 mm</td>
<td>1,282,085</td>
<td>6,878,819</td>
<td>0.8382</td>
<td>0.563053</td>
</tr>
<tr>
<td>3 mm</td>
<td>1,910,865</td>
<td>10,260,583</td>
<td>0.83847</td>
<td>0.567828</td>
</tr>
</tbody>
</table>

Figure 4. The static mesh simulation of the underwater robot model.

In the PMM experiment, first, pure surging is generated, as shown in Figure 5a. Given the surging velocity with amplitude $a$ and frequency $\omega$, the motion is governed by the following equations:

Figure 5. Oscillatory motion in the PMM experiments: (a) trajectory of the robot during pure surging; (b) trajectory of the robot during heave test; (c) the robot during yaw motion.
where \( X_0 \) is a constant. The acting force \( X \) is sampled as the PMM produces the oscillatory motion. Due to the intrinsic characteristics of thrusters and their layout, the practical translational motions (heave and sway) are always coupled with surging. Thus, during the heaving test, the robot model oscillates in the vertical plane and keeps moving at a constant surging velocity as the trajectory is displayed in Figure 5b. The dynamic equation is written as:

\[
\begin{align*}
    u &= a \omega \cos \omega t \\
    X &= X_0 - X_a \dot{u} + X_s u + X_s |u| u \\
    \omega &= - + + \omega \\
\end{align*}
\]

The robot’s swaying motion is simulated similarly to the heaving motion. In the rotational motion, regarding yaw, the robot model rotates in the XOY plane under opposing constant flow velocity in Figure 5c. The dynamic equation is given by

\[
\begin{align*}
    r &= b \omega \cos \omega t \\
    Z &= Z_0 - Z_\omega \dot{w} + Z_\omega w + Z_\omega |w| w \\
\end{align*}
\]

Therefore, the rotational motion in pitch is induced by the PMM experiment. To numerically simulate PMM, the User Defined Function (UDF) is employed to code the motion of the robot model. Figure 6 exhibits the inner region of the overset mesh model containing the underwater robot inside two fluid computational domains: with a size of 4 m × 2 mm × 2 m for translational motion and a size of 1.5 m × 1.5 mm × 1.5 m for rotational motion separately. As the dynamic mesh is restricted by the principle of dynamics, the robot model, enclosed by the overset mesh, is translated or rotated in the flow fields. The rest of the settings remain unchanged. All simulations were performed on a server with Intel-core i7 processors and 32 GB of internal memory, with a time step of 0.02 s and lasting for 15 s. To record the robot performing the PMM experiments in a steady state, Figure 7 shows the velocity contour corresponding to the robot motion of each DOF from 9 s to 12 s. We obtained 75 groups of data in different motions and used the nonlinear fitting in MATLAB to predict the hydrodynamic coefficients, presented in Table 3. The R-square of the fitting is greater than 0.993, indicating a good fit. We also carried out similar PMM experiments to estimate the hydrodynamic coefficients of the separated son robots separately, and they are summarized in Tables 4 and 5.

Figure 6. Overset mesh simulation of the underwater robot model.
Figure 7. Velocity contour of PMM experiments in a steady state: (a) during pure surging; (b) during sway test; (c) during heave test; (d) during pitch test; (e) during yaw test.

Table 3. Hydrodynamic coefficients of the hybrid propulsion-actuated underwater robot.

<table>
<thead>
<tr>
<th>Added Mass Coefficients</th>
<th>Value</th>
<th>Linear Drag Coefficients</th>
<th>Value</th>
<th>Quadratic Drag Coefficients</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_u$</td>
<td>−2.677</td>
<td>$X_u$</td>
<td>0.2234</td>
<td>$X_{u</td>
<td>u}$</td>
</tr>
<tr>
<td>$Y_v$</td>
<td>−7.64</td>
<td>$Y_v$</td>
<td>16.18</td>
<td>$Y_{v</td>
<td>v}$</td>
</tr>
<tr>
<td>$Z_w$</td>
<td>−19.35</td>
<td>$Z_w$</td>
<td>48.38</td>
<td>$Z_{w</td>
<td>w}$</td>
</tr>
<tr>
<td>$M_q$</td>
<td>−0.4535</td>
<td>$M_q$</td>
<td>0.0492</td>
<td>$M_{q</td>
<td>q}$</td>
</tr>
<tr>
<td>$N_r$</td>
<td>−0.03828</td>
<td>$N_r$</td>
<td>0.1027</td>
<td>$N_{r</td>
<td>r}$</td>
</tr>
</tbody>
</table>
Table 4. Hydrodynamic coefficients of the propeller-actuated son robot.

<table>
<thead>
<tr>
<th>Added Mass Coefficients</th>
<th>Value</th>
<th>Linear Drag Coefficients</th>
<th>Value</th>
<th>Quadratic Drag Coefficients</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{\dot{u}}$</td>
<td>-1.17</td>
<td>$X_u$</td>
<td>0.2988</td>
<td>$X_{\dot{u}b}$</td>
<td>6.116</td>
</tr>
<tr>
<td>$Y_{\dot{v}}$</td>
<td>-9.053</td>
<td>$Y_v$</td>
<td>9.421</td>
<td>$Y_{\dot{v}b}$</td>
<td>18.21</td>
</tr>
<tr>
<td>$Z_{\dot{w}}$</td>
<td>-5.481</td>
<td>$Z_w$</td>
<td>10.05</td>
<td>$Z_{\dot{w}b}$</td>
<td>30.59</td>
</tr>
<tr>
<td>$M_{\dot{q}}$</td>
<td>0.01626</td>
<td>$M_q$</td>
<td>0.2101</td>
<td>$M_{\dot{q}b}$</td>
<td>0.2392</td>
</tr>
<tr>
<td>$N_r$</td>
<td>-0.01039</td>
<td>$N_r$</td>
<td>0.1121</td>
<td>$N_{r</td>
<td>b}$</td>
</tr>
</tbody>
</table>

Table 5. Hydrodynamic coefficients of the waterjet-actuated son robot.

<table>
<thead>
<tr>
<th>Added Mass Coefficients</th>
<th>Value</th>
<th>Linear Drag Coefficients</th>
<th>Value</th>
<th>Quadratic Drag Coefficients</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{\dot{u}}$</td>
<td>0.08322</td>
<td>$X_u$</td>
<td>0.5505</td>
<td>$X_{\dot{u}b}$</td>
<td>5.437</td>
</tr>
<tr>
<td>$Y_{\dot{v}}$</td>
<td>-1.956</td>
<td>$Y_v$</td>
<td>10.53</td>
<td>$Y_{\dot{v}b}$</td>
<td>0.02338</td>
</tr>
<tr>
<td>$Z_{\dot{w}}$</td>
<td>-1.694</td>
<td>$Z_w$</td>
<td>10.27</td>
<td>$Z_{\dot{w}b}$</td>
<td>0.02383</td>
</tr>
<tr>
<td>$M_{\dot{q}}$</td>
<td>0.04801</td>
<td>$M_q$</td>
<td>0.03435</td>
<td>$M_{\dot{q}b}$</td>
<td>0.05548</td>
</tr>
<tr>
<td>$N_r$</td>
<td>0.06808</td>
<td>$N_r$</td>
<td>0.03851</td>
<td>$N_{r</td>
<td>b}$</td>
</tr>
</tbody>
</table>

3. Trajectory Tracking Control

Based on the above design of the underwater robot, its hybrid propulsion mode not only helps the robot adapt to complex environments but also its multibody structure allows the robot to be divided into three son robots. However, these advantages make it challenging to design a robust trajectory control method, as it faces unknown environmental disturbances as well as the varying parameters in the robot model. Considering the state variables of the underwater robot varying on the time scale, Figure 8 displays a double-loop architecture of our proposed trajectory tracking control, which includes an outer loop for the robot’s kinematic control and an inner loop for its dynamic control. Depending on Lyapunov theory, a virtual velocity command is devised in the outer-loop kinematic control regarding the robot simplified 5-DOF model, while a full-state feedback L1-adaptive control [29,30] is exploited in the inner-loop dynamic control to design the control inputs (generalized control forces and moments) so as to keep up with the virtual velocity command.
Figure 8. The double-loop control architecture of trajectory tracking.

3.1. Kinematic Control

Consider that the robot tracks a smooth and bounded trajectory \( \eta_d = [x_d, y_d, z_d, \theta_d, \psi_d]^T \), where its first derivative is also bounded. The kinematic tracking error relative to the robot’s body-fixed frame is expressed as

\[
\eta_e = \begin{bmatrix}
    x - x_d \\
y - y_d \\
z - z_d \\
\theta - \theta_d \\
\psi - \psi_d
\end{bmatrix}
\]

where it is equivalent to transforming the actual error of the trajectory \( \eta_e \) in the inertial frame to the body-fixed frame. To make the error approach to zero, we take the time derivative of the error, and its dynamic \( \eta_e \) can be described.

\[
\begin{align*}
\dot{x}_e &= y_e r \cos \theta - q_z e + u_d (\sin \theta \sin \theta_d + \cos \theta \cos \theta_d \cos \psi_e) - u \\
\dot{y}_e &= -x_e r \cos \theta - z_e \sin \theta + u_d \cos \theta \sin \theta_d \sin \psi_e + v_d \cos \psi_e - v \\
\dot{z}_e &= q_x e + y_e \sin \theta + u_d (\cos \theta \sin \theta_d \cos \psi_e - \cos \theta \sin \theta_d) - w \\
\dot{\theta}_e &= q_x e - q - w_d \sin \theta \sin \psi_e \\
\dot{\psi}_e &= q \psi_e \tan \theta \sec^2 \theta - r \frac{1}{\cos \theta} - \frac{1}{\cos^2 \theta}
\end{align*}
\]

where \( \eta_e = [u_d, v_d, w_d, \theta_d, \psi_d]^T = J(\eta_d)^T \eta_d \). Based on the Lyapunov candidate function \( V_1 = \frac{1}{2} x^2_e + \frac{1}{2} y^2_e + \frac{1}{2} z^2_e + \frac{1}{2} \theta^2_e + \frac{1}{2} \psi^2_e \), the virtual velocity command is designed with positive constants \( k_1, k_2, k_3, k_4, k_5 \) as

\[
\nu_c = \begin{bmatrix}
u_x \\
u_y \\
u_z \\
u_{\theta} \\
u_{\psi}
\end{bmatrix} = \begin{bmatrix}
    k_1 x_e + u_d (\sin \theta \sin \theta_d + \cos \theta \cos \theta_d \cos \psi_e) - v_d (\cos \theta \sin \theta_d \cos \psi_e - \cos \theta \sin \theta_d) \\
k_2 y_e + u_d \cos \theta \sin \theta_d \sin \psi_e + v_d \cos \psi_e + w_d \sin \theta \sin \psi_e \\
k_3 z_e + u_d (\cos \theta \sin \theta_d \cos \psi_e - \cos \theta \sin \theta_d) - v_d \sin \theta \sin \psi_e \\
k_4 \theta_e + q_d \\
k_5 \psi_e \cos^2 \theta + r_d \frac{\cos \theta}{\cos \theta_d} + q \psi_e \tan \theta
\end{bmatrix}
\]

so that \( \dot{V}_1 = -k_1 x^2_e - k_2 y^2_e - k_3 z^2_e - k_4 \theta^2_e - k_5 \psi^2_e \leq 0 \). Obviously, the monotonically decreasing \( V_1 \) is bounded, which ensures the boundness of \( \eta_e \) as well as the position and attitude of the underwater robot \( \eta \) with the assumption of the desired trajectory. Consequently, it is concluded that the virtual velocity command Equation (12), designed by the outer-loop kinematic control, is also bounded. According to the Barbalat Lemma [31], \( \dot{V}_1 \) is uniformly continuous due to the boundness of
\[ \dot{V}_1 = -2k_1 x \hat{x} - 2k_2 y \dot{y} - 2k_3 z \hat{z} - 2k_s \theta \dot{\theta} - 2k_\psi \psi \dot{\psi}, \]
which gives rise to the limit \( \lim_{t \to \infty} \dot{V}_1 = 0 \) and eventually suggests that the trajectory errors converge asymptotically to zero.

3.2. Dynamic Control

In the presence of the time-varying model parameters and the external disturbances, we choose the L1-adaptive control of the inner loop, given the velocity command generated by the outer loop, because it ensures fast adaptation with guaranteed robustness.

In the case of our study, the hybrid propulsion-actuated robot is a light underwater vehicle of small size (less than 5 kg). Thus, its slow dynamics make it appropriate to neglect the Coriolis term due to low velocities. As a result, the dynamics can be rewritten as:

\[ \dot{v} = -\frac{D(v)}{M} v + \frac{1}{M} (\tau + \tau_d) \]  
(13)

Due to the existence of the model uncertainties, a self-tuning item \( \delta^T v \) is added to compensate for the unknown varying parameters associated with the state variables. Then, Equation (13) can be rewritten as the desired dynamics in terms of a Hurwitz matrix

\[ A = -\frac{D^*}{M^*}, \quad B = \frac{1}{M^*}, \quad \text{a control input } \bar{u} \quad \text{and unknown disturbances } \sigma: \]

\[ \dot{v} = Av + B(\bar{u} + \delta^T v + \sigma) \]  
(14)

where the varying parameters \( D(v) \) and \( M \) are both replaced by \( D^* \) and \( M^* \) so that the model uncertainties as well as the external disturbances can be compensated by the unknown \( \delta \) and \( \sigma \) that needs to be adapted. With the same system structure as Equation (14), we consider the following state predictor:

\[ \hat{\dot{v}} = A\hat{v} + B(\bar{u} + \hat{\delta}^T \hat{v} + \hat{\sigma}) \]  
(15)

where \( \hat{\dot{v}}, \hat{\delta} \) and \( \hat{\sigma} \) are the estimates \( v, \delta \) and \( \sigma \), respectively. Before designing the adaptive laws for the estimates \( \hat{\delta} \) and \( \hat{\sigma} \), let the unknown parameters \( \delta \) and \( \sigma \) be assumed to be uniformly bounded and uniformly bounded in their rate of variation.

To estimate the unknown \( \hat{\delta} \) and \( \hat{\sigma} \) online, the L1-adaptive control used full-state feedback to calculate the errors \( \hat{v} \) between the measured states and predicted ones. The error dynamics is

\[ \hat{\dot{v}} = A\hat{v} + B(\hat{\delta}^T \hat{v} + \hat{\sigma}) \]  
(16)

Define \( \hat{v} = \hat{v} - v, \hat{\delta} = \hat{\delta} - \delta, \hat{\sigma} = \hat{\sigma} - \sigma \). Consider the below Lyapunov function candidate:

\[ V_2 = \hat{v}^T P \hat{v} + \Gamma^{-1}(\hat{\delta}^T \hat{\delta} + \hat{\sigma}^T \hat{\sigma}) \]  
(17)

Let the symmetrical matrix \( P > 0 \) be the solution of the Lyapunov equation

\[ A^T P + PA = -Q, \quad \bar{Q} = Q^T > 0 \quad \text{and} \quad \Gamma > 0 \]

is the adaptation gain. Based on the Lyapunov function, the adaptive laws are updated:

\[
\begin{cases}
\dot{\hat{\delta}} = -\Gamma \hat{v}^T PB \\
\dot{\hat{\sigma}} = -\Gamma (PB)^T \hat{v}
\end{cases}
\]  
(18)
which makes \( \dot{V}_2 = -\ddot{u}'Q\ddot{u} \leq 0 \). Under the above three assumptions, the Barbalat Lemma guarantees the boundness of the estimates \( \hat{\delta} \) and \( \hat{\sigma} \), as well as the asymptotic convergence of the state predictor.

Next, we formulated the control input \( \ddot{u} \) in terms of fast adaption and robustness. The Laplace transform of the control signal is

\[
\ddot{u} = F(s)\left( -\frac{1}{A^*B}v + \hat{\delta}v - \hat{\sigma} \right)
\]

(19)

where \( F(s) = \frac{k}{s+k}, k > 0 \) is a low-pass filter with a strictly proper stable transfer function and the DC gain equals one, which aims to obtain a uniform performance bound for the control input. The full-state feedback L1-adaptive control is designed through Equations (16), (18) and (19), and satisfies the L1-norm condition.

\[
\max \sum_{\mu} \left\| \left( (sI - A)^{-1}B \right) \left( \frac{s}{s+k} \right) \right\| < 1
\]

(20)

Given the L1-adaptive control architecture, the following closed-loop reference system is introduced as the ideal form of Equation (13).

\[
\dot{\ddot{u}}_{ref}(t) = A\ddot{u}_{ref}(t) + B(\ddot{u}_{ref}(t) + \hat{\delta}(t)v_{ref}(t) + \sigma(t)), v_{ref}(0) = v_0
\]

\[
\ddot{u}_{ref} = \frac{k}{s+k}\left( -\frac{1}{A^*B}v - \hat{\delta}v_{ref} - \sigma \right)
\]

(21)

According to [32], the above reference system has been proven to be BIBO (bounded-input bounded-output) stable.

4. Simulation Results and Discussion

In this section, we have evaluated the 3D-trajectory control performance of the devised double-loop control architecture in the MATLAB simulation environment. Based on the mathematical model of the underwater robot, firstly, the hydrodynamic coefficients obtained from the above CFD simulations were used to build the controlled system model in Simulink. Then, the controller was configured using the parameters in Table 6. The kinematic control parameters were selected by trial-and-error method to make the robot track the reference trajectory without being too aggressive. The dynamic control parameter \( k \) was set to satisfy the L1-norm condition, and the adaptation gain \( \Gamma \) should be large enough to ensure stability. To test the controller, a 3D-reference trajectory, including straight line, circular and helical curves, was generated by using the following:
The initial position of the robot was a 1 m offset from the start point on the reference trajectory in the Z-direction. In the following simulation, three scenarios were run for the underwater robot in terms of its robustness to external disturbances and uncertainties related to the model.

4.1. Tracking Performance in Disturbance-Free Environment

Without any environmental disturbance, Figure 9 shows that the hybrid propulsion-based underwater robot can track the complex trajectory in three dimensions of 30 m × 30 m × 30 m space within 500 s. The tracking errors of the robot’s position and orientation in each DOF are depicted in Figure 10. Since the robot’s initial position is set 1 m below the reference trajectory in the Z-axis, it is observed that the tracking error \( e_Z \) induces the controller to adjust the robot’s pitch angle promptly so as to ascend for eliminating \( e_Z \) and \( e_\theta \). After 80 s, the robot makes two large turns of 180° in succession with good accuracy. The yaw tracking performance (maximum error 5.3°) guarantees that the robot makes hard turns successfully and then it continues to follow the helical-curved trajectory. Even though the position errors fluctuate around zero in surging and swaying directions, their maximums are no more than 1 m and finally incline to converge to zero.
Figure 9. The reference and the underwater robot trajectories in three dimensions.
Figure 10. The trajectory tracking errors of the robot position and orientation in each DOF.

Moreover, Figure 11 plots the errors between the virtual velocity command and the robot’s actual velocity. The most noticeable errors occur in the yaw angular velocity and converge to zero within 5 s. The errors are no more than 0.0735 rad/s and occur when the robot enters and exits the circular and helical curves. The dynamical control errors in the rest DOF are close to zero, except for the initial errors in surging, heaving velocities and pitch angular velocity; however, they decline quickly due to the L1-adaptive controller output forces and torques, displayed in Figure 12. The forces in surge motion keep the robot moving forward, and the torques are generated instantly based on the errors that occurred in the yaw angular velocity to follow the circular and helical curves, while the forces in the sway direction are also produced to prevent drifting in sway when taking turns. In the disturbance-free environment, the above results verify that the full-state feedback L1-adaptive control is qualified to deal with the time-varying parameters in the model. Considering the reference trajectory is not generated of a particular type, the tracking performance reveals that our proposed control architecture succeeds in maneuvering the hybrid propulsion-based underwater robot to track any sufficiently smooth path parameterized by time, which indicates a good match and fast response under the L1-adaptive control.
4.2. Tracking Performance with Environmental Disturbance

Since the small underwater robot is more sensitive to disturbances in the complex environment, the controller is required to be robust enough to reject unexpected disturbances. Thus, two types of environmental disturbances were prepared to apply to the robot. One was the punctual disturbance given by impulsive velocity signals (with an amplitude of 0.1 m/s) in the robot’s surging and swaying motions to check whether the controller could regulate the robot back to its position. Such a situation corresponds to the case, such as hitting an obstacle or another vehicle. As shown in Figure 13, the impulse signals are applied at 250 s and immediately result in errors of magnitude of 0.087 m/s and 0.086 m/s in the surge and sway directions, respectively. It is the adaptive laws in dynamic control that allow the unknown disturbances to be estimated based on the errors, taking 12 s and 8 s to eliminate them. Figure 14 illustrates that the robot’s position eventually converges to the desired trajectory.

The other type of disturbance is periodic signals, such as the wind or waves. As such, we applied periodic forces to the robot’s surging, swaying and heaving motions, respectively. The period of all force signals is 100 s, and the force amplitudes along the surge and sway are 0.5 N, while the amplitude of force in the heave is 0.3 N. To reject such unknown disturbances, the designed state predictor of the L1-adaptive controller can online estimate these external uncertainties efficiently, as shown in Figure 15. Using the updated adaptive laws, Figure 16 shows that the errors in dynamic control are suppressed
instantly so that the L1-adaptive controller is not severely affected by the periodic disturbances. Consequently, the robot controller is robust enough to track the desired trajectory accurately under external periodic disturbances, as exhibited in Figure 17.

**Figure 13.** The errors between the virtual velocity command and the robot velocity under impulsive disturbance signals.

**Figure 14.** The underwater robot trajectory under impulsive disturbance signals.
Figure 15. The estimates of unknown periodic disturbances.

Figure 16. The errors between the virtual velocity command and the robot’s velocity under periodic disturbance signals.
4.3. Tracking Performance with Parameter Uncertainty

As the hybrid propulsion-based underwater robot is designed with a multibody structure, which allows it to be divided into three sub-robots, it requires the devised trajectory tracking controller to be robust enough with respect to the variation of parameters when dealing with each sub-robot. Therefore, we built the mathematical model of each sub-robot in Simulink using the parameters in Tables 4 and 5. All simulation setups, including the double-loop trajectory tracking controller, the reference trajectory and the initial positions, are the same. Figure 18 makes a comparison of the three spatial trajectories generated by the single propeller-actuated sub-robot, the waterjet-actuated sub-robot and the hybrid propulsion-actuated robot. Obviously, the variation of the model parameters has little influence on the robot tracking performance. In particular, both the state predictor and the adaptive laws play an important role in the trajectory tracking controller, making it robust and fast enough to compensate for parameter uncertainties.

5. Conclusions

To adapt to the complex and unpredictable underwater environment, a hybrid propulsion-based underwater robot was designed with a multibody structure equipped with a pair of propeller-based thrusters for efficient open-water cruising and a waterjet-based thruster for confined space traversing. Before providing a robust trajectory tracking
control for the robot, we formulated the simplified mathematical model of the robot in 5-DOF and used the PMM experiment in the CFD simulation to estimate the hydrodynamic coefficients. Considering the model uncertainties and environmental disturbances, a double-loop trajectory tracking control architecture with self-tuning adaptive laws is established in this paper. Based on the Lyapunov theory, the outer-loop kinematic control can generate the virtual velocity command to eliminate position and orientation tracking errors, while the inner-loop dynamic control uses a full-state feedback L1-adaptive control to make the robot velocity match the virtual velocity command. In terms of the variation of the parameters in the model and the unknown disturbances, three scenarios were prepared for the simulation of the underwater robot tracking a 3D-reference trajectory. The results show good tracking performance, though there were small position tracking errors when the robot made large turns; however, these errors were eventually eliminated as the asymptotic convergence was proven in the kinematic control. Moreover, the state predictor and the adaptive laws contribute to making the dynamic controller robust enough to compensate for the model’s uncertainties and environmental disturbances. In the future, the outer-loop kinematic control method needs to be improved since the results indicate that the asymptotical convergence of the tracking errors is not fast enough compared to that of inner-loop dynamic control, while the actuator saturation issue regarding the control inputs also needs to be taken into account.

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References
4. RTsys. Comet-300 Miro-AUV. Available online: https://www2.whoi.edu/site/osl/vehicles/remus-600/?_gl=1*1mgt5o2*_ga*MjYyN-TEyOTg5LjE2NzAyMzcxMTU._ga_HLKFX9jZK*MTY5MDE4OTEzNC41LjAuMTY5MDE4OTEzNC4wLjAuMA (accessed on 10 August 2023).
5. Woods Hole Oceanographic Institution. REMUS-600 AUV. Available online: https://www2.whoi.edu/site/osl/vehicles/remus-600/?_gl=1*_mgt5o2*_ga*MjYyN-TEyOTg5LjE2NzAyMzcxMTU._ga_HLKFX9jZK*MTY5MDE4OTEzNC41LjAuMTY5MDE4OTEzNC4wLjAuMA (accessed on 10 August 2023).


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